Panel Session: "Optimization Techniques in Voltage Collapse Analysis," IEEE/PES Summer Meeting, San Diego, July 14, 1998.

Applications of Optimization to Voltage Collapse Analysis

Claudio A. Cañizares
University of Waterloo
E&CE Department
Waterloo, ON, Canada, N2L 3G1
claudio@iliniza.uwaterloo.ca

***Abstract*—This paper describes several applications of optimization for voltage stability analysis (VSA) of power systems. Voltage stability problems, particularly those related to voltage collapse, are presented here as optimization problems, formally demonstrating, with the help of bifurcation theory, that well-known voltage collapse techniques are basically equivalent to some typical optimization methodologies. Recently developed analysis techniques and new research directions that have sprung up from bifurcation analysis of power systems and the application of optimization techniques to VSA are also presented in this paper. Throughout the paper, the advantages and disadvantages of using different optimization techniques for VSA in practical systems are discussed as well.**

Keywords: Voltage stability, voltage collapse, optimization, bifurcations.

I. Introduction

Most power systems today are operated close to their steady-state stability limits, as demonstrated by several voltage collapse events throughout the world, such as some of the disturbances of North America's Western interconnection in 1996 [1, 2]. Thus, voltage collapse studies, an integral part of voltage stability analysis (VSA) of power systems, have become rather important for the design and operation of power systems [3].

Most of the numerical tools used in voltage collapse studies are based on concepts and/or techniques developed from bifurcation analysis of power systems [3, 4, 5, 6]. More recently, however, new optimization-based tools have been developed to study voltage collapse problems in power systems. The main purpose of this paper is to show the similarities and strong ties between tools developed for the computation of collapse points from bifurcation theory and those based on optimization techniques. Furthermore, based on these similarities, the paper discusses possible improvements to existent optimization techniques used for optimal active and reactive power dispatch in power systems, as well as proposing new applications of optimization for the determination of cost-effective operating conditions that account for the voltage collapse problem.

The paper starts with a review in Section II of a few basic concepts in bifurcation analysis of power systems, to establish the basic background needed for the discussions presented in Section III regarding the association between bifurcation theory and optimization techniques used in voltage collapse studies. Section IV presents some possible research directions on optimization and bifurcation studies of power systems, and Section V summarizes the main ideas presented in this paper.

II. Voltage Collapse

As the ties between existent voltage collapse analysis tools developed from bifurcation theory and optimization techniques can only be explained based on certain bifurcation conditions, typically referred to as transversality conditions, this section concentrates on briefly introducing bifurcation theory.

A discussion on generic power system models is presented first to help explain bifurcation theory and its association with voltage collapse. Most of the material presented here is discussed in detail in [3].

A. System Models

Nonlinear dynamical systems, such as those obtained from certain power system models, can be generically described by the following ordinary differential equations (ODE):

$$\dot{x} = f(x, \lambda, p) \tag{1}$$

where $x \in \Re^n$ corresponds to the state variables; $\lambda \in \Re^\ell$ represents a particular set of "non-controllable" parameters that drive the system to a bifurcation in a quasi-static manner, i.e., as λ changes, the system steadily moves from equilibrium point to equilibrium point; $p \in \Re^k$ represents a series of "controllable" parameters associated with control settings; and $f : \Re^n \times \Re \times \Re^k \mapsto \Re^n$ is a nonlinear vector function [7, 8].

Typical power system models used in stability analysis, and hence VSA, are modeled with a set of differential and algebraic equations (DAE) of the form

$$\begin{bmatrix} \dot{x} \\ 0 \end{bmatrix} = \begin{bmatrix} f(x, y, \lambda, p) \\ g(x, y, \lambda, p) \end{bmatrix} = F(z, \lambda, p) \tag{2}$$

where $x \in \Re^n$ typically stands for state variables corresponding to a series of system devices and their controls, such as generators; $\lambda \in \Re^\ell$ stands for slow varying parameters that are typically associated with variable loading levels, over which operators have no direct control; $p \in \Re^k$ represents the control settings that operators directly or

indirectly control, such as AVR reference set points or shunt compensation; $f : \Re^n \times \Re^m \times \Re \times \Re^k \mapsto \Re^n$ corresponds to the nonlinear vector field directly associated with the state variables x; vector $y \in \Re^m$ represents the set of algebraic variables defined by the nonlinear algebraic function $g : \Re^n \times \Re^m \times \Re \times \Re^k \mapsto \Re^m$, which typically correspond to load bus voltages and angles, depending on the load models used; and $F(\cdot) = (f(\cdot), g(\cdot))$ and $z = (x, y) \in \Re^N$ $(N = n + m)$.

