Oscillatory Stability Limit Prediction Using Stochastic Subspace Identification

Hassan Ghasemi, *Student Member, IEEE,* Claudio Cañizares, *Senior Member, IEEE,* and Ali Moshref

***Abstract*— Determining stability limits and maximum loading margins in a power system is important and can be of significant help for system operators for preventing stability problems. In this paper, stochastic subspace identification is employed to extract the critical mode(s) from the measured ambient noise without requiring artificial disturbances (e.g. line outages, generator tripping and adding/removing loads), so that the identified critical mode may be used as an on-line index to predict the closest oscillatory instability. The proposed index is not only independent of system models and truly represents the actual system, but it is also computationally efficient. The application of the proposed index to several realistic test systems is examined using a transient stability program and PSCAD/EMTDC, which has detailed models that can capture the full dynamic response of the system. The results show the feasibility of using the proposed identification technique and index for on-line detection of proximity to oscillatory stability problems.**

***Index Terms*— System identification, subspace methods, oscillatory stability, stability indices, bifurcations.**

I. Introduction

POWER systems are being operated closer to their static or dynamic stability limits due to increase in power demand, lack of transmission expansion, and the fact that new power plants are not being built close to loads due to environmental and economic constraints. Deregulation of electricity markets is also pushing these systems to be increasingly stressed, since the main objective is to transfer power from generation surplus areas to generation deficit points, thus leading to transmission congestion problems. Therefore, nowadays stability problems associated with voltage stability and poorly damped oscillations, leading in some cases to major blackouts have become more prevalent (e.g. August 2003 North-East blackout [1], September 2003 Italy blackout [2], August 1996 WSCC blackout [3]). Hence, identification of stability limits associated with voltage stability, angle stability or a combination of both is of great interest to Independent System Operators (ISO). If proper on-line monitoring tools are available, this can help operators determine the available security margins of the system when planing and operating the electricity market and its associated system, thus allowing them to take proper actions in real time in the case of system contingencies.

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H. Ghasemi and C. Cañizares are with the Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, Ontario, N2L 3G1, Canada (email: hassan@thunderbox.uwaterloo.ca, ccanizar@uwaterloo.ca).

A. Moshref is with Powertech Labs Inc., Surrey, BC, V3W 7R7, Canada (email: ali.moshref@powertechlabs.com).

A. Stability Indices

Voltage collapse and oscillatory phenomena have been directly associated with bifurcation theory in nonlinear systems. Saddle-node bifurcations (SNB) and limit-induced bifurcations (LIB) have been directly related to voltage stability, and a fair amount of research work has been conducted to detect and predict the corresponding stability limits [4], [5]. On the other hand, oscillatory phenomena have been associated with Hopf bifurcations (HB) [6], [7].

Some research has been carried on detection of HB points in power systems as well as their prediction and control [8], [9], [10]. All the previous research is based on the linearization of the power system's differential-algebraic equations (DAE's), and the manipulation of the corresponding matrices in order to calculate the modes of the system. These DAE models are based on approximate system representation and data; however, obtaining models and data (e.g. system controllers) for a system with several thousand buses is rather cumbersome. Therefore, during the last decade, there has been a trend to use identification techniques based on the real time response of the power system in order to analyze its behavior.

Various techniques have been recently developed based on wide area measurements (WAM) to provide information of poorly damped modes, identify causal factors using sensitivity analysis, and propose actions for real time operation and off-line studies to alleviate the problematic system oscillations [11], [12], [13]. However, these tools are not designed to provide predictions of the available stability limits when system parameters change (e.g. loading); this is one of the main objectives of this paper.

B. System Identification

Prony method is a well-known system identification method that has been widely used in power systems to determine modal content, develop equivalent linear models of power systems, and tuning controllers using system measurements [14], [15], [16], [17]. This method, when compared to modal analysis, is a close duplicate of the latter; however, it needs high signal-to-noise (SNR) ratio in order to get accurate results. Thus, it cannot be readily applied to the response of a normally operated power system, since it needs "large enough" perturbations (e.g. line or generator outages), which are usually not available. There are linear time invariant models, such as auto-regressive (AR), auto-regressive moving-average (ARMA), and stochastic state-space, which is a transformed representation of ARMA, that can be employed to analyze the measured response of power systems under normal operating conditions. In this case, the input is white noise and can be interpreted as random load variations during the day, assuming that the system does not change and stays at its equilibrium point [18], [19]. ARMA models are more general than AR, but they are computationally expensive. Although it is possible to use AR by removing the moving average term of ARMA, the system order needed to model the signal would be high.

There are different approaches to solve for unknown parameters of models, such as prediction error methods (PEM's) and correlation methods that include instrumental variable (IV) methods. PEM's are basically based on nonlinear iterative optimization techniques, with related problems such as convergence, the existence of the local minima, and high computational burden. Correlation methods are numerically robust, but they need the calculation of covariance. Stochastic subspace methods have also been recently used to estimate the unknown parameters of stochastic state-space models [20]. They are numerically robust, as they utilize well known techniques such as QR-factorization, least square and singular value decomposition (SVD). Furthermore, they are data driven without requiring the computation of covariances and are hence faster. Thus, in this paper, a stochastic subspace identification method is used to extract the critical modes of the system from the simulation results of test cases that are excited by random load switching. The behavior of the critical modes as the system loading changes is studied as a possible on-line index to predict oscillatory stability margins. A comparison of the proposed subspace method with respect to state-space and ARMA models using PEM is also presented. Several test cases are used to illustrate the application of the proposed technique, and corresponding observations are reported. To study the feasibility of the proposed index in a real power system, a test case is also simulated using PSCAD/EMTDC [21] to represent the full dynamic response of the system.

The paper is structured as follows: In Section II, a brief review of the stability index adopted in this paper is presented. In Section III, the subspace identification algorithm is explained in detail. The results of applying the proposed subspace method to identify the critical modes and index are discussed in Section IV for a variety of test cases, from a 3-bus to a 14,000-bus system; a comparison of the proposed identification method and PEM based techniques is then presented. Finally, Section V summarizes the main contributions of this paper.

