11.1 INTRODUCTION TO AMPERE'S LAW

Moving charges (currents) create magnetic fields that rotate around the source

The Biot-Savart Law lets us find $\vec{B}$ for any current distribution

But Ampère's Law relates $\vec{B}$ directly to its source!

↳ While Gauss Law relates $\vec{E}$ to enclosed charge...

Ampère's Law relates $\vec{B}$ to enclosed current within a closed loop!

↳ Ampère's Law states that if we calculate the line integral of $\vec{B}$ around a closed loop, the result is equal to μ_0 times the enclosed current:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

11.2 CURRENT DENSITY

When charges are moving within a conductor, we can define the current $I(t)$ as:

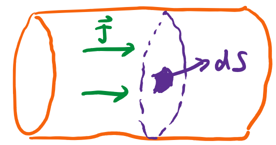
$$\boxed{I(t) = \frac{dQ(t)}{dt}}$$

where $dQ(t)$ is the charge passing through a small part dS at a given time t

↳ units of $C/s = A$

Current: a measure of the rate at which charges are crossing a surface S

But what if the charges are moving through some volume?



→ We define a volume current density $\vec{J}$ at a point

→ We construct a small area dS relative to the current flow

→ We consider a current dI passing through surface S

Case #1: dS ⊥ to Current Flow

↳ $|\vec{J}| = \frac{dI}{dS}$ $[A/m^2]$ for some current flowing through some surface

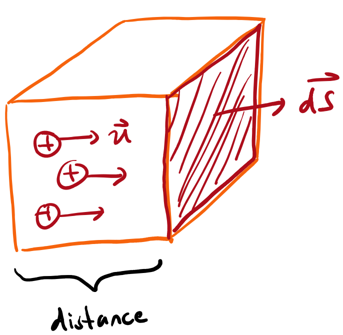
↳ $\vec{J}$ is in the direction identical to that of current flow

Case #2: dS not ⊥ to Current Flow

↳ If the current dI flows through an arbitrary surface, then the current through dS is: $dI = \vec{J} \cdot d\vec{S}$ ⇒ Total Current: $\boxed{I = \int_S \vec{J} \cdot d\vec{S}}$

What if the charges have a uniform velocity?

→ Let's say that the charges are moving at a steady drift velocity $\vec{u}$



→ We construct a solid rectangular volume of $(\vec{u}dt)(dS)$ with $d\vec{S}$ ⊥ to current flow

→ Through time dt, all charges within the box will pass through dS

→ If there are N charges per unit volume, each of charge q, then the current is:

distance

$d = u\,dt$

→ If there are N charges

charge q, then the current is:

$$dI = \frac{dQ(t)}{dt} = \frac{Nq\,(dS\,\vec{u}\,dt)}{dt} = Nq\,dS\,\vec{u}$$

→ This means the volume current density is:

$\vec{J} = \frac{dI}{dS} = Nq\vec{u}$ but we can let $\rho_v = Nq$ so we simplify!

$$\boxed{\vec{J} = Nq\vec{u} = \rho_v \vec{u}}$$

Can we relate $\vec{J}$ to $\vec{E}$?

→ The velocity $\vec{u}$ of moving charges depends on the applied electric field $\vec{E}$

→ This means $\vec{J}$ is proportional to $\vec{E}$ so that: $\boxed{\vec{J} = \sigma \vec{E}}$

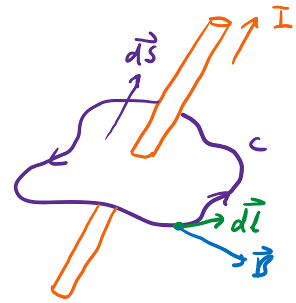
σ = conductivity $[S/m] = [\mho/m]$ mho

11.3a AMPERE'S LAW

Ampère's Law: states that the closed loop integral of the magnetic field will always be proportional to the total current passing through the cross-section enclosed by the loop

$$\boxed{\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I_{encl} = \mu_0 \int_S \vec{J}\cdot d\vec{S}}$$

AMPÈRE'S LAW



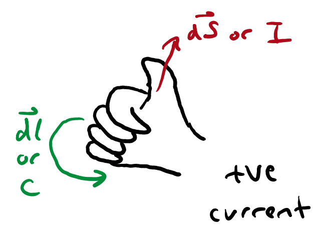
→ We call curve C an Amperian Loop is the boundary of a region that current flowing through it

→ We use the right-hand thumb rule to determine whether the current is +ve or -ve

↳ thumb points along $d\vec{S}$

↳ fingers curl along $d\vec{l}$

$\Big\}$ current along $d\vec{S}$ is +ve when $d\vec{l}$ is counterclockwise



11.3b EXAMPLE #1

You do an experiment and find that $\oint \vec{B} \cdot \vec{dl} = 0$. Does this mean that the magnetic field $\vec{B}$ is 0 in the region?

