Probabilistic Optimal Power Flow in Electricity Markets Based on a Two-Point Estimate Method

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***Abstract*— This paper presents an application of a two-point estimate method (2PEM) to account for uncertainties in the optimal power flow (OPF) problem in the context of competitive electricity markets. These uncertainties can be seen as a by-product of the economic pressure that forces market participants to behave in an "unpredictable" manner; hence, probability distributions of locational marginal prices are calculated as a result. Instead of using computationally demanding methods, the proposed approach needs $2n$ runs of the deterministic OPF for n uncertain variables to get the result in terms of the first three moments of the corresponding probability density functions. Another advantage of the 2PEM is that it does not require derivatives of the nonlinear function used in the computation of the probability distributions. The proposed method is tested on a simple 3-bus test system and on a more realistic 129-bus test system. Results are compared against more accurate results obtained from MCS. The proposed method demonstrates high level of accuracy for mean values when compared to the MCS; for standard deviations, the results are better in those cases when the number of uncertain variables is relatively low or when their dispersion is not large.**

***Index Terms*— Probabilistic optimal power flow, electricity markets, two-point estimate method, probability distribution, uncertainty.**

I. Introduction

DEREGULATION and privatization have changed the way power systems around the world are being operated nowadays; this has had significant impact on the planning and the operation of power systems. For example, market participants' behavior has become somewhat unpredictable, which can be considered as one of the main factors for electricity price volatility in some markets. Another "by-product" of deregulation is the reduction in power system stability margins as a result of reduced capital investment in infrastructure and changes in dispatch and loading patterns.

In order to cope with the increased uncertainty imposed by the introduction of electricity markets, the application of stochastic tools should be useful to study and thus understand the system and its associated market [1], [2]. For a power system operator, a tool that is able to take into account uncertainties in some parameters should be useful. In markets based on optimal power flows (OPF) to calculate electricity prices, one may use stochastic analysis tools for market studies to see the impact of participants' behavior on electricity prices. Since OPF is a deterministic tool, it would have to be run many times to encompass all, or at least the majority, of possible operating conditions. More accurate MCS, which are able to handle "complex" random variables, are an option but are computationally more demanding, and as such of limited use for on-line types of applications. Computationally effective but still accurate and reliable methods are therefore of significant interest. There are also other uncertainties that can be considered in the problem such as equipment outages; however, this paper concentrates on studying the effect of bid and load uncertainty in prices, which is an issue of significant relevance at this point in electricity markets (e.g. Ontario).

In the literature, several approximate methods that can be used for analysis of engineering systems under uncertainty have appeared. Examples of these methods include the truncated Taylor series expansion method [3]; the discretization method [4]; the common uncertain source method [5], [6]; the first-order second-moment method [1], [7], which is basically a variant of the Taylor series expansion method; the cumulant method [2], [8], [9]; and the point estimate method [10]–[12]. The main idea behind these methods is to use approximate formulas for calculating the statistical moments of a random quantity that is a function of n random variables, as opposed to a more accurate Monte Carlo approach, which is computationally more demanding.

In this paper, stochastic behavior of market participants is introduced in the OPF calculation by means of a two-point estimate method (2PEM). Every uncertain variable is replaced with only two deterministic points placed on each side of the corresponding mean, which enables the use of the deterministic OPF. The results are the moments of the variables of interest, which in this paper are locational marginal prices (LMPs). One of the main advantages of the proposed approach is reduced computational burden, since only $2n$ runs of the deterministic OPF are needed for n uncertain variables.

The paper is organized as follows: The probabilistic optimal power flow (P-OPF) problem is discussed in Section II, including an overview of the 2PEM. The results obtained on a 3-bus test system and on a more realistic 129-bus test system are presented and discussed in Section III. Finally, in Section IV the main contributions of the paper are summarized.

II. Probabilistic Optimal Power Flow

Power systems are stochastic in nature. This renders power-systems analysis tools, at least to some extent, inaccurate when deterministic data are used. To account for uncertainties inherent to power systems, probabilistic techniques have been used

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since the early seventies [13], where the uncertainty in system demand was first considered in a standard power flow problem. Several different approaches have been proposed to address the problem, such as the formulation of the problem as a general probabilistic linear transformation, first-order second-moment method (FOSMM) when the problem is treated as a general nonlinear problem, or fast Fourier transforms to account for different types of distributions.

The problem of optimal dispatch is considered as a probabilistic problem in [14], where the authors use Gram-Charlier series to represent the probability density function (PDF) of the system load, which is the only uncertainty considered in this case. In [15], the authors propose a more general approach to account for uncertainties in all the OPF variables based on the same Gram-Charlier series technique. In [1], the term P-OPF is proposed; optimality conditions are treated in this case as a general nonlinear probabilistic transformation, and FOSMM is used to find the statistical characteristics of the output variables, which are bus voltages and angles, and active and reactive powers. In [1], bus loading is considered as the only uncertain parameter.

The cumulant method for probabilistic power system simulation is discussed in [8]. In [2] and [9], the authors propose the cumulant method for the P-OPF. They make use of the fact that the inverse of the Hessian in the logarithmic barrier interior point method can be seen as a linear mapping. For inelastic demand, which is considered uncertain, the method gives cumulants for unknown system variables, which are bus voltages and angles, and active and reactive powers. The problem is basically the same as in [1] and results are compared in [7], where the authors show that the results using the FOSMM exactly match the results using the cumulant method for some combinations of independent Gaussian distributions.

Although some of the above references note that the proposed approaches could be used for any OPF variable, none of them actually considered the LMPs as uncertain variables; however, the LMPs are important in the context of electricity markets. This problem of bidding uncertainty is discussed in Section III to show some problems that are not observed when only load uncertainty is considered. This paper studies these specific issues, proposing a computationally efficient technique to calculate the PDFs of the LMPs as a result of uncertain behavior of market participants.

A. Problem Formulation

This paper considers a typical deterministic OPF-based market model, as discussed in [16], i.e. the following security constrained optimization problem is studied:

$$
\begin{aligned}
\text{Min. } & G = -(C_D^T P_D - C_S^T P_S) && \rightarrow \text{social welfare} \quad (1)\\
\text{s.t. } & F(\delta, V, Q_G, P_S, P_D) = 0 && \rightarrow \text{power flow equations}\\
& 0 \le P_S \le P_{S_{\max}} && \rightarrow \text{supply bid limits}\\
& 0 \le P_D \le P_{D_{\max}} && \rightarrow \text{demand bid limits}\\
& |P_{ij}(\delta, V)| \le P_{ij_{\max}} && \rightarrow \text{“security” limits}\\
& |P_{ji}(\delta, V)| \le P_{ji_{\max}} && \\
& I_{ij}(\delta, V) \le I_{ij_{\max}} && \rightarrow \text{thermal limits}\\
& I_{ji}(\delta, V) \le I_{ji_{\max}} && \\
& V_{\min} \le V \le V_{\max} && \rightarrow \text{voltage limits}\\
& Q_{G_{\min}} \le Q_G \le Q_{G_{\max}} && \rightarrow \text{generator limits}
\end{aligned}
$$

where C_S and C_D are vectors of supply and demand bids in $\$/\mathrm{MWh}$, respectively; Q_G stand for the generator reactive powers; V and δ represent the bus phasor voltages and angles, respectively; P_{ij} and P_{ji} represent the power flowing through the lines in both directions, and are used to model system security by limiting the transmission line power flows, together with line current I_{ij} and I_{ji} thermal limits and bus voltage limits; and P_S and P_D represent bounded supply and demand power bids in MW, respectively. In this model, which is typically referred to as a security constrained OPF, P_{ij} and P_{ji} limits are obtained by means of off-line angle and/or voltage stability studies. Here, the objective is to maximize social welfare, which is basically the sum of consumers’ and producers’ surplus, respectively. In case of inelastic demand, demand powers P_D are known, which can be represented by setting C_D to 0 in (1).

