An Affine Arithmetic Based Methodology for Reliable Power Flow Analysis in the Presence of Data Uncertainty

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***Abstract*—Power flow studies are typically used to determine the steady state or operating conditions of power systems for specified sets of load and generation values, and is one of the most intensely used tools in power engineering. When the input conditions are uncertain, numerous scenarios need to be analyzed to cover the required range of uncertainty. Under such conditions, reliable solution algorithms that incorporate the effect of data uncertainty into the power flow analysis are required. To address this problem, this paper proposes a new solution methodology based on the use of Affine Arithmetic, which is an enhanced model for self-validated numerical analysis in which the quantities of interest are represented as affine combinations of certain primitive variables representing the sources of uncertainty in the data or approximations made during the computation. The application of this technique to the power flow problem is explained in detail, and several numerical results are presented and discussed, demonstrating the effectiveness of the proposed methodology, especially in comparison to previously proposed interval arithmetic's techniques.**

***Index Terms*—Power flow analysis, uncertain systems, affine arithmetic, interval mathematic**

I. Introduction

Power flow analysis is one of the most fundamental and most heavily used tools in power engineering studies, such as network optimization, voltage control, state estimation and others [1]. It allows power system analysts to determine the steady state or operating conditions of a power system for a specified set of values such as load and generation levels. For the most common formalization of the power flow problem, referred here to as a deterministic power flow, all input data are specified using deterministic variables resulting either from a snapshot of the system or defined by the analyst based on several assumptions about the system under study (e.g. expected/desired generation/load profiles). This approach provides the solution for a single system state which is deemed representative of the limited set of system conditions corresponding to the data assumptions. Thus, when the input conditions are uncertain, numerous scenarios need to be analyzed. To address this problem solution algorithms that incorporate the effect of data uncertainties into the power flow analysis have been proposed (e.g. [2-4]).

Reliable power flow solution algorithms allow analysts to gain insights into the level of confidence of power flow studies by effectively performing sensitivity analyses to estimate the rate of change in the power flow solutions with respect to changes in input data. Conventional methodologies available in the literature propose the use of probabilistic methods for these types of studies [5], accounting for the variability and stochastic nature of the input data used. In particular, uncertainty propagation studies based on sampling based methods such as Monte Carlo's require several model runs that sample various combinations of input values. Since the number of simulations may be rather large, the needed computational resources for these types of studies could be prohibitively expensive [6]. Furthermore, as discussed in [1], [2], [4], [7], these techniques present other shortcomings, such as the statistical dependence of the input data, and the problems associated with accurately identifying probability distributions for some input data, as in the case of power generated by wind or photovoltaic generators. These issues could lead to complex computations that may limit the use of these methods in practical applications, especially for the study of large networks.

In order to overcome some of the aforementioned limitations of sampling- and statistical-based methods, the use of soft-computing-based methodologies for uncertainty representation in power flow studies has been proposed in the literature. For example, the application of Fuzzy set theory to represent imprecise information rather than using uncertainty associated with a frequency of occurrence has been proposed in several papers [8-10]. In this paradigm, the power flow input data are modeled by fuzzy numbers, which are special types of fuzzy sets.

Other studies reported in the literature have proposed the employment of self-validated computing for uncertainty representation in power flow analysis. The main advantage of self-validated computations is that their resolution algorithm itself keeps track of the accuracy of the computed quantities,

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as part of the process of computing them, without requiring information about the type of uncertainty in the parameters [11]. The simplest and most popular of these models is Interval Mathematics (IM), which allows for numerical computation where each quantity is represented by an interval of floating point numbers without a probability structure [12]. Such intervals are added, subtracted, and/or multiplied in such a way that each computed interval is guaranteed to contain the unknown value of the quantity it represents.

Many authors consider IM as a subset of the fuzzy theory, since ordinary real intervals can be considered as a special case of fuzzy numbers. However, defining the connection between IM and fuzzy set theory is not a trivial task [13]. More recently, in [14], the ideas of fuzzy sets and interval analysis are both connected to a general topological theory. Similarly, in [15] it is argued that the theory of fuzzy information granulation, the rough set theory and interval analysis can all be considered as subsets of a conceptual and computing paradigm of information processing called Granular Computing. An essential aspect of this paradigm is that its constituent methodologies are complementary and symbiotic rather than competitive and exclusive. Based on these principles, the present paper main contributions lie on the application of advanced IM based solution approaches to power flow analysis with data uncertainties, considering that:

- the search of new techniques for achieving more efficient computational processes, more reliable solution algorithms, and better enclosures of solution sets are still open problems;
- IM has the inherent ability of bounding all numerical computations and is suitable for large scale applications [2];
- self validated computing could play an important role in the development of efficient computational paradigms based on the integration of fuzzy sets and interval analysis (interval fuzzy theory), which is an emerging trend of computational intelligence [16].

The application of “standard” IM, referred here as interval arithmetic (IA), to power flow analysis has been investigated by various authors [4], [7], [17]. However, the adoption of this solution technique present many drawbacks derived mainly by the so called “dependency problem” and “wrapping effect” [11], [18]. In particular, the use of the Interval Gauss elimination in the power flow solution process leads to realistic solution bounds only for certain special classes of matrices (e.g. M-matrices, H-matrices, diagonally dominant matrices, tri-diagonal matrices) [19]. The problem of excessive conservatism in interval linear equation solving could be overcome by using Krawczyk's method or the Interval Gauss Seidel iteration procedure; however, in these cases, the linearized power flow equations should be preconditioned by an M-matrix in order to guarantee convergence [17]. These drawbacks make the application of IA to power flow analysis complex and time consuming. In the current paper these issues are addressed by proposing a new methodology for reliable power flow analysis in the presence of data uncertainty based on the use of Affine Arithmetic (AA) [11], [20]. AA is an enhanced model for self validated numerical analysis in which the quantities of interest are represented as affine combinations (affine forms) of certain primitive variables, which stand for sources of uncertainty in the data or approximations made during the computation. Unlike IA, it keeps track of correlations between computed and input quantities, and is therefore resistant to the loss of precision often observed in long interval computations [11].

