A Robust Optimization Approach for Planning the Transition to Plug-in Hybrid Electric Vehicles

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***Abstract*—This paper proposes a new technique to analyze the electricity and transport sectors within a single integrated framework to realize an environmentally and economically sustainable integration of plug-in hybrid electric vehicles (PHEVs) into the electric grid, considering the most relevant planning uncertainties. The method is based on a comprehensive robust optimization planning that considers the constraints of both the electricity grid and the transport sector. The proposed model is justified and described in some detail, applying it to the real case of Ontario, Canada, to determine Ontario's grid potential to support PHEVs for the planning horizon 2008-2025.**

***Index Terms*—PHEV, transportation, electric grid, planning, data uncertainty, robust optimization, Ontario.**

I. Glossary of Terms and Nomenclature

Abbreviations

ADI	Average Deviation Index.
AER	All-Electric Range.
AFV	Alternative-Fuel Vehicle.
AMPL	A Modeling Language for Mathematical Programming.
CAD	Canadian Dollar.
CHP	Combined Heat and Power.
GV	Gasoline Vehicle.
HOEP	Hourly Ontario Energy Price.
IESO	Independent Electricity System Operator.
LDV	Light-Duty Vehicle.
MILP	Mixed Integer Linear Programming.
NE	North East.
NP	Non-deterministic Polynomial-time.
NW	North West.
OPA	Ontario Power Authority.
PHEV	Plug-in Hybrid Electric Vehicle.
SCC	Social Cost of Carbon.
SUV	Sport Utility Vehicle.
SW	South West.

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Indexes

c	Index for vehicle type.
e	Index for constraint under uncertainty.
i, j	Index for zones.
l	Index for voltage angle block.
m	Index for Monte Carlo simulation.
v	Index for uncertain parameter.
y	Index for year.

Sets

E	Set of constraints subject to uncertainty.
U	Set of uncertainty.
V	Set of total uncertain parameters.
V_e	Set of uncertain parameters in constraint e.
VT	Set of different types of light-duty vehicles.
X	Set of mixed integer feasible solution.
Y	Set of planning years.
Y_1	Set of planning years excluding the first year.
Z	Set of zones.
Ω	Set of transmission lines.

Parameters

DR	Discount rate (8%).
DT	PHEVs' daily trip running on battery (km).
E_{CO2}	Constant value of CO_2 emissions from burning gasoline (2.3 kg/liter).
ER_{chp_i}	Emission rate of CHP plants (ton/MWh).
ER_{coal_i}	Emission rate of coal plants (ton/MWh).
$FE_{\text{gv}_{cy}}$	Fuel economy of the gasoline vehicle (km/liter).
h	Number of off-peak hours.
M	Number of Monte Carlo simulations.
$N_{\text{ldv}_{iy}}$	Total number of light-duty vehicles.
$P_{\text{e}_{iy}}$	Zonal base-load electricity demand (MW).
$\overline{P}_{\text{g}_{iy}}$	Maximum base-load generation power (MW).
$\underline{P}_{\text{g}_{iy}}$	Lower bound of zonal generation power (MW).
$\overline{P}_{\text{ga}_{iy}}$	Maximum capacity of non-polluting base-load generation resources (MW).
$\overline{P}_{\text{gb}_{iy}}$	Maximum capacity of non-polluting plus CHP generation resources (MW).
$\overline{P}_{\text{im}_{iy}}$	Upper bound of imported power (MW).
$\underline{P}_{\text{im}_{iy}}$	Lower bound of imported power (MW).
$\overline{P}_{\text{ex}_{iy}}$	Upper bound of exported power (MW).
$\underline{P}_{\text{ex}_{iy}}$	Lower bound of exported power (MW).
$P_{\text{ch}_{iy}}$	Total maximum PHEVs charging power.

$\overline{P}_{\mathrm{d}ijy}$, $\overline{P}_{\mathrm{r}ijy}$	Maximum capacity of transmission corridor for direct and reverse power flows, respectively (MW).
s_{ev}	Scaled deviation.
$SC_{CO_2}\mathrm{p}$	Social cost of CO_2 emission in the population area (CAD/ton).
$SC_{CO_2}\mathrm{g}$	Social cost of CO_2 emission of generation (CAD/ton).
VS_c	Percent share of vehicle.
Γ_e	Budget of uncertainty.
ϵ_a, ϵ_b	Small positive numbers.
ε_e	Constraint violation probability (%).
λ	A random parameter.
$\bar{\mu}_y$	Maximum possible PHEV penetration (%).
π_y	Internal or hourly Ontario energy price (CAD/MWh).
π_{im_y}	Import electricity price (CAD/MWh).
π_{ex_y}	Export electricity price (CAD/MWh).
ρ	Size of the relative perturbation (%).

Variables

FF_{iy}	Feasibility factor.
K_{liy}	Binary variable to denote if the average zonal generation power is located in the lth segment of the emission cost curve.
p_e, q_{ev}, r_v	Additional continuous variables used to develop robust counterpart problems.
$P_{\mathrm{g}_{iy}}$	Average zonal generation power (MW).
$P_{\mathrm{g}_{liy}}$	Continuous auxiliary variable for the average zonal generation power ($l = 1, 2, 3$).
P_{ijy}	Power flow between zones (MW).
$P_{\mathrm{im}_{iy}}$	Zonal imported power (MW).
$P_{\mathrm{ex}_{iy}}$	Zonal exported power (MW).
P_{loss}	Power loss (MW).
V_{liy}	Continuous auxiliary variable for $K_{liy}P_{g_{iy}}$.
W	A continuous variable used to represent all the uncertain parameters in the constraints.
δ_{iy}	Voltage angle (rad).

II. Introduction

CHALLENGES related to the supply of oil have led governments and companies around the world to invest in new transportation technologies and incentive programs to encourage the purchase of alternative-fuel vehicles (AFVs), such as PHEVs, due to their environmental benefits and the technical advantages of electric drives. Although approaches such as distributed generation and demand-side management are being widely implemented, the optimal utilization of the existing energy infrastructure in the context of its utilization for transportation purposes must still be addressed. For example, during off-peak hours, namely 11 pm to 7 am, the underused electric grid infrastructure could be an efficient means of charging batteries for PHEVs. In return, battery storage in PHEVs could provide dispersed storage capacity for the whole grid, which would help address the intermittent nature of renewable energy sources (e.g., wind and solar) and also facilitate the further development of environmentally friendly energy resources. The impact of PHEVs on the electric grid should therefore be analyzed in detail, and appropriate quantitative tools and optimization planning models for a transition to these vehicles should be developed in order to maximize their environmental benefits and minimize their corresponding costs and negative effects.

