Sequential Methods in Solving Economic Power Flow Problems

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Abstract

This paper compares two linear interior point programming algorithms and an interior point quadratic programming algorithm that are used to solve the optimal power flow problem. The paper focuses on the numerical oscillations that occur because of the sequential linearization of the problem. Two methods to reduce the oscillations are discussed and implemented on a six bus test system.

Keywords: Optimal Power Flow, Sequential Programming, Interior Point Methods.

I. Introduction

Since the introduction of interior point methods [1], it has become widely accepted that linear programming problems can be effectively solved by approaching the optimal solution through the feasible region. Interior point algorithms have since been successfully applied to power system problems as discussed in [2], [3] and [4].

Because of the nonlinear nature of the electrical power system, Optimal Power Flow (OPF) problems are nonlinear programming problems. Therefore, sequential linear programming (SLP) and sequential quadratic programming (SQP) algorithms can be used to solve these problems. The application and comparison of sequential linear and quadratic programming to solve the OPF economic dispatch problem are reviewed in this paper.

In sequential programming, the original problem is solved by successively approximating the original problem using Taylor series expansions at the current operating point and then moving in an optimal direction until the solution converges. The linearization of the system of equations introduces the possibility of numerical oscillation around the optimal solution. In this paper, two methods are reviewed to reduce these oscillations for both the sequential linear and quadratic programming methods.

The sequential algorithms are tested and compared using a six bus system based on the IEEE 6 bus test system composed of two generators, two under-load tap changing transformers and two variable reactance capacitors.

II. Optimal Power Flow and System Modeling

The objective of the economic dispatch problem is to minimize the instantaneous operating costs in a transmission system. The problem examines the scheduling of system generators to operate the system at a minimal cost. The problem is subject to the operating constraints that the generated power must equal the system load and system losses, maximum and minimum voltage magnitude levels, and the maximum power flow through a transmission line. The economic dispatch problem can be defined by the optimization problem:

$$\text{min.} \quad f = \sum_{i=1}^{m} Cost_i(P_{Gi}) \tag{1}$$

subject to (s.t.):

$$\sum_{i=1}^{n} P_{Gi} = P_{SystemLoad} + P_{SystemLosses} \tag{2}$$
$$\sum_{i=1}^{n} Q_{Gi} = Q_{SystemLoad} + Q_{SystemLosses}$$
$$P_{Gi_{min}} \leq P_{Gi} \leq P_{Gi_{max}}$$
$$Q_{Gi_{min}} \leq Q_{Gi} \leq Q_{Gi_{max}}$$
$$V_{i_{min}} \leq V_i \leq V_{i_{max}}$$
$$|S_{ij}| \leq S_{im_{max}}$$

where m is the number of generators in the system.

In the economic dispatch problem, the generator cost is usually represented using a quadratic function (3).

$$f = \sum_{i=1}^{m} a_i + b_i P_{Gi} + c_i P_{Gi}^2 \tag{3}$$

When using sequential methods the objective function (1) and constraints (2) can be linearized as follows:

$$\min \Delta f = \sum_{i=1}^{m} (b_i + 2c_i P_{Gi}) \Delta P_{Gi} \tag{4}$$
$$\text{s.t. } \boldsymbol{A \Delta x} = \boldsymbol{d}$$

where $\boldsymbol{A}$ is the Jacobian of the system constraints, $\boldsymbol{\Delta x}$ is the optimal step change in the system variables, and $\boldsymbol{d}$ is the difference between the original constraints and the value of the variables at the current operating point.

III. Interior Point Algorithms

The linear programming algorithms used in the analysis were implemented in $Matlab^{TM}$ [5] based on the algorithms presented in [3, 4, 6, 7]. The $Matlab^{TM}$ [5] based quadratic programming software package QP-solve [8] was used in implementing the SQP analysis.

A. *Dual Affine Scaling Algorithm*

The Dual-Affine scaling algorithm determines the solution to the primal LP problem:

$$\begin{aligned} \text{min.} \quad & c^t x \\ \text{s.t.} \quad & Ax = b, \ x \geq 0 \end{aligned} \tag{5}$$

by finding the solution to the dual problem:

$$\begin{aligned} \text{max.} \quad & b^t y \\ \text{s.t.} \quad & A^T x \ \leq \ c \ , \ r \geq 0 \end{aligned} \tag{6}$$

The algorithm is composed of two phases; the first phase is an initialization phase used to generate a good feasible point for the dual problem, and the second phase determines the optimal solution to the dual problem. The first phase was solved using the method outlined in [9], where an artificial variable is introduced in the objective function. The second phase calculates the optimal search directions by obtaining the dual-variable direction. A sequence of such projections are done starting from the solution of the previous projection until the optimal solution is found. The full details of the algorithm are given in [4].

B. *Primal-Dual Algorithm*

The Primal-Dual Algorithm solves both the primal (5) and the dual (6) problems. If x is a feasible solution for the primal problem and y is a feasible solution for the dual problem, then $b^t y \leq c^t x$. If $b^t y = c^t x$ then x is optimal for the primal problem and y is optimal for the dual problem [6]. Defining the duality gap as the difference between the primal and the dual objective functions, the optimal solution is found when the duality gap is reduced to zero or a small epsilon.