When some of the input variables are uncertain, (1) becomes probabilistic. By solving the OPF problem (1) using an interior point method technique, (1) is transformed into a nonlinear equation of the general form (2), which is better known as a Lagrangian function. Therefore, generally speaking, the OPF can be seen as a multivariate nonlinear function h of the form:

$$Y = h(X) \tag{2}$$

where capital letters denote random variables. In this paper, the input vector X can be written as:

$$X = [C_D \ C_S \ P_{D_{max}} \ P_{S_{max}}] \tag{3}$$

and the output vector Y can be written as:

$$Y = [\delta \ V \ Q_G \ P_S \ P_D \ \rho] \tag{4}$$

where ρ is a vector of Lagrangian multipliers. In the context of electricity markets, the Lagrangian multipliers of the power flow equations are of special interest, since they can be directly associated with the system LMPs [16], [17]. It has to be noted that uncertain input vector renders all output variables uncertain as well; however, this paper mostly concentrates on the analysis of the uncertainty associated with the LMPs.

B. Two-Point Estimate Method

In order to account for uncertainties in the P-OPF, a 2PEM [12], which is basically a variation of the original point estimate method (PEM) described in [10], [11] is used to decompose (2) into several sub-problems by taking only two deterministic values of each uncertain variable placed on both sides of the corresponding mean. The deterministic OPF is then run twice for each uncertain variable, once for the value below the mean, and once for the value above the mean, with other variables kept at their means. This method is described in detail below.

Suppose that $Y = h(X)$ is a general nonlinear multivariate function as in (2). The goal is then to find the PDF $f_Y(y)$ of

Y when the PDF $f_X(x)$ is known, where $x \in X$ and $y \in Y$. As mentioned before, there are several approximate methods to address this problem. One of the disadvantages of both the truncated Taylor series expansion method [3] and the FOSMM [1], [7] is that these require the evaluation of the derivatives of $h(X)$ with respect to X. The discretization method [4], on the other hand, uses discrete probability distributions to replace continuous probability distributions; this method is simple to use, but it may be computationally intensive (e.g. suppose that the underlying nonlinear function $Y = h(X)$ consists of 20 variables and each of them is replaced by three discrete values only; hence, to estimate the probability distribution of the output, one needs more than 3^{20} evaluations). Finally, the common uncertain source method [5], [6] assumes that the input variables are dependent and normally or lognormally distributed.

The point estimate method (PEM) is a simple to use numerical method for calculating the moments of the underlying nonlinear function. The method was developed by Rosenblueth in the 1970's [10], [11] and is used to calculate the moments of a random quantity that is a function of one or several random variables. Although the moments of the output variables are calculated, one has no information on the associated probability distribution (PD). Generally speaking, this PD can be any PD with the same first three moments; however, when the PD of the input variables is known, the output variables tend to have the same PD, as illustrated in this paper for the OPF problem, where both input and output variables are normally distributed. However, in some cases, the discrete behavior of the OPF results in PD of the output variables that is not normal anymore, as illustrated in Section III.

Let X denote a random variable with PDF $f_X(x)$; for $Y = h(X)$, the PEM uses two probability concentrations to replace $h(X)$ by matching the first three moments of $h(X)$. When Y is a function of n random variables, the PEM uses 2^n probability concentrations located at 2^n points to replace the original joint probability density function of the random variables by matching up to the second and third-order non-crossed moments. The moment of Y, i.e. $E(Y^l)$, $l = 1, 2$, where $E(\cdot)$ is the expectation, is then calculated by weighting the value of Y to the power of l evaluated at each of the 2^n points. When n becomes large, the use of 2^n probability concentrations is not economical. Hence, a simplified method that makes use of only $2n$ estimates was proposed in [12]; referred to as a 2PEM. More recently, a 2PEM was applied to power systems in [18], where the authors use this method to assess the power transfer capability uncertainty.

In this paper, the method originally proposed in [12] is applied to the OPF problem in the context of competitive electricity markets, since the proposed 2PEM shows significant advantages over other methods that use $2n$ rather than 2^n point concentrations. Thus, the method used here considers also the skewness of the PDF, shown in [12] to be more accurate than its rivals.

C. Computational procedure

As discussed in [10]–[12], when one is not interested in the distribution of Y but only in an approximation to its first few moments, the X's PDF can be ignored by using only the corresponding moments; the solution in this case will be independent of the distribution assigned to X. Any distribution having the same first moments as the given distribution will furnish the exact solution when Y is a linear function of X. If Y is nonlinear but sufficiently smooth, the solution will be sufficiently accurate in the neighborhood of the expectation of X, provided X's dispersion is not too large. It has to be noted that this approach works for all distribution of X with identical first three moments. The full derivation of the basic formulas on which the method is based is given in the Appendix.

Note that the method can be used for any number of concentrations depending on the assumed PDF of input variables. The order of the point estimate method used in the paper is chosen based on the assumption that the probability distributions of input variables are normal; therefore, the 2-point estimate method suffices. If the distributions of input variables were of a higher order, a higher order point estimate method would be needed. However, when one is interested in only the first three moments, namely the mean, standard deviation and skewness, only two concentrations for each uncertain variable suffice. Note that, since a normal distribution for input variables is assumed here, the third moment equals zero.