II. On-Line Stability Index

A power system model can be represented using the following DAE model:

$$\begin{aligned} \dot{x} &= f(x,\ z,\ \lambda,\ \rho) \\ 0 &= g(x,\ z,\ \lambda,\ \rho) \\ y &= h(x,\ z,\ \lambda,\ \rho) \end{aligned} \tag{1}$$

where $x \in \Re^n$ is a vector of state variables that represents the state variables of generators, loads and other system controllers; $z \in \Re^m$ is a vector of steady state algebraic variables that result from neglecting fast dynamics in some load phasor voltage magnitudes and angles; $\lambda \in \Re^\ell$ is a set of uncontrollable parameters such as active and reactive power load variations; $\rho \in \Re^a$ is a set of controllable parameters such as tap or AVR set points; and $y \in \Re^l$ is a vector of output variables such as power through the lines and generators' output power. The nonlinear functions $f : \Re^n \times \Re^m \times \Re^\ell \times \Re^a \mapsto \Re^n$, $g : \Re^n \times \Re^m \times \Re^\ell \times \Re^a \mapsto \Re^m$, and $h : \Re^n \times \Re^m \times \Re^\ell \times \Re^a \mapsto \Re^l$ stand for the differential equations, algebraic constraints and output variable measurements, respectively.

For slowly varying parameters λ, the power system model (1) has been shown to present the following local bifurcations, on which most stability indices in the current literature are based [22]:

1) *Saddle-node Bifurcations (SNB):* When the system state matrix A:

$$A = D_x f \mid_o - D_z f \mid_o D_z g \mid_o^{-1} D_x g \mid_o \tag{2}$$

has one zero eigenvalue with unique nonzero eigenvectors, the equilibrium point $(x_o,\ z_o,\ \lambda_o,\ \rho_o)$ is typically referred to as SNB point (other transversality conditions must also be met). In power systems, this bifurcation point is associated with voltage stability problems due to the local merger and disappearance of equilibria (operating points) as λ changes.

2) *Limit-induced Bifurcations (LIB):* LIBs occur at an equilibrium point where the eigen-system of A undergoes a discrete change due to the fact that system states or algebraic variables reach a limit. There are various types of LIBs, of which a saddle LIB (SLIB) can be associated in power systems with voltage stability problems due to the local merger and disappearance of equilibrium points as λ changes.

3) *Hopf Bifurcations (HB):* In this case, two complex conjugate eigenvalues of A cross the imaginary axis as λ changes. This results in limit cycles that may lead to oscillatory instabilities in power systems, as it has been observed in practice [3], [23].

A. Existent Stability Index

The index, referred in [8] as Hopf bifurcation index (HBI), predicts the distance to the closest SNB or HB stability point, given a known load change direction. It is based on the system state matrix A and the "critical" eigenvalues $\mu_c = \alpha_c \pm j\beta_c$, i.e. the eigenvalues that eventually cross the imaginary axis at a SNB or HB point as λ changes. Thus, the HBI is defined as:

$$\text{HBI}(A, \beta_c) = sv_{min}(A_m) \tag{3}$$

where sv_{min} is the minimum singular value of the modified state matrix A_m, which is defined as:

$$A_m = \begin{bmatrix} A & +\beta_c I_n \\ -\beta_c I_n & A \end{bmatrix}_{2n} \tag{4}$$

At an SNB or HB point, sv_{min} becomes zero. In [8], the authors also compare the HBI to an eigenvalue index (EVI) which is defined as the real part of the critical eigenvalue. It is worth mentioning that almost in all test cases, both HBI and EVI present a fairly linear behavior, but obtaining the HBI for a large system is not as easy as obtaining the EVI.

B. Proposed Stability Index

Computing the HBI in (3) requires making assumptions about the model, so that the matrix A can be explicitly determined. However, if the "output" deviations $\Delta y(t) \in \Re^l$ of (1), i.e.

$$\begin{aligned} \Delta\dot{x}(t) &= A\,\Delta x(t) \\ \Delta y(t) &= C\,\Delta x(t) \end{aligned} \quad (5)$$

is measured, the modal content in the output signal $\Delta y(t)$ is the same as the modal content in matrix A if the system "modes" are excited; hence, these modes can be calculated using system identification techniques. If the eigenvalues $\mu_i(A) = \alpha_i + j\beta_i$ of the system can be identified, one may define a new diagonal state matrix $\Lambda = diag\{\mu_1, \mu_2, \ldots, \mu_n\}$, which can be used to define the following new modified state matrix:

$$\Lambda_m = \begin{bmatrix} \Lambda & +\beta_c I_n \\ -\beta_c I_n & \Lambda \end{bmatrix}_{2n} \quad (6)$$

In this case, using basic singular value decomposition concepts, the sv_{min} for Λ_m can be calculated using:

$$sv_{min}(\Lambda_m) = \min_{i=1,n} \left\{ \sqrt{\alpha_i^2 + (\beta_c - \beta_i)^2} \right\} \quad (7)$$

If the closest eigenvalue to the imaginary axis is the critical one, then

$$sv_{min}(\Lambda_m) = |\alpha_c| \quad (8)$$

Since multiple eigenvalues can be readily determined using system identification tools applied to measured signals like generator speed or power, and assuming that the critical eigenvalue can be monitored, a System Identification Stability Index (SISI) is defined here as:

$$\text{SISI} = |\alpha_c| \quad (9)$$

Observe that this index does not require assumptions regarding the system modeling as the HBI would require. Furthermore, based on the observations in [8] regarding the behavior of the HBI and EVI, one would expect this index to behave "linearly" close to a HB point.

In practice, several modes need to be monitored, i.e. an index value would be generated for each mode, so that the mode that eventually crosses the imaginary axis is properly captured. This can be accomplished by measuring signals in different parts of the system, where the modes of interest are best observed, as discussed in more detail in Section IV. However, in some cases, the critical mode is known from previous knowledge of the system (e.g. the 0.28 Hz mode in the WECC system which can be easily observed on transmission corridors between the north and south of the system), in which case, this is the mode used to define the SISI.