The stability of DAE systems is thoroughly discussed in [9]. The main idea in that paper is that if $D_y g(x, y, \lambda, p)$ can be guaranteed to be nonsingular along system trajectories of interest, the behavior of system (2) along these trajectories is primarily determined by the local ODE reduction

$$\dot{x} = f(x, y^{-1}(x, \lambda, p), \lambda, p)$$

where $y = y^{-1}(x, \lambda, p)$ comes from applying the Implicit Function Theorem to the algebraic constraints $g(x, y, \lambda, p) = 0$ on the trajectories of interest. The reader is referred to [10, 11] for a full discussion on the cases where the nonsingularity of $D_y g(\cdot)$ cannot be guaranteed over the full range of operating conditions. In the current paper, $D_y g(\cdot)$ is assumed to be nonsingular for all system equilibria of interest.

Another issue that one must be aware of in stability analysis is that system equilibria defined by $F(z_o, \lambda_o, p_o) = 0$ are actually obtained from a subset of equations

$$G(\tilde{z}_o, \tilde{\lambda}_o, \tilde{p}_o) = G|_o = 0 \quad \subset \quad F(z_o, \lambda_o, p_o) = F|_o = 0$$

typically referred to as the power flow equations, where $(\tilde{z}, \tilde{\lambda}, \tilde{p})$ are a subset of (z, λ, p). Hence, the typical procedure is to first solve the power flow equations and, based on the corresponding solutions, find equilibrium points of the dynamic model before proceeding with the stability analysis of the full dynamical system represented by (2). For some particular dynamic and power flow models, all power flow solutions may correspond to actual system equilibria and vice versa [12]. However, for most $F(\cdot)$ and $G(\cdot)$ models, the solution of the power flow equations does not guarantee an initial system equilibria, as solutions of $G|_o = 0$ do not necessarily guarantee that $F|_o = 0$ exists, and vice versa; this is particularly important for the discussions in Section IV regarding the development of new optimization techniques for power system studies. In this paper, for the most part, the proper system equilibria are assumed to be available, i.e., equilibrium points are assumed to be computed from the dynamic model $F(\cdot)$.

B. Bifurcation Theory

Bifurcation theory deals with the study of the stability of ODE systems (1) that move from equilibrium to equilibrium as the parameters λ slowly change [7, 8]. There are several types of bifurcations associated with the changes of λ, some are local and some are global, depending on the behavior of the system dynamic manifolds and equilibrium points. Many cases of actual voltage collapse in power systems have been associated with saddle-node bifurcations [3]; hence, this paper deals with these types of local bifurcations only.

Saddle-node bifurcations, also known as turning points or fold bifurcations, are generic, codimension one, local bifurcations of nonlinear dynamical systems represented by (1). These bifurcations are typically identified by a couple of equilibrium points merging at the bifurcation point as the slow varying parameters λ approach a maximum local value λ_*. Thus, a saddle-node bifurcation corresponds to an equilibrium point (x_*, λ_*, p_*), i.e., $f(x_*, \lambda_*, p_*) = 0$, where the corresponding system Jacobian $D_x f|_* = D_x f(x_*, \lambda_*, p_*)$ has a unique zero eigenvalue, and the following transversality conditions hold [8, 13]:

$$\begin{aligned} D_x f|_* \, \mathrm{v} = D_x^T f|_* \, \mathrm{w} &= 0 \\ \mathrm{w}^T D_\lambda f|_* &\neq 0 \\ \mathrm{w}^T [D_x^2 f|_* \mathrm{v}] \mathrm{v} &\neq 0 \end{aligned}$$

where v and w are properly normalized right and left "zero" eigenvectors in $\Re^n$ of $D_x f|_*$, respectively. These transversality conditions are important, as these allow, by means of center manifold theory, to describe the meaningful dynamic phenomena that characterize the system behavior close to the bifurcation point [7, 8, 14]; thus, the signs and magnitudes of the second and third conditions determine locally how the system bifurcates. These conditions also allow for the development of robust numerical techniques to determine the location of saddle-node bifurcation points.

The nonsingularity assumption of the algebraic constraints Jacobian is used in [13] to demonstrate that these transversality conditions, at a saddle-node bifurcation point (z_*, λ_*, p_*) of system (2), are equivalent to

$$D_z F|_* \, v = D_z^T F|_* \, w = 0 \tag{3}$$

$$w^T \, D_\lambda F|_* \neq 0 \tag{4}$$

$$w^T [D_z^2 F|_* v] v \neq 0 \tag{5}$$

where v and w are the corresponding right and left zero eigenvectors in $\Re^N$ of $D_z^T F|_*$, properly normalized. Thus, the methods described in this paper apply to both ODE and DAE systems under a nonsingularity assumption for $D_y g|_*$.

Saddle-node bifurcations are generic, i.e., they are expected to occur in nonlinear systems with one slow varying parameter, as opposed to other types of "singular" local bifurcations such as transcritical and pitchfork, which require certain specific symmetries in the system to occur [8]. In particular, systems that have a constant $D_\lambda f(\cdot)$ or $D_\lambda F(\cdot)$ matrix, which is the typical case in power systems with constant PQ load models, can be expected to bifurcate through a saddle-node, as condition (4) is generically met [15].

III. Current Use of Optimization

As saddle-node bifurcations have been directly associated to certain voltage collapse problems, a saddle-node bifurcation point is also referred to as the collapse point. Hence, determining the location of this point in the (λ, p) parameter space, i.e., computing the value of (z_*, λ_*, p_*), is of practical importance in power systems.