This paper intends to bring the following main contributions to the existing literature:

1) It shows that the use IA in power flow analysis leads to over-pessimistic estimation of the solution hull that are not useful in most practical applications. This is mainly due to the inability of IA to keep track of correlations between the power systems state variables that are highly correlated.
2) It proposes the employment of AA to represent the uncertainties of the power flow state variables. In this approach each state variable can be expressed by a first degree polynomial composed by a central value, i.e. the nameplate value, and a number of partial deviations that represent the correlation among various variables. The adoption of AA for uncertainty representation allows to expressing the power flow equations in a more convenient formalism compared to the traditional and widely used linearization frequently adopted in interval Newton methods.
3) Finally, it proposes a solution methodology based on AA for power flow analysis with data uncertainties. By using the proposed methodology, a reliable estimation of the power flow solution hull can be computed taking into account the parameter uncertainty interdependencies as well as the diversity of uncertainty sources. The main advantage of this solution strategy is that it requires neither derivative computations nor interval systems, being thus suitable in principle for large scale power flow studies, where robust and computationally efficient solution algorithms are required.

The rest of the paper is organized as follows: Section II presents a brief review the power flow problem with data uncertainties. In Section III, the proposed AA formulation and solution procedure is described in detail. To assess the effectiveness of the proposed methodology, detailed simulation studies for a realistic system, namely, the IEEE 57-bus test system, are presented and discussed in Section IV. Finally, Section V summarizes the main conclusions and contributions of the paper.

II. PROBLEM FORMULATION

Power flow analysis deals mainly with the calculation of the steady-state voltage phasor angle and magnitude for each network bus for a given set of parameters such as load demand and real power generation, under certain assumptions such as balanced system operation. Based on this information, the network operating conditions, in particular real and reactive power flows on each branch, power losses and generator reactive power outputs, can be determined. Thus, the input (output) variables of the power flow problem are typically: the real and reactive power (voltage magnitude and angle) at each load bus (a.k.a. PQ buses); the real power generated and the voltage magnitude (reactive power generated and voltage angle) at each generation bus (a.k.a. PV buses); and the voltage magnitude and angle (the real and reactive power generated) at the slack bus (a.k.a. reference bus).

A. Power Flow Equations

The equations typically used to solve the power flow problem are the real power balance equations at the generation and load buses, and the reactive power balance at the load buses. These equations can be written as:

$$\begin{aligned} P_i^{SP} &= V_i \sum_{j=1}^{N} V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) \quad i \in nP \\ Q_j^{SP} &= V_j \sum_{k=1}^{N} V_k Y_{jk} \sin(\delta_j - \delta_k - \theta_{jk}) \quad j \in nQ \end{aligned} \tag{1}$$

where:

N is the total bus number;

nQ is the list of the buses in which the reactive power is specified;

nP is the list of the buses in which the active power is specified;

P_i^{SP} and Q_j^{SP} are the real and reactive power injections specified at i-th and j-th bus;

$\vec{V} = V_i \angle \delta_i$ is the i-th bus voltage (in polar coordinates);

$\vec{Y}_{ij} = Y_{ij} \angle \theta_{ji}$ is the ij-th element of the bus admittance matrix.

Due to the nonlinear nature of the power flow equations, numerical methods are employed to obtain a solution that is within an acceptable tolerance. This solution is referred to as an "unconstrained" solution since it has been obtained without taking into account the limits on the output variables, in particular max/min values of the reactive power at generation buses, max/min voltages module at load buses, etc. Therefore, if the obtained unconstrained solution is not feasible, a new solution satisfying the limits on the output variables, namely, a "constrained" feasible solution, should be computed. In this context, the feasibility of the reactive power limits at the generation buses is probably the most relevant issue that needs to be considered [21]. To solve this problem, the typical solution strategy is to carry out a bus-type "switching", which consists on converting a PV-bus into a PQ-bus with the reactive power set at the limiting value if the corresponding limits are violated; if at any consequent iteration, the voltage magnitude at that bus is below or above its original set point, depending on whether the generator is respectively underexcited or overexcited, the bus is then reverted back to a PV-bus.

B. Sources of Uncertainty in Power Flow Analysis

Uncertainties in power flow studies stem from several sources both internal and external to the power system. The most relevant uncertainties are related to the complex dynamics of the active and reactive power supply and demand, which may vary due to:

- overall economic activities and population in the analysed area (long term);
- weather conditions (short term);
- price of electricity based on competing energy sources (short and medium term);
- improvements on the energy end use (long term).

Forecasting these variations involves uncertainty, which could be significant, especially in the medium and long term.

One main source of uncertainty nowadays is the variable nature of generation patterns due to competition [3], as well as weather and location which significantly affect wind- and solar-power sources [22]. Other uncertainties are induced by model errors, which derive from the approximations in the values of the resistances, reactances and shunts in the models used to represent transmission lines and transformers [23]. These types of uncertainties are less significant than those associated with input active and reactive power variations, and hence are not considered in the proposed AA methodology discussed next.

III. AA-BASED METHODOLOGY FOR POWER FLOW ANALYSIS

A. Elements of AA

AA, introduced in [11] and [20], is a method for range analysis that allows for the manipulation of sources of error both external (e.g. imprecise or missing input data or uncertainty in the mathematical modeling) and internal (e.g. round off and truncation errors). AA is similar to standard IM (IA) but, in addition, keeps track of correlations between the input and computed quantities. This extra information allows to provide much tighter bounds in the computing process, reducing the likelihood of error explosion problems observed in large IA computations [20].