A review of the literature (e.g., [1]–[6]) shows that transmission system constraints have not yet been included in the analysis of the impact of PHEVs on the grid. At the generation level, previous work has attempted either to derive the grid potential for the present support of PHEVs or to determine the gross generation capacity required for supporting a target value of PHEV penetration at a specified future time. However, these studies do not consider the penetration of PHEVs as a transitional process, and a mathematical programming approach, especially with the consideration of parameter uncertainty, has not been applied to study the grid potential for supporting PHEVs.

This paper explores the technical and economic feasibility of improving the utilization of the electric grid during off-peak hours for PHEVs, and presents an optimization model for planning the transition to these types of vehicles. Since these studies require the estimation of a variety of parameters, estimation errors may substantially influence the optimal solution and the optimal value. Therefore, the application of a robust optimization approach is proposed, based on a sensitivity analysis using Monte Carlo simulations to determine the impact of estimation errors on the parameters of the planning model and identify the most relevant parameters to be considered in the robust model. In the adopted robust methodology, decision makers can properly adjust the level of conservatism, which is not possible in the classic robust optimization approach in which robustness is ensured at the cost of significant loss of optimality. The practical application of the proposed optimization framework is demonstrated by applying it to Ontario, Canada, with the purpose of determining the optimal potential penetration of PHEVs into Ontario's transport sector by 2025, without jeopardizing the reliability of the grid or requiring the development of new infrastructure, and with consideration of the most relevant uncertainties. The proposed model builds on the authors' previous work in [7] to offer the following new contributions:

1) A model of the emission costs of generation.
2) A comprehensive sensitivity analysis to identify the parameters that most influence the optimal value.
3) Application of a robust optimization approach to derive robust optimal penetrations of PHEVs into the transport sector, which properly considers data uncertainty.

The remainder of the paper is structured as follows: Section III discusses the environmental effects of PHEVs with respect to both the transport sector and generation sites, and presents a model of generation emission costs. Section IV introduces the optimization model used for planning the transition to PHEVs. Section V discusses the impact of parameter uncertainty on optimization models along with the methodologies used to deal with this problem; the robust optimization approach adopted in this study to address data uncertainty is also described in

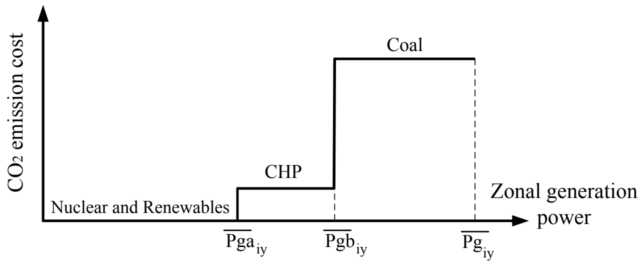

Fig. 1. Emission cost function of generation.

this section. The application of the proposed methodology to Ontario, Canada, is presented in Section VI. Finally, Section VII summarizes the main conclusions and contributions of this study.

III. Environmental Aspects

A. PHEVs

Gasoline and diesel vehicles and fossil fuel plants release CO_2 into the atmosphere. Based on the social cost of carbon emissions (SCC), the monetary value of the damage to the environment can be estimated [8]. This study, therefore, assigns an environmental credit for each PHEV introduced into the transport sector, depending on the SCC, the fuel economy of the light-duty vehicle (LDV) to be replaced by a PHEV, and the known value of CO_2 emissions from burning gasoline (2.3 kg/liter). An environmental cost is also assigned to each power generation plant, based on the SCC, the power generation level, and the CO_2 emission rates of the plant. Disregarding this environmental cost may lead to an overestimation of the environmental benefits of PHEVs, because extra power that may increase the CO_2 emissions must be generated to cover the energy requirements of PHEVs.

High penetration of PHEVs into society's vehicle fleet would shift emissions from highly populated areas to a limited number of central generation power plants. Studies show that PHEVs result in lower emissions than conventional gasoline vehicles even for regions with high CO_2 levels from electric generation [2], [3]. The present study thus differentiates the impact of CO_2 emissions based on their point of origin, as per [9], and considers different values for social costs in densely populated areas versus sparsely populated areas, where power generation plants are typically located. Note that CO_2 emission is used as an indicator metric to represent the suite of emissions from power generation including NO_x, SO_x, volatile organic compound (VOC), particulate matter (PM) as well as climate change gases such as CO_2. In view of all these considerations, different CO_2 emission costs/credits in the range of 50 to 125 CAD/ton are assumed in the populated area where urban air quality is an issue, and the CO_2 emission cost of generation is assumed to be 10 CAD/ton [8].

B. Emission Cost Model of Generation

Figure 1 represents a typical emission cost function during off-peak hours (e.g., Ontario), when the contributing base-load generation resources include nuclear, renewables, CHP and coal plants.

To incorporate the cost function in Fig. 1 into the optimization model, three sets of binary variables K_{1iy}, K_{2iy} and K_{3iy} corresponding to three segments of the cost function are defined. Also, three sets of continuous auxiliary variables $P_{\mathrm{g}_{1iy}}$, $P_{\mathrm{g}_{2iy}}$ and $P_{\mathrm{g}_{3iy}}$ for the average zonal generation power $P_{\mathrm{g}_{iy}}$ are needed, and are defined as follows:

$$P_{\mathrm{g}_{iy}} = \begin{cases} P_{\mathrm{g}_{1iy}}: & \text{if } K_{1iy} = 1, \\ P_{\mathrm{g}_{2iy}}: & \text{if } K_{2iy} = 1, \\ P_{\mathrm{g}_{3iy}}: & \text{if } K_{3iy} = 1. \end{cases} \tag{1}$$

This results in the following CO_2 generation emission costs to be added to the objective function:

$$\begin{aligned} EC^* = \sum_y \sum_i \Big\{ & ER_{\mathrm{chp}_i} \Big[K_{2iy} P_{\mathrm{g}_{iy}} - K_{2iy} \overline{P_{\mathrm{ga}}}_{iy} + K_{3iy} \Big(\overline{P_{\mathrm{gb}}}_{iy} - \overline{P_{\mathrm{ga}}}_{iy} \Big) \Big] \\ & + ER_{\mathrm{coal}_i} \Big(K_{3iy} P_{\mathrm{g}_{iy}} - K_{3iy} \overline{P_{\mathrm{gb}}}_{iy} \Big) \Big\} SC_{CO_2\mathrm{g}} \times h \times 365 \end{aligned} \tag{2}$$

