Bulk Power System Dynamics and Control - VI, August 22-27, 2004, Cortina D'Ampezzo, Italy

# Prediction of Instability Points Using System Identification

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***Abstract*— Determining maximum loading margins is an important issue in power system operation, as system operators must take proper preventive actions to avoid stability problem. In this paper, an index based on the identification of critical modes of the system is presented. The proposed index does not need a system model as it is based on actual field measurements. Compared to other existent indices, the proposed index yields more accurate stability margins as if accounts for the full dynamic response of the system, at lower computational costs, which would make it an adequate tool for on-line stability monitoring.**

## I. Introduction

Power systems are very complex nonlinear systems; hence, proper modeling of such systems is an issue. Furthermore, the quick increase in power demand and the lack of transmission system expansion is forcing these systems to operate close to their stability limits, which is leading more often nowadays to system collapses following severe faults or contingencies. Hence, there is a need for proper on-line monitoring tools to warn operators of impending stability problem.

Voltage collapse and oscillatory phenomena have been directly associated with many of the stability problems that power systems have faced in the recent past (e.g. August 1996 WSCC black out [1], August 2003 North East black out [2]). These are stability problems due to local or global bifurcations; in particular, Hopf bifurcations (HB), saddle-node bifurcations (SNB) and limit-induced bifurcations (LIB) have been directly associated with these problems [3]. A wide range of research has been carried out on detection and control of these types of bifurcations (e.g. [4], [5], [6], [7], [8], and several indices have been proposed to determine the proximity of the current operating point to the "closest" bifurcation point [3], [9], [10], [11]. However, all of these indices suffer from one or more of the following shortcomings:

- are computationally expensive;
- are based on approximate power system models;
- and/or do not capture the full dynamic behavior of the system.

In this paper, a method based on the Prony identification technique is used to calculate a new stability index that does not need of a particular power system model, and can be readily and quickly computed to determine the distance from the current operating point to an instability point, based on actual measured signals such as generator rotor speed, angle or power through the lines.

Prony method as a system identification method that has been widely used in power systems to determine modal content, develop equivalent linear models of power systems, and tuning controllers using system measurement [12], [13], [14], [15]. In this paper, it is used to calculate the critical modes of the system, with the real part being used as an index to determine the distance to instability.

This paper is structured as follows: In Section II, a brief description of bifurcation theory and the mathematical background of the proposed index are given; it is also shown that using a similarity transformation, the index proposed in [10] and the proposed index are similar under the system identification context proposed in this paper. In Section III, the Prony method used as the base for the proposed index is described, and important issues regarding the application of this method are discussed. To show the use and features of the proposed index in power systems, in Section IV, the result of aplying the new stability index to 2 test systems are compared to the ones obtained for a previously proposed index. Finally, Section V summarizes the main contributions of this paper as well as identifying future research directions.

## II. Proposed Stability Index

### A. Background

A power system model can be represented using the following DAE:

$$\begin{aligned} \dot{x} &= f(x,\, y,\, \lambda,\, p) \\ 0 &= g(x,\, y,\, \lambda,\, p) \end{aligned} \qquad (1)$$

where $x \in \Re^n$ is a vector of state variables that represents the state variables of generators, loads and other system controllers; $y \in \Re^m$ is a vector of steady state algebraic variables that result from neglecting fast dynamics such as some load phasor voltage magnitudes and angles; $\lambda \in \Re^l$ is a set of uncontrollable parameters such as active and reactive power load variations; and $p \in \Re^k$ is a set of controllable parameters such as tap settings, AVR, SVC or STATCOM set points.

For slowly varying parameters $\lambda$, the power system model (1) has been shown to present the following local bifurcartions, on which most stability indices in the current literature are based [3]:

1) **Saddle-node Bifurcations (SNB):** When the jacobian of the system state matrix $A = D_x f \mid_o - D_y f \mid_o D_y g \mid_o^{-1} D_x g \mid_o$ has one zero eigenvalue with unique nonzero eigenvectors, the equilibrium point ($x_o$, $y_o$, $\lambda_o$, $p_o$) is typically referred to as SNB point (other transversality conditions must also be met). In power systems, this bifurcation point is associated with voltage stability problem due to the local merger and disappearing of equilibria or operating points as $\lambda$ changes.
2) **Limit-induced Bifurcations (LIB):** LIBs occur at an equilibrium point where the eigen-system of $A$ undergoes a discrete change due to the fact that system states or algebraic variables reach a limit. There are various types of LIBs, of which a saddle LIB (SLIB) can be associated in power systems with voltage stability problems due to the local merger and disappearing of equilibrium or operating points as $\lambda$ changes.
3) **Hopf Bifurcations (HB):** In this case, two complex conjugate eigenvalues of $A$ cross the imaginary axis as $\lambda$ changes. This results in limit cycles that may lead to oscillatory instabilities in power systems, as it has been observed in practice [1], [16].