For systems with p fixed at a value of p_o, and a single parameter $\lambda \in \Re$, two techniques, namely, direct methods and continuation methods, were originally developed to determine bifurcation points of ODE systems (1) [7], and have been successfully applied to the computation of collapse points in power systems [3, 16]. For multivariable parameter problems, alternative techniques have been developed to study collapse problems. Thus, for systems with fixed controllable parameters $p = p_o$ and multiple parameters $\lambda \in \Re^\ell$, direct and iterative techniques have been developed to determine the minimum value of λ with respect to an initial operating condition, also known as the minimum distance to collapse [17, 18]. For systems with controllable parameters p free to change, direct and continuation methods have been proposed to determine the optimal values of p that maximize the distance to collapse, i.e., maximize the value of λ with respect to an initial operating condition [14]; the practical implications associate with this problem have been discussed in [19, 20].

This section briefly discusses all of these problems, and shows, using bifurcation theory, how solution methodologies can be developed from optimization theory.

A. Direct Methods

Direct methods, also known in power system applications as Point of Collapse (PoC) methods [21], were developed to compute saddle-node bifurcations of nonlinear systems with a scalar parameter $\lambda \in \Re$ (λ is a scalar and not a vector in this case). In DAE systems, the method consists of solving, for a given value p_o of the controllable parameters p, equations

$$\begin{aligned} F(z, \lambda, p_o) &= 0 \\ D_z^T F(z, \lambda, p_o) w &= 0 \\ ||w||_\infty &= 1 \end{aligned} \tag{6}$$

for the variables z, a scalar parameter λ, and the vector w, to directly obtain the collapse point (z_*, λ_*, p_o) [16, 22, 23]. These three equations correspond to the steady-state equations, the singularity conditions at the collapse point, and the nonzero left eigenvector requirement, in that order. Other equations with the right eigenvector and/or different nonzero eigenvector requirement may be used; however, equations (6) present the best numerical characteristics for large systems [16], as confirmed below by the results of applying optimization techniques to solve this problem.

Similar equations to (6) may be obtained by representing the problem as an optimization problem, as initially proposed in [24]. Thus, the collapse problem may be stated as

$$\begin{aligned} \text{Min.} \quad & -\lambda \\ \text{s.t.} \quad & F(z, \lambda, p_o) = 0 \end{aligned} \tag{7}$$

This problem may be solved using the Lagrangian

$$\mathcal{L}(z, \lambda, w) = -\lambda + w^T F(z, \lambda, p_o)$$

where w corresponds to the Lagrangian multipliers. Hence, necessary conditions to obtain a solution of (7) are given by $\nabla \mathcal{L} = 0$, i.e.,

$$\begin{aligned} D_w \mathcal{L} &= F(z, \lambda, p_o) = 0 \\ D_z \mathcal{L} &= D_z^T F(z, \lambda, p_o) w = 0 \\ \frac{\partial \mathcal{L}}{\partial \lambda} &= w^T \frac{\partial F}{\partial \lambda}(z, \lambda, p_o) - 1 = 0 \end{aligned} \tag{8}$$

These equations are basically the same as (6), with the exception of the third one, which is just another way of guaranteeing a nonzero w, and corresponds to transversality condition (4).

Stating the collapse problem as an optimization problem allows for the use of several well known optimization techniques to compute the collapse point, as discussed in [25]. One particular technique that is especially appealing due to its limit handling capabilities is Interior Point Methods, which has been successfully applied to the computation of the collapse point [26, 27].

B. Continuation Method

The continuation method consists basically on successive computations of equilibrium points for different values of system parameters λ and p. For a scalar parameter $\lambda \in \Re$ and a fixed $p = p_o$, this method allows the user to trace, by automatically changing the value of λ, the complete equilibrium profile or bifurcation manifold (this corresponds to the PV curves for some system models), which consists on one dimensional manifold or curve in the (z, λ, p) space. An approximation of the bifurcation point (z_*, λ_*, p_o) may be obtained by monitoring the value of λ until it reaches its maximum value λ_* at this point; hence, it can be readily used as an alternative method to compute the collapse point. The advantages of this method with respect to the direct method are that it produces additional information regarding the behavior of the system variables z and that limits can be reliably enforced. For power system applications, however, this method, although reliable and very informative, is computationally expensive, especially for large systems with multiple limits [16].

