# Elimination of Algebraic Constraints in Power System Studies

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## Abstract

A methodology for eliminating algebraic equations from power system models is proposed. The proposed model has applications in steady state power system analysis and transient stability studies. The steady state solution of the proposed model is compared to traditional load flow solution techniques using a sample system. The transient response of the proposed model is also compared to a traditional transient stability model by simulating a three phase fault in the system.

Keywords: power system modeling, transient stability, bifurcation analysis

## I. Introduction

Voltage stability problems in power systems may occur for a variety of reasons, from voltage control problems with automatic voltage regulators (AVR) and under-load tap-changer (ULTC) transformers, to instabilities created by different types of bifurcations. Several conference proceedings, for example, [1], summarize most of the voltage stability problems and discuss techniques and models proposed by several researchers relating to the area of bifurcation theory.

This paper examines a generalized algorithm to remove all nonlinear algebraic equations from the system model. In the current literature of bifurcation analysis, either the generator or the load are modeled using differential equations, but the transmission system is generally modeled using algebraic equations. A model for incorporating transmission line dynamics into bifurcation analysis is presented in [2], but the model can not be applied to all power system problems.

In Section II the traditional system is review. The proposed dynamic model is then presented, using a generalized format so that it can be applied to any load flow and stability problem in Section III. A comparison between the traditional and proposed algorithms is done using a sample system in Section IV. Conclusions are given in Section V.

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## II. Traditional System Model

In power system analysis, the time frame associated with the dynamics being considered can vary dramatically. The time response of exciter and some controllers are fast, whereas mechanical devices such as boiler controllers have very slow transients. Components of the power system can be described dynamically, but in most models some dynamics are excluded. For example, when analyzing the switching of power electronic devices, which have very fast dynamics, the slow dynamics of transformer taps and boilers are ignored. When analyzing components with fast dynamics the analysis is done for only a short period of time, usually measured in milliseconds. When considering components with slower dynamics, the dynamics of faster components are usually ignored. When the dynamics of a component are ignored, the differential equation used to model that component is transformed into an algebraic constraint. The system is then described using a differential algebraic model:

$$\dot{x} = f(x, y, \lambda) \tag{1}$$
$$0 = g(x, y, \lambda) \tag{2}$$

If all the dynamics of the power system are considered, the equations (1) and (2) can be written as:

$$\dot{x} = f(x, y, \lambda) \tag{3}$$
$$\dot{y} = g(x, y, \lambda) \tag{4}$$

In stability analysis, models are generally composed of differential-algebraic equations which can be written in the form of equations (1) and (2). Typically, the dynamics associated with transmission lines are ignored, since the time response of the transmission line is very fast as compared to the other system components. The transmission system is modeled by introducing algebraic constraints which typically correspond to the classical power flow model. This leads, for certain loading conditions, to numerical difficulties, as the constraints may become singular, rendering the model unsolvable. The proposed system model incorporates the dynamics associated with the transmission line and impedance loads, *without the need for additional data.*

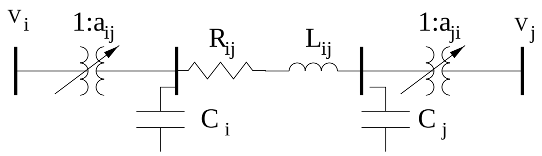


Fig. 1. Transmission line single line diagram

## III. Proposed System Model

Most power system stability analyses make the assumption that equation (2) is non-singular and, therefore, can be directly or indirectly substituted into (1). Intuitively this approach does not seem acceptable in all cases. For example, if the transmission system is approaching the traditional maximum power transfer limit, the interaction between the dynamically modeled load and/or generators with the transmission system may play a significant role in resulting bifurcation problems.