In AA, a partially unknown quantity x is represented by an affine form $\hat{x}$ which is a first degree polynomial of the form:

$$\hat{x} = x_0 + x_1\varepsilon_1 + x_2\varepsilon_2 + .. + x_n\varepsilon_n \tag{2}$$

where x_0 is the central value and the x_i's are known real coefficients that represents various partial deviations. The variables ε_i ("noise") stand for independent sources of uncertainty, each contributing to the total uncertainty of the quantity x, and are assumed to lie in the interval [-1,1]. The uncertainty source may be external if related to uncertainty in some inputs, or internal if related to round off and truncation errors in the computation of $\hat{x}$. The coefficients x_i represent the magnitude of the corresponding uncertainty.

A key feature of AA models is that the same noise ε_i may appear in different quantities thus representing the same source of uncertainty in the given process. In AA, each elementary real-number operation is replaced by a corresponding affine form operation, with the final result being in affine form as well. Thus, for a general operation $z = f(x, y)$, the corresponding AA operation $\hat{z} = f(\hat{x}, \hat{y})$ is a procedure that computes the affine form $\hat{z}$ for z that is consistent with the affine forms $\hat{x}$ and $\hat{y}$. If f represents an affine operation, the affine representation for z is obtained by expanding and rearranging into an affine form the noises ε_i; thus:

$$\hat{x} \pm \hat{y} = (x_0 \pm y_0) + (x_1 \pm y_1)\varepsilon_1 + (x_2 \pm y_2)\varepsilon_2 + \\ .. + (x_n \pm y_n)\varepsilon_n \quad (3)$$

$$\alpha\hat{x} = (\alpha x_0) + (\alpha x_1)\varepsilon_1 + (\alpha x_2)\varepsilon_2 + .. + (\alpha x_n)\varepsilon_n \quad \forall \alpha \in \mathrm{R} \quad (4)$$

$$\hat{x} \pm \lambda = (x_0 \pm \lambda) + x_1\varepsilon_1 + x_2\varepsilon_2 + .. + x_n\varepsilon_n \qquad \forall \lambda \in \mathrm{R} \quad (5)$$

On the other hand, if f is a non-affine operation, $\hat{z}$ cannot be express exactly as an affine combination of the noise symbols ε_i, thus:

$$\hat{z} = f(\hat{x}, \hat{y}) = f(x_0 + x_1\varepsilon_1 + x_2\varepsilon_2 + .. + x_n\varepsilon_n, y_0 + \\ + y_1\varepsilon_1 + y_2\varepsilon_2 + .. + y_n\varepsilon_n) = f^*(\varepsilon_1, \varepsilon_2, .., \varepsilon_n) \quad (6)$$

In this case the problem is the identification of an affine function:

$$f^a(\varepsilon_1, \varepsilon_2, .., \varepsilon_n) = z_0 + z_1\varepsilon_1 + ... + z_n\varepsilon_n \quad (7)$$

that approximates the function $f^*(\varepsilon_1, \varepsilon_2, .., \varepsilon_n)$ reasonably well over its domain, so that:

$$\hat{z} = f^a(\varepsilon_1, \varepsilon_2, .., \varepsilon_n) + z_k\varepsilon_k = \\ = z_0 + z_1\varepsilon_1 + ... + z_n\varepsilon_n + z_k\varepsilon_k \quad (8)$$

where the term $z_k\varepsilon_k$ represents the residual or approximation error:

$$e^*(\varepsilon_1, \varepsilon_2, .., \varepsilon_n) = f^*(\varepsilon_1, \varepsilon_2, .., \varepsilon_n) - f^a(\varepsilon_1, \varepsilon_2, .., \varepsilon_n) \quad (9)$$

The noise ε_k in (8) is distinct from all other ε_i, and the coefficient $\hat{z}_k$ is an upper bound on the absolute magnitude of e^*, i.e.:

$$|z_k| > \max\{|e^*(\varepsilon_1, \varepsilon_2, .., \varepsilon_n)| : (\varepsilon_1, \varepsilon_2, .., \varepsilon_n) \in U\} \quad (10)$$

The affine approximation function f^a in (9) could assume different structures, depending on the desired degree of accuracy and the available computational resources. A good approximation function that is reasonably accurate and not very expensive to compute is the following affine combination of the input forms $\hat{x}$ and $\hat{y}$:

$$f^a(x) = \alpha\hat{x} + \beta\hat{y} + \varsigma \quad (11)$$

where the unknown function coefficients α, β and ζ can be identified based on Chebyshev's approximation theory for univariate functions [20], which furnishes an algorithm for finding the optimum coefficients of the affine approximation function.

B. An AA Approach for Solving the Power Flow Equations

AA can be effectively adopted for uncertainty representation in power flow analysis. Thus, each state variable, i.e. the voltage magnitude of the load buses and the voltage phase of all nodes but the slack, is expressed by a central value and a set of partial deviations. These deviations are associated with as many noise variables as those which describe the effect of the various phenomena affecting the system state variables. Without loss of generality, and as discussed in Section II-B, the uncertainties considered here are those associated with the active and reactive power in loads and the active power in generators[1]. Therefore, the affine forms representing the power system state variables are:

$$V_i = V_{i,0} + \sum_{j \in nP} V^P_{i,j}\varepsilon_{Pj} + \sum_{k \in nQ} V^Q_{i,k}\varepsilon_{Qk} \quad \text{for } i \in nQ \\ \delta_i = \delta_{i,0} + \sum_{j \in nP} \delta^P_{i,j}\varepsilon_{Pj} + \sum_{k \in nQ} \delta^Q_{i,k}\varepsilon_{Qk} \quad \text{for } i \in nP \quad (12)$$

where:

ε_{Pj} is the noise representing the uncertainty of the active power injection at the j-th bus;

ε_{Qk} is the noise representing the uncertainty of the reactive power injection at the k-th bus;

$V_{i,0}$ is the central value of the i-th bus voltage magnitude;

$\delta_{i,0}$ is the central value of the i-th bus voltage angle;

$V^P_{i,j}$ is the partial deviation of the i-th bus voltage magnitude due to the active power injected at the j-th bus;

$V^Q_{i,j}$ is the partial deviation of the i-th bus voltage magnitude due to the reactive power injected at the j-th bus;

$\delta^P_{i,j}$ is the partial deviation of the i-th bus voltage angle due to the active power injected at the j-th bus;

[1] Further noise variables describing other uncertainty sources and/or more complex correlations between the affine forms could be assumed. For example it could be possible to share the same noise variables for clusters of statistically dependent loads.