And in the following additional set of constraints:

$$\begin{aligned} & 0 \le P_{\mathrm{g}_{1iy}} \le K_{1iy} \overline{P_{\mathrm{ga}}}_{iy} \\ & K_{2iy} \left(\overline{P_{\mathrm{ga}}}_{iy} + \epsilon_a \right) \le P_{\mathrm{g}_{2iy}} \le K_{2iy} \left(\overline{P_{\mathrm{gb}}}_{iy} + \epsilon_a \right) \\ & K_{3iy} \left(\overline{P_{\mathrm{gb}}}_{iy} + \epsilon_a + \epsilon_b \right) \le P_{\mathrm{g}_{3iy}} \le K_{3iy} \left(\overline{P_{\mathrm{g}}}_{iy} + \epsilon_a + \epsilon_b \right) \\ & K_{1iy} + K_{2iy} + K_{3iy} \le 1 \\ & P_{\mathrm{g}_{iy}} = P_{\mathrm{g}_{1iy}} + P_{\mathrm{g}_{2iy}} + P_{\mathrm{g}_{3iy}} \\ & \forall i \in Z \wedge y \in Y \end{aligned} \tag{3}$$

To remove non-linear terms in (2), i.e., $K_{2iy} P_{\mathrm{g}_{iy}}$ and $K_{3iy} P_{\mathrm{g}_{iy}}$, to keep the model a mixed integer linear programming (MILP) problem, these terms are replaced by the variables V_{2iy} and V_{3iy}, respectively, plus the following constraints:

$$\begin{aligned} & V_{2iy} \ge P_{\mathrm{g}_{iy}} - \overline{P_{\mathrm{g}}}_{iy} \left(1 - K_{2iy} \right) \\ & V_{3iy} \ge P_{\mathrm{g}_{iy}} - \overline{P_{\mathrm{g}}}_{iy} \left(1 - K_{3iy} \right) \\ & V_{2iy} \ge \underline{P_{\mathrm{g}}}_{iy} K_{2iy} \\ & V_{3iy} \ge \underline{P_{\mathrm{g}}}_{iy} K_{3iy} \\ & \forall i \in Z \wedge y \in Y \end{aligned} \tag{4}$$

IV. Optimization Model

With the goal of determining the potential optimal penetration of PHEVs into the transport sector for each year of the planning horizon, an MILP model is developed as explained next.

A. Objective Function

The model's objective is to minimize the present value of the net electricity and emission costs. Thus, the objective function consists of electricity generation and imported/exported power cost/revenue components, and emission cost and credit components for generation facilities and population areas, respectively. The objective function is therefore expressed as follows:

$$\min \sum_{y \in Y} \frac{1}{(1+DR)^{y-y_1}} \left(C_{1y} - C_{2y} + C_{3y} \right) \tag{5}$$

where $C_{iy}, i \in \{1, ..., 3\}$ are different cost or revenue components as follows:

1) Net Electricity Cost: C_{1y} represents the net total electricity costs in year y. Assuming that batteries are charged during a similar time frame on both weekdays and weekends, this component can be expressed as follows:

$$C_{1y} = \sum_{i \in Z} \left(P_{\mathrm{g}_{iy}} \pi_y + P_{\mathrm{im}_{iy}} \pi_{\mathrm{im}_y} - P_{\mathrm{ex}_{iy}} \pi_{\mathrm{ex}_y} \right) \times h \times 365 \quad (6)$$

2) Environmental Credit: C_{2y} represents the environmental credits assigned to PHEVs in year y and is stated as:

$$C_{2y} = \sum_{i \in Z} \Bigg\{ FF_{iy}.\bar{\mu}_y.N_{\mathrm{ldv}_{iy}}.DT.SC_{CO_2}\mathrm{p}.E_{CO_2} \times 0.365 \sum_{c \in VT} \left(\frac{VS_c}{FE_{\mathrm{gv}_{cy}}} \right) \Bigg\} \quad (7)$$

Note that appropriate credit in the objective function is given only to the charge depletion share of vehicle miles traveled, denoted by DT and also referred to as the all-electric range (AER). It is to be emphasized that $\bar{\mu}_y$ is fixed by the system planner based on an assumed transition curve to account for the time needed for the development of the required infrastructure [7]. Also, FF_{iy} determines the percentage of the penetration levels set by the transition curve that is achievable due to electricity grid constraints.

3) Emission Cost of Generation: C_{3y} represents the environmental costs of generation in year y. Based on the mathematical formulation (Section III-B), this cost component is expressed as follows:

$$C_{3y} = \sum_{i \in Z} \Big\{ ER_{\mathrm{chp}_i} \Big[V_{2_{iy}} - K_{2iy} \overline{P_{\mathrm{ga}}}_{iy} + K_{3iy} \left(\overline{P_{\mathrm{gb}}}_{iy} - \overline{P_{\mathrm{ga}}}_{iy} \right) \Big] + ER_{\mathrm{coal}_i} \left(V_{3_{iy}} - K_{3iy} \overline{P_{\mathrm{gb}}}_{iy} \right) \Big\} SC_{CO_2}\mathrm{g} \times h \times 365 \quad (8)$$

B. Constraints

1) Transmission System: Given that the presented planning study, requires only an approximate representation of the grid, a dc optimal power flow model that accounts for the transmission system losses is adopted. Following the method proposed in [10], a linear approximation of power losses in year y can be obtained using piecewise linear blocks for voltage angle differences. This method results in the following zonal-power-balance constraints:

$$\begin{gathered} P_{\mathrm{g}_{iy}} - P_{\mathrm{e}_{iy}} - FF_{iy}\overline{\mu}_y P_{\mathrm{ch}_{iy}} + P_{\mathrm{im}_{iy}} - P_{\mathrm{ex}_{iy}} \\ - \sum_{(i,j) \in \Omega} \left[\frac{1}{2} P_{\mathrm{loss}}(\delta_{iy}, \delta_{jy}) - P_{ijy}(\delta_{iy}, \delta_{jy}) \right] = 0 \\ \forall i \in Z \wedge y \in Y \end{gathered} \quad (9)$$

plus several other constraints related to the power loss linearization as discussed in [10] and [11].

2) Transmission Capacity Constraints: Based on the transmission model, these constraints are defined as follows:

$$\begin{gathered} -P_{ijy}(\delta_{iy}, \delta_{jy}) + \frac{1}{2} P_{\mathrm{loss}}(\delta_{iy}, \delta_{jy}) \le \overline{P_{\mathrm{d}}}_{ijy} \\ P_{ijy}(\delta_{iy}, \delta_{jy}) + \frac{1}{2} P_{\mathrm{loss}}(\delta_{iy}, \delta_{jy}) \le \overline{P_{\mathrm{r}}}_{ijy} \\ \forall (i,j) \in \Omega \wedge y \in Y \end{gathered} \quad (10)$$

The maximum limits $\overline{P_{\mathrm{d}}}_{ijy}$ and $\overline{P_{\mathrm{r}}}_{ijy}$ are obtained based on standard thermal and stability considerations.

3) Zonal Power Generation Limits: These limits are the minimum and maximum effective generation capacities (including the capacity factors) available in each zone during the planning years.

$$\underline{P_{\mathrm{g}}}_{iy} \le P_{\mathrm{g}_{iy}} \le \overline{P_{\mathrm{g}}}_{iy} \quad \forall i \in Z \wedge y \in Y \quad (11)$$

The lower bounds $\underline{P_{\mathrm{g}}}_{iy}$ may be based on the operational considerations for base-load generation resources in each zone.