Figure 1 illustrates a generic dq-axis single line diagram for a branch between two buses. ULTC transformers have been placed at each bus to make the algorithm more general. If no transformer is present, then the tap setting is fixed at unity and the differential equation used to model the transformer is excluded. Kirchhoff's voltage law for this circuit in a dq reference frame[3] gives:

$$a_{ij}v_{q_i} - \frac{v_{q_j}}{a_{ji}} = Ri_q + \omega X_{ij} i_q + X_{ij}\tfrac{d}{dt}i_q \qquad (5)$$

$$a_{ij}v_{d_i} - \frac{v_{d_j}}{a_{ji}} = Ri_d - \omega X_{ij} i_d + X_{ij}\tfrac{d}{dt}i_d \qquad (6)$$

Equations (5) and (6) may be written in the standard format of equation (1) as follows:

$$\dot{i}_q = \tfrac{1}{X_{ij}}(a_{ij}v_{q_i} - \tfrac{v_{q_j}}{a_{ji}} - Ri_q - \omega X_{ij} i_q) \qquad (7)$$

$$\dot{i}_d = \tfrac{1}{X_{ij}}(a_{ij}v_{d_i} - \tfrac{v_{d_j}}{a_{ji}} - Ri_d + \omega X_{ij} i_d) \qquad (8)$$

In order to numerically isolate variables, shunt capacitance is incorporated into the model. This prevents complicated expressions from arising due to combining equations from one transmission line into another or with a load. The bus voltages is hence described by the following linear differential equation:

$$\dot{v}_q = \tfrac{1}{C}i_q - \omega i_d \qquad (9)$$

$$\dot{v}_d = \tfrac{1}{C}i_d + \omega i_q \qquad (10)$$

where $v$ is the voltage across the capacitor, $i$ is the current thought the capacitor and $C$ is the shunt capacitance. In most cases, shunt capacitance are part of the existing model, but if there is no capacitance present, a value of $10^{-5}$ pu is used without a significant impact on the circuit. A shunt resistor is used at generator buses to define the voltage for the automatic voltage regulator. The EPRI simulation package, EMTP, uses a similar numerical method in defining voltages at generator buses [4].

Including equations (7) through (10) into the system model increases the number of differential equations in the model by two for each line and bus.

To further eliminate non-linear algebraic equations, the system generators are modeled using differential equations. In typical stability studies, generator models include dynamic behavior for the stator d and q axis current, the field current, the field voltage, automatic voltage regulator and governor models. Non-slack generators are modeled without a governor as the mechanical power is fixed for these machines, since they represent constant power-voltage machines. A detailed generator model can be found in [3]

Non-slack bus generators are incorporated into the model by considering their frequency characteristics. In steady state, the frequency of each of the generators must be equal to the frequency of the system which is controlled by the slack bus. If a generator's frequency is greater than the system frequency, the mechanical power input of this generator is greater than its electrical power output. As the generator increases in speed, the angle between this generator's d-q reference will increase relative to the system's d-q reference. The opposite applies when the non-slack generator's frequency is less than the system frequency. In balanced steady state operation, the rate of increase between the relative angles of the reference frames is zero, but the value of the angle may not be zero. That is:

$$\dot{\delta}_{21} = 0 \qquad (11)$$

$$\delta_{21} = c \qquad (12)$$

$$\delta_{21} = \delta_2 - \delta_1 \qquad (13)$$

where the subscripts 12 are used to represent the angle $\delta$ between the two reference frames denoted as 1 and 2, and $c$ is a constant.

To accurately model frequency deviations, it is important that reference frame transformations are included to transform parameters from the system d-q axis to the non-slack generators' d-q axis frame. The standard transformation between reference frames is [3] :

$$f''_{qd} =' K'' f'_{qd} \qquad (14)$$

$$'K'' = \begin{bmatrix} cos(\delta_{21}) & -sin(\delta_{21}) \\ sin(\delta_{21}) & cos(\delta_{21}) \end{bmatrix} \qquad (15)$$

where $f$ represents any of the electrical variables (current, voltage, flux linkage). Because of its structure, equation (14) is invertible for any value of $\delta_{21}$, as the determinant of equation (15) is always equal to one.