$\delta^{Q}_{i,j}$ is the partial deviation of the i-th bus voltage angle due to the reactive power injected at the j-th bus.

The central values of the affine forms (12) are calculated by solving the conventional power flow equations (1) for the "nominal" operating point defined by:

$$\begin{cases} P_i^{SP} = mid\left(\left[P^{SP}_{i,\min}, P^{SP}_{i,\max}\right]\right) = \dfrac{P^{SP}_{i,\max} - P^{SP}_{i,\min}}{2} \quad \text{for } i \in nP \\ Q_i^{SP} = mid\left(\left[Q^{SP}_{i,\min}, Q^{SP}_{i,\max}\right]\right) = \dfrac{Q^{SP}_{i,\max} - Q^{SP}_{i,\min}}{2} \quad \text{for } i \in nQ \end{cases} \tag{13}$$

A first estimation of the partial deviations of the affine forms (12) can be calculated by means of the sensitivities of the desired voltage magnitudes and angles with respect of the uncertain inputs at the "nominal" operating point. Thus:

$$\begin{array}{ll} V^P_{i,j} = \left.\dfrac{\partial V_i}{\partial P_j}\right|_0 \Delta P_j \quad V^Q_{i,k} = \left.\dfrac{\partial V_i}{\partial Q_k}\right|_0 \Delta Q_k \text{ for } j \in nP \quad k, i \in nQ \\ \delta^P_{i,j} = \left.\dfrac{\partial \delta_i}{\partial P_j}\right|_0 \Delta P_j \quad \delta^Q_{i,k} = \left.\dfrac{\partial \delta_i}{\partial Q_k}\right|_0 \Delta Q_k \text{ for } i, j \in nP \quad k \in nQ \end{array} \tag{14}$$

Observe that if the power flow equations would contain only affine expressions, i.e. be a linear system of equations, the obtained affine forms would be the exact solution. However, these equations are nonlinear expressions, and hence the obtained affine forms are usually an underestimation of the exact result [24]. Thus, to guarantee the inclusion of the solution domain, each partial deviation is multiplied by an amplification coefficient [24]. Starting from this initial affine solution, a "domain contraction" based method for narrowing its bounds is proposed here as explained next.

The proposed solution algorithm first starts by plugging (12), with the initial partial deviation approximations defined in (14), in the right-hand side of the power flow equations (1) to compute the following AA form of the injected powers:

$$\begin{array}{l} \hat{Q}_i = Q_{i,0} + \sum\limits_{j\in nP} Q^P_{i,j}\varepsilon_{Pj} + \sum\limits_{k\in nQ} Q^Q_{i,k}\varepsilon_{Qk} + \sum\limits_{h\in nN} Q_{i,h}\varepsilon_h \quad \text{for } i \in nQ \\ \hat{P}_i = P_{i,0} + \sum\limits_{j\in nP} P^P_{i,j}\varepsilon_{Pj} + \sum\limits_{k\in nQ} P^Q_{i,k}\varepsilon_{Qk} + \sum\limits_{h\in nN} P_{i,h}\varepsilon_h \quad \text{for } i \in nP \end{array} \tag{15}$$

where

$\hat{Q}_i$, $\hat{P}_i$ are the affine forms of the calculated active and reactive power injections in the i-th bus;

ε_h are new noise variables introduced in the computational process due to the presence of non affine operations (nN denotes the set of these new noise variables);

$Q_{i,0}, Q^P_{i,j}, Q^Q_{i,j}, Q_{i,h}, P_{i,0}, P^P_{i,j}, P^Q_{i,j}, P_{i,h}$ are the computed central values (13) and the partial deviations of the affine forms of the calculated active and reactive powers injected in the i-th.

Note that the AA operators (3)-(5) and affine approximations of the sinusoidal functions described in [20] are used to obtain $Q^P_{i,j}, Q^Q_{i,j}, Q_{i,h}, P^P_{i,j}, P^Q_{i,j}, P_{i,h}$.

The obtained affine forms (15) can be arranged in the following matrix form:

$$\begin{bmatrix} \hat{Q}_1 \\ \ldots \\ \hat{Q}_{nQ} \\ \hat{P}_1 \\ \ldots \\ \hat{P}_{nP} \end{bmatrix} = \begin{bmatrix} Q_{1,0} \\ \ldots \\ Q_{nQ,0} \\ P_{1,0} \\ \ldots \\ P_{nP,0} \end{bmatrix} + \begin{bmatrix} Q^P_{1,1} & \ldots & Q^P_{1,nP} & Q^Q_{1,1} & \ldots & Q^Q_{1,nQ} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ Q^P_{nQ,1} & \ldots & Q^P_{nQ,nP} & Q^Q_{nQ,1} & \ldots & Q^Q_{nQ,nQ} \\ P^P_{1,1} & \ldots & P^P_{1,nP} & P^Q_{1,1} & \ldots & P^Q_{1,nQ} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ P^P_{nP,1} & \ldots & P^P_{nP,nP} & P^Q_{nP,1} & \ldots & P^Q_{nP,nQ} \end{bmatrix} \begin{bmatrix} \varepsilon_{P_1} \\ \ldots \\ \varepsilon_{P_{nP}} \\ \varepsilon_{Q_1} \\ \ldots \\ \varepsilon_{Q_{nQ}} \end{bmatrix} + \begin{bmatrix} Q_{1,1} & \ldots & Q_{1,nN} \\ \ldots & \ldots & \ldots \\ Q_{nQ,1} & \ldots & Q_{nQ,nN} \\ P_{1,1} & \ldots & P_{1,nN} \\ \ldots & \ldots & \ldots \\ P_{nP,1} & \ldots & P_{nP,nN} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \ldots \\ \ldots \\ \ldots \\ \ldots \\ \varepsilon_{nN} \end{bmatrix} \tag{16}$$