The non-negativity constraints are incorporated into the objective function of the dual problem with the introduction of a logarithmic function (7). The dual problem is re-written in equality form by introducing the stack variations, r.

$$\begin{aligned} \text{max.} \quad & b^t y \ - \ \mu \sum_{j=1}^{n} \ln x_j \\ \text{s.t.} \quad & A^T y \ + \ r \ = \ c \ , \ r \geq 0 \end{aligned} \tag{7}$$

In order to find the optimal solution of the problem a Lagrangian approach is used. The Lagrangian function for (7) is as follows:

$$L(x, y, r; \mu) = b^t y - \mu \sum_{j=1}^{n} \ln x_j - x^t (A^t y + z - c) \tag{8}$$

The necessary conditions for the optimal solution are found by determining the critical points of the Lagrangian function. This is found by setting the partial derivatives of the Lagrangian equal to zero and solving for the Lagrangian variables (x,y,r), here μ is considered constant. Therefore, an optimal solution must satisfy the following:

$$\begin{aligned} Ax \ &= \ b \\ A^T y \ + \ r \ &= \ c \\ Xre \ &= \ \mu e \end{aligned} \tag{9}$$

where X=diag(x); R=diag(r); and e=(1,1, ..., 1).

For μ equal to zero the duality gap is also reduced to zero [6]. Therefore, an approximate solution to (9) is found by applying one step of Newton's Method:

$$\begin{bmatrix} A & 0 & 0 \\ 0 & A^t & I \\ Z & 0 & X \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta r \end{bmatrix} = \begin{bmatrix} \rho \\ \sigma \\ \phi \end{bmatrix} \tag{10}$$

where $\rho = b - Ax$, $\sigma = c - A^t y - r$, and $\phi = \mu E - XZe$. Thus, starting from an initial point, an increment can be found by solving (10). The increments are scaled using the algorithm presented in [6]. After the states are updated, the process is repeated with a reduced value of μ until convergence.

IV. Simulation Results

The algorithms were implemented and tested on a six bus system based on the IEEE 6 bus test system, which is depicted in Fig. 1.

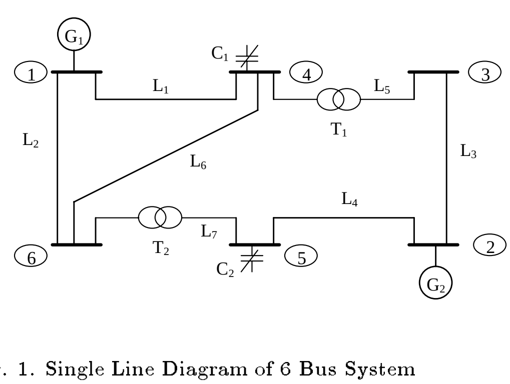

Fig. 1. Single Line Diagram of 6 Bus System

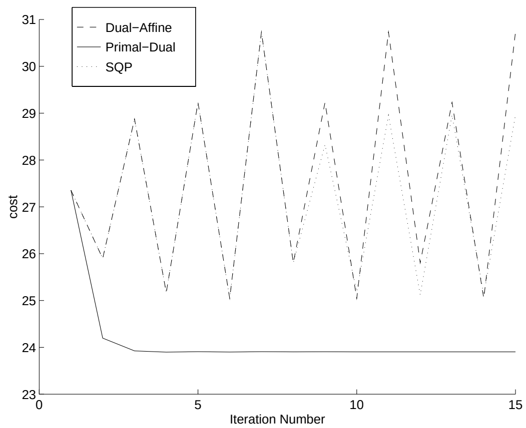

Fig. 2. Illustration of Numerical Oscillations

Figure 2 illustrates the convergence of the economic dispatch problem using the three sequential methods. The oscillatory behavior of the solution was expected because of the sequential linearization of the problem. This behavior is also discussed in [3]. Further simulations starting from various initial conditions resulted in the same initial behavior. The increased oscillations detected in the Dual-Affine algorithm can be attributed to the different convergence criteria of the methods. The convergence criterion of the Primal-Dual method is tighter than the Dual-Affine. Attempts to tighten the convergence of the Dual-Affine method resulted in numerical difficulties.

Two approaches to control the oscillations were tested, the first being to limit the maximum step change in control parameters and the second to scale the step size.

The results of scaling the step size is shown in Fig. 3. The damping of the oscillations is increased for the primal-dual algorithm, converging to the same steady state solution shown in Fig. 2. The Dual-Affine algorithm still displayed undamped oscillations, but of smaller magnitude than the original solution with no step size scaling. The step size scaling algorithm had the greatest effect on the convergence of the SQP algorithm. The level of oscillation was greatly reduced, although it is still undamped.

The result of limiting the step change in generation to a maximum magnitude of 0.1 per unit is seen in Fig. 4. It can be seen that the limiting of the step size had almost no effect on the Primal-Dual algorithm, but did partially stabilize the numerical oscillations of the Dual-Affine and the SQP algorithms. The final cost that the Primal-Dual algorithm converged to was the same as the case with no modifications.