The computational strategy used in this method is illustrated in Fig. 1, which can be basically summarized in the following two main steps:

1. *Predictor*: A known equilibrium point (z_1, λ_1, p_o) is used to compute the direction vector Δz_1 and a change $\Delta \lambda_1$ of the system parameter. This first step

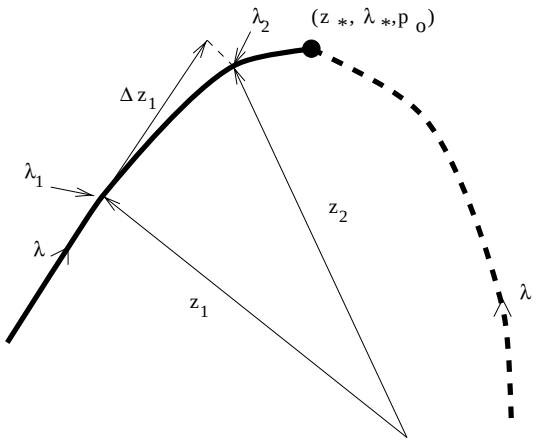

Fig. 1. Continuation method.

is known as the predictor, since it generates an initial guess $(z_1 + \Delta z_1, \lambda_1 + \Delta\lambda_1, p_o)$ for the corrector step. A way of computing this direction is by computing the tangent direction, or an approximation thereof, to the bifurcation manifold at the equilibrium point (z_1, λ_1, p_o) with respect to the variable parameter λ, i.e.,

$$\frac{\Delta z_1}{\Delta\lambda_1} \approx \left.\frac{\mathrm{d}z}{\mathrm{d}\lambda}\right|_1 = -\left[D_z F|_1\right]^{-1} \left.\frac{\mathrm{d}F}{\mathrm{d}\lambda}\right|_1$$

Hence, one may choose the step $\Delta\lambda_1$ to then compute the change Δz_1 in z_1 as follows:

$$\begin{aligned} \Delta\lambda_1 &= \frac{\alpha}{\|\mathrm{d}z/\mathrm{d}\lambda|_1\|} \\ \Delta z_1 &= \Delta\lambda_1 \left.\frac{\mathrm{d}z}{\mathrm{d}\lambda}\right|_1 \end{aligned} \tag{9}$$

where α is a positive scalar that is chosen to control the "length" of the step; typically, $\alpha = 1$ is an adequate choice. Observe that the closer to the bifurcation point, the smaller the step, as this is normalized with respect to the norm of the tangent vector, which gets larger the closer the system is to the bifurcation point.

2. *Corrector*: Once the new approximate point $(z_1 + \Delta z_1, \lambda_1 + \Delta\lambda_1, p_o)$ is determined in the predictor step, the actual equilibrium point (z_2, λ_2, p_o) must then be computed. In order to be able to solve this problem, an additional equation is required, as there is one more variable than number of equations. Thus, the following problem must be solved:

$$\begin{aligned} F(z, \lambda, p_o) &= 0 \\ \varphi(z, \lambda) &= 0 \end{aligned}$$

for (z, λ), using as initial guess the approximate solution generated by the predictor step.

Several techniques have been proposed to chose the additional equation $\varphi(z, \lambda) = 0$ [3]; however, the simplest approach is to fix one of the variables in z, or more typically λ itself, at the value yielded by the predictor step. The problem with the latter is that the resulting system of equations may not have a solution, especially if the chosen value $\lambda_2 = \lambda_1 + \Delta\lambda_1$ exceeds the corresponding value at the bifurcation point, i.e., $\lambda_2 > \lambda_*$. By cutting the step length when this occurs, i.e., reducing the value of α in (9), convergence problems can be avoided.

These two steps are cyclically applied until the changes in λ are smaller than certain tolerance ϵ, i.e., until $\Delta\lambda_1 < \epsilon$. However, one must be aware that, since the Jacobian $D_z F|_*$ is singular at the bifurcation point, a parameterization is sometimes needed in the predictor and/or corrector steps, depending on the value of ϵ and the numerical techniques used, to avoid convergence difficulties. In practice, this is not required when step-cutting techniques are utilized [16].

By restating the collapse problem as an optimization problem as follows:

$$\begin{aligned} \text{Min.} \quad & \mathcal{F}(u) = -\lambda \\ \text{s.t.} \quad & c(u) = F(z, \lambda, p_o) = 0 \end{aligned} \tag{10}$$

where $u = (z, \lambda)$, the continuation methods can be readily demonstrated to be equivalent to a generalized reduced-gradient (GRG) method [14].

The GRG method is a well-known nonlinear optimization technique that consists on following the constraint manifold defined by $c(u) = 0$, using predictor-corrector steps based on an "optimal" direction obtained from tangent information at known points on this manifold [28, 29]. Thus, using similar notation as in the description of the continuation method, this method consists of the following steps:

1. *Predictor*: Assuming that a set of initial values for $u = (z, \lambda)$ satisfying the constraints is known, say $u_1 = (z_1, \lambda_1)$ such that $c(u_1) = F(z_1, \lambda_1, p_o) = 0$, the initial vector u_1 is changed by taking a step $\Delta u_1 = (\Delta z_1, \Delta\lambda_1)$ in the direction of the steepest descent on the tangent plane to $c(u) = 0$ at u_1. Thus,

$$\underbrace{\begin{bmatrix} z_2 \\ \lambda_2 \end{bmatrix}}_{u_2} = \underbrace{\begin{bmatrix} z_1 \\ \lambda_1 \end{bmatrix}}_{u_1} + \alpha \underbrace{\begin{bmatrix} \Delta z_1 \\ \Delta\lambda_1 \end{bmatrix}}_{\Delta u_1}$$