The transformation presented above is then used to transform voltages and currents from non-slack generator dq reference frames to the transmission system's dq reference frame, which is determined by the slack generator's frequency. To define the angle $\delta_{21}$, the rotor speed of the non-slack generator is required, i.e.

$$\dot{\omega}_r \quad = \frac{\omega_e}{2H}(T_e - T_m) \tag{16}$$

where $T_e$ and $T_m$ are the electro-magnetic torque and mechanical torque respectively; $\omega_e$ is the base angular frequency; and $\omega_r$ is the angular speed of the rotor. As described above, the behavior of the angle $\delta_{21}$ is a function of the difference between the non-slack generator speed $\omega_r$ and the angular frequency of the system $\omega_1$, i.e.:

$$\dot{\delta}_{21} = \omega_r - \omega_1 \tag{17}$$

## IV. Numerical Example

Numerical simulations were performed on a three bus, two generator power system illustrated in Fig. 2. The generator located at BUS 1 acts as the system slack bus, and the generator at BUS 2 is a constant voltage-power controlled bus. The parameters for each of the generators are given in Table I. An impedance load is connect to each of the buses, with the greatest load occurring at BUS 3. The parameters for the impedance loads are given an Table II, and the parameters for the transmission system are given in Table III.

The steady state results are shown in Table IV. To ensure the reliability of the proposed algorithm the solution was also found using a traditional load flow algorithm. The percent difference between the results of the two algorithms, shown in the last column of Table IV, demonstrates that the algorithms converge with only minor differences to the same solution.

The time domain simulation of the system when a three phase ground fault is applied at BUS 2 is shown in Figs 3 and 4. The fault was applied for 150 milliseconds before being cleared. Figure 3 is the time domain response using the proposed algorithm, and Fig. 4 is the response using the typical transient stability model of the same system. The transient stability model makes the assumption that the frequency of the system does not vary significantly and uses phasors to model the

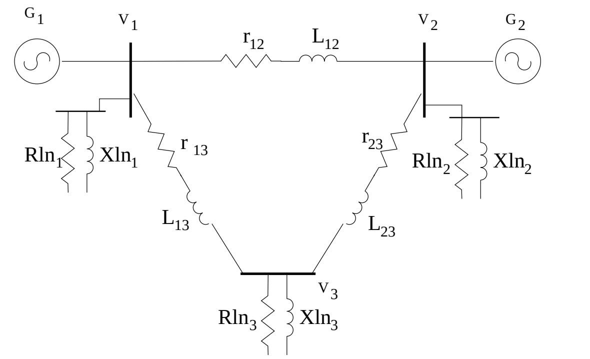


Fig. 2. Transmission Line Single Line Diagram

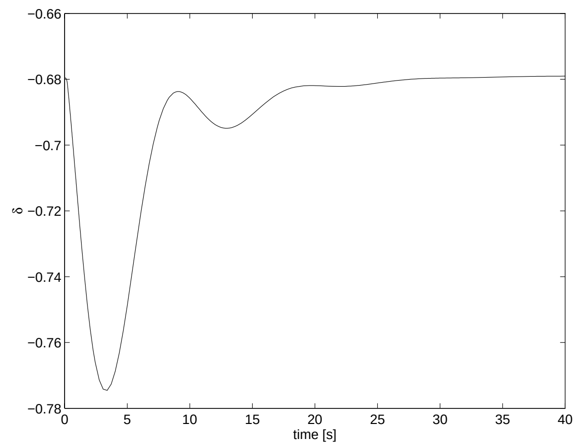


Fig. 3. Transient response of Generator 2 machine angle $\delta_{21}$ with a three phase ground fault applied at Bus 2 at $t = 0.1s$ for 150 milliseconds.

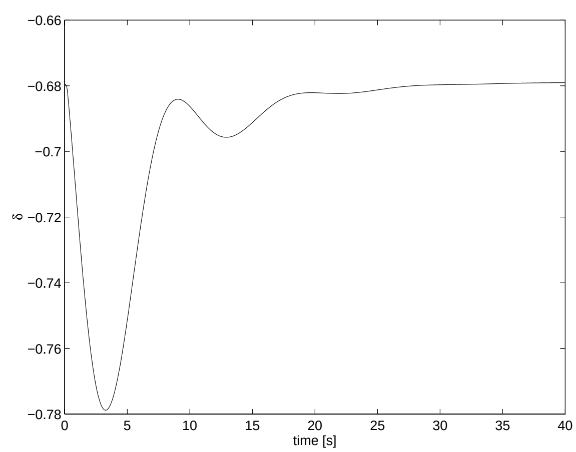


Fig. 4. Proposed models response of Generator 2 machine angle $\delta_{21}$ with a three phase ground fault applied at Bus 2 at $t = 0.1s$ for 150 milliseconds