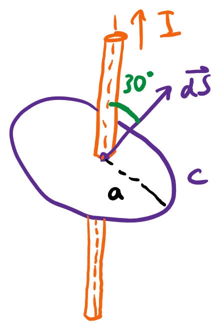
Ans: $\oint \vec{B} \cdot \vec{dl} = \underbrace{\mu_0}_{\neq 0} \underbrace{I_{encl}}_{= 0} = 0$ → only conclusion is that the total current is 0

NO, $\vec{B}$ doesn't have to be 0 in the region

11.3c EXAMPLE #2

You make a circular loop around a very long wire carrying a current I as shown in the diagram. The surface normal on the area enclosed within the loop makes an angle of $30°$ with the wire and the radius of the loop is a. What is the value of $\oint \vec{B} \cdot \vec{dl}$?

Diagram:



Ans: $\oint \vec{B} \cdot \vec{dl} = \underline{\mu_0 I_{enc}}$

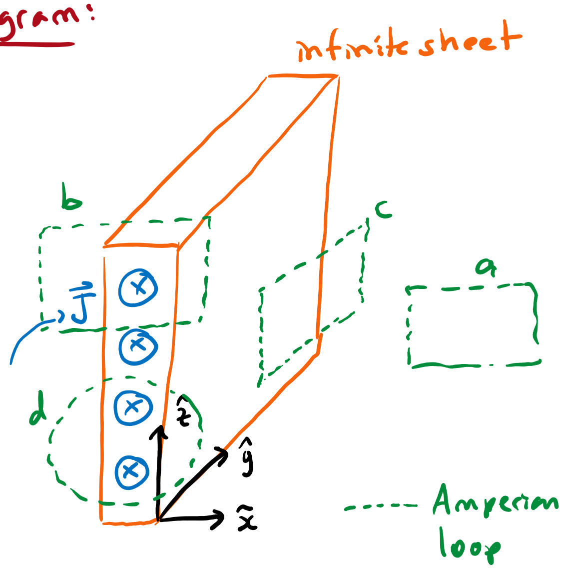
Why is this important?

→ When we take measurement, we don't have to worry about the device being calibrated at a specific angle

11.3d EXAMPLE #3

We have a large infinite sheet with uniform current density $\vec{J}$ flowing through it as shown in the diagram. List all Amperian loops for which Ampere's Law is valid.

Diagram:



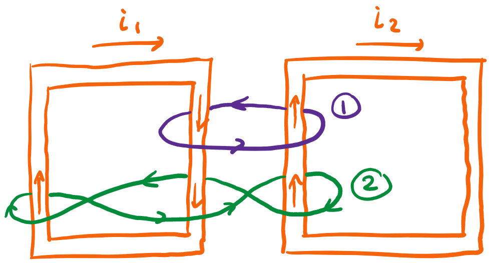
Ans: Ampère's Law only requires a CLOSED LOOP

↳ a, b, c, d are all valid because they are all closed loops

11.3e EXAMPLE #4

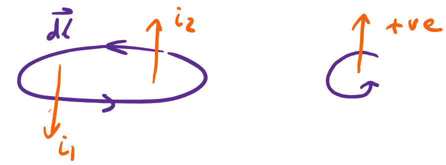
The figure shows two closed loops wrapped around currents i_1 and i_2. What is the value of the integral $\oint \vec{B} \cdot \vec{dl}$?

Diagram:



Ans:

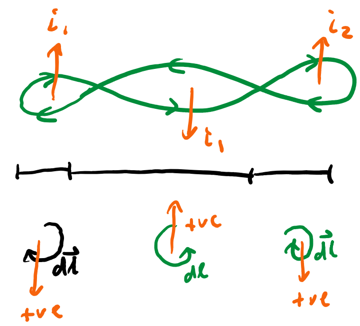
→ Loop 1:



$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{encl}$$

$$= \mu_0 (i_2 - i_1)$$

→ Loop 2:



$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{encl}$$

$$= \mu_0 (-i_1 - i_1 - i_2)$$

$$= \mu_0 (-2i_1 - i_2)$$

11.4 CALCULATION OF MAGNETIC FIELD WITH AMPERE'S LAW

Ampère's Law allows us to treat certain magnetostatic problems involving symmetry to obtain the magnetic field $\vec{B}$.

STEPS TO APPLYING AMPÈRE'S LAW:

① Recognize symmetry and sketch magnetic field lines

→ think of a mathematic loop where we can guarantee $\vec{B}$ is either:

↳ fully parallel to $\vec{dl}$ or the field lines

↳ fully ⊥ to $\vec{dl}$ in areas that are not parallel

→ this means that B is a constant we can factor out:

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{encl}$$

$$\underline{B} \oint dl = \mu_0 I_{encl} \quad \text{but} \quad I_{encl} = \int_S \vec{J} \cdot d\vec{S}$$

② Rearrange and solve for $|\vec{B}|$

$$B = \frac{\mu_0 I_{encl}}{\oint dl} = \frac{\mu_0 \int_S \vec{J} \cdot \vec{dS}}{\oint dl}$$

③ Determine direction of $\vec{B}$ by inspection