4) Zonal Import/Export Power Limits:

$$\begin{gathered} \underline{P_{\mathrm{im}}}_{iy} \le P_{\mathrm{im}iy} \le \overline{P_{\mathrm{im}}}_{iy} \\ \underline{P_{\mathrm{ex}}}_{iy} \le P_{\mathrm{ex}iy} \le \overline{P_{\mathrm{ex}}}_{iy} \\ \forall i \in Z \wedge y \in Y \end{gathered} \quad (12)$$

5) Generation Emission Constraints: These constraints are expressed by (3) and (4) (Section III-B).

6) Penetration Constraints: These constraints are defined as:

$$0 \le FF_{iy} \le 1 \quad \forall i \in Z \wedge y \in Y \quad (13)$$

$$FF_{iy}\overline{\mu}_y - FF_{iy-1}\overline{\mu}_{y-1} \ge 0 \quad \forall i \in Z \wedge y \in Y_1 \quad (14)$$

where (13) states that the penetration levels in each year cannot exceed the limits $\overline{\mu}_y$ set by the assumed transition curve, and (14) enforces the increase of the ultimate penetration levels over time. For uniform PHEV penetration in all zones, the following constraints should also be considered:

$$FF_{iy} - FF_{jy} = 0 \quad \forall i,j \in Z \wedge y \in Y_1 \quad (15)$$

V. Robust Optimization

Optimization models often rely on some input parameters whose values must be estimated. The estimated parameter values, however, may be different from the true values because of data limitations, biased data, unrealistic assumptions, numerical errors in the estimation process or the nonstationary nature of the data. On the other hand, errors in estimating input parameters may severely affect the obtained optimal solution and its actual performance. Thus, as the data take values different than nominal or expected ones, several constraints may be violated, and the optimal solution yielded by the nominal data may no longer be optimal or even feasible. Therefore, due to the impact of data uncertainty on the quality and feasibility of the optimization models, methodologies should be adopted that appropriately deal with the uncertainty of the parameters in the model [12], [13].

Sensitivity analysis and stochastic programming are the classical approaches for dealing with parameter uncertainty in optimization models [14], [15]. Sensitivity analysis measures the sensitivity of a solution to stochastic changes in the input parameters; however, it provides no mechanism by which this sensitivity can be controlled [15]. In the stochastic programming approach, probability distributions of the uncertain parameters are needed which are difficult to estimate in practice. Furthermore, only an expected value of the objective function is optimized.

In view of all the difficulties with sensitivity analysis and classical stochastic programming approaches, one of the most attractive approaches to deal with parameter uncertainty in the optimization process is *robust optimization* [16]. The classic robust optimization method yields an uncertainty-immunized solution, which remains feasible in all realizations of the input data, and the value of the objective function at this solution is the guaranteed value of the uncertain objective. The main tools in the classic robust optimization framework are *uncertainty sets* and a *robust counterpart problem*. Thus, uncertainty in the input data is described through uncertainty sets, which contain all or most possible values that may be realized for the uncertain parameters. Also, a deterministic problem, which is called a robust counterpart problem, is associated with the uncertain problem [17]. Given the nonempty uncertainty sets U_e, the robust optimization yields a solution that optimizes the worst-case performance when the input data belong to the uncertainty sets. More specifically, robust optimization solves the following problem:

$$\begin{aligned} \min \quad & \max \quad f_0(\boldsymbol{x}, \boldsymbol{d}_0) \\ \text{s.t.} \quad & f_e(\boldsymbol{x}, \boldsymbol{d}_e) \geq 0, \forall e \in E, \forall \boldsymbol{d}_e \in U_e, \end{aligned} \tag{16}$$

where for $e \in E \cup \{0\}$, the set U_e is the uncertainty set of the parameter $\boldsymbol{d}_e$.

Although this approach provides immunization to parameter uncertainty, its results are perceived to be too conservative for real applications, i.e., robustness is ensured at the cost of significantly losing optimality [13], [18]. To rectify this shortcoming of robust optimization, it has been suggested to intelligently shrink the uncertainty set (e.g., [19]). One of such techniques was proposed in [12], which is also applicable to discrete optimization models [20]. The main feature of this formulation, which is used here as explained next, is that it does not lead to nonlinear models; therefore, the tractability of the problem is not affected. Also, this approach offers full control on the degree of conservatism desired for any constraint [12], [13], [20].

Consider the following general linear programming model:

$$\begin{aligned} \min_{\mathbf{x} \in X} \quad & \mathbf{c}'\mathbf{x} \\ \text{s.t.} \quad & \tilde{\mathbf{a}}_e'\mathbf{x} \geq b_e, \quad \forall e \end{aligned} \tag{17}$$

where X includes all mixed integer feasible solutions, and uncertainty is assumed without loss of generality to affect only the constraint coefficients $\tilde{\mathbf{a}}_e$. Every element of the vector $\tilde{\mathbf{a}}_e$, i.e., $\tilde{a}_{ev}$, $v \in \{1, 2, ..., n\}$, is assumed to be subject to uncertainty and belongs to a symmetrical interval $[\hat{a}_{ev} - \Delta a_{ev}, \hat{a}_{ev} + \Delta a_{ev}]$ known by the decision-maker, where $\hat{}$ and Δ are used to represent nominal values and deviation magnitudes, respectively. This interval is centered at the point forecast $\hat{a}_{ev}$, while Δa_{ev} measures the precision of the estimate. The scaled deviation s_{ev} of parameter $\tilde{a}_{ev}$ from its nominal value can then be defined as:

$$s_{ev} = \frac{\tilde{a}_{ev} - \hat{a}_{ev}}{\Delta a_{ev}}, \tag{18}$$

which belongs to [-1,1]. The aggregate scaled deviation for constraint e, $\sum_{v=1}^{n} |s_{ev}|$, which is more accurate than individual ones, can take any value between 0 and n; however, it is unlikely that all of the coefficients $\tilde{a}_{ev}$ take their worst cases simultaneously. Consequently the true value of $\sum_{v=1}^{n} |s_{ev}|$ can be assumed to be in a narrower range, i.e.,

$$\sum_{v=1}^{n} |s_{ev}| \leq \Gamma_e, \forall e. \tag{19}$$

where $\Gamma_e \in [0, n]$, referred to as the *budget of uncertainty* of constraint e, is used to adjust the robustness against the level of conservatism of the solution. Its value reflects the attitude of the decision-maker toward uncertainty. Thus, Γ_e=0 indicates no "protection" against uncertainty, and Γ_e=n yields a very conservative solution since it can be interpreted as all the uncertain parameters' taking their worst-case values at the same time. For any values between 0 and n, the decision-maker makes a trade-off between the protection level of the constraint and the degree of conservatism of the solution.