TABLE I
SYNCHRONOUS GENERATOR PARAMETERS

| Variables | Generator 1 | Generator 2 |
|---|---|---|
| $H$ | 2.3700 s | 2.8000 s |
| $\omega_{base}$ | 377 rad/s | 377 rad/s |
| $R_s$ | 0.0014 pu | 0.0014 pu |
| $R_{fd}$ | 0.0079 pu | 0.0706 pu |
| $X_{ls}$ | 0.0535 pu | 0.0535 pu |
| $X_{fd}$ | 0.0706 pu | 0.0706 pu |
| $X_q$ | 0.0461 pu | 0.0461 pu |
| $X_d$ | 0.0695 pu | 0.0695 pu |

TABLE II
IMPEDANCE LOAD PARAMETERS

| Bus Number | R | L |
|---|---|---|
| Bus 1 | 7.0727 pu | 0.0281 s |
| Bus 2 | 5.4080 pu | 0.0191 s |
| Bus 3 | 1.2252 pu | 0.0054 s |

TABLE III
TRANSMISSION LINE PARAMETERS

| Sending Bus | Receiving Bus | $R_{line}$ | $L_{line}$ |
|---|---|---|---|
| Bus 1 | Bus 2 | 0.0100 pu | 1.5915e-04 s |
| Bus 1 | Bus 3 | 0.0800 pu | 6.3662e-04 s |
| Bus 2 | Bus 3 | 0.0600 pu | 4.7746e-04 s |

transmission system. The figures indicate that the proposed model can be used to accurately perform transient stability analysis.

Because the time constants of the proposed model associated with the transmission system are very small, numerical integration is computationally intensive. The use of explicit integration methods, such as the Runge-Kutta was not feasible in this case due to time and memory requirements; implicit methods methods were successfully applied.

## V. CONCLUSIONS

In this paper, a new model is proposed for modeling power systems in bifurcation studies. The model was also shown to be accurate for transient stability studies. The typical transient stability model is a subset of the proposed model, since transient stability models consider dynamics of generators and loads but not dynamics of the transmission system. The "quasi-steady-state" phasor models used in transient stability studies to represent the transmission system has been replaced by dynamic variables. The proposed model makes no assumptions about fluctuation in the system frequency, which are ignored in typical transient stability models.

The disadvantage of the proposed model is some numerical integration problems due to the large differences in time constants of the different element modes, although implicit integration techniques can be used to partly overcome this problem. However, the proposed model can be used to eliminate "singular" algebraic constraints.

TABLE IV
LOAD FLOW RESULTS - DYNAMIC ALGORITHM

| Variable | Value | % diff. to Traditional Algorithm |
|---|---|---|
| Bus 1 q-axis Voltage | 1.0296 pu | n/a |
| Bus 1 d-axis Voltage | 0.0303 pu | n/a |
| Bus 1 Voltage Magnitude | 1.0300 pu | 0 |
| Bus 1 Voltage Angle | 0° | 0 |
| Bus 2 q-axis Voltage | 1.0391 pu | n/a |
| Bus 2 d-axis Voltage | 0.0431 pu | n/a |
| Bus 2 Voltage Magnitude | 1.0400 pu | 0 |
| Bus 2 Voltage Angle | 0.6924° | 1.5 |
| Generator 2 Machine Angle | 1.3592° | n/a |
| Bus 3 q-axis Voltage | 0.9539 | n/a |
| Bus 3 d-axis Voltage | 0.0943 | n/a |
| Bus 3 Voltage Magnitude | 0.9586 | 0 |
| Bus 3 Voltage Angle | 3.9612° | 0.15 |

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