The procedure for computing the moments of the output variables for the OPF problem can be summarized in the following steps:

1) Determine the number of uncertain variables n.
2) Set $E(Y) = 0$, and $E(Y^2) = 0$.
3) Set $k = 1$.
4) Determine the locations of concentrations $\xi_{k,1}$ and $\xi_{k,2}$, and the probabilities of concentrations $P_{k,1}$ and $P_{k,2}$:

$$\xi_{k,1} = \sqrt{n} \tag{5}$$
$$\xi_{k,2} = -\sqrt{n} \tag{6}$$
$$P_{k,1} = P_{k,2} = \frac{1}{2n} \tag{7}$$

5) Determine the two concentrations $x_{k,1}$ and $x_{k,2}$:

$$x_{k,1} = \mu_{X,k} + \xi_{k,1}\sigma_{X,k} \tag{8}$$
$$x_{k,2} = \mu_{X,k} + \xi_{k,2}\sigma_{X,k} \tag{9}$$

where $\mu_{X,k}$ and $\sigma_{X,k}$ are the mean and the standard deviation of X_k, respectively.
6) Run the deterministic OPF for both concentrations $x_{k,i}$ using $X = [\mu_{X,1}, \mu_{X,2}, \ldots, x_{k,i}, \ldots, \mu_{X,n}]$.
7) Update $E(Y)$ and $E(Y^2)$:

$$E(Y) \cong \sum_{k=1}^{n}\sum_{i=1}^{2}(P_{k,i}h([\mu_{X,1}\ldots x_{k,i}\ldots\mu_{X,n}])) \tag{10}$$
$$E(Y^2) \cong \sum_{k=1}^{n}\sum_{i=1}^{2}(P_{k,i}h([\mu_{X,1}\ldots x_{k,i}\ldots\mu_{X,n}])^2) \tag{11}$$

8) Calculate the mean and the standard deviation:

$$\mu_Y = E(Y) \tag{12}$$

$$\sigma_Y = \sqrt{E(Y^2) - \mu_Y^2} \tag{13}$$

9) Repeat steps 4 to 8 for $k = k + 1$ until the list of uncertain variables is exhausted.

III. Results

The proposed method was tested on a 3-bus test system and on a more realistic 129-bus test system. For a 3-bus test system, both demand bids and supply bids were considered uncertain and were, therefore, represented with appropriate PDFs. For the 129-bus test system, demand was considered inelastic, since this is the typical case in most electricity markets nowadays (e.g. Ontario). The 3-bus test system was chosen because it is simple enough to give a clear insight into the nature of the problem while still retaining some of the features of "real" power systems. The 129-bus test system is used here as it has been used for market studies before [17], [19], [20], since this hypothetic system model has a realistic number of players and a realistic representation of a transmission system.

Normal distribution with the mean value set at the initial value was assumed in all cases, since this is typically the case. Notice, however, that other types of distributions (e.g. gamma, lognormal) could be readily used. Several different standard deviations were used to represent market participants' behavior. All the results were obtained in MATLAB, using PSAT [21] to solve the required OPFs.

A. Comparison of the Results

For all scenarios, the mean and the standard deviation of the 2PEM are compared with the corresponding values obtained with the MCS, which were considered "accurate", and are calculated as:

$$\mu_{\text{MCS}} = \frac{1}{N} \sum_{i=1}^{N} x_i \tag{14}$$

$$\sigma_{\text{MCS}} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \mu_{\text{MCS}})^2} \tag{15}$$

where N is number of Monte Carlo samples, and x is the variable for which the mean μ_{MCS} and the standard deviation σ_{MCS} are calculated. The errors for the mean and standard deviation, respectively, are therefore defined as:

$$\epsilon_\mu = 100(\mu_{\text{MCS}} - \mu_{\text{2PEM}})/\mu_{\text{MCS}}\ [\%] \tag{16}$$

$$\epsilon_\sigma = 100(\sigma_{\text{MCS}} - \sigma_{\text{2PEM}})/\sigma_{\text{MCS}}\ [\%] \tag{17}$$

As shown in the examples below, the output variables tend to have the same PD as the input variables, which in this paper is a normal distribution. Therefore, a normal distribution is fitted to the MCS results using the mean (14) and the standard deviation (15) to highlight the difference between the two sets of results, particularly throughout the plots depicted in this section. However, in some of the cases discussed bellow, the "discrete" behavior of the OPF skew the PD of the output variables. In such cases, the normal distribution fitted to the MCS results does not fit very well, which also affects the comparison of the results.

There are also other possible ways of comparing the results. One option is to compare the corresponding cumulative density functions (CDF), as for example in [22]. However, this does not allow to readily compare the results for all output variables, especially when dealing with a relatively large number of them. Therefore, in this paper, the comparisons are made using the corresponding mean and standard deviation of the 2PEM and MCS results, respectively, as in [2], which works reasonably well in most cases, given the fact that output variables tend to be normally distributed. This approach also gives an idea of how good the estimates for the actual mean and standard deviation are.

B. 3-bus Test System

The 3-bus test system represents three generation companies (GENCOs) and two supply companies (ESCOs) that provide supply and demand bids. The test system is extracted from [23], where a complete data set can be found. Since this test system is not based on a real system, no particular attention was paid to the representation of bidders' behavior, in the sense that the probability distributions used do not necessarily reflect the actual behavior of market participants; the bids were chosen to simply test the proposed technique and illustrate a possible use of the proposed analysis techniques. In order to simplify the analysis, the same standard deviation was assumed for all supply bidders and for all demand bidders, respectively. Several different scenarios were considered. Thus, for both suppliers and bidders standard deviation of corresponding bids was assumed to be 0, 0.1, 0.5, 1.0, and 1.5; the bids' means correspond to the base-case values, which can be found in [23]. With 5 different probability distributions for suppliers and bidders, one gets 25 different scenarios.

The results were compared against MCS with 10,000 samples. Although fewer numbers of samples could be used to obtain reasonable results with MCS, the computational burden is not really an issue with the 3-bus test system, thus the relatively large number of MCS samples. The error in the standard deviation obtained for the proposed 2PEM with respect to the MCS results was well below 10% in most cases, whereas the error in the mean values never exceeded 0.33%. However, there are some scenarios where the estimated standard deviation differed from the MCS value by almost 40%. By inspecting these scenarios more closely, it can be observed that the normal distribution fitted to the MCS results was far from accurate, as explained in more detail below. The assumption of the demand bids' standard deviation being equal to zero, i.e. not dispersed or purely deterministic, which can be expected in the case of inelastic demand, yielded LMPs with very small standard deviation values, or even zero in some cases.

Three representative sets of results are shown in Figs. 1, 2, and 4. Figure 1 shows the result for the LMP at bus 3 where the proposed method gives good results. Three plots are shown: The first one is a result of the 2PEM; the histogram is a result of the MCS; and the dotted curve is the curve fitted to the MCS results (MCS fit). The same standard deviation of 0.5 was assumed for both the GENCOs' and the ESCOs' bids. Observe that the MCS and the 2PEM yield almost the same results. This can be explained based on the fact that the OPF, which is basically a nonlinear function, is reasonably smooth, i.e. it does not experience sharp jumps. However, this is not always the case, as depicted in Fig. 2 and Fig. 4 for the LMP at bus 3.

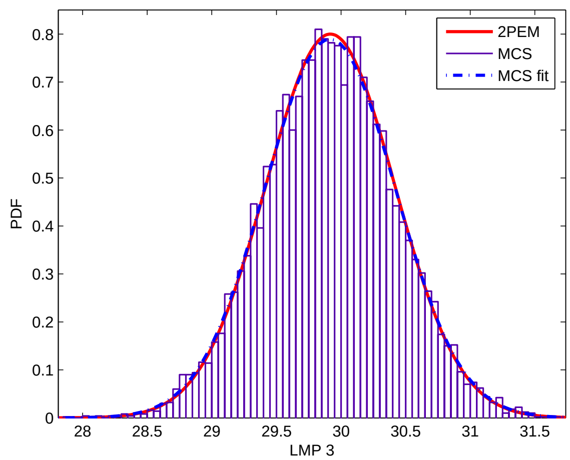

Fig. 1. LMP at Bus 3 for 3-bus system; $\sigma(C_S) = 0.5$, $\sigma(C_D) = 0.5$.