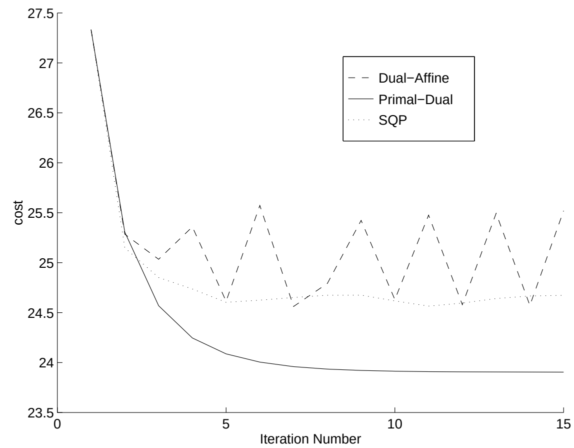

Fig. 3. Effect of Scaled Step Size on Oscillations

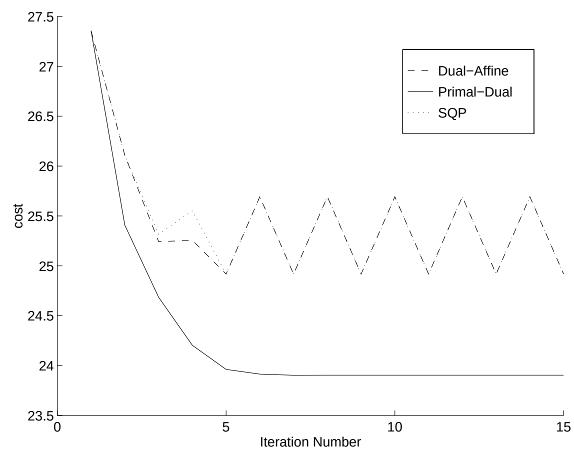

Fig. 4. Effect of Maximum Step Length Control on Oscillations - Generation Step Only

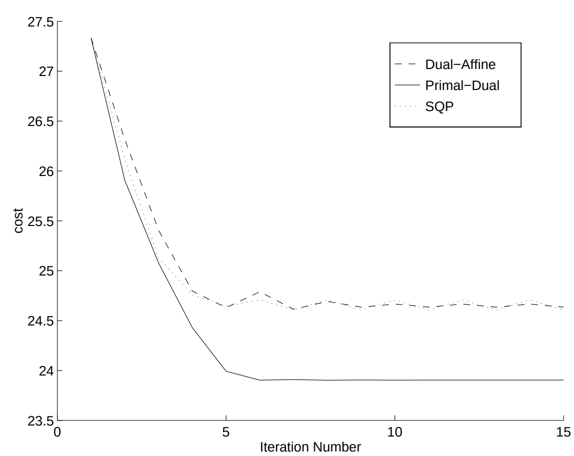

Fig. 5. Effect of Maximum Step Length Control on Oscillations

Finally, Fig. 5 shows the effect of controlling the step change magnitude of all the control parameters, i.e., generation level, transformer tap position, bus voltage magnitude and reactive compensation. The stabilizing effect of this technique can make it very popular, but a concern is the possibility of getting trapped in a local minimum, since the optimal solution will only be found if it exists in the neighborhood of the current operating point. If the economic dispatch problem is being computed in real time to optimize how a system should respond to changing loads, then limiting parameter shifts is very important as it is undesirable to make large changes in electrical system parameters. Therefore, the desired solution would be the local minimum.

Placing small non-zero values on the diagonal of the quadratic cost term in the SLP algorithms did not have an effect on the oscillations. This was done to improve the condition number of the quadratic cost matrix.

Since the Dual-Affine algorithm considers only the dual problem, the solution found may not be the optimal solution for both the primal and the dual problems. This explains the higher optimal steady state solution of the Dual-Affine method.

V. Conclusions

In this paper Sequential Linear and Quadratic Programming Algorithms are applied to the economic dispatch problem of the Optimal Power Flow Problem. It was found that the Primal-Dual algorithm performed better than the Dual-Affine algorithm.

Because of the very sparse nature of the quadratic cost matrix, it was found that Sequential Quadratic Programming may not be well suited for solving economic dispatch problems. If larger test systems are examined, where the ratio of generator buses to non-generator buses is small, increased oscillations due to the sparse quadratic term are expected.

Considering the cases presented in this paper, sequential linear programming appears to be a better tool than sequential quadratic programming for solving the economic dispatch problem. It is recommended that different quadratic algorithms be investigated, since a difference in the convergence criteria may be causing the limitations on this form of programming.

Further tests will be made on larger benchmark systems to compare the three methods. Some attention will be placed on developing methods to improve the Dual-Affine stopping criteria. Flexible A.C. Transmission System (FACTS) devices may be incorporated into the system to better define their role in system optimization. The optimal power flow problem examined in this paper can then be extended to study the use of FACTS devices to minimize operating costs.

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