Or in a more general form:

$$f(X) = AX + B \tag{17}$$

where:

$$A = \begin{bmatrix} Q^P_{1,1} & \ldots & Q^P_{1,nP} & Q^Q_{1,1} & \ldots & Q^Q_{1,nQ} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ Q^P_{nQ,1} & \ldots & Q^P_{nQ,nP} & Q^Q_{nQ,1} & \ldots & Q^Q_{nQ,nQ} \\ P^P_{1,1} & \ldots & P^P_{1,nP} & P^Q_{1,1} & \ldots & P^Q_{1,nQ} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ P^P_{nP,1} & \ldots & P^P_{nP,nP} & P^Q_{nP,1} & \ldots & P^Q_{nP,nQ} \end{bmatrix} \tag{18}$$

$$X = \begin{bmatrix} \varepsilon_{P_1} \\ \ldots \\ \varepsilon_{P_{nP}} \\ \varepsilon_{Q_1} \\ \ldots \\ \varepsilon_{Q_{nQ}} \end{bmatrix} \tag{19}$$

$$B = \begin{bmatrix} Q_{1,0} \\ \dots \\ Q_{nQ,0} \\ P_{1,0} \\ \dots \\ P_{nP,0} \end{bmatrix} + \begin{bmatrix} Q_{1,1} & \dots & Q_{1,nN} \\ \dots & \dots & \dots \\ Q_{nQ,1} & \dots & Q_{nQ,nN} \\ P_{1,1} & \dots & P_{1,nN} \\ \dots & \dots & \dots \\ P_{nP,1} & \dots & P_{nP,nN} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \dots \\ \dots \\ \dots \\ \dots \\ \varepsilon_{nN} \end{bmatrix} \quad (20)$$

Here: A is a matrix of computed real coefficients; X is the vector that need to be contracted, with initial values for each of its components set at [-1,1]; and B is an interval vector, since the new noise variables ε_h vary in the interval [-1,1] and hence it is not possible to contract them, as these represent internal noise introduced by the AA computational process.

The power flow solution can then be obtained by contracting the vector X so that:

$$AX + B = f^{SP} \quad (21)$$

where f^{SP} is the following interval vector defining the specified range of the active and reactive powers:

$$f^{SP} = \begin{bmatrix} [Q^{SP}_{1\min}, Q^{SP}_{1\max}] \\ \dots \\ [Q^{SP}_{NQ\min}, Q^{SP}_{NQ\max}] \\ [P^{SP}_{1\min}, P^{SP}_{1\max}] \\ \dots \\ [P^{SP}_{NP\min}, P^{SP}_{NP\max}] \end{bmatrix} \quad (22)$$

The problem thus is reduced to solving the IA problem:

$$\begin{aligned} AX &= f^{SP} - B \\ &= C \end{aligned} \quad (23)$$

where A is a real, known matrix. This linear IA problem can be effectively solved using the following N_P+N_Q constrained linear optimization problems, where N_P and N_Q represent the number of PV nodes and PQ nodes, respectively:

$$\begin{aligned} &\min\,(\varepsilon_{Qk}, \varepsilon_{Pj}) \qquad \text{for } k \in nQ,\, j \in nP \\ &\text{s.t.} \\ &-1 \le \varepsilon_{Qk} \le 1 \\ &-1 \le \varepsilon_{Pj} \le 1 \\ &\inf(C_i) \le \sum_{j \in nP} A_{ij}\varepsilon_{Pj} + \sum_{k \in nQ} A_{ik}\varepsilon_{Qk} \le \sup(C_i) \\ &\text{for } i = 1,2,..N_P + N_Q \end{aligned} \quad (24)$$

$$\begin{aligned} &\max\,(\varepsilon_{Qk}, \varepsilon_{Pj}) \qquad \text{for } k \in nQ,\, j \in nP \\ &\text{s.t.} \\ &-1 \le \varepsilon_{Qk} \le 1 \\ &-1 \le \varepsilon_{Pj} \le 1 \\ &\inf(C_i) \le \sum_{j \in nP} A_{ij}\varepsilon_{Pj} + \sum_{k \in nQ} A_{ik}\varepsilon_{Qk} \le \sup(C_i) \\ &\text{for } i = 1,2,..N_P + N_Q \end{aligned} \quad (25)$$

These are standard linear programming (LP) problems which can be readily and efficiently solved by using an LP solver such as CPLEX. The desired power flow solution is then obtained as:

$$\begin{aligned} V_i &= V_{i,0} + \sum_{j \in nP} V^P_{i,j}[\varepsilon_{Pj,\min}, \varepsilon_{Pj,\max}] + \sum_{k \in nQ} V^Q_{i,k}[\varepsilon_{Qk,\min}, \varepsilon_{Qk,\max}] \quad \text{for } i \in nQ \\ \delta_i &= \delta_{i,0} + \sum_{j \in nP} \delta^P_{i,j}[\varepsilon_{Pj,\min}, \varepsilon_{Pj,\max}] + \sum_{k \in nQ} \delta^Q_{i,k}[\varepsilon_{Qk,\min}, \varepsilon_{Qk,\max}] \ \text{for } i \in nP \end{aligned} \quad (26)$$

Observe that the propped solution procedure represents an alternative to the traditional and widely used linearization formalism adopted in IA approaches, which is based on the Interval Newton method and consist on solving the following IA problem:

$$f(x_0 + \Delta x) \in f(x) + J(x_0)\Delta x \quad \forall x \in x_0 \quad (27)$$

where x_0 is a vector of intervals, the Jacobian matrix $J(x_0)$ is an interval matrix, and $f(x)$ is a real vector defined by x, which is typically the midpoint of x_0. Solving (27) requires the "inversion" of the interval matrix $J(x_0)$, which is a nontrivial problem [25], [26]. As pointed out in [17], [19], this is the main impediment in the application of IA to power flow studies. The solution of (23) does not require an interval matrix inversion, making it computationally efficient and hence readily applicable to real size systems.