In Fig. 2, the GENCOs' bids are considered deterministic, i.e. their standard deviation is zero, while the ESCOs' bids have a rather large standard deviation of 1.5. Notice that beyond a certain point, the LMP does not increase any further, which causes a sharp jump in its PDF. This is due to the fact that, no matter how much an ESCO is willing to pay, the LMP cannot increase any further, because ESCO's demand is at its upper limit. This is basically the case of inelastic demand. If the ESCO's demand limit were higher, the LMP would increase further depending on the marginal GENCO. It has to be noted, however, that in realistic systems with a large number of supply bids, this phenomenon is expected to be much less pronounced than in this simple example where the number of bidders is very small. In spite of this, the PDF estimation is still quite accurate because the spike in the PDF somewhat compensates for the "missing tail" beyond the maximal LMP. This problem can be better explained based on the simplified market clearing mechanism depicted in Fig. 3, where, for the sake of simplicity, only the bid price of ESCO 2 is considered uncertain, and congestion and system losses are neglected. The dashed line corresponds to the bid's lower and upper limits. As long as the bid of ESCO 2 is lower than the bid of GENCO 3, the market is cleared at 150 MW with GENCO 1 being the marginal generator (point O). Above that point, GENCO 3 becomes marginal, which results in the market clearing at 200 MW (point B). The market clearing price (MCP) varies between the values corresponding to points A and B, respectively. Observe that the MCP cannot increase any further than MCP at point B because ESCO 2 is at its upper limit, regardless of the ESCO's bid; i.e. there is an upper limit to the MCP, as shown in Fig. 2.

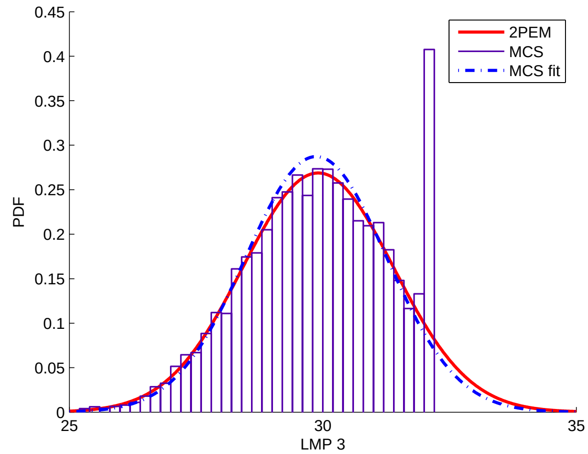

Fig. 2. LMP at Bus 3 for 3-bus system; $\sigma(C_S) = 0.0$, $\sigma(C_D) = 1.5$.

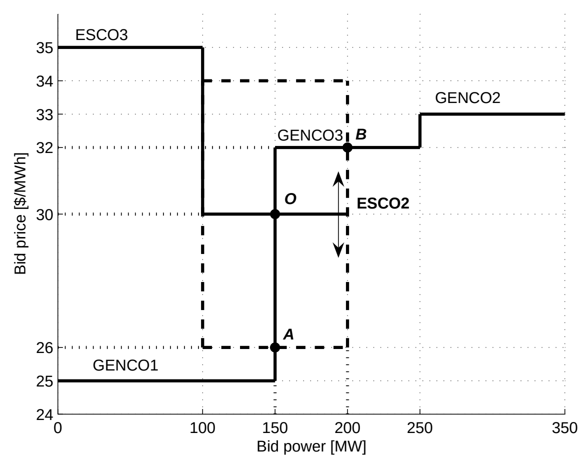

Fig. 3. Market clearing for uncertain ESCO 2's bid price for 3-bus system.

Figure 4 illustrates the case where GENCOs' bids have a large standard deviation of 1.5, while the ESCOs' bids are assumed to be "narrow", with a relatively small standard deviation of 0.1. This figure shows a different phenomenon than in the previous case; here, the LMP has a long "tail" on the left-hand side of the LMP's PDF. This can be explained using the simplified market clearing description illustrated in Fig. 5. In this case, only GENCO 3's bid price is considered uncertain to simplify the problem. Note that, as long as the bid of GENCO 3 remains above the bid of ESCO 2, the MCP will remain at point O, with market clearing at 150 MW. The MCP will change only if the bid of GENCO 3 is lower than the bid of ESCO 2. In that case, the MCP is between points A and O, and market is cleared at 200 MW. Note that point A in Fig. 5 corresponds to the lowest possible bid of GENCO 3, denoted by the dashed line. Observe that if ESCO 2's bid varies in a narrow range, the MCP follows accordingly, which explains the narrow variation around the mean shown in Fig. 4.

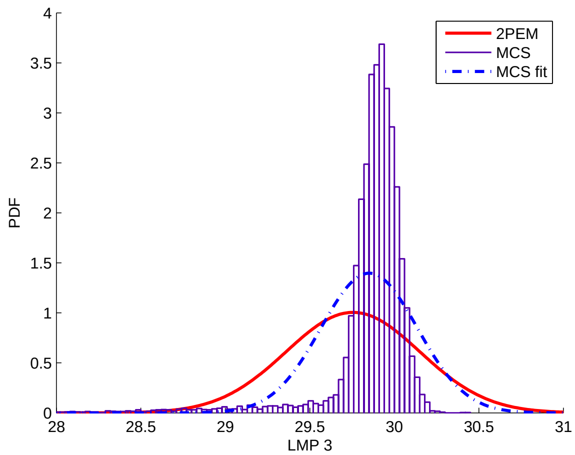

Fig. 4. LMP at Bus 3 for 3-bus system; $\sigma(C_S) = 1.5$, $\sigma(C_D) = 0.1$.

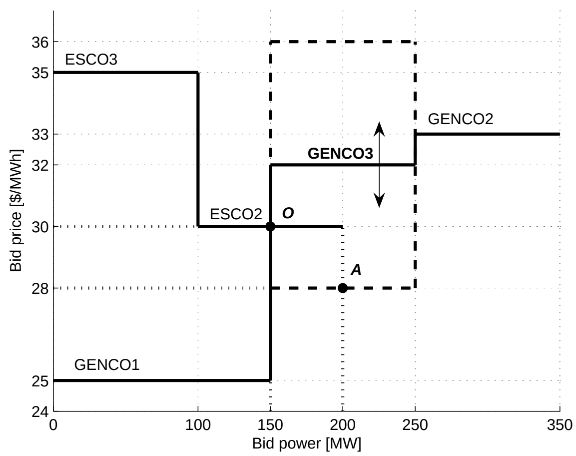

Fig. 5. Market clearing for uncertain GENCO 3's bid price for 3-bus system.