### B. Existent Stability Index

Several methods have been proposed to detect SNB, LIB and HB points based on a variety of indices [3], [10], [11]. All of these indices are based on approximate models of the power system, which do not necessarily represent the exact behavior of the real system.

The index proposed in [10] has the advantage over other indices of being predictable, and having a quasi-linear profile if divided by its derivative, which makes it useful as a predictor of an impending instability. This index, referred in [10] as HBI, is based on the system state matrix A and the "critical" eigenvalues $\mu_c = \alpha_c \pm j\beta_c$, i.e. the eigenvalues that eventually cross the imaginary axis at an HB point as $\lambda$ changes, to predict the distance to the instability point, and is defined as:

$$\text{HBI}(A, \beta_c) = \sigma_{min}(A_m) \tag{2}$$

where $\sigma_{min}$ is the minimum singular value of the modified state matrix $A_m$, which is defined as:

$$A_m = \begin{bmatrix} A & +\beta_c I_n \\ -\beta_c I_n & A \end{bmatrix}_{2n} \tag{3}$$

At an HB or SNB point, $\sigma_{min}$ becomes zero. In most cases this index may be approximated, as shown by the results in [10], as:

$$\text{HBI}(A, \beta_c) \cong |\alpha_c| \tag{4}$$

### C. Proposed Stability Index

System identification methods are based on the linearized version of (1) with respect to an equilibrium point, which leads to:

$$\Delta\dot{x}(t) = A\ \Delta x(t) \tag{5}$$

$$u(t) = C\ x(t) \tag{6}$$

The modal content in the measured or output signal $u(t)$ is basically the same as the modal content in matrix $A$, and can be calculated using system identification techniques. Hence, if the eigenvalues $\mu_i(A) = \alpha_i + j\beta_i$ of the system can be identified, one may define a new diagonal state matrix $\Lambda = diag\{\mu_1, \mu_2, \ldots, \mu_n\}$ which can be used to define the following new modified state matrix:

$$\Lambda_m = \begin{bmatrix} \Lambda & +\beta_c I_n \\ -\beta_c I_n & \Lambda \end{bmatrix}_{2n} \tag{7}$$

In this case, the $\sigma_{min}$ for $\Lambda_m$ can be calculated using:

$$\sigma_{min}(\Lambda_m) = \min_{i=1,n} \left\{ \sqrt{\alpha_i^2 + (\beta_c - \beta_i)^2} \right\} \tag{8}$$

If the closest eigenvalue to the imaginary axis is the critical one, then

$$\sigma_{min}(\Lambda_m) = |\alpha_c| \tag{9}$$

Since multiple eigenvalues can be readily determined using system identification tools applied to measured signals like generator speed or angle, and assuming that the critical eigenvalue can be monitored, a System Identification Stability Index (SISI) is defined here as:

$$\text{SISI} = |\alpha_c| \tag{10}$$

In the following section, the well known Prony method for system identification is discussed, and important issues regarding its use in the computation of the proposed SISI are presented.

## III. System Identification

As previously discussed, a stability index can be defined based on the modal content of the system states. Hence, field measurements like generator speed, power through lines, etc., can be used to extract the "dominant" eigenvalues of the system using system or signal identification techniques.

The Prony method has been widely used to determine the modal content or equivalent linear models of power systems, as well as tuning controllers based on certain measured signals [12], [13], [14], [15]. Thus, consider that a signal $u(t)$ is sampled at a frequency larger than the Nyquist frequency, this signal then can be represented in continuous and discrete time respectively as:

$$u(t) \;=\; \sum_{i=1}^{n} \bar{R}_i \, e^{\lambda_i t} \tag{11}$$

$$u(k) \;=\; \sum_{i=1}^{n} \bar{R}_i \, Z_i^k \tag{12}$$

where $\bar{R}_i$ is an output residues; $Z_i = e^{\lambda_i T}$; $T$ is the sampling time; and $k$ is integer time. Assuming that $u(k)$ is the function of past values of $u$ and using a difference equation in the form of an Auto-Regressive (AR) model, then:

$$u(k) = -a_1\, u(k-1) - a_2\, u(k-2) - \ldots - a_n\, u(k-n) \tag{13}$$

Equation (13) can be written in a matrix form as:

$$U \;=\; D\,\theta \tag{14}$$

where

$$U = [u_{k+n}\; u_{k+n+1}\; u_{k+n+2}\; \ldots\; u_{k+N}]^t_{N-n} \tag{15}$$

$$\theta = [-a_1\; -a_2\; \ldots\; -a_n]^t \tag{16}$$

$$D = \begin{bmatrix} u_{k+n-1} & u_{k+n-2} & \cdots & u_k \\ u_{k+n} & u_{k+n-1} & \cdots & u_{k-n+1} \\ \vdots & & & \\ u_{k+N-1} & u_{k+N-2} & \cdots & u_{k+N-n} \end{bmatrix}_{(N-n)\times(2n)} \tag{17}$$

where $N$ is the number of samples. The least square method can be used to compute $\theta$; this leads to the eigenvalues of the system, which are the roots of the system characteristic equation:

$$z^n \;+ a_1\, z^{n-1} + a_2\, z_{n-2} + \ldots + a_n \;=\; 0 \tag{18}$$

The order of the system $n$ is usually unknown. In [13], the "rule-of-thumb" of an initial guess for $n = N/6$ is proposed. Once the roots of the system characteristic equation are obtained, the poles corresponding to high frequencies which are known not to be present in power systems are discarded. This technique, however, may lead to results that depend heavily on the user.

A more accurate technique for choosing $n$ proposed in [17] for communication systems is applied here to power systems. This method is based on monitoring the singular values of matrix $D$ for $n = N/2$ . Although, the maximum possible order which can be chosen for the system is $N/2$, the practical order of a matrix is equal to the number of its largest singular values. Thus, for $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n \geq \ldots \geq \sigma_{N/2}$, if $\sigma_{n+1} \approx \sigma_{n+2} \approx ... \approx \sigma_{N/2} \approx 0$, then $n < N/2$ is the order of the matrix $D$.

It is important to note that before starting the Prony method, in order to obtain a correct response, the mean value of the signal and the high frequency components have to be removed.

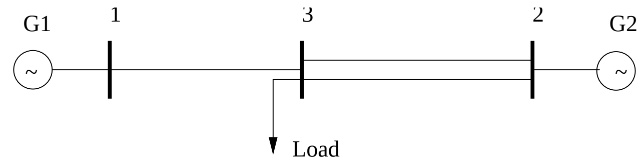


Fig. 1. IEEE-3Bus test system.

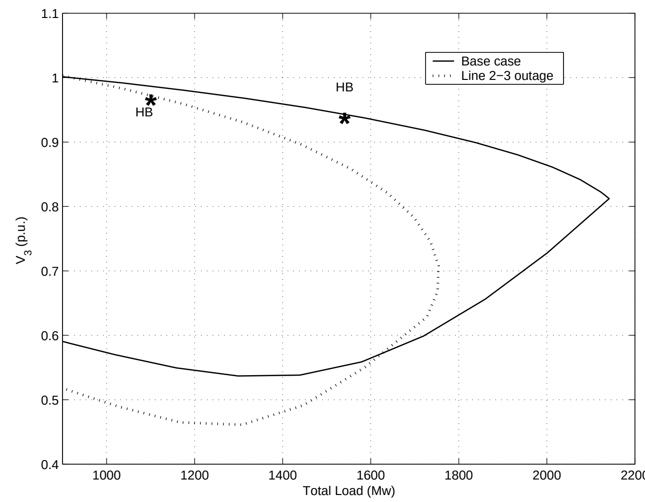


Fig. 2. PV curves at bus 3 for the IEEE-3bus system, for the base case and for a line 2-3 outage.

## IV. Examples

The IEEE 3-bus system of Fig. 1 is used here to test the application of the proposed index. There is 900 MW and 300 Mvar load at bus 3 modeled as a constant PQ load. The PV curves for the base system and for a line 2-3 outage are given in Fig. 2; observe that the system presents HBs at certain loading conditions before the maximum loading point. The rotor speed of generator $G_1$ for a line 2-3 outage is shown in Fig. 3 for 2 loading conditions; notice that at higher loading, the contingency results in unstable oscillations associated with the HB depicted in Fig. 2.