The uncertainty set U in this context becomes:

$$U = \{(\tilde{a}_{ev}) \mid \tilde{a}_{ev} = \hat{a}_{ev} + \Delta a_{ev} s_{ev}, \forall e, v, s_{ev} \in S_e\}, \tag{20}$$

where:

$$S_e = \left\{ \mathbf{s}_e = [s_{e1}, s_{e2}, ..., s_{en}] \mid |s_{ev}| \leq 1, \forall v, \sum_{v=1}^{n} |s_{ev}| \leq \Gamma_e \right\}. \tag{21}$$

A robust optimal solution can now be obtained from the following counterpart problem:

$$\begin{aligned} \min_{\mathbf{x} \in X} \quad & \mathbf{c}'\mathbf{x} \\ \text{s.t.} \quad & \hat{\mathbf{a}}_e'\mathbf{x} + \min_{\mathbf{s}_e \in S_e} \sum_{v=1}^{n} \Delta a_{ev} x_v s_{ev} \geq b_e, \quad \forall e. \end{aligned} \tag{22}$$

Through the application of strong duality, Theorem 1 of [12] proves that (22) is equivalent to the following linear programming model:

$$\begin{aligned} \min_{\mathbf{x} \in X} \quad & \mathbf{c}'\mathbf{x} \\ \text{s.t.} \quad & \hat{\mathbf{a}}_e'\mathbf{x} - \Gamma_e p_e - \sum_{v \in V_e} q_{ev} \geq b_e, && \forall e \in E, \\ & p_e + q_{ev} \geq \Delta a_{ev} r_v, && \forall e \in E \wedge v \in V_e, \\ & -r_v \leq x_v \leq r_v, && \forall v \in V, \\ & p_e \geq 0, && \forall e \in E, \\ & q_{ev} \geq 0, && \forall e \in E \wedge v \in V_e, \\ & r_v \geq 0, && \forall v \in V. \end{aligned} \tag{23}$$

Observe that this robust formulation requires the determination of a budget of uncertainty $\Gamma_e \in [0, |V_e|]$ for each constraint e subject to uncertainty, as well as the definition of new decision variables p_e, q_{ev} and r_v.

In [12], it is shown that for constraint e with n uncertain parameters to be violated with a probability of at most ε_e, it is sufficient to choose a budget of uncertainty Γ_e at least equal to $1 + \Phi^{-1}(1 - \varepsilon_e)\sqrt{n}$, where Φ is the cumulative distribution of a standard normal. Alternatively, the violation probability of constraint e at a given budget of uncertainty Γ_e can be calculated as $1 - \Phi\left(\frac{\Gamma_e - 1}{\sqrt{n}}\right)$.

Note that Problem (17) can be rewritten as follows:

$$
\begin{aligned}
\min_{\mathbf{x}\in X} \quad & W \\
\text{s.t.} \quad & W - \mathbf{c}'\mathbf{x} \geq 0, \\
& \mathbf{A}\mathbf{x} \geq \mathbf{b}.
\end{aligned} \tag{24}
$$

This representation transfers the uncertainty in the objective function to the constraints that allows the use of the aforementioned robust optimization framework.

VI. Application to Ontario, Canada

This section discusses the application of the robust optimization model to Problem (24) for the real-case of Ontario, Canada, considering the issue of data uncertainty.

A. Ontario's Electric Power System Data

1) Transmission and Generation: Ontario's IESO represents the Ontario's electricity network with 10 zones [21]. These zones include Bruce, West, SW, Niagara, Toronto, East, Ottawa, Essa, NE, and NW. This same representation is used in this study to develop the 10-zone simplified model of Ontario's network which considers the main grid load and generation centers and transmission corridors. The model is mostly a 500 kV network, with a 230 kV interconnection between the northern zones (NE and NW). Hence, the parameters of this network are based on typical values of 230 kV and 500 kV systems, considering the approximate distances between zones, and transmission capacities, as per general information provided by the IESO [21] and line loading limits in percent of surge impedance loading. Also, based on the existing and planned projects provided by the OPA, an approximate pattern of transmission capacity enhancements are taken into account for this simplified model with the details provided in [7]. Note that the dc optimal power flow and zonal models used here to represent the Ontario's electricity system are adequate for the long-term planning horizon under consideration, with these kinds of models being widely used in practice for these types of studies. More detailed transmission system models such as ac optimal power flow and nodal-based models are typically used in short-term planning and dispatch, and lead to rather complicated mixed integer nonlinear programming problems in the context of the large multi-year model considered here, while resulting only on marginal improvements in the accuracy of the results, if at all solvable.

The maximum and minimum base-load generation power in each zone were found based on a zonal pattern of base-load generation capacity development between 2008 and 2025 contributing to base-load energy in Ontario [11]. The mix of base-load generation resources in this model includes nuclear, wind, hydro (only those units with limited dispatch capability and small-scale units less than 10 MW), CHP, coal, and conservation and demand management. The generation capacities allowed finding the maximum capacity of polluting and non-polluting base-load generation resources as required for the emission cost function. Also, as per [22], the average CO_2 emission rates of coal plants in the West and SW zones were considered to be 0.9553 and 0.9881 ton/MWh, respectively. Moreover, two different emission rates of 1.1536 and 1.1966 ton/MWh were found for the two coal plants located in the NW zone. For simplicity, an average figure of 1.1751 ton/MWh was used for the coal plants in this zone. Also, according to [23], [24], an approximate CO_2 emission rate of 0.25 ton/MWh was assumed for the CHP plants in Ontario. All these emission rates are considered to be constant during the planning years.

2) Electricity Demand: As per [25], [26], the average base and peak-demand values in Ontario from 2007 to 2025 are expected to increase by 21% and 24%, respectively. On average, these load increases translate into 1.11% and 1.26% of base and peak-demand annual growth rates, respectively, for all of Ontario. Considering these figures and based on a method discussed in [11], the annual base-load growth rate of the whole Ontario was decomposed into different zones. These zonal base-load annual growth rates along with the Ontario's average demand in 2007 (for the time period between 23:00-7:00) were used to obtain the expected base-load demands in different zones of Ontario from 2008-2025.

3) Electricity Imports and Exports: Power import and export possibilities exist in six of Ontario's zones: West, Niagara, East, Ottawa, NE and NW. In general, there is a 4,000 MW import capability in Ontario from Manitoba, Quebec, Michigan, New York and Minnesota, and an additional 1,000-1,250 MW from Quebec is being developed [21], [27]. No limitation on power export is considered in this study; however, a maximum of 750 MW of power imports from New York and Quebec between 2015-2022 was assumed for both the Ottawa and East zones. Moreover, a maximum of 500 MW of imports in both NE and NW were assumed during the same period. Also, a maximum of 300 MW of imported power in NW was assumed in other years within the planning period. These assumptions are consistent with the operational import limits found in [27]; however, an upgrade of the import capability in NE is also assumed in this study. If no upgrade on the import capability of NE takes place by 2015, the assumed import power for this zone from 2015-2022 in excess of the import limit was assumed to be covered by local dispatchable hydro units.