From these examples, it can be seen that the accuracy of the proposed method largely depends on the "smoothness" of the OPF seen as a general nonlinear probabilistic function. This behavior is difficult to predict in advance, since it depends on the bidders' behavior, i.e. on the submitted bids and their uncertainty.

C. 129-bus Test System

The 129-bus test system used in this paper is taken from [17], where more details about the system can be found; the system is a simplified model of a European system. It was assumed that 32 generators participate in the market auction, and demand was considered inelastic. Bids were assumed to be around 30-40$/MWh based on the average prices in the European electricity markets in the last few years. Fixed generation was assumed to be around 65% of total demand, with the remaining capacity being offered on an OPF-based market.

In the case of the 3-bus system, no assumptions with regards to risk-proneness of market participants were made, i.e. the same standard deviation was assumed for all generators and loads, respectively. In reality, this assumption is not realistic. Therefore, in the case of this test system, the market participants that sell most of the energy by long-term contracts were assumed to be able to take greater risk ("large" standard deviation). On the other hand, those participants who offer most of their energy on the spot market were assumed to be more risk-averse ("small" standard deviation).

Three different scenarios were considered. In Scenario 1, all generators offer some capacity on the market. In Scenarios 2 and 3, some of the most expensive generators can exercise market power due to insufficient cheap generation. In Scenario 2, supply bid prices of the six most expensive generators were assumed uncertain, while in Scenario 3, supply bid prices of the same six generators were assumed deterministic but higher than in the base case. With regards to loads, in Scenarios 1 and 2, loads were assumed inelastic and fixed, while in Scenario 3, loads were assumed to be inelastic but varying with a small standard deviation (this is similar to what has been observed in the Ontario market). Results obtained with the 2PEM were compared against results obtained with 1,000-sample MCS. Since the MCS were used only for comparison purposes, no attempt has been made to determine the optimal number or reduce the number of MCS samples, nor any other techniques, such as variance reduction techniques, have been used to achieve a desired accuracy level. By observing the convergence of the mean and standard deviation (see Fig. 6), a number of 1,000 samples was determined to be sufficient in order to obtain an adequate level of accuracy. No simulations resulted in divergent OPFs.

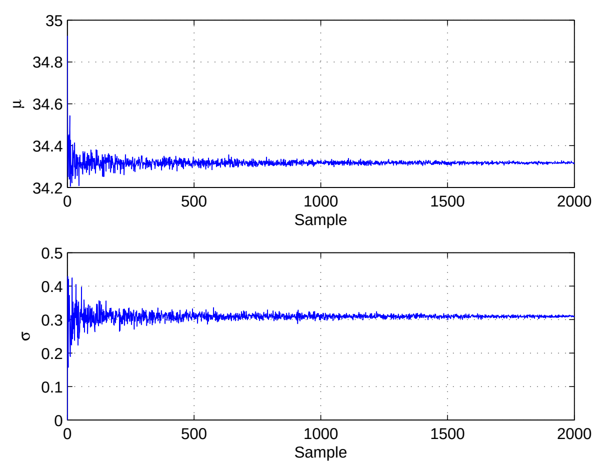

Fig. 6. MCS samples analysis.

1) Scenario 1: In the first scenario, those participants that sell most of the energy by long-term contracts are assumed to take greater risk on the spot market, since their income mostly depends on fixed contracts. Therefore, their bids vary more compared to the bids of those generators that offer more capacity on the spot market. These latter players are considered more risk-averse, since their income depends more on their success on the spot market. Thus, Table I summarizes fixed generation P_{G_o}, supply bid block P_S, supply bid price C_S, and standard deviation of supply bid price $\sigma(C_S)$, respectively, for all GENCOs. Standard deviation for bids varied between 0.24 $/MWh and 1.4 $/MWh, depending on the risk taken by the suppliers as explained above.

Figure 7 depicts the LMPs at the bus with the best (Bus 4) and worst PDF approximations (Bus 21), respectively (due to large number of buses, only LMPs at two representative buses are shown), and the error of estimating the mean and the standard deviation of the LMP distribution using the 2PEM, considering that the results obtained with a 1,000-sample MCS are "accurate" and therefore used as a benchmark. Observe that the results for the mean value of the LMP distribution using the 2PEM are well within 1% of the corresponding MCS values. In the case of standard deviation, the 2PEM results don't agree that well with the MCS results; in some cases, the error can be up to 35%. The reason for poor performance of the 2PEM in this case is a relatively large number of uncertain variables in the P-OPF problem, since the 2PEM calculates the impact of players before linearly combing all the impacts to come up with the final estimate. Thus, according to (5), the position of each of the 2PEM concentrations $\xi_{k,i}$ depends on the number of uncertain variables n; as n increases, concentrations $\xi_{k,i}$ are taken further away from the mean of the corresponding variable. Sometimes, from a certain point on, "the impact" of a given variable does not increase any more, hence the bigger error. Therefore, if the bids' standard deviation is reduced to values between 0.04 $/MWh and 0.21 $/MWh, the error in estimating the LMP's mean and standard deviation is greatly reduced as shown in Fig 7.

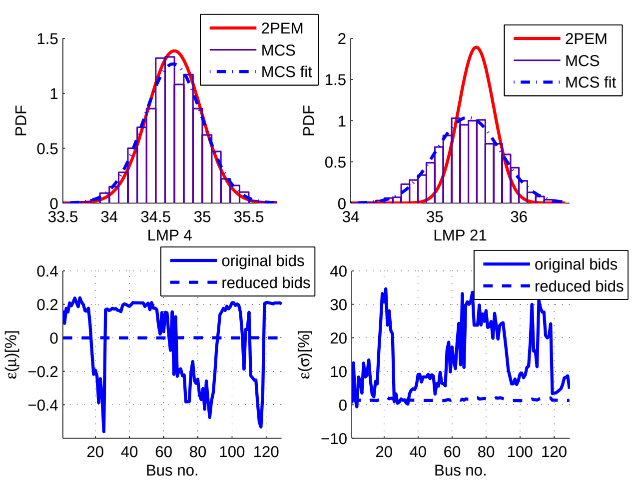

Fig. 7. Scenario 1 for the 129-bus system: LMP at Bus 4 and Bus 21 with the best and worst mean and standard deviation estimation errors $\epsilon(\mu)$ and $\epsilon(\sigma)$, respectively, for a wide range of bid variations and for a reduced variation of supply bids.

2) Scenario 2: Here, the available capacity of the cheapest generators was reduced by 2875 MW, thus simulating conditions where more expensive generators can exercise market power. As opposed to the first scenario, only bids of the six most expensive generators were assumed uncertain here, as shown in Table II. In this scenario, Bus 1 has the best, and Bus 83 worst PDF approximations, respectively, as illustrated in Fig. 8. Observe that the most expensive generators can indeed take advantage of lack of cheap generation, since increasing their bids leads to prices that are considerably higher than in the first scenario, which is clear when comparing Figs. 7 and 8.