where

$$\begin{bmatrix} \Delta z_1 \\ \Delta\lambda_1 \end{bmatrix} = -UU^T \nabla_u^T \mathcal{F}(u_1)$$

and the matrix $U \in \Re^{(N+1)\times 1}$, a basis for the null space of $D_u c(u_1) \in \Re^{N\times(N+1)}$, i.e., $D_u c|_1 U = 0$, is defined as

$$\begin{aligned} U &= \begin{bmatrix} -[D_{u^{(1)}} c(u_1)]^{-1} D_{u^{(2)}} c(u_1) \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} -[D_z F|_1]^{-1} \left.\frac{\mathrm{d}F}{\mathrm{d}\lambda}\right|_1 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} \left.\frac{\mathrm{d}_z}{\mathrm{d}\lambda}\right|_1 \\ 1 \end{bmatrix} \end{aligned}$$

assuming that $u = (u^{(1)}, u^{(2)}) = (z, \lambda)$, where $u^{(1)} = z$ are the dependent (computed) variables, and $u^{(2)} = \lambda$ is the independent (defined) variable. Hence,

$$\begin{bmatrix} \Delta z_1 \\ \Delta \lambda_1 \end{bmatrix} = -\begin{bmatrix} \left.\frac{\mathrm{d}z}{\mathrm{d}\lambda}\right|_1 \\ 1 \end{bmatrix} \begin{bmatrix} \left.\frac{\mathrm{d}z}{\mathrm{d}\lambda}\right|_1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} \left.\frac{\mathrm{d}z}{\mathrm{d}\lambda}\right|_1 \\ 1 \end{bmatrix} \quad (11)$$

The scalar $\alpha > 0$ is used to change the size of the step, so that $\mathcal{F}(u_2) < \mathcal{F}(u_1)$ $(\lambda_2 > \lambda_1)$, and to guarantee that the corrector step yields a solution, as discussed below.

2. *Corrector*: The value predicted by "linearized" step (11), must now be corrected to ensure that the constraints are satisfied. A modified Newton-Raphson iteration is used to obtain a solution for z so that $c(u) = F(z, \lambda, p_o) = 0$, by using as an initial guess

$$\underbrace{z_2^0}_{u_0^{(1)}} = z_1 + \alpha \Delta z_1 = z_1 + \alpha \left.\frac{\mathrm{d}z}{\mathrm{d}\lambda}\right|_1$$

and keeping constant

$$\underbrace{\lambda_2}_{u_0^{(2)}} = \lambda_1 + \alpha \Delta \lambda_1 = \lambda_1 + \alpha$$

Thus, the corrector step is defined as

$$\underbrace{z_2^{i+1}}_{u_{i+1}^{(1)}} = \underbrace{z_2^i}_{u_{i+1}^{(1)}} - \beta \left[D_{u^{(1)}} c(u_i^{(1)}, u_0^{(2)}) \right]^{-1} c(u_i^{(1)}, u_0^{(2)}) = z_2^i - \beta [D_z F(z_2^i, \lambda_2, p_o)]^{-1} F(z_2^i, \lambda_2, p_o) \quad (12)$$

where $i = 1, 2, \ldots$ stands for the iteration number. The scalar constant $\beta > 0$ is used to ensure a solution to $c(u) = 0$ [30]. Hence, if no solution z_2 is encountered as the value of β is reduced up to certain reasonable tolerance, the vector generated by the predictor step (11) is changed by reducing the value of α. This process is repeated until convergence of (12) is attained.

The predictor and corrector steps (11) and (12), respectively, are applied sequentially until $\mathcal{F}(u) = -\lambda$ is minimized within an acceptable tolerance, i.e., λ approximately reaches its maximum value λ_*. Observe that the GRG method described above is basically a continuation method with a step-cutting technique, with some slight modifications on the predictor and corrector steps with respect to the continuation method described above.

An advantage of the continuation technique as compared to the direct method is that it is relatively simple to introduce operational limits, as these can be enforced "on-the-fly" as the solution is approached. However, one must be aware of the possibility of reaching certain limits that significantly change the "shape" of the solution manifold (as defined by $c(u) = F(z, \lambda, p_o) = 0$), which could generate nontrivial changes in the corresponding tangent surfaces, resulting on convergence problems during the continuation process. A simple example of this type of problem in power systems is the "Q-limit" instabilities [16, 31]. Finally, as with any GRG-type method, one must be also aware of the possibility of "zigzagging" during the computation process [28], which considerably slows down the convergence process.