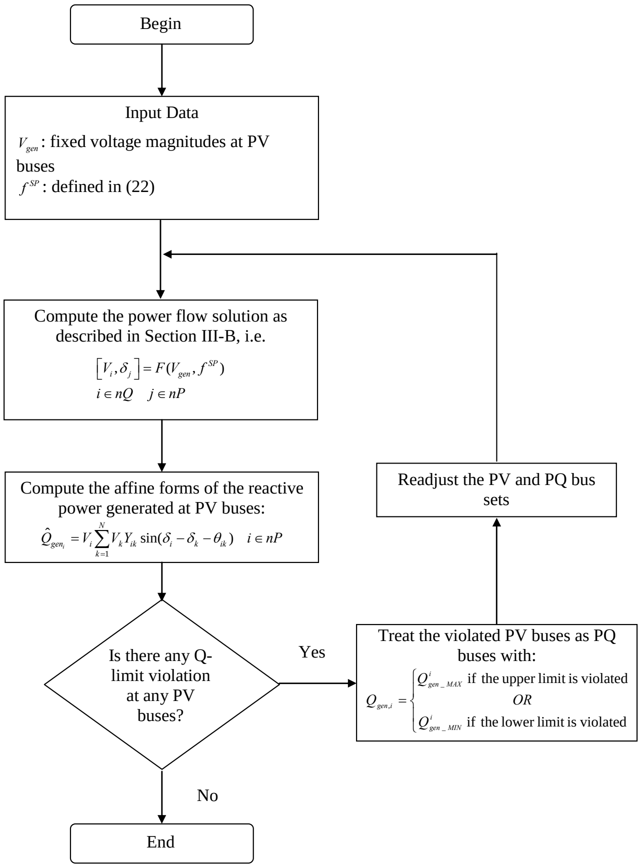
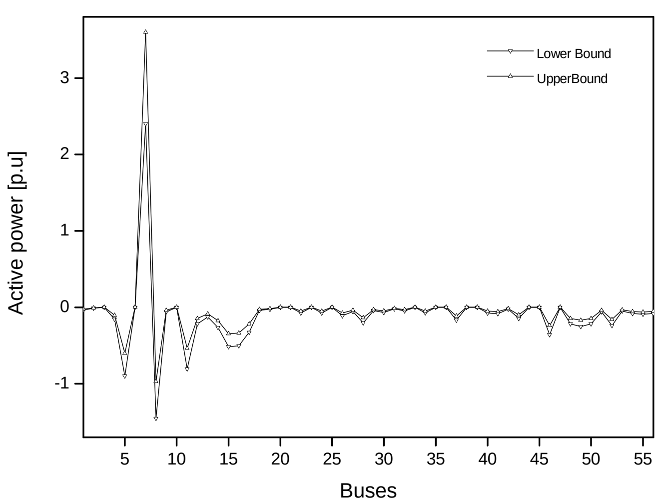


Fig.1: Solution algorithm adopted to manage the limits on the reactive power at PV buses.

The described AA-based solution methodology can be improved to account for reactive power limits and properly model the generators' voltage regulators. This is done here by using the standard PV- and PQ-bus switching as described in Section II-A, which results in the iterative procedure depicted in Fig. 1. Convergence is attained when all the reactive power limits at PV buses are satisfied.



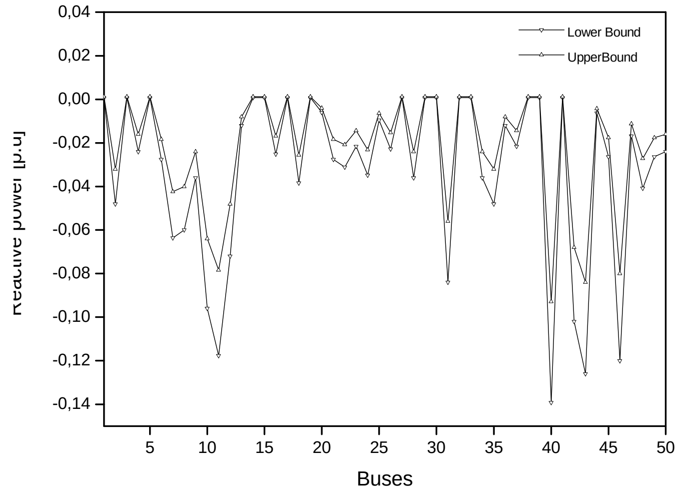


Fig. 2: Active and reactive power bounds.

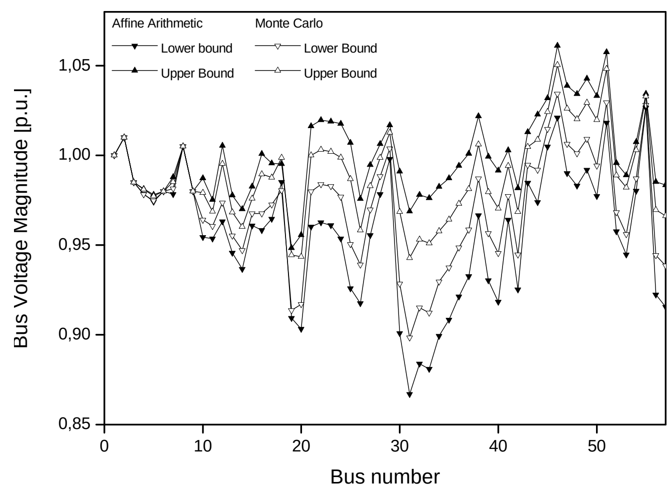


Fig. 3: Bus voltage magnitude bounds.