As in the previous case, Fig. 8 also shows the error for the mean and the standard deviation of the LMP distribution using the 2PEM. It is interesting to observe that the error for standard deviation is much lower that in the previous scenario and barely exceeds 3%; the error for the mean is again very low. The reason for much performance of the 2PEM in this case is the lower number of uncertain variables in the P-OPF problem.

TABLE I

129-BUS TEST SYSTEM, SUPPLY BIDS FOR SCENARIO 1

Bus	P_{G_o} [MW]	P_S [MW]	C_S [$/MWh]	$\sigma(C_S)$ [$/MWh]
1	517.63	604	30	0.58
2	374.38	444	34.16	0.64
3	265.61	280	33.6	0.74
4	511.96	592	32.52	0.58
5	393.63	512	30	0.62
6	241.65	296	37.96	0.72
7	183.23	224	34	0.78
8	256.81	296	35.2	0.72
9	308.02	124	30	0.94
10	236.8	296	36.52	0.72
11	494.44	592	35.04	0.58
12	879.24	296	34.84	0.72
13	764.04	788	34.12	0.54
14	76.82	884	33.64	0.52
15	1383.12	1200	33.04	0.48
16	254.41	37	39	1.36
17	192	65.6	30	1.14
18	838.39	296	37.76	0.72
19	981.6	1200	32.88	0.48
20	2285.65	824	32.8	0.54
21	387.19	532	35.52	0.6
22	1640.88	600	36.6	0.58
23	471.17	480	30	0.62
24	599.27	816	39.12	0.54
25	251.97	275.2	36.44	0.74
32	719.2	296.67	32	0.72
34	540.8	223.08	32	0.78
87	221.6	91.41	34	1.04
95	217.6	89.76	32	1.04
108	150.4	62.04	34	1.16
120	451.2	186.12	32	0.84
129	8	3.3	32	2.8

TABLE II

129-BUS TEST SYSTEM, SUPPLY BIDS FOR SCENARIO 2

Bus	P_{G_o} [MW]	P_S [MW]	C_S [$/MWh]	$\sigma(C_S)$ [$/MWh]
6	241.65	296	38.96	1
10	236.8	296	37.52	1
16	254.41	37	40	1
18	838.39	296	38.76	1
24	599.27	816	40.12	1
25	251.97	275.2	37.44	1

3) Scenario 3: In this scenario, the available capacity of the cheapest generators was again reduced by 2875 MW, thus

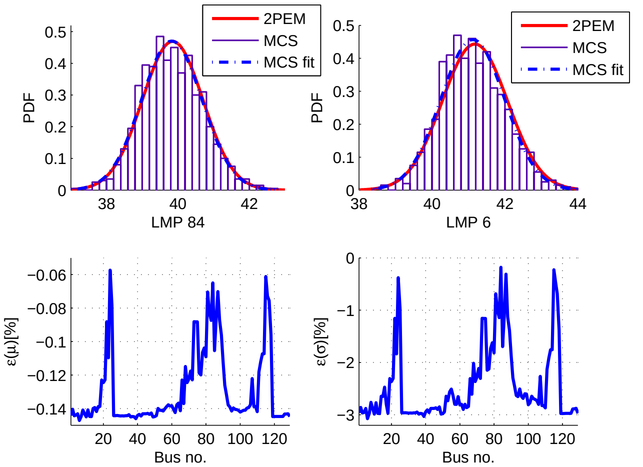

Fig. 8. Scenario 2 for the 129-bus system: LMP at Bus 84 and Bus 6 with the best and worst mean and standard deviation estimation errors $\epsilon(\mu)$ and $\epsilon(\sigma)$, respectively.

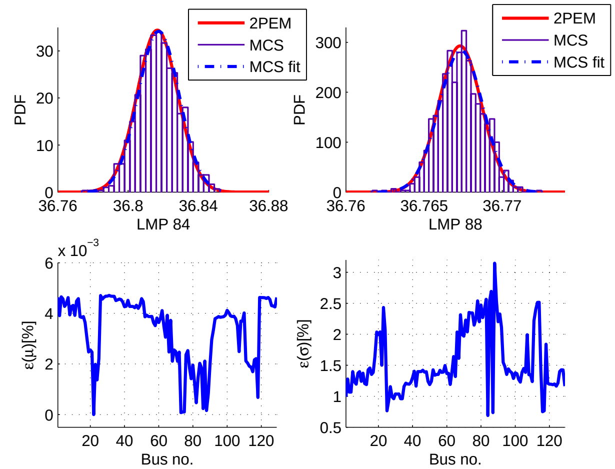

Fig. 9. Scenario 3 for the 129-bus system with uncorrelated demand: LMP at Bus 84 and Bus 88 with the best and worst mean and standard deviation estimation errors $\epsilon(\mu)$ and $\epsilon(\sigma)$, respectively.

simulating similar market conditions as in Scenario 2. As opposed to the first two scenarios, only demand power at all load buses was assumed uncertain, with a standard deviation of 2%. Bids of the six most expensive generators (6, 10, 16, 18, 24, and 25) were assumed fixed and 2 \$/MWh higher than those used in Scenario 1.

In the first set of results, demand power levels were considered uncorrelated, while in the second set they were considered correlated. The assumption of the demand being correlated is reasonable since some factors (e.g. weather) likely influence large groups of consumers in the same way. Note that the original 2PEM cannot handle correlated variables; however, if demand is assumed correlated in such a way that the demand of a given node would be high if the demand of any other node is high as well, one can assume that all demand power levels are scaled by one variable. This new variable is then treated as a new uncertain variable in (2). The use of only one variable for demand power levels also reduces the total number of variables, thus reducing the error of the 2PEM and the computational burden.

As in the previous cases, Figs. 9 and 10 depict buses with the best and worst PDF approximations, respectively. Note that in Fig. 10, two curves are fitted to the MCS results for LMP at Bus 1; the first one (MCS fit 1) considers full MCS results, while the second one (MCS fit 2) neglects "the outliers" in the MCS results. The outliers are due to the fact that, in some cases, different generators set the price, depending on the loading level. If the load is high, some more expensive generators are dispatched; on the other hand, when the load is low, there is enough cheap generation which results in a lower price. Observe in Fig. 9 that if demand power levels are uncorrelated, the LMPs vary only slightly, since the total demand does not change much. However, if the demand is correlated, the LMPs change more, as observed in Fig. 10, where a slight variation in demand power levels produces significant variations in LMPs (this is a phenomenon that has been observed in the Ontario market).

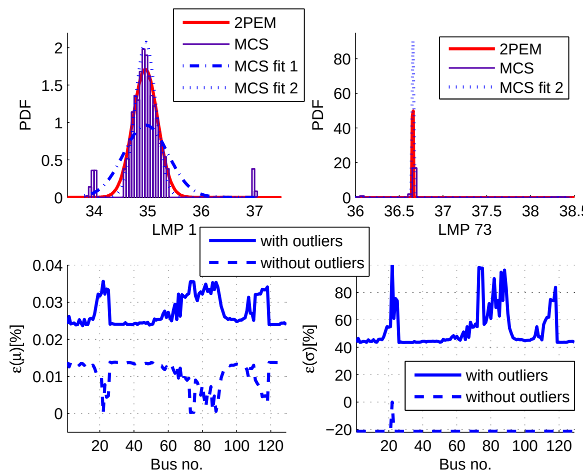

Fig. 10. Scenario 3 for the 129-bus system with correlated demand: LMP at Bus 1 and Bus 73 with the best and worst mean and standard deviation estimation errors $\epsilon(\mu)$ and $\epsilon(\sigma)$, respectively, with and without "outliers".