4) Electricity Prices: Calculated average electricity prices in Ontario between 2003-2008 for the time period of 23:00-7:00 shows, in general, a declining trend (with the exception of 2005) [11]. However, the average electricity price in 2008 was conservatively assumed to be the expected value of the HOEP for all of the planning period. Also, the expected values of importing and exporting electricity prices were assumed to be the same as the HOEP.

B. Ontario's Transport Sector Data

1) Number of LDVs: In order to determine the Ontario's grid potential for charging the PHEVs, it is first necessary to determine the number of LDVs during the planning period, which in turn requires the zonal population levels and the per capita number of vehicles during this period. Therefore, the population of cities and towns of more than 10,000 inhabitants were used to find the population of each zone, considering the geographical location of the zones. The population of each zone was then scaled up such that the sum of zonal population would equal to the 12,861,940 population estimate for Ontario on January 1, 2008, as per [28]. The annual base-load growth rate for each zone was approximately used to find the zonal population in the study period. Also, based on [29], a per capita number of vehicle equal to 0.55 was assumed to be valid for Ontario from 2008-2025. These figures were used to estimate the expected number of LDVs in different zones of Ontario during the planning horizon.

2) Vehicle Types: As per [29], six types of LDVs including compact and mid-size sedan (each with 29% share), mid-size and full-size SUVs (each with 4% share), vans (16% share), and pick-up trucks (18% share) were assumed for the Ontario's LDV fleet.

3) Fuel Economy: The approximate fuel economies of the different types of LDVs assumed to be replaced by PHEVs were calculated based on [30]. Thus, the fuel economy of compact sedan, mid-size sedan and mid-size SUV were determined to be 16.02, 12.28 and 9.43 km/liter, respectively. Also, full-size SUVs, vans and pickup trucks were assigned a similar approximate fuel economy of 7.73 km/liter. Furthermore, in order to not overestimate the environmental credits of PHEVs, it was assumed that these fuel economies improve by 20% by the end of the planning horizon.

4) PHEV Charging Time: It is assumed that the charging takes place during off-peak hours from 23:00-7:00 in both weekdays and weekends.

5) PHEV Charging Demand: In order to evaluate the PHEVs charging demand, a 30-km-all-electric average daily trip (referred to as PHEV30km), a 70% maximum allowable depth of discharge, a 1.4-kW connection power level, and an 85% charging efficiency were assumed. Based on these assumptions and the specific energy (kWh/km) required for each type of LDVs [30]–[32], the battery-charging requirements for different types of PHEV30km were calculated.

The interested readers are also referred to [7], [11] for further details regarding the Ontario's electric grid and transport sector data.

C. Sensitivity Analysis

The optimization model for planning the transition to PHEVs in Ontario, involves many parameters with inherent estimation errors. A common approach to investigate the impact of estimation errors on the optimal solution and the corresponding optimal value is to represent the errors as perturbations to the data in order to perform a sensitivity analysis on the optimal solution [33]. The results reveal the uncertain parameters that most influence the optimal value, which can then be used for developing robust counterpart problems.

A sensitivity analysis using Monte Carlo simulation was carried out to find the impact of estimation errors in the parameters of the model developed in Section IV. More precisely, given a parameter A in an optimization problem, M perturbations ΔA are generated to represent the estimation errors in the parameter A. Here, it is assumed that the perturbations have independent normal distributions, i.e.,

$$\Delta A = \rho\,\lambda A, \tag{25}$$

where $\rho \in (0,1]$ indicates the size of the relative perturbation and λ is a random parameter, which is assumed to follow a normal distribution with zero mean and unit standard deviation. Throughout this section, ρ and M are fixed at 10% and 1000, respectively, to represent a reasonable perturbation level, and since 1000 perturbations produce satisfactory results that do not change significantly with increased simulations. Also, only one parameter at a time is assumed to be perturbed while all other parameters remain at their nominal or expected values.

In this sensitivity analysis, the following three quantities are considered [34]:

1) *True optimal value*: It is the value of the objective function (OF) using an unperturbed parameter A at the optimal solution obtained from the unperturbed parameter A; this is denoted by $OF_{\mathrm{A}}\left(X_{\mathrm{A}}\right)$.
2) *Actual optimal value*: It is the value of the objective function using an unperturbed parameter A at the optimal solution obtained from the perturbed parameter $A+\Delta A$; this is denoted by $OF_{\mathrm{A}}\left(X_{\mathrm{A}+\Delta\mathrm{A}}\right)$.
3) *Estimated optimal value*: It is the value of the objective function using a perturbed parameter $A+\Delta A$ at the optimal solution obtained from the perturbed parameter $A+\Delta A$; this is denoted by $OF_{\mathrm{A}+\Delta\mathrm{A}}\left(X_{\mathrm{A}+\Delta\mathrm{A}}\right)$.

To quantitatively compare the sensitivity of the optimal value to perturbations of different parameters, the following *Average Deviation Index (ADI)* measure is proposed:

$$ADI = \frac{1}{M}\sum_{m=1}^{M}\left|OF_{\mathrm{A}}\left(X_{\mathrm{A}+\Delta\mathrm{A}_m}\right) - OF_{\mathrm{A}}\left(X_{\mathrm{A}}\right)\right|, \tag{26}$$

where $OF_{\mathrm{A}}\left(X_{\mathrm{A}+\Delta\mathrm{A}_m}\right)$ is the actual optimal value in simulation m. Calculated ADI values for different parameters of the optimization model are ranked to identify the most influential parameters that can then be used to develop the robust counterpart problem. A comprehensive discussion regarding the use of actual optimal value instead of estimated one for the calculation of ADI in (26) can be found in [11], [34]. It is also worth mentioning that typical methods used in the literature for sensitivity analysis are valid for small perturbations [35], allowing the *estimated* optimal value to be determined approximately. However, they are not appropriate for obtaining the *actual* optimal value, in particular, given the larger range of parameter variations considered here.

The sensitivity analysis was performed considering 19 of the most relevant single or group parameters of the model. AMPL [36] and CPLEX [37] were used to solve all MILP problems and determine the true and actual optimal values. Note that MILPs are classified in the category of *NP-hard* problems, and consequently one faces computational challenges when solving large-size MILP problems which can be addressed by a trade-off between solution quality and computational time; this is realized by appropriately defining the optimality gap value. In this paper, in order to achieve a high degree of precision, the optimality gap was fixed at 0.02% for all the simulations.