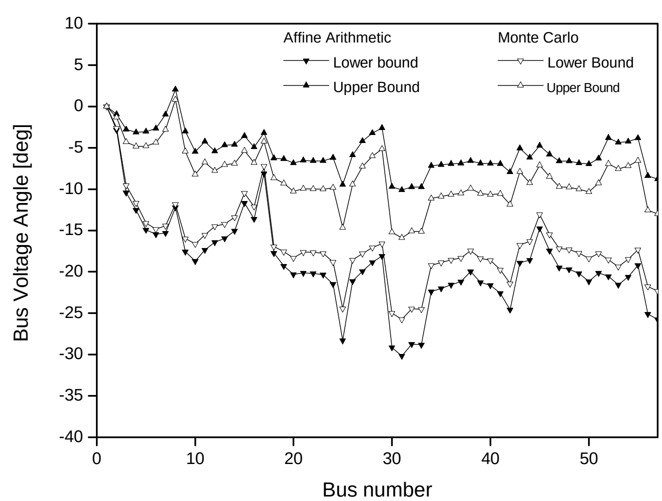


Fig. 4: Bus voltage angle bounds.

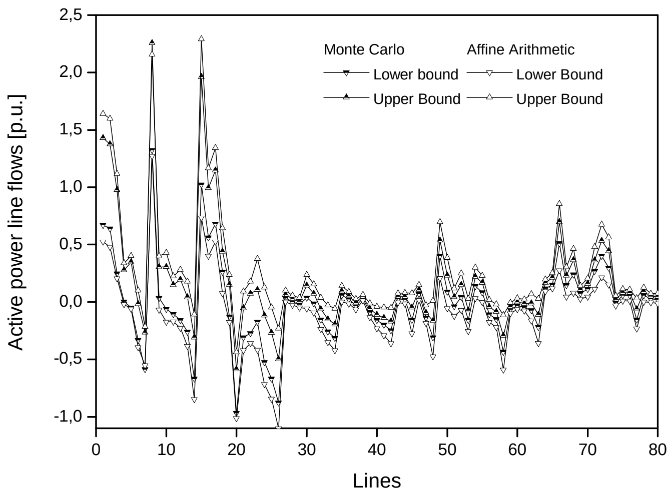


Fig. 5: Bounds of the active power line flows.

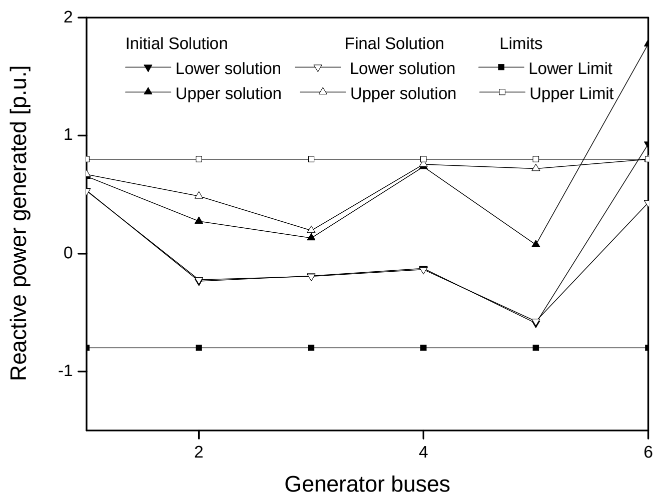


Fig. 6: Reactive power bounds at PV buses.

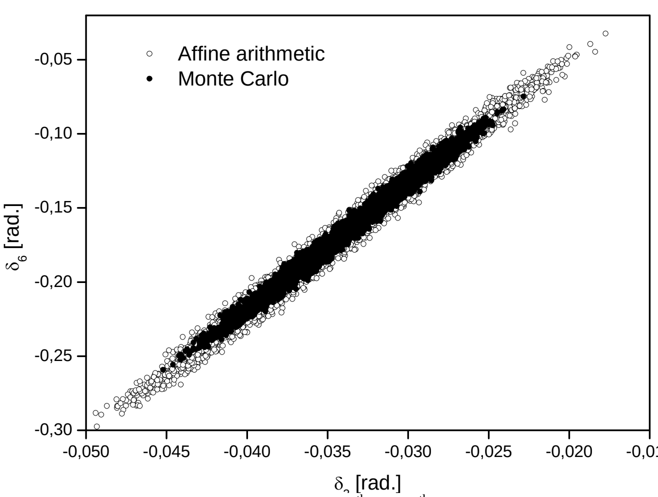


Fig. 7: Solutions boundary of the 2th and 6th bus voltage angles.

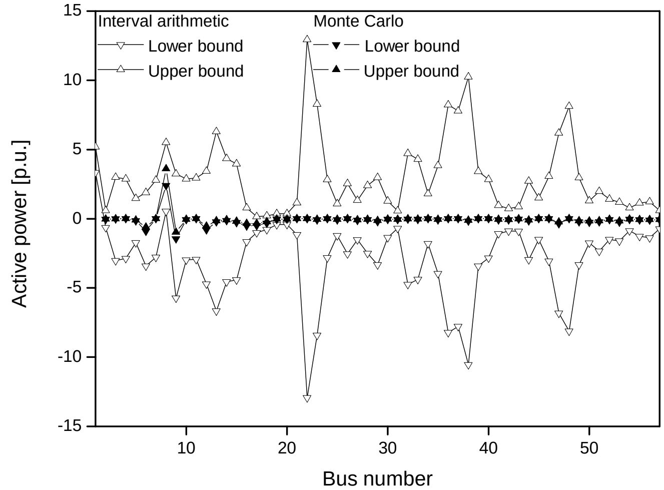


Fig. 8: Active powers at the PV and PQ buses obtained by IA-based processing of the state variable bounds computed by the Mote Carlo approach.