Figures 9 and 10 also show the error for the mean and the standard deviation of the LMP distribution using the 2PEM for uncorrelated and correlated demands, respectively. Observe that the 2PEM gives good results if demand is uncorrelated, while for the correlated demand the error can be as high as 100%. The higher error is due to the "outliers" in the MCS results. This phenomenon can be observed for Bus 73 in Fig. 10 where the LMP is almost discrete except the "outliers", which skew the results. If the 2PEM results are compared to the MCS results that neglect "the outliers", the results are much better. It is interesting to observe that the error for standard deviation is almost the same for all the buses, except for Bus 22, where the error is zero because the LMP has zero standard deviation at this bus and both MCS and the 2PEM yield the correct result.

4) Computational Issues: One of the advantages of the proposed method is computational efficiency. Table III summarizes the computational "burden" for different scenarios for the

129-bus test system, given in terms of the number of OPF runs, since these are the main computational steps in MCS and the proposed technique, hence allowing for platform-independent comparisons. Bare in mind that the computational burden of the 2PEM is directly proportional to the number of uncertain variables, since the proposed method needs exactly two runs of the deterministic OPF for each uncertain parameter, whereas for MCS this is proportional to the number of samples used. Note that the number of MCS samples could be reduced given the desired level of accuracy; however, this particular issue is not considered in this paper.

Observe in Table III that the proposed approach is computationally significantly faster then MCS, especially if the number of uncertain variables is low. If the number of uncertain variables is large, MCS can be a viable alternative, given its accuracy.

TABLE III
COMPARISON OF COMPUTATIONAL BURDEN FOR THE 129-BUS TEST SYSTEM

Scenario	OPF runs	
	2PEM	MCS
1	64	1000
2	12	1000
3	2	1000

IV. CONCLUSIONS

In this paper, a new approach using the 2PEM for accounting for uncertainties in bidding in the OPF problem has been proposed. Bids of market players were considered uncertain and the first two moments of the LMP's PDF were calculated as a result. The approach was tested on a simple 3-bus test system with demand and supply-side bidding and on a more realistic 129-bus test system with inelastic demand.

The proposed approach is accurate provided that the OPF is "well-behaved" and that the number of uncertain parameters is not "too large". In large systems with many market players, the first condition tends to be fulfilled given the large number of market players. If the number of players is small, as in the case of the 3-bus test system, the results may not be sufficiently accurate due to "discrete" behavior of the OPF in some cases. In larger systems, with many market players, the 2PEM does not perform well if the number of uncertain variables is too large. With lower numbers of uncertain variables, the performance is adequate.

The proposed method is computationally significantly faster than using an MCS approach. This is especially true when the number of uncertain parameters is low, since the computational time is directly proportional to the number of uncertain variables. When the number of random variables is large, MCS is a better alternative, given its accuracy.

V. ACKNOWLEDGMENTS

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APPENDIX
TWO-POINT ESTIMATE METHOD

For sake of completeness, this appendix provide the full derivation of the basic formulas used in this paper. Thus, suppose that $Y = h(X)$ is a general nonlinear multivariate function. The goal is to find the PDF $f_Y(y)$ of Y when the PDF $f_X(x)$ is known.

A. *Function of One Variable*

First, a fictitious distribution of X is chosen in such a way that the first three moments exactly match the first three moments of the given PDF of X. In order to estimate the first three moments of Y, one can choose a distribution of X having only two concentrations placed unsimetrically around the X's expectation. If that is the case, one has enough parameters to take into account the first three moments of X and to obtain a third-order approximation to the first three moments of Y. A particularly simple function satisfying these requirements consists in two concentrations, P_1 and P_2, of the probability density function $f_X(x)$, respectively at $X = x_1$ and x_2:

$$f_X(x) = P_1\delta(x - x_1) + P_2\delta(x - x_2) \tag{18}$$

where the lower-case letters denote specific values of a random variable, and $\delta(\cdot)$ is Dirac's delta function.

Choosing $\xi_i = |x_i - \mu_X|/\sigma_X$, $i = 1, 2$, where μ_X and σ_X are the mean and the standard deviation of X, respectively, one can calculate the first three moments of $f_X(x)$. Thus, the j^{th} moment is defined as:

$$M_j(X) = \int_{-\infty}^{+\infty} x^j f_X(x)dx \quad j = 1, 2, \ldots \tag{19}$$

Alternatively, the central moments are:

$$M'_j(X) = \int_{-\infty}^{+\infty} (x - \mu_X)^j f_X(x)dx \quad j = 1, 2, \ldots \tag{20}$$

The zeroth and the first moment always equal 1 and 0, respectively. The zeroth and the first three central moments of (18) are then:

$$M'_0 = 1 = P_1 + P_2 \tag{21}$$
$$M'_1 = 0 = \xi_1 P_1 - \xi_2 P_2 \tag{22}$$
$$M'_2 = \sigma_X^2 = \sigma_X^2(\xi_1^2 P_1 + \xi_2^2 P_2) \tag{23}$$
$$M'_3 = \nu_X \sigma_X^3 = \sigma_X^3(\xi_1^3 P_1 - \xi_2^3 P_2) \tag{24}$$

where ν_X is the skewness of X.

Using the Taylor series expansion of $h(X)$ about μ_X yields:

$$h(X) = h(\mu_X) + \sum_{j=1}^{\infty} \frac{1}{j!} g^{(j)}(\mu_X)(x - \mu_X)^j \tag{25}$$

where $g^{(j)}(\cdot)$, $j = 1, 2, \ldots$, denotes the j^{th} derivative of $h(\cdot)$ with respect to x. The mean value of Y can be calculated by taking the expectation of the above equation, resulting in

$$\begin{aligned} \mu_Y &= E(h(X)) = \int_{-\infty}^{+\infty} h(x) f_X(x)dx = \\ &= h(\mu_X) + \sum_{j=1}^{\infty} \frac{1}{j!} g^{(j)}(\mu_X) M'_j(X) \end{aligned} \tag{26}$$

Let $x_i = \mu_X + \xi_i \sigma_X$, $i = 1, 2$, denote the i^{th} location, where ξ_1 and ξ_2 are constants to be determined. Let P_i be the probability concentrations at location x_i, $i = 1, 2$. Multiplying (25) by P_i with x_i, $i = 1, 2$, and summing them up leads to

$$\begin{aligned} P_1 h(x_1) + P_2 h(x_2) &= h(\mu_X)(P_1 + P_2) \\ &+ \sum_{j=1}^{\infty} \frac{1}{j!} g^{(j)}(\mu_X)(P_1 \xi_1^j + P_2 \xi_2^j)\sigma_X^j \end{aligned} \tag{27}$$