C. *Minimum Distance to Collapse*

When the bifurcation parameter λ is multidimensional, i.e., there is more than one parameter that drives the system to bifurcation, such as in the case of several loads in a power system changing independently of each other, an issue that is of interest is to determine the value of λ that would yield the closest bifurcation to a given operating or equilibrium point (z_o, λ_o, p_o) [17]. Assuming that the controllable parameters p remain fixed at a given value p_o, the problem can be stated as the following optimization problem [14]:

$$\begin{aligned} \text{Min.} \quad & \mathcal{F}(u) = -\frac{1}{2}\|\lambda - \lambda_o\|_2 \\ \text{s.t.} \quad & c(u) = \begin{bmatrix} F(z, \lambda, p_o) \\ D_z^T F(z, \lambda, p_o)\, w \end{bmatrix} = 0 \end{aligned} \quad (13)$$

where $u = (z, \lambda, w)$, and w is a properly normalized "zero" eigenvector. Thus, the idea is to determine the minimum distance from a given operating point defined by λ_o to a collapse point defined by λ_*. Observe that the constraints force the solution to be a "singular" bifurcation, as these constraints essentially correspond to transversality conditions (3) and (4).

This problem can be solved in several ways, as indicated in [17]. The simplest approach to solving it would be to use the Lagrangian function

$$\mathcal{L}(z, \lambda, w, \mu, \gamma) = -\frac{1}{2}(\lambda - \lambda_*)^T(\lambda - \lambda_*) + \mu^T F(z, \lambda, p_o) + \gamma^T D_z^T F(z, \lambda, p_o) w \quad (14)$$

Thus, based on transversality conditions (3), (4) and (5) at the saddle-node bifurcation point (z_*, λ_*, p_o), the Lagrange multipliers that solve the necessary optimality conditions given by $\nabla \mathcal{L} = 0$, can be shown to be $\mu_* = w_*$ and $\gamma_* = 0$ [14]. These values lead to equations (15), where the last equation ensures a nonzero left eigenvector w_* based on transversality condition (4).

$$\begin{aligned} F(z_*, \lambda_*, p_o) &= 0 \\ D_z^T F(z_*, \lambda_*, p_o)\, w_* &= 0 \\ D_\lambda^T F(z_*, \lambda_*, p_o)\, w_* &= \lambda_* - \lambda_o \end{aligned} \quad (15)$$

Equations (15) basically correspond to a direct method to compute the desired solution; however, other optimiza-

tion techniques such as Interior Point methods, with better convergence characteristics especially for systems with inequality constraints (limits on the system variables), can be used in this case. For example, a continuation method could be developed by applying the predictor and corrector steps of the GRG method described above to optimization problem (13). This requires a proper choice of dependent $u^{(1)}$ and independent $u^{(2)}$ variables, as indicated in [14]. Thus, one may chose

$$u^{(1)} = \begin{bmatrix} z \\ \hat{w} \\ \lambda_l \end{bmatrix} \qquad u^{(2)} = \begin{bmatrix} w_k \\ \hat{\lambda} \end{bmatrix}$$

were $w_k \in \Re$ is a non-zero element of the "zero" left eigenvector w, and $\lambda_l \in \Re$ is an element of the set of parameters λ; thus, $w = (\hat{w}, w_k)$ and $\lambda = (\hat{\lambda}, \lambda_l)$. This choice guarantees a non-zero w and proper convergence characteristics.

Another solution method proposed in [17], is to apply an iterative technique based on the property that the left eigenvector at the solutions point w_* is colinear with the vector $\lambda_* - \lambda_o$, i.e.,

$$w_* = k(\lambda_* - \lambda_o)$$

where k is a scalar constant. This iterative techniques, as well as the direct method, have been shown to yield reasonable results in realistic power system examples in [18].

D. Maximizing the Distance to Collapse

As saddle-node bifurcations have catastrophic effects on the stability of nonlinear systems, mechanisms must be developed to avoid them. One possible way of avoiding these stability problems would be to maximize the distance to the saddle-node bifurcation with respect to the slow varying parameters λ. This maximization can be achieved by changing the controllable parameters p in the nonlinear system. This problem can be stated as an optimization as follows [14]:

$$\begin{aligned} \text{Min.} \quad & \mathcal{F}(u) = -\frac{1}{2}\|\lambda - \lambda_o\|_2 \\ \text{s.t.} \quad & c(u) = \begin{bmatrix} F(z,\lambda,p) \\ D_z^T F(z,\lambda,p)\, w \end{bmatrix} = 0 \end{aligned} \tag{16}$$

where $u = (z, \lambda, p, w)$, w is a properly normalized "zero" eigenvector, and λ_o is assumed to defined the initial operating point used as the reference point the for optimization process. Observe that the basic difference between (13) and this optimization problem is that p is allowed to change in this case. Hence, the same Lagrangian function (14), together with transversality conditions (3), (4) and (5), can be used to obtain the following solution to (16):

$$\begin{aligned} F(z_*, \lambda_*, p_*) &= 0 \qquad (17) \\ D_z^T F(z_*, \lambda_*, p_*) w_* &= 0 \\ D_p^T F(z_*, \lambda_*, p_*) w_* &= 0 \\ D_\lambda^T F(z_*, \lambda_*, p_*) w_* &= \lambda_* - \lambda_o \qquad (18) \end{aligned}$$

The last equation properly normalizes the "zero" left eigenvector $w_* \in \Re^N$. Equations (17) basically define a direct method to solve this problem.