The results of the sensitivity analysis are illustrated in Table I; observe in this table that all the parameters considered are classified into two categories based on their corresponding ADI values and ranking. Based on these results, the electricity

TABLE I
AVERAGE DEVIATION INDEX OF THE PHEV TRANSITION MODEL FOR DIFFERENT UNCERTAIN PARAMETERS.

Rank	Uncertain parameter	ADI [CAD]
1	Price of export power	5.0598e+006
2	Hourly Ontario Energy Price (HOEP)	5.0523e+006
3	Annual growth rate of LDVs in Toronto	3.3158e+006
4	Price of import power	2.4700e+006
5	Annual growth rate of LDVs in SW	2.2047e+006
6	Annual growth rate of LDVs in Ottawa	1.4607e+006
7	Annual growth rate of LDVs in West	1.0384e+006
8	Annual growth rate of LDVs in Essa	8.6978e+005
9	Annual growth rate of LDVs in East	3.2441e+005
10	Annual growth rate of LDVs in NE	1.4591e+005
11	Annual growth rate of LDVs in Niagara	1.4146e+005
12	Annual growth rate of LDVs in Bruce	1.4817e+004
13	Annual growth rate of LDVs in NW	1.1417e+004
14	Average fuel economy of GVs (compact sedan)	0
15	Average fuel economy of GVs (mid-size sedan)	0
16	Average fuel economy of GVs (mid-size SUV)	0
17	Average fuel economy of GVs (full-size SUV)	0
18	Average fuel economy of GVs (van)	0
19	Average fuel economy of GVs (pick-up truck	0

prices and annual growth rates of vehicles in four zones (Toronto, SW, Ottawa and West) are found to be the most influential parameters. Observe in Table I that the average fuel economy of different types of gasoline vehicles have zero ADI values, which means that a 10% fluctuation of these parameters have no impact on the optimal solution. This is expected, as the share of each type of gasoline vehicles from the total number of light-duty vehicles is limited. The large ADI values of the electricity prices compared to other parameters can be explained by the fact that the electricity cost and revenue terms in the objective function of the model are much larger than the CO_2 emission cost and credit terms.

Given the limitation of the robust optimization techniques including the one used here, uncertain parameters such as annual growth rates of LDVs which simultaneously appear in multiple constraints and the objective function cannot be handled by this methodology, hence, nominal values of these parameters were used for the robust optimization studies. In addition, electricity prices are subject to more fluctuations and their forecasting is significantly more challenging compared to other parameters such as annual growth rates of LDVs. For these reasons, only electricity prices in the first category of Table I are considered for developing the robust counterpart problem.

D. Robust Optimization Model

Based on the discussions in the previous sections, the robustness of the PHEV transition model is investigated here with respect to perturbations in electricity prices. As HOEP as well as export and import electricity prices are defined for each individual year of the planning span (2008-2025), there are in total 54 parameters whose uncertainties can influence the optimal value. These parameters can be expressed as follows:

$$0.9\widehat{\pi}_y \leq \widetilde{\pi}_y \leq 1.1\widehat{\pi}_y,\ \Delta\pi_y = 0.1\widehat{\pi}_y,\ \forall y \in Y \tag{27}$$

$$0.9\widehat{\pi}_{\mathrm{im}_y} \leq \widetilde{\pi}_{\mathrm{im}_y} \leq 1.1\widehat{\pi}_{\mathrm{im}_y},\ \Delta\pi_{\mathrm{im}_y} = 0.1\widehat{\pi}_{\mathrm{im}_y},\ \forall y \in Y \tag{28}$$

$$0.9\widehat{\pi}_{\mathrm{ex}_y} \leq \widetilde{\pi}_{\mathrm{ex}_y} \leq 1.1\widehat{\pi}_{\mathrm{ex}_y},\ \Delta\pi_{\mathrm{ex}_y} = 0.1\widehat{\pi}_{\mathrm{ex}_y},\ \forall y \in Y \tag{29}$$

In this case, based on (24), the following constraints should be added to those of the PHEV transition model discussed in Section IV:

$$W - \sum_{y\in Y} \frac{1}{(1+DR)^{y-y_1}} \left(C_{1y} - C_{2y} + C_{3y}\right) - \Gamma_1 p_1 - \sum_{v\in V} q_{1v} \geq 0 \tag{30}$$

where C_{1y}, which include the influential uncertain parameters is defined as follows, and C_{2y} and C_{3y} are the same as the ones previously represented in (7) and (8):

$$C_{1y} = \sum_{i\in Z} \left(P_{\mathrm{g}_{iy}} \widehat{\pi}_y + P_{\mathrm{im}_{iy}} \widehat{\pi}_{\mathrm{im}_y} - P_{\mathrm{ex}_{iy}} \widehat{\pi}_{\mathrm{ex}_y}\right) \times 8 \times 365 \tag{31}$$

$$p_1 + q_{1v} \geq \Delta a_{1v} r_v, \quad \forall v \in V = V_1 = \{1, ..., 54\}, \tag{32}$$

where the deviations of the uncertain parameters are determined to be as follows:

$$\Delta a_{1v} = \begin{cases} \frac{8\times 365}{(1+DR)^{v-1}} \Delta\pi_{y_1+v-1} : & \forall v \in \{1, ..., 18\}, \\ \frac{8\times 365}{(1+DR)^{v-19}} \Delta\pi_{\mathrm{im}_{y_1+v-19}} : & \forall v \in \{19, ..., 36\}, \\ \frac{8\times 365}{(1+DR)^{v-37}} \Delta\pi_{\mathrm{ex}_{y_1+v-37}} : & \forall v \in \{37, ..., 54\}. \end{cases}$$

$$-r_v \leq \sum_{i\in Z} P_{\mathrm{g}_{i,y_1+v-1}} \leq r_v, \quad \forall v \in \{1, ..., 18\} \tag{33}$$

$$-r_v \leq \sum_{i\in Z} P_{\mathrm{im}_{i,y_1+v-19}} \leq r_v, \quad \forall v \in \{19, ..., 36\} \tag{34}$$

$$-r_v \leq \sum_{i\in Z} P_{\mathrm{ex}_{i,y_1+v-37}} \leq r_v, \quad \forall v \in \{37, ..., 54\} \tag{35}$$

The performance of the robust solution as a function of the protection level (Γ) for a CO_2 emission cost/credit of 125 CAD/ton is illustrated in Fig. 2, which demonstrates how optimality is affected as the budget of uncertainty or protection level of the constraint with uncertain parameters increases. Observe in Fig. 2 that increasing the protection level above Γ=30 has a negligible impact on the loss of optimality, showing that almost 56% of the uncertain parameters in the PHEV transition model greatly influence the optimal solution. The trade-off between optimality and robustness is also illustrated in Fig. 3; observe that lower losses of optimality correspond to higher probabilities of constraint violation, as expected. Note that by allowing an 8% loss of optimality, it is possible to make the probability of constraint violation less than 5% when emission constraints for generation are considered. It is also interesting to note that the impact of emission constraints for generation becomes more conspicuous for lower violation probabilities.