One can match the first four terms of the right side of (26) and (27), which results in

$$\begin{aligned} P_1 + P_2 &= M_0'(X) = 1 \\ P_1\xi_1 + P_2\xi_2 &= M_1'(X)/\sigma_X = \lambda_{X,1} \\ P_1\xi_1^2 + P_2\xi_2^2 &= M_2'(X)/\sigma_X^2 = \lambda_{X,2} \\ P_1\xi_1^3 + P_2\xi_2^3 &= M_3'(X)/\sigma_X^3 = \lambda_{X,3} \end{aligned} \tag{28}$$

This system of four equations has four unknowns, i.e. P_1, P_2, ξ_1, ξ_2. The solution of the system is

$$\begin{aligned} \xi_1 &= \lambda_{X,3}/2 + \sqrt{1 + (\lambda_{X,3}/2)^2} \\ \xi_2 &= \lambda_{X,3}/2 - \sqrt{1 + (\lambda_{X,3}/2)^2} \\ P_1 &= -\xi_2/\zeta \\ P_2 &= \xi_1/\zeta \end{aligned} \tag{29}$$

where $\zeta = \xi_1 - \xi_2 = 2\sqrt{1 + (\lambda_{X,3}/2)^2}$. Hence, for a normal distribution where $\lambda_{X,3} = 0$,

$$\begin{aligned} \xi_1 &= 1 \\ \xi_2 &= -1 \\ P_1 &= P_2 = 1/2 \end{aligned} \tag{30}$$

Now, from (27) and (28) one has that

$$\begin{aligned} h(\mu_X) + \sum_{i=1}^{3} \frac{1}{i!} g^{(i)}(\mu_X)\lambda_{X,i}\sigma_X^i &= \\ P_1 h(x_1) + P_2 h(x_2) - \sum_{i=4}^{\infty} \frac{1}{i!} g^{(i)}(\mu_X)(P_1\xi_1^i + P_2\xi_2^i)\sigma_X^i \end{aligned} \tag{31}$$

Substituting (31) into (26) gives

$$\begin{aligned} \mu_Y &= P_1 h(x_1) + P_2 h(x_2) \\ &+ \sum_{j=4}^{\infty} \frac{1}{j!} g^{(j)}(\mu_X)(\lambda_{X,j} - (P_1\xi_1^j + P_2\xi_2^j))\sigma_X^j \end{aligned} \tag{32}$$

Therefore,

$$\mu_Y \cong P_1 h(x_1) + P_2 h(x_2) \tag{33}$$

is a third-order approximation. If $h(X)$ is a third-order polynomial, meaning that the derivatives of the order higher than three are zero, the 2PEM gives the exact solution to μ_Y.

Similarly, one can show that the second and the third-order moment of Y, respectively, can be approximated by

$$E(Y^2) \cong P_1 h(x_1)^2 + P_2 h(x_2)^2 \tag{34}$$

$$E(Y^3) \cong P_1 h(x_1)^3 + P_2 h(x_2)^3 \tag{35}$$

B. Function of Several Variables

Let Y denote a random quantity that is a function of n random variables, i.e. $Y = h(X) = h(X_1, X_2, \ldots, X_n)$. Let $\mu_{X,k}$, $\sigma_{X,k}$ and $\nu_{X,k}$ denote the mean, standard deviation and skewness of X_k, respectively, where the variables are uncorrelated. Let $P_{k,i}$ denote the concentrations (or weights) located at $X = [\mu_{X,1}, \mu_{X,2}, \ldots, x_{k,i}, \ldots, \mu_{X,n-1}, \mu_{X,n}]$, where

$$x_{k,i} = \mu_{X,k} + \xi_{k,i}\sigma_{X,k}, \quad i = 1, 2, \quad k = 1, 2, \ldots, n \tag{36}$$

One can expand $Y = h(X)$ in a multivariate Taylor series about the mean values of X. Thus, similar to the case of a function of one variable, one can establish 3 equations for each of the random variable X_k, by matching the first 3 moments of the probability density function of X_k, leading to

$$\begin{aligned} P_{k,1}\xi_{k,1} + P_{k,2}\xi_{k,2} &= M_1'(X_k)/\sigma_{X,k} = \lambda_{X,k,1} \\ P_{k,1}\xi_{k,1}^2 + P_{k,2}\xi_{k,2}^2 &= M_2'(X_k)/\sigma_{X,k}^2 = \lambda_{X,k,2} \\ P_{k,1}\xi_{k,1}^3 + P_{k,2}\xi_{k,2}^3 &= M_3'(X_k)/\sigma_{X,k}^3 = \lambda_{X,k,3} \end{aligned} \tag{37}$$

where $k = 1, 2, \ldots, n$. Since the sum of the concentrations is one, one has

$$\sum_{k=1}^{n} (P_{k,1} + P_{k,2}) = 1 \tag{38}$$

By specifying that the sums of the concentrations satisfy

$$P_{k,1} + P_{k,2} = 1/n, \quad k = 1, 2, \ldots, n \tag{39}$$

four equations with 4 unknowns for each random variable X_k are established, yielding the solution

$$\begin{aligned} \xi_{k,1} &= \lambda_{k,3}/2 + \sqrt{n + (\lambda_{k,3}/2)^2} \\ \xi_{k,2} &= \lambda_{k,3}/2 - \sqrt{n + (\lambda_{k,3}/2)^2} \\ P_{k,1} &= -\frac{\xi_{k,2}}{n\zeta_k} \\ P_{k,2} &= \frac{\xi_{k,1}}{n\zeta_k} \end{aligned} \tag{40}$$

where $\zeta_k = 2\sqrt{n + (\lambda_{k,3}/2)^2}$, and $k = 1, 2, \ldots, n$. For symetric probability distributions, where skewness $\nu_{k,3}$, and as a result $\lambda_{k,3}$, equal zero, (40) further simplifies into

$$\begin{aligned} \xi_{k,1} &= \sqrt{n} \\ \xi_{k,2} &= -\sqrt{n} \\ P_{k,1} &= P_{k,2} = \frac{1}{2n} \end{aligned} \tag{41}$$

where $k = 1, 2, \ldots, n$.

The first three moments can then be approximated by

$$E(Y) \cong \sum_{k=1}^{n} \sum_{i=1}^{2} (P_{k,i} \times h([\mu_{X,1}, \ldots, x_{k,i}, \ldots, \mu_{X,n}])) \tag{42}$$

$$E(Y^2) \cong \sum_{k=1}^{n} \sum_{i=1}^{2} (P_{k,i} \times h([\mu_{X,1}, \ldots, x_{k,i}, \ldots, \mu_{X,n}])^2) \tag{43}$$

$$E(Y^3) \cong \sum_{k=1}^{n} \sum_{i=1}^{2} (P_{k,i} \times h([\mu_{X,1}, \ldots, x_{k,i}, \ldots, \mu_{X,n}])^3) \tag{44}$$

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