A continuation methodology can also be developed following the same GRG method as before, and defining the dependent and independent variables as follows [14, 19, 20]:

$$u^{(1)} = \begin{bmatrix} z \\ \hat{w} \\ p_l \end{bmatrix} \qquad u^{(2)} = \begin{bmatrix} w_k \\ \hat{p} \\ \lambda \end{bmatrix}$$

where, once more, $w_k \in \Re$ is defined to guarantee a non-zero w vector, which in turn requires $p_l \in \Re$ to become a computed variable; thus, $w = (\hat{w}, w_k)$ and $p = (\hat{p}, p_l)$.

Other optimization techniques with better limit handling capabilities and improved convergence characteristics, such as Interior Point methods, can be used in this case to avoid the pitfalls of the proposed direct and continuation methods. The practical implications of applying these two methods to power system studies are presented in [14].

IV. New Applications of Optimization

Based on the experience acquired through the study of saddle-node bifurcation problems in power systems and their close relation with various optimization techniques, several possible improvements to existent optimization tools are discussed in this section.

A. Improved Models for Optimization Studies

Typical optimization studies are based on simple power flow models. Thus, these techniques try to solve optimization problems of the general form

$$\begin{aligned} \text{Min.} \quad & \mathcal{F}(\tilde{z}, \tilde{\lambda}, \tilde{p}) \\ \text{s.t.} \quad & G(\tilde{z}, \tilde{\lambda}, \tilde{p}) = 0 \end{aligned}$$

plus several inequality constraints to model limits on various power flow variables, such as bus voltages and generator reactive powers. Usually, $\tilde{p}$ represents the active power injections in PV (generator) buses and/or reactive power injections from shunt compensation in the system.

When these techniques are applied to study stability problems, as in this case of bifurcation studies, power flow models are inadequate, since controls and their corresponding limits are rather important [32], particularly for new system elements such as Flexible AC Transmission System (FACTS) controllers [19, 20]. A proper optimization problem in this case would be represented by

$$\begin{aligned} \text{Min.} \quad & \mathcal{F}(z, \lambda, p) \\ \text{s.t.} \quad & F(z, \lambda, p) = 0 \end{aligned}$$

plus any other inequality constraints representing limits on the system variables. Observe that $F(z, \lambda, p)$ in this case corresponds to the actual equations and variables, or a proper subset thereof, of the full dynamical system,

so that the actual equilibrium equations and system variables, with their corresponding limits, are used in the optimization process.

A power system dynamic model, based on a simple second order generator model with no realistic representation of control variable limits, is used for an optimal power dispatch problem in [33, 34], with some additional stability criteria represented through transient energy functions (TEF) in the optimization process. It would be interesting to apply new optimization techniques such as Interior Point methods to study a similar problem, and to address the issue of optimal reactive power dispatch, but using detailed generator and controller (e.g., FACTS) models to analyze their effect on the optimization process.

B. Optimal P and Q Dispatch to Maximize Stability

The problem of accounting for system stability in optimal active power dispatch of power systems is studied in [33, 34], as indicated above. In these two papers, the objective function is modified to include a measure of the "stability level" of the system by adding a TEF measurement to the objective function. The proposed approach may be summarized using the following multi-objective optimization problem:

$$\begin{aligned} \text{Min.} \quad & [\mathcal{F}_1(z,\lambda,p), \mathcal{F}_2(z,\lambda,p)] \\ \text{s.t.} \quad & F(z,\lambda,p) = 0 \end{aligned} \tag{19}$$

where $\mathcal{F}_1(\cdot)$ stands for the classical generation cost function and $\mathcal{F}_2(\cdot)$ corresponds to a TEF-based measure of stability of the system.

It has been shown that the system stability is tightly coupled to the distance to collapse, i.e., the "closer" to the point of collapse, the "less stable" the system is [14, 35]. Thus, it should be possible to set up a similar optimization problem as (19) where $\mathcal{F}_2(\cdot)$ is replaced by a measure of the distance to collapse, say

$$\mathcal{F}_2(z,\lambda,p) = \frac{1}{2}\|\lambda - \lambda_o\|_2$$

and the constraints are modified accordingly. A possible outcome of such optimization process could be to determine an optimal power dispatch that maximizes the distance to collapse.

V. Conclusions

This paper demonstrated the strong ties between bifurcation and optimization techniques for the analysis of voltage collapse problems. It is shown that by simply restating the bifurcation problem as an optimization one, typical techniques used to compute bifurcation points are basically the same as certain optimization techniques. Although several computational methods based on bifurcation theory have been shown to be efficient tools for VSA, using optimization techniques for these types of studies present several advantages, especially due to their limit handling capabilities.

Optimization techniques are widely developed tools that are primarily used in power systems to determine optimal active and reactive power dispatch patterns. However, these optimization techniques can be significantly improved by using some concepts developed in bifurcation theory, especially regarding the importance of accurate modeling of system controls and their limits. Furthermore, not much work has been done on accounting for the distance to collapse in these kinds of tools. Hence, computing P and Q dispatch patterns that not only minimize costs but also increase distance to collapse, thus enhancing system stability, while using adequate device models to accurately represent the system with all its control limits, is the purpose of our current research work.