It is important to mention that the trend of the index, and more specifically its slope, may change when the system changes due to large disturbances as well as changes in loading and dispatching [8]. Figure 1 illustrates an example of a possible index profile for a real power system as system conditions vary. Notice that as the system operating conditions change, the index changes as well; when this index reaches a predetermined threshold, which could be readily associated with a minimum system damping, a warning or emergency alarm would be triggered.

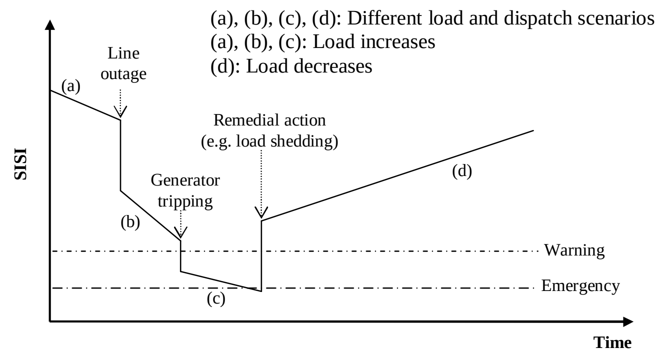

Fig. 1. SISI profile for a real power system with varying operating conditions.

III. Stochastic Subspace Identification

Subspace methods have become popular because of their numerical simplicity, robustness of the techniques that are used in the algorithms, and their state-space form which is convenient for control purposes [20]. Some of the main advantages of these methods over previously used identification methods are:

1) *Model order selection*: For PEM based techniques, different criteria have been proposed in order to select the best order of the model, such as Akaike Information Criterion (AIC) and Final Prediction Error (FPE) [24]. These techniques are defined based on prediction error variance. For instance, to choose the orders p and q of an ARMA(p,q) model, first a range is selected for p and q, and then, for each (p,q) pair, the parameters of the model are estimated; the pair obtaining the lowest value of AIC or FPE criterion is selected as the best estimate of the model order. This approach results in high computational costs, since in order to obtain the prediction error variance, all the possible models in the selected range have to be first estimated. On the other hand, in subspace methods, there is the possibility to estimate the model order without having to estimate the parameters of a variety of different models. Order estimation in this case is one step of the algorithm, and only requires a SVD.
2) *Handling large data*: When the amounts of data are large, order selection with PEM based techniques is just not feasible [25]. This is the case of the application being considered in this paper, which requires long "blocks" of data in order to truly estimate the power system mode(s).
3) *On-line application*: If for any reason, the system dynamics change, the model needs to be updated; the subspace methods are a better choice in this case due to the superiority of subspace over other methods from a computational point of view [25].

The state-space matrices are identified in this case by using measured outputs and robust numerical techniques, such as

QR-factorization, least square and SVD without the need to calculate correlations, i.e. it is data driven instead of covariance driven. Therefore, these types of techniques are computationally more efficient.

A. Stochastic State-Space Model

Consider the discrete stochastic state-space model:

$$\begin{aligned}\Delta x_{k+1} &= A_d\,\Delta x_k + w_k, \quad A_d \;=\; e^{A\,T_s} \\ \Delta y_k &= C\,\Delta x_k + v_k\end{aligned} \tag{10}$$

where Δx_{k+1} is the discrete time state vector; w_k and v_k are zero mean noise $E[w_k] = E[v_k] = 0$, and independent from Δx_k, i.e. $E[\Delta x_k.w_k^t] = 0$, $E[\Delta x_k.v_k^t] = 0$; Δy_k is the output vector; and T_s is the sampling time. This is slightly different from (5) as it accounts for process noise w_k and measurement noise v_k (normal load switching in power systems can be interpreted as process noise). The output measurements are put into a block Hankel matrix and divided into a past and future parts; thus,

$$\begin{aligned}Y_{0|2i-1} &= \left(\begin{array}{cccc}\Delta y_0 & \Delta y_1 & \dots & \Delta y_{j-1} \\ \Delta y_1 & \Delta y_2 & \dots & \Delta y_j \\ \dots & \dots & \dots & \dots \\ \Delta y_{i-1} & \Delta y_i & \dots & \Delta y_{i+j-2} \\ \hline \Delta y_i & \Delta y_{i+1} & \dots & \Delta y_{i+j-1} \\ \Delta y_{i+1} & \Delta y_{i+2} & \dots & \Delta y_{i+j} \\ \dots & \dots & \dots & \dots \\ \Delta y_{2i-1} & \Delta y_{2i} & \dots & \Delta y_{2i+j-2}\end{array}\right) \\ &= \left(\frac{Y_{0|i-1}}{Y_{i|2i-1}}\right) = \frac{Y_p}{Y_f}\end{aligned} \tag{11}$$

where i is typically equal to $2M/l$, with M representing the "expected" maximal order of the system; and j is the number of samples of each measured output signal. The subscripts of $Y_{0|2i-1}$, $Y_{0|i-1}$ and $Y_{i|2i-1}$ are the first and last element in the first column of the block Hankel matrix. Matrices Y_p^+ and Y_f^- are defined by shifting the border between past and future one row down in (11), i.e. $Y_p^+ = Y_{0|i}$, and $Y_f^- = Y_{i+1|2i-1}$.

There are different algorithms to calculate the system matrices A_d and C. The canonical variate algorithm (CVA) is used here, as explained below.

B. Canonical Variate Algorithm [20]

Once the block Hankel matrix and its submatrices are formed, the following steps are taken:

1) Calculate the projection matrices $\mathcal{O}_i$ and $\mathcal{O}_{i-1}$:

$$\mathcal{O}_i = Y_f / Y_p \tag{12}$$
$$\mathcal{O}_{i-1} = Y_f^- / Y_p^+ \tag{13}$$

where "Y_f/Y_p" denotes a shorthand for the projection of the row space of the future matrix Y_f on the row space of past matrix Y_p, i.e. $Y_f Y_p^t (Y_p Y_p^t)^\dagger Y_p$ where the symbol $\dagger$ denotes the pseudo inverse of the matrix. Robust QR-factorization methods may be used to calculate the projection matrices.