For a sampling frequency of 40 Hz, 800 samples of the generator $G_1$ rotor speed at 900 MW were obtained. The 5 largest singular values of the matrix D are $\{0.0125, 0.0116, 0.0000095, 0.0000035, 0.0000017\}$; hence, the practical order of the system is $n = 2$, as there is a large drop from the second to the third singular value. The characteristic equation can then be formulated and the modal content of the signal is obtained using (18), which leads to 2 critical eigenvalues $\mu_c = -0.3893 \pm j9.3729$. The small signal stability program of the Power System Toolbox (PST) [18] was also used to obtain the critical eigenvalues of the post-contingency system, determining $\mu_c = -0.3959 \pm j9.2983$, which are practically identical to the ones obtained using Prony.

By varying the load at bus 3, the eigenvalues of the base system and the system under a line 2-3 outage were computed using PST and Prony; the eigenvalue profiles

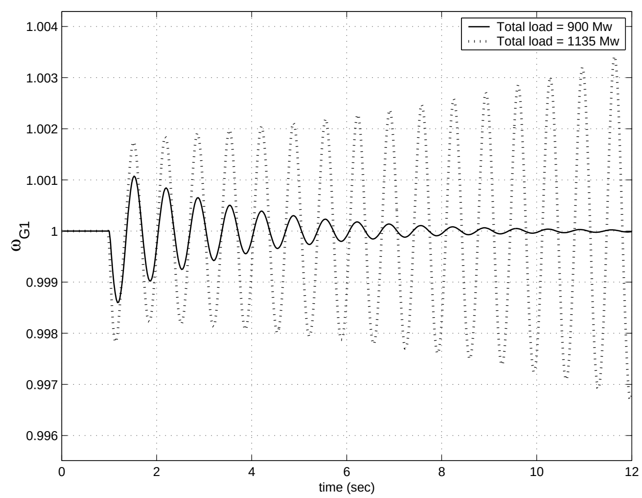


Fig. 3. Generator $G_1$ speed following a line 2-3 outage.

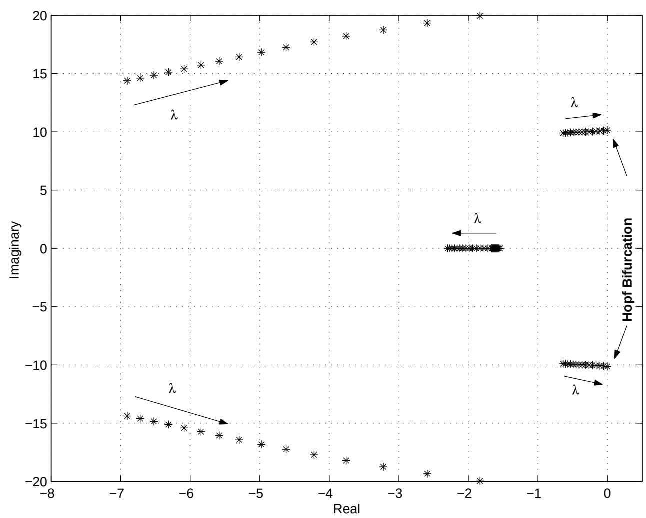


Fig. 4. Eigenvalue profiles with respect to load changes in the IEEE-3bus system.

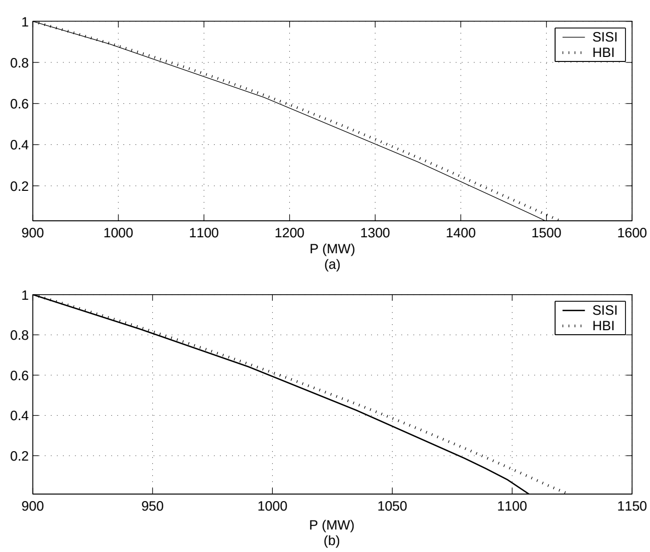


Fig. 5. Normalized stability index for IEEE-3bus system: (a) base system; (b) line 2-3 outage.