IV. Simulation Results

This section discusses the application of the proposed methodology to the power flow analysis with uncertainties of the IEEE 57-bus test system [27]. The power flow solution bounds obtained by the proposed AA-based technique are compared to those calculated using a Monte Carlo simulation with a uniform distribution, which is typically assumed to yield the “correct” solution intervals. For the latter, 5000 different values of the input variables within the assumed input bounds were randomly selected, and a conventional power flow solution was obtained for each one; this procedure yielded the desired interval solutions defined by the largest and the smallest values of the bus voltage magnitudes and angles as well as line flows. It should be noted that increasing the number of Monte Carlo simulations beyond 5000 did not yield any significant changes to the solution intervals.

All computational tasks were performed using Matlab. For the representation of affine forms, a vectorial based approach was adopted, which is computationally more efficient in Matlab. Thus, each affine form is represented as a vector whose first element represents the central value while the other vector components describe the partial deviations with respect to the corresponding noise variable as follows:

$$\hat{x} = x_0 + x_1\varepsilon_1 + x_2\varepsilon_2 + .. + x_n\varepsilon_n \Leftrightarrow \hat{x} = [x_0, x_1, x_2, .., x_n] \quad (28)$$

Without loss of generality, a ± 20% tolerance on load and generator powers was assumed. Observe that this would define an interval wide enough to properly evaluate the proposed method. The assumed power profiles are shown in Fig. 2. Since the system data does not contain information regarding reactive power limits at PV buses, these were assumed to be ± 0.8 p.u. to be able to test this particular feature of the proposed algorithm.

Based on the assumed load and generator power bounds to represent input data uncertainty, the proposed AA-based methodology was applied to estimate the bounds of the power flow solution. After the first iteration, the solution algorithm detected that the upper bound of the reactive power on the 6th generator, connected at the 12th bus, violated the upper limit. Consequently, the solution algorithm (*i*) switched this bus from PV to PQ; (*ii*) fixed the corresponding reactive power to the maximum allowable value; and (*iii*) proceeded to calculate another power flow solution. The solution algorithm converged to a feasible solution in two iterations.

The computed AA-based solution was compared with that obtained by using the Monte Carlo approach. The corresponding profiles are shown in Fig. 3-6, with Fig. 3 depicting the bus voltages magnitude bounds; Fig. 4 shows the bus voltages angle bounds; Fig. 5 illustrates the bounds of the active power line flows; and Fig. 6 depicts the reactive power generated at the PV buses before and after the bus-type switch, i.e. the initial and the final solutions. Observe that the AA-based methodology gives fairly good approximations of the power flow solution bounds when compared to the benchmark intervals obtained with the Monte Carlo approach; this is mainly due to the intrinsic characteristic of AA that keeps track of correlations between the power systems state variables.

Notice also that the solution bounds are slightly conservative, which is due to the fact that AA, like IA, yields "worst case" bounds, which take into account any uncertainties in the input data as well as all internal truncation and roundoff errors. This is to be expected, since, as stated in [28], the random, uniformly distributed variation of parameters (with mean equal zero) assumed in the Monte Carlo approach tends to underestimate the worst case variations. This can be considered an advantage of the proposed approach, since no assumptions regarding the probability distribution of load and generator power variations are required.

In order to assess the benefits of uncertainty representation by AA compared to IA, further studies aimed at characterizing the solution domain of the power flow equations were performed.

These results are summarized in Fig. 7 and 8, with Fig. 7 showing the solution domain for the 2nd and 6th bus voltage angles assessed by AA and the Monte Carlo approach, whereas Fig. 8 depicts the active power bounds at the PV and PQ buses obtained by applying IA to the "correct" power flow interval computed using Monte Carlo. Observe that AA yields a more realistic elliptical approximation of the solution domain compared to the typical "hyper box" (rectangle) obtained using IA, which may result on missing some salient features of the actual variations of the power systems state variables.

This in turn leads to a large overestimation of the complex power bus injections, as confirmed by the wide IA-based bounds shown in Fig. 8. Note that, although these bounds were obtained by processing the best available approximation of the state variable solution bounds, the calculated active power bus injection bounds are significantly larger than the assumed ± 20 % interval.

As far as the computational requirements are concerned, the AA based solution strategy required about 7 seconds (on a 1.2 GHz CPU with 2 GB of RAM) to converge to a suitable power flow solution for this case study. This is about 3% of the simulation time required by the Monte Carlo approach.

V. CONCLUSIONS

This paper proposed a new methodology for reliable power flow analysis in the presence of data uncertainty based on use of AA, allowing to better handle uncertainty compared to the traditional and widely used IA approaches. The latter are based on interval Newton methods that require inverting an interval matrix and thus presenting a major impediment for its practical application. Based on the proposed new AA formalism, the power flow solution bounds were shown to be simply obtained by solving a power flow plus two straight forward LP problems. It was shown with the help of tests run on a realistic power system that using AA allows addressing effectively the "wrapping effect" and the "dependency problem" of IA, that leads to a better characterization of the effects of input data uncertainty in power flow solutions, and a more realistic approximation of the solution domain compared to the typical "hyper box" form obtained with IA approaches. The presented analyses and results demonstrate that the proposed AA-based approach is well suited for the assessment of uncertainty propagation in power flow solutions, and that it can be effectively applied to study large systems, independent of the types and levels of uncertainties in the input data.

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