2) Calculate the SVD of the weighted projection:

$$\begin{aligned}W_1 \mathcal{O}_i W_2 &= U S V^t \\ &= (U_1 \;\; U_2)\begin{pmatrix} S_1 & 0 \\ 0 & 0\end{pmatrix}\begin{pmatrix} V_1^t \\ V_2^t\end{pmatrix}\end{aligned} \tag{14}$$

where weighting matrices $W_1 = \left(\frac{1}{j} Y_f Y_f^t\right)^{-\frac{1}{2}}$, and $W_2 = I_j$ (identity matrix $j \times j$); and U and V are unitary matrices resulting from the SVD process. The order of the system n is determined by inspecting the singular values in S, i.e. determine the $n \times n$ submatrix S_1, which are the cosine of the principal angles between the row space of past outputs Y_p and future outputs Y_f. In practice, however, due to noise and rounding errors, the structure of S is not as shown in (14), i.e. the zero diagonal matrix is not really zero. In this case, a "large" reduction in the singular values of S are used to define S_1 (the system order n), as discussed in detail for the test systems in Section IV.

3) Determine the extended observability matrices Γ_i and Γ_{i-1} as follows:

$$\Gamma_i = W_1^{-1} U_1 S_1^{1/2} \tag{15}$$
$$\Gamma_{i-1} = \underline{\Gamma_i} \tag{16}$$

where $\underline{\Gamma_i}$ is obtained by removing the last l rows of Γ_i with l being the number of outputs in (5).

4) Find the Kalman filter state sequences $\hat{X}_i$ and $\hat{X}_{i+1}$ as follows:

$$\hat{X}_i = \Gamma_i^\dagger \mathcal{O}_i \tag{17}$$
$$\hat{X}_{i+1} = \Gamma_{i-1}^\dagger \mathcal{O}_{i-1} \tag{18}$$

5) Then, A_d and C are determined as:

$$\begin{pmatrix} A_d \\ C \end{pmatrix} = \begin{pmatrix} \hat{X}_{i+1} \\ Y_{i|i} \end{pmatrix} \hat{X}_i^\dagger \tag{19}$$

Once A_d is calculated, the eigenvalues $z_i = z_{R_i} + j z_{I_i}$ of matrix A_d can be transformed onto the s-plane $s_i = \lambda_{R_i} + j\lambda_{I_j}$ using a bilinear transformation, i.e. $s_i = \frac{2}{T_s}(z_i - 1)/(z_i + 1)$.

IV. Case Studies

System identification techniques are typically applied to actual field measurements to measure damping and detect critical eigenvalues. Since these types of signals were not readily available for the variety of test cases used here to study the proposed identification technique and index, the authors have used signals generated through time domain simulations of models of the desired test systems. For the small IEEE 3-bus system used in this paper, detailed PSCAD/EMTDC [21] modeling and simulation was feasible, so that the resulting signals would be as close to field measurements as possible. However, this was certainly not feasible for the larger test system used here (IEEE 14-bus system, 2-area benchmark system and a real system with 14,000 buses). In this case, transient stability models and programs were used to generate the desired signals (Power System Toolbox-PST [26]). Since for the 3-bus test system, the errors incurred by neglecting "fast" network transients were small, as expected [27], using these approximate signals to test the proposed technique and index is arguably reasonable.

The desired signals for the test cases were obtained by using independent random load modulation with a magnitude of 1% of both active and reactive power at load buses; white Gaussian noise was added to the output signals as measurement noise v_k, such that the signal-to-noise ratio (SNR) is 20 db. A Monte-Carlo type of simulation was used to test the feasibility and accuracy of the identified modes by simulating 20 independent cases at each operating condition. Assuming a direction for load change and generation dispatching, critical modes were computed at different operating conditions using both small signal stability analysis of a "linearized" transient stability model and the proposed stochastic subspace identification method applied to signals generated by time domain simulation. The proposed SISI stability index was then obtained and for all test cases analyzed.

Attention should be paid to selecting proper signals, i.e. signals in which the critical mode is observable. Hence, the observability index $OI = |C_m r_i|$ was used here to indicate the observability of the critical mode in a given measured signal $\Delta y_m = C_m \Delta x \in \Delta y$, where r_i is the right eigenvector corresponding to μ_c. Relative angle deviation, generator speed deviation and power deviation signals can all be used [28]. In this paper, the chosen output signal is the power of one of the generators in each area in which the critical mode is best observable, since the influence of very low frequencies on power due to governors' effect is very small. In order to calculate the OI's, one needs both A and C matrices; since these matrices are not available during the system identification process until the proper signals are selected, this observability analysis should be performed off-line based on system modeling and previously gathered data.

The signals were recorded for 4 minutes at 40 Hz rate and passed through Chebyshev low-pass and high-pass filters with cut-off frequency of 2 Hz and 0.05 Hz, respectively, and then resampled at 10 Hz rate; the proposed subspace method was then applied to these signals to extract the critical mode. Due to the nature of filters, these affect both the magnitude and phase of the measured signals and hence there would be a time delay between the filtered and originally measured signals [29]. These time delays play an important role on state estimation, protection and control systems; however, since the main aim of the present work is the detection and monitoring of modal content (frequency and damping), these delays should not have a direct effect on the identified modes.

The order of the system was determined by inspecting the singular values of S in (14), as explained in Section III.B; thus, n is chosen based on a "large" reduction in these singular values. For the different test cases, Fig. 2 shows the singular values obtained from simulations for all the test systems. Thus, from this figure, $n = 14$ for the IEEE 3-bus system, $n = 14$ for the IEEE 14-bus system, and $n = 13$ for the 2-area benchmark system. Experience has shown that order overspecification results in better estimates of modes; hence, n was selected to be 18, 20, and 16 for these three test cases, respectively.