Studies showed that, in spite of the loss of optimality with the increase in the budget of uncertainty, uniform PHEV penetration and the total number of PHEVs by 2025 are not impacted by the constraint protection level and are equal

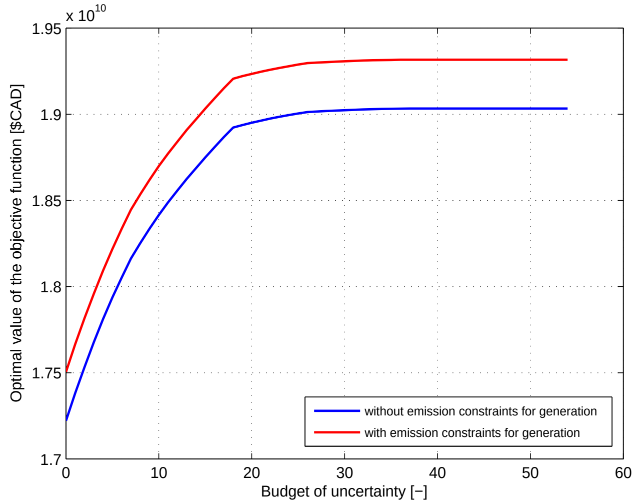

Fig. 2. Impact of the budget of uncertainty Γ on the optimal value of the robust PHEV transition model.

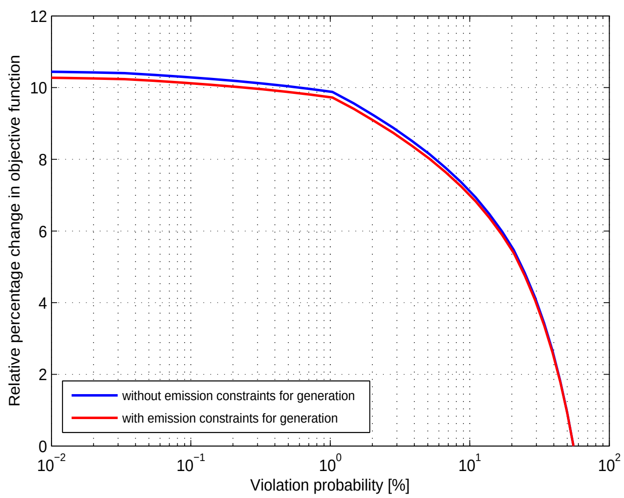

Fig. 3. Relative change in optimal value of the robust PHEV transition model with respect to the probability bound of constraint violation.

to 10.59% and 912,106, respectively. To investigate whether uniform PHEV penetrations remain independent of budget of uncertainty for lower emission costs, further robust analyses were performed using different values for social cost of emissions in the population area. These studies reveal that the independence of the uniform PHEV penetration from the budget of uncertainty holds for even relatively large deviations (about 50%) of the social cost of emissions with regard to the initially assumed 125 CAD/ton. However, larger than 50% reductions in the social cost causes the budget of uncertainty to greatly influence uniform PHEV penetration. This is demonstrated in Table II where the robust results are found based on a social cost of emission equal to 50 CAD/ton. Observe that under full protection (Γ=54) of the constraint with uncertain parameters, the optimal value is increased by 10.14%; hence reducing the chance of constraint violation to zero can be realized at the cost of losing 10.14% optimality. In this case, the total number of PHEVs is significantly reduced to 58,284. It is

TABLE II
SAMPLE RESULTS OF DETERMINISTIC AND ROBUST PHEV TRANSITION MODEL WITH $SC_{CO_2 p}$=50 CAD/TON, DISREGARDING EMISSION CONSTRAINTS FOR GENERATION. (DM: DETERMINISTIC MODEL; RM: ROBUST MODEL.)

	Γ [-]	Violation probability [%]	Optimal value [CAD]	Change [%]	Uniform penetration [%]	# of PHEVs by 2025
DM	-	-	17,749,875,901	0	9.91	853,397
RM	0	55.41	17,749,875,901	0	9.91	853,397
	5	29.31	18,462,420,204	4.01	9.91	853,397
	10	11.03	18,942,348,114	6.72	9.91	853,397
	14	3.84	19,213,595,806	8.25	9.61	827,938
	24	0.09	19,513,035,695	9.93	0.68	58,284
	34	3.5490e-004	19,548,443,500	10.13	0.68	58,284
	54	2.7489e-0011	19,550,124,730	10.14	0.68	58,284

TABLE III
SAMPLE RESULTS OF DETERMINISTIC AND ROBUST PHEV TRANSITION MODEL WITH $SC_{CO_2 p}$=50 CAD/TON, INCLUDING EMISSION CONSTRAINTS FOR GENERATION.

	Γ [-]	Violation probability [%]	Optimal value [CAD]	Change [%]	Uniform penetration [%]	# of PHEVs by 2025
DM	-	-	18,024,967,839	0	7.93	682,994
RM	0	55.41	18,024,967,839	0	7.93	682,994
	5	29.31	18,732,050,321	3.92	7.93	682,994
	10	11.03	19,208,575,792	6.57	7.93	682,994
	14	3.84	19,475,404,857	8.05	1.55	133,548
	24	0.09	19,772,818,146	9.70	0	0
	34	3.5490e-004	19,808,225,951	9.89	0	0
	54	2.7489e-0011	19,809,907,181	9.90	0	0

interesting to note that even setting the budget of uncertainty at Γ=14, i.e., assuming 14 out of 54 uncertain parameters take their worst-case values at the same time, results in a significantly low value of violation probability (3.84%). Table III presents similar results for the case of considering emission constraints for generation. Observe that setting the budget of uncertainty at Γ=14 results in a reasonable trade-off between optimality and conservatism, since the constraint with uncertain parameters is protected with a probability of 96.16% at the cost of losing 8.05% of optimality. In this case, at least 1.55% uniform PHEV penetration can be obtained which translates into 133,548 PHEVs that can be introduced into Ontario's transport sector by 2025.

VII. CONCLUSION

This study examined the optimal utilization of the electric grid infrastructure during off-peak times for charging PHEVs. Considering environmental issues, a comprehensive optimization model for the transition to PHEVs was developed to determine the optimal electric grid potential that would support these vehicles within a planning horizon. The issue of parameter uncertainty was incorporated into the model, and a methodology based on Monte Carlo simulation was proposed for identifying the parameters that most influence the optimal solution. Using these uncertain parameters, a robust optimization approach with the capability of adjusting the level of conservatism/risk was then applied to derive robust optimal penetration levels of PHEVs into the transport sector. The usefulness of the proposed robust analysis methodology was demonstrated by applying it to the real-case of Ontario, Canada.

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