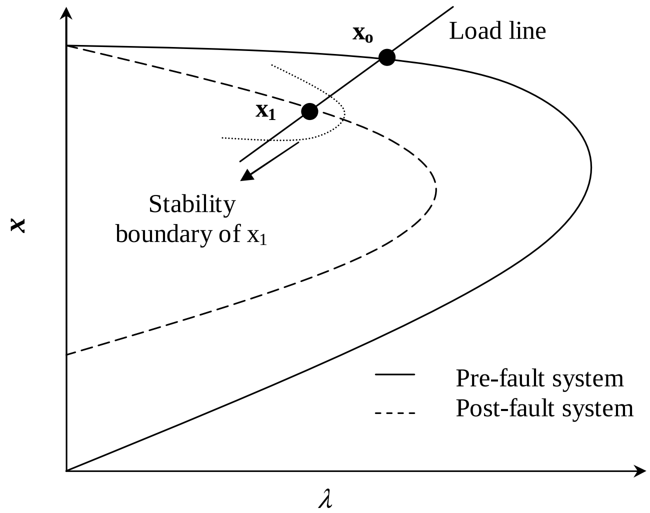


Fig. 6. Stability of post-fault system regarding large and small disturbance ( $x_o$ is out of stability region of $x_1$ which is locally stable).

obtained with PST with respect to load changes in the base system are illustrated in Fig. 4. For each system, the normalized stability indices were obtained and are depicted in Fig. 5. Observe that all indices predict a HB at about 1500 MW for the base system and 1100 MW for a line 2-3 outage, with both HBI and SISI indices presenting a rather linear profile. Notice that near the maximum loading point, the SISI index and HBI indices diverge; this is due to the difference between the nature of large disturbances and small disturbances. Thus, in the case of large disturbance, the system initial state $x_o$, as shown in Fig. 6, is out of the stability region of the post-disturbance equilibrium point $x_1$, which leads the system to instability. Observe that, the SISI index obtained using real time data is able to detect this phenomenon, which is an interesting feature of the proposed index.

The IEEE 14-bus benchmark system shown in Fig. 7 is used for more realistic tests of the proposed index [19]. It has 5 generators, 2 of them provide providing both active and reactive power at buses 1 and 2; generators at buses 3, 6 and 8 are basically synchronous condensors. The generators are modeled using subtransient models and loads are modeled as constant PQ loads. There is an HB point for both the base system and for a line 2-4 outage before the "nose" point, as shown in the corresponding PV curves depicted in Fig. 8. The base system at a 259 MW load yields the critical eigenvalues $\mu_c = 0.35 \pm j9.36$ (1.49 Hz frequency and 0.037 damping ratio), which is the eigenvalue that eventually crosses the imaginary axis at the HB point, as shown in the eigenvalue profiles depicted in Fig. 9.

The stability indices for both base and line 2-4 outage condition are depicted in Fig. 10. Observe that both indices predict an HB at about 350 MW for the base system and 328 MW for a line 2-4 outage. For light loading conditions, the two indices HBI and SISI are basically the same; however,

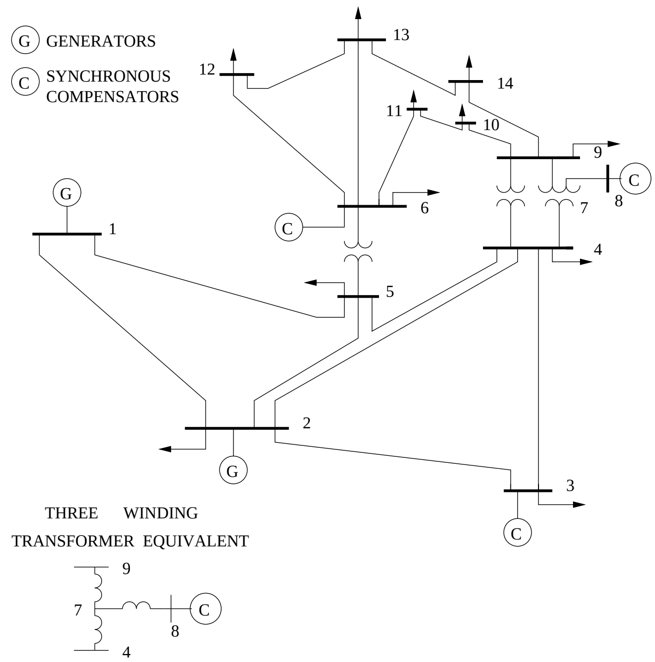


Fig. 7. IEEE 14-Bus test system.