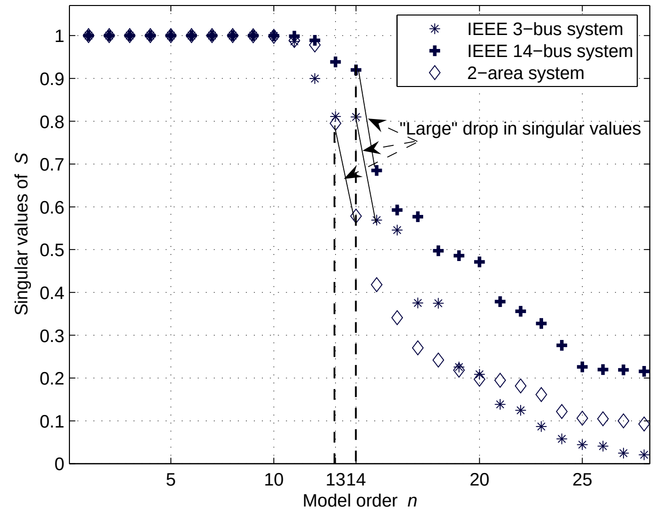

Fig. 2. Singular values of weighted projection (14).

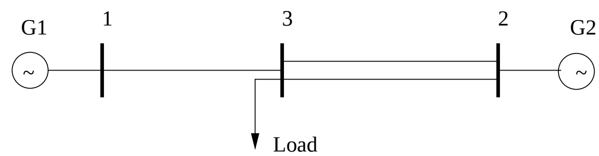

Fig. 3. IEEE 3-bus test system.

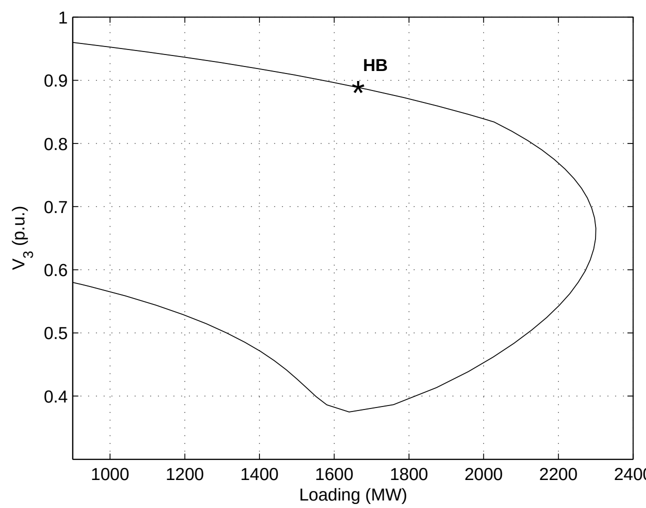

Fig. 4. PV curve at Bus 3 for the IEEE 3-bus system.

A. IEEE 3-bus Test System

A single line diagram of the test system is shown in Fig. 3 [30]. There is a 900 MW and 300 MVar load at Bus 3 modeled as a constant impedance load. Each machine has a simple exciter, and a simple governor is used for the machine at Bus 1. Observe that, from the voltage profile depicted in Fig. 4, the system presents an HB point at about 1650 MW loading level before the nose point; this HB was detected by computing the eigenvalues of the corresponding linearized transients stability model of the system.

The system was also simulated using PSCAD/EMTDC to capture the full dynamic response of a detailed model of the

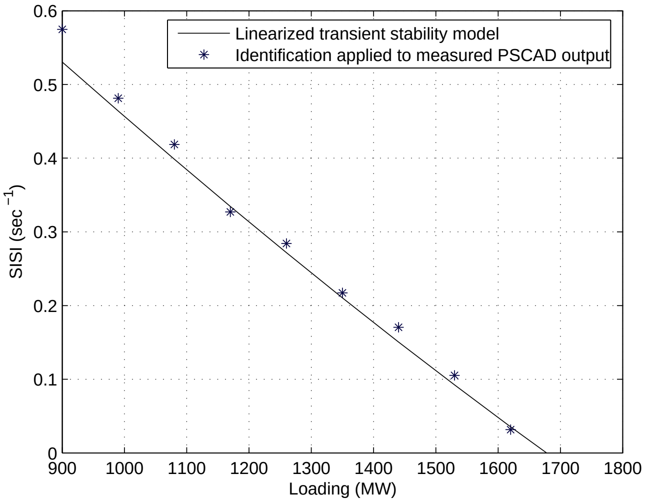

Fig. 5. SISI for the IEEE 3-bus system.

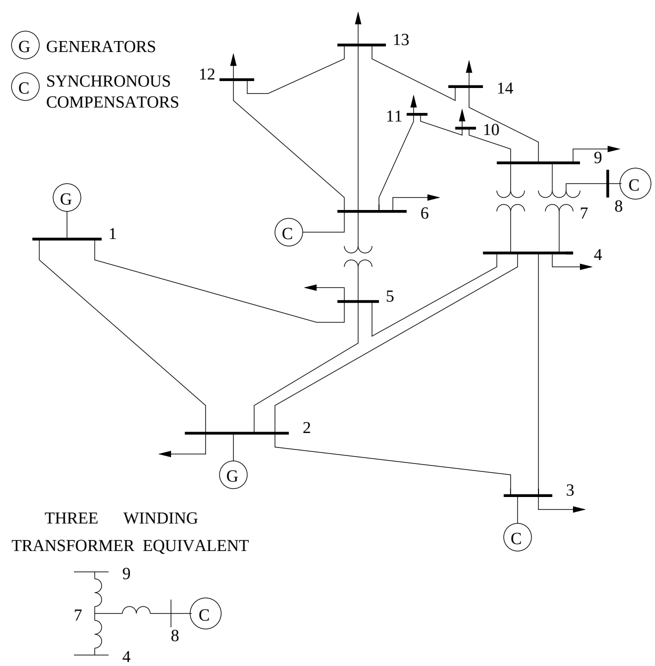

Fig. 6. IEEE 14-bus test system.

system, which accounts for “fast” network transients. In order to perturb the system in PSCAD, a pseudo-random binary signal (PRBS) [31], which approximates white noise, was used to represent a 1% load modulation at Bus 3. The proposed stochastic subspace technique applied to the generator powers yielded the results shown in Fig. 5. Observe that both PSCAD “detailed” models and linearized transient stability models in PST yeild fairly similar results. The maximum error in this case, between the identified and modeled SISI is 0.047 sec^{-1}, which is small.

B. IEEE 14-bus Test System

The test system shown in Fig. 6 has 5 generators; two of them are providing both active and reactive power at Buses 1 and 2, and generators at Buses 3, 6 and 8 are basically synchronous condensers [32]. The generators are modeled using subtransient models and loads are modeled as constant impedance loads in PST. There is an HB point at about 360 MW loading level, as shown in the corresponding PV curve depicted in Fig. 7. The SISI results of applying the proposed subspace method to 4-minute blocks of P_{G_1} and P_{G_2} (a one-minute block of the output power P_{G_2} of generator 2 is depicted in Fig. 8) as the system loading changes are shown in Fig. 9; this figure depicts the mean value (Mean SISI) and the confidence limit of one standard deviation (σ SISI) of the SISI. Observe that as the critical mode moves towards the right half plane and hence becomes less damped, the standard deviation decreases; this has been previously observed for ARMA models as well in [18]. As loads are increased, some of the generators reach their maximum Var limit; notice that no discontinuity was observed in the index, as opposed to SNB indices that usually show this when control limits are reached [4]. The maximum error between the mean value of the identified and modeled SISI is 0.0121 sec^{-1}, which shows that the proposed identified SISI is a good measure of proximity to oscillatory instabilities.

C. Two-area Benchmark System

A single line diagram of the system is shown in Fig. 10 [27]. There is an inter-area mode that eventually crosses the imaginary axis as loads are increased. Generators G_2 and G_3 are rescheduled proportionally to their base power to respond to load increases at Bus 7 and 9. There is an HB point before the nose point as shown in the corresponding PV curve depicted in Fig. 11. Loads are modeled as constant PQ loads and are increased with a constant power factor. The inter-area mode, $\mu_c = -0.4204 \pm j4.9123$ (damping 8.53% and frequency 0.7818 Hz), crosses the imaginary axis at about 3100 MW of total loading. The mean value and confidence limit of one standard deviation of the SISI obtained using the proposed identification technique and the SISI using the linearized transient stability model associated with the inter-area mode are depicted in Fig. 12; observe the fairly linear profile of these SISIs. As critical mode becomes less damped, it is more likely to capture it accurately, i.e. the method works better for less damped modes, since the oscillations die out slower once the mode is excited. The maximum error between the mean value of the identified and modeled SISI is 0.0462 sec^{-1}, showing the good accuracy of the proposed identification method.

D. Real System (14,000 buses)

In order to investigate the shape of the proposed index for a real power system, a 14,000-bus system was studied (the details of the system are confidential information). To move this system closer to an instability condition, several generator outages were modeled, and a low damped 0.33 Hz mode appears due to the simulated contingencies; the power transfer among some areas in the system was then increased until the appearance of an HB point associated with this mode at a 39.6 GW loading level, approximately. It is interesting to

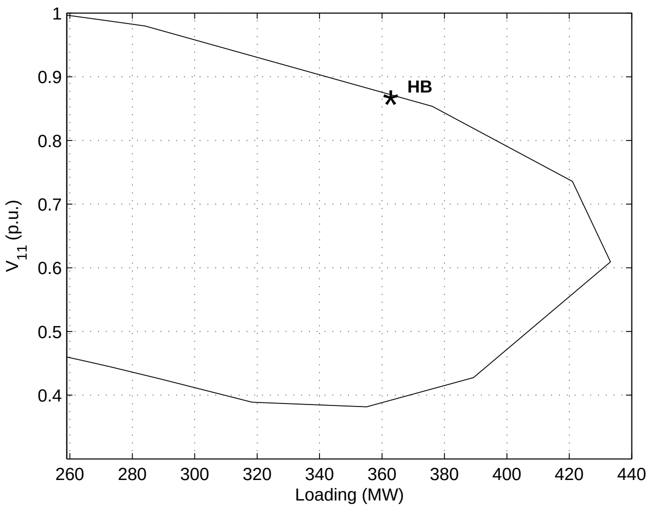

Fig. 7. PV curve at Bus 11 for the IEEE 14-bus system.

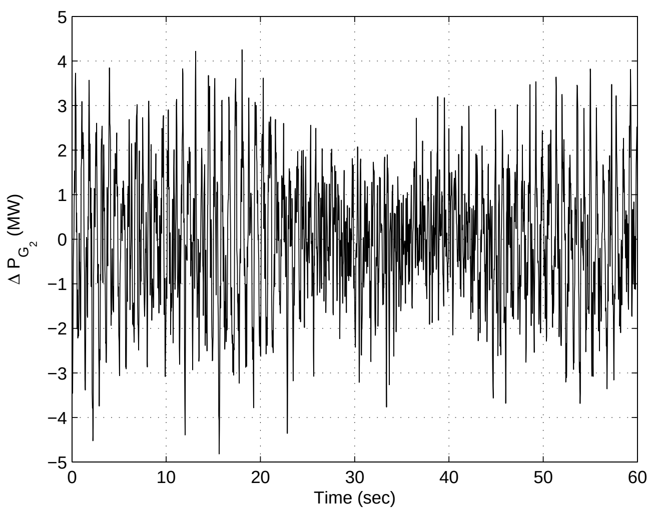

Fig. 8. One-minute block measurement of the change in generator G_2's power at 285 MW loading level.

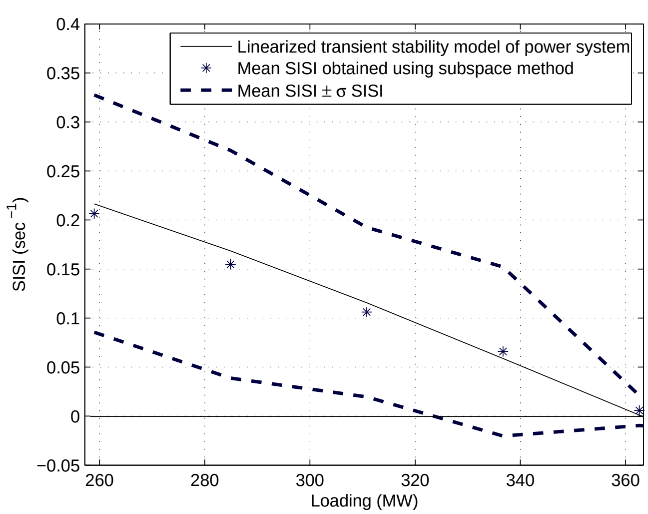

Fig. 9. SISI for the IEEE 14-bus system.

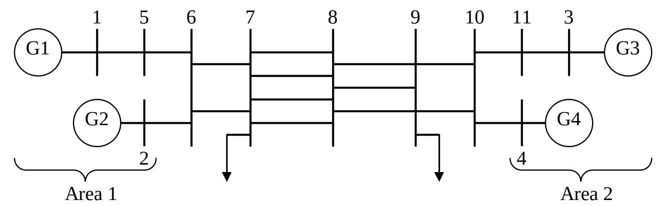

Fig. 10. Two-area benchmark system.

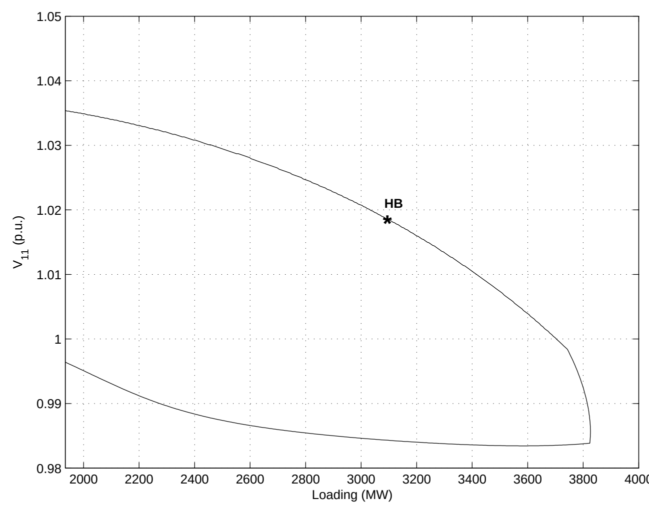

Fig. 11. PV curve at Bus 11 for the 2-area benchmark system.

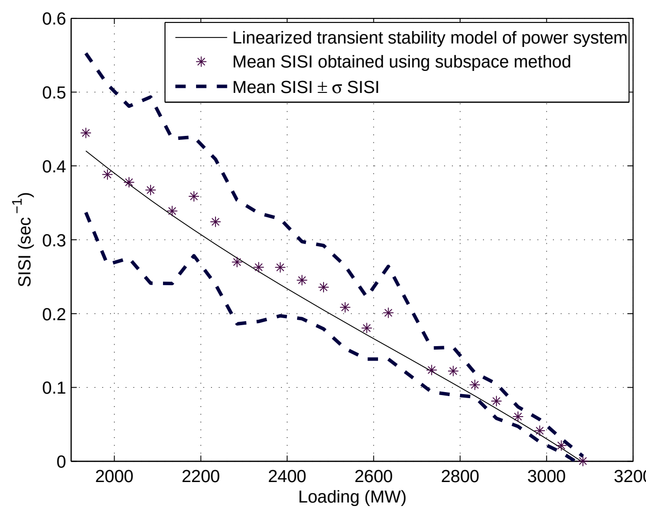

Fig. 12. SISI for the 2-area benchmark system.

highlight the fact that this example is somewhat similar to the Western Electricity Coordinating Council, WECC (previously known as WSCC) 1996 blackout, where several contingencies triggered a 0.28 Hz mode that led the system to instability [3].

Due to the large size of the system and the constraints imposed by the transient stability analysis package used (TSAT [33]), implementing load switching as in previous test cases was not possible; hence, the system in this case was perturbed using two 50 MW load perturbations in different areas to obtain the desired signals and excite the desired mode. By analyzing the 0.33 Hz mode shape, two generators' output

TABLE I

CPU TIME IN SEC FOR DIFFERENT METHODS AND SYSTEM ORDERS

Method	MATLAB function	n=13	n=16	n=19	n=22
Subspace	`n4sid(n)`	0.63	0.83	1.00	1.16
ARMA	`armax(n,n)`	6.99	9.96	14.08	19.44
State-space	`pem(n)`	11.17	13.99	24.67	36.20

power signals in the main areas that oscillate against each other were selected to identify the mode. The SISIs obtained from system identification and the linearized transient stability model are shown in Fig. 13; the maximum error between the identified and modeled SISI is 0.0042 $\sec^{-1}$. Notice that in this case, the index is not quite as linear as in the previous cases, and hence prediction of maximum loading margins at light loading levels are not that accurate; however, no discontinuity was observed while increasing the power transfer between areas, even though several generators reached their Var limits.

E. Computational Issues

One of the assumptions in stochastic subspace identification is that the w_k and v_k are independent of Δx_k, i.e. $E[\Delta x_k.w_k^t] = 0$, $E[\Delta x_k.v_k^t] = 0$. It is typically assumed that the data blocks are long enough and ergodic, i.e. $j \to \infty$ and the underlying system model is not changing, so that the expectation operator E can be replaced with the average over one experiment run for infinite time. This is why subspace methods work well for long data blocks even in the presence of noise [20]. The asymptotic properties of the proposed subspace method are also discussed in [34], where it is shown that for a white noise input, CVA and PEM are asymptotically equivalent.

In this section, the effect of using different sizes of data blocks on the accuracy of the method was investigated for the 2-area benchmark system, and the asymptotic behavior of the proposed subspace method was compared to an ARMA model and a state-space model whose parameters were estimated using PEM. For this purpose, the following built in functions of MATLAB were used: function `n4sid()` for the subspace method; `armax()` for ARMA; and `pem()` for the state-space model. The data was generated for an inter-area mode $-0.1228 \pm j4.7824$ (damping 2.57% and frequency 0.7611 Hz). The standard deviation of the estimated SISI and frequency for different size of data blocks are depicted in Fig. 14 and Fig. 15, respectively; these were obtained based on a Monte-Carlo type of simuation with 20 independent cases. Notice that for short data blocks, the ARMA and state-space models yield better performance. On the other hand, for long data blocks, as explained before, the proposed subspace method and ARMA and state-space models behave similarly.

From the computational burden point of view, the previously mentioned MATLAB built in functions were used to estimate the parameters of the corresponding models for the same 4-minute block data. Table I shows the CPU times obtained for each function as the system order n changes; observe that the subspace method is significantly faster than the others.

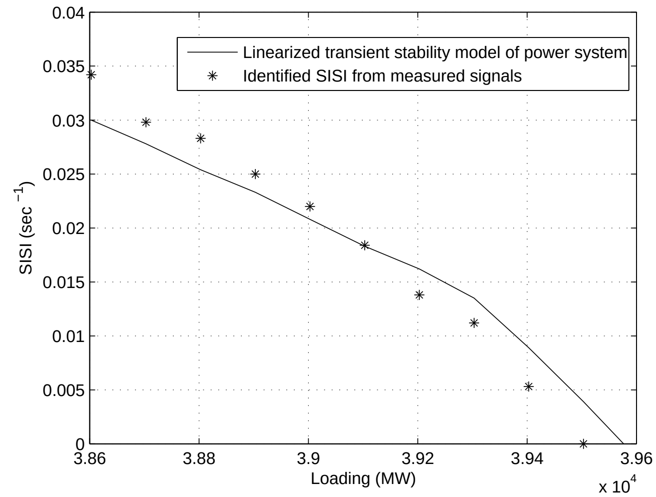

Fig. 13. SISI for the 14,000-bus real system.

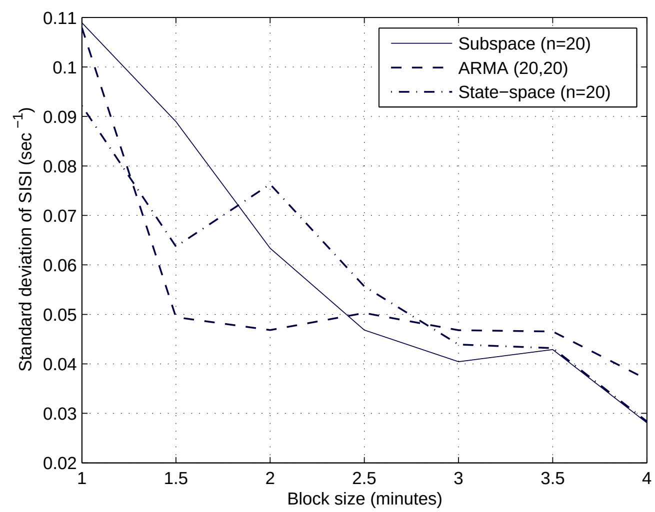

Fig. 14. Standard deviation of the estimated SISI for 2-area benchmark system.

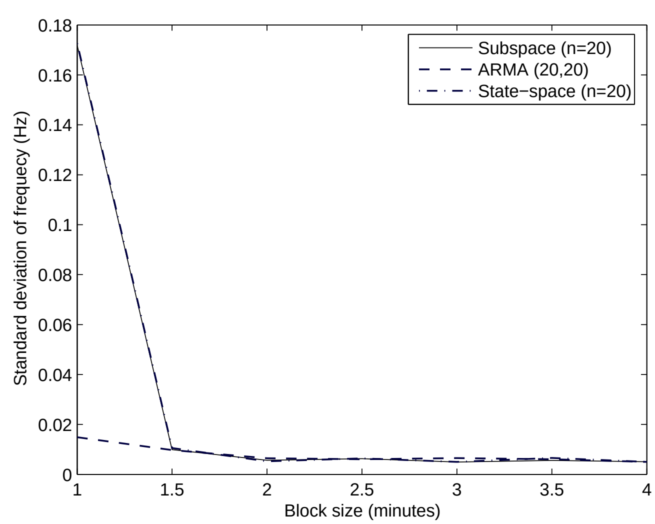

Fig. 15. Standard deviation of the estimated frequency of the inter-area mode for the 2-area benchmark system.

V. Conclusions

A stochastic subspace identification method has been proposed in this paper to capture critical modes of power systems without requiring large disturbances. The method was applied to the time response of various test cases obtained by exciting the system loads randomly, which is what typically occurs during the operation of real power systems, demonstrating that it is possible to identify the desired critical modes with acceptable accuracy. For various lengths of data blocks, the accuracy of the critical modes estimated using the proposed subspace method was also investigated, showing that the longer the data blocks, the more accurate the estimation. For long data blocks and white noise input, the proposed subspace method and PEM based methods were shown to behave similarly, i.e. they are asymptotically equivalent; however, the subspace methods were shown to be computationally more efficient.

Based on the identified modes, an index was proposed to predict the closest oscillatory instability (HB point), showing it to be fairly linear and without discontinuities with respect to system load increases. Furthermore, the proposed index is based on on-line field measurements, thus capturing the real behavior of the system without the need of modeling approximations, which is a drawback of similar previously proposed indices. Hence, given the quasi-linear profile of the proposed index as well as the lower computational costs associated with the system identification technique used to compute it, this index and associated identification methodology may be used as an effective on-line tool to help system operators monitor system stability.

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Hassan Ghasemi (S'01) received his B.Sc. and M.Sc. degree from the University of Tehran, Iran, in 1999 and 2001, respectively. He did research on analysis and design of machine drives during his master program. He also worked in Jovain Electrical Machines Co. (JEMCO) 2000-2001 as a part-time electrical engineer. He has been in the Ph.D. program at the Electrical and Computer Engineering Department, University of Waterloo, Canada, since 2002 working in the field of power system modeling and application of system identification to stability analysis of power systems.

Claudio A. Cañizares (SM'00) received in April 1984 the Electrical Engineer diploma from the Escuela Politécnica Nacional (EPN), Quito-Ecuador, where he held different teaching and administrative positions from 1983 to 1993. His MS (1988) and PhD (1991) degrees in Electrical Engineering are from the University of Wisconsin-Madison. Dr. Cañizares has held various academic and administrative positions at the E&CE Department of the University of Waterloo since 1993 and is currently a full professor. His research activities concentrate in the study of stability, modeling, simulation, control and computational issues in power systems in the context of electricity markets.

Ali Moshref Ph.D. in Electrical Engineering with over 25 years of experience in consulting, research and software development. He is a principal engineer at Powertech Labs. Inc. His technical activities include power system planning and operation, development of operating guidelines, computer applications in power system analysis.