References

[1] "Planning against voltage collapse," technical report of tf 38-01-03, CIGRE, October 1986.

[2] "Power systems outages that occurred in the Western interconnection on July 1996," disturbance report, Western Systems Coordinating Council (WSCC), September 1996.

[3] P. Kundur, editor, "Voltage stability assessment, procedures and guides," technical report Final draft, IEEE/PES Power Systems Stability Subcommittee, 1998. Available at http://www.power.uwaterloo.ca.

[4] L. H. Fink, editor, *Proc. Bulk Power System Voltage Phenomena II—Voltage Stability and Security*, ECC Inc., Fairfax, VA, August 1991.

[5] L. H. Fink, editor, *Proc. Bulk Power System Voltage Phenomena—Voltage Stability and Security*, EL-6183, EPRI, January 1989.

[6] L. H. Fink, editor, *Proc. Bulk Power System Voltage Phenomena III—Voltage Stability and Security*, ECC Inc., Fairfax, VA, August 1994.

[7] R. Seydel, *Practical Bifurcation and Stability Analysis—From Equilibrium to Chaos*. Springer-Verlag, New York, second edition, 1994.

[8] J. Guckenheimer and P. Holmes, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*. Springer-Verlag, New York, 1986.

[9] D. J. Hill and I. M. Y. Mareels, "Stability theory for differential/algebraic systems with application to power systems," *IEEE Trans. Circuits and Systems*, vol. 37, no. 11, November 1990, pp. 1416–1423.

[10] V. Venkatasubramanian, H. Schättler, and J. Zaborszky, "A taxonomy of the dynamics of the large power systems with emphasis on its voltage stability," pp. 9–52 in [4].

[11] V. Venkatasubramanian, X. Jiang, H. Schättler, and J. Zaborszky, "Current status of the taxonomy theory of large power system dynamics—DAE systems with hard limits," in [6].

[12] I. Dobson, "The irrelevance of load dynamics for the loading margin to voltage collapse and its sensitivities," pp. 509–518 in [6].

[13] C. A. Cañizares, "Conditions for saddle-node bifurcations in ac/dc power systems," *Int. J. of Electric Power &*

Energy Systems, vol. 17, no. 1, February 1995, pp. 61–68.

[14] C. A. Cañizares, "Calculating optimal system parameters to maximize the distance to saddle-node bifurcations," *IEEE Trans. Circuits and Systems–I*, vol. 45, no. 3, March 1998, pp. 225–237.

[15] C. A. Cañizares and S. Hranilovic, "Transcritical and Hopf bifurcations in ac/dc systems," pp. 105–114 in [6].

[16] C. A. Cañizares and F. L. Alvarado, "Point of collapse and continuation methods for large ac/dc systems," *IEEE Trans. Power Systems*, vol. 8, no. 1, February 1993, pp. 1–8.

[17] I. Dobson, "Computing a closest bifurcation instability in multidimensional parameter space," *J. Nonlinear Science*, vol. 3, 1993, pp. 307–327.

[18] F. L. Alvarado, I. Dobson, and Y. Hu, "Computation of closest bifurcations in power systems," *IEEE Trans. Power Systems*, vol. 9, no. 2, May 1994, pp. 918–928.

[19] C. A. Cañizares and Z. T. Faur, "Analysis of SVC and TCSC controllers in voltage collapse," IEEE/PES PE-200-PWRS-0-2-1998, accepted for publication in *IEEE Trans. Power Systems*, March 1998.

[20] C. A. Cañizares, A. Berizzi, and P. Marannino, "Using FACTS controllers to maximize available transfer capability," *Proc. Bulk Power Systems Dynamics and Control IV–Restructuring*, Santorini, Greece, August 1998.

[21] F. L. Alvarado and T. H. Jung, "Direct detection of voltage collapse conditions," pp. 5.23–5.38 in [5].

[22] C. A. Cañizares, F. L. Alvarado, C. L. DeMarco, I. Dobson, and W. F. Long, "Point of collapse methods applied to ac/dc power systems," *IEEE Trans. Power Systems*, vol. 7, no. 2, May 1992, pp. 673–683.

[23] V. Ajjarapu, "Identification of steady state voltage stability in power systems," *Int. J. of Electric Power & Energy Systems*, vol. 11, 1991, pp. 43–46.

[24] T. Van Cutsem, "A method to compute reactive power margins with respect to voltage collapse," *IEEE Trans. Power Systems*, vol. 6, no. 1, February 1991, pp. 145–156.

[25] J. Lu, C. W. Liu, and J. S. Thorp, "New methods for computing a saddle-node bifurcation point for voltage stability analysis," *IEEE Trans. Power Systems*, vol. 10, no. 2, May 1995, pp. 978–989.

[26] C. J. Parker, I. F. Morrison, and D. Sutanto, "Application of an optimization method for determining the reactive margin from voltage collapse in reactive power planning," *IEEE Trans. Power Systems*, vol