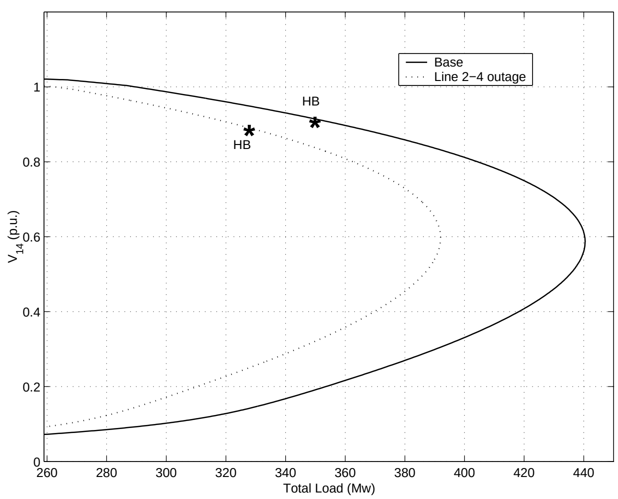


Fig. 8. PV curves for IEEE-14bus system at bus 14, for the base case and for a line 2-4 outage.

the closer to the instability point, the larger the difference between them, as observed in the previous example.

## V. Conclusions

A quasi-linear stability index SISI that can be used to predict represents the distance to the closest instability point with respect to system load changes is proposed. This index is based on on-line field measurements, thus capturing the real behavior of the system without modeling approximations, which is a drawback of similar previously proposed indices. Given its characteristics, this index may be used as an effective tool to help system operators take proper action following system contingencies to avoid stability problems.

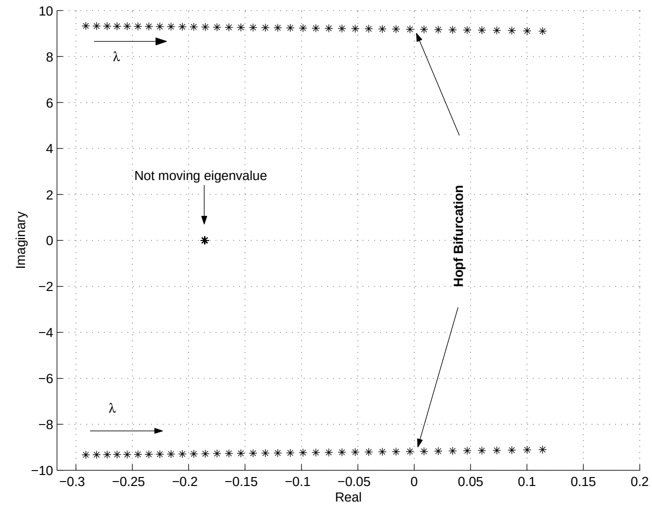


Fig. 9. Eigenvalue profiles with respect to load changes in the IEEE-14bus system.

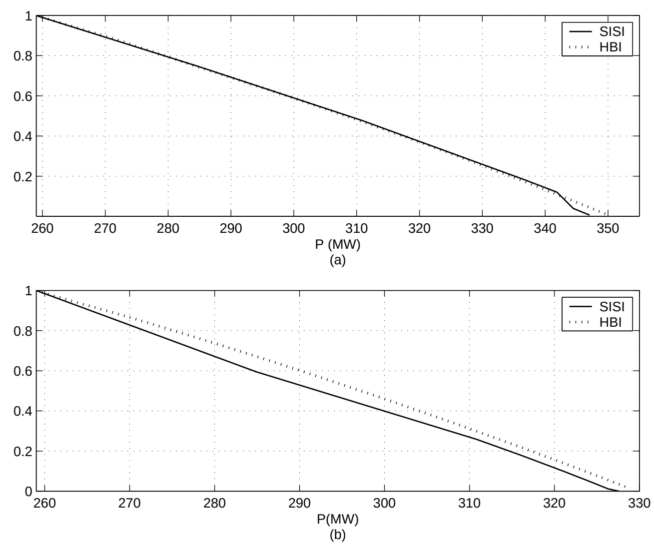


Fig. 10. Normalized stability index for IEEE-14bus system: (a) base system; (b) line 2-4 outage

To test the validity and performance of the proposed index in an actual operating environment, similar studies have to be performed in detailed EMTP type of models, to reproduce the actual signals that would be used to obtain the desired index. In this way, issues associated with the presence of higher frequency signals in the actual measurements, which are ignored in the present paper, can be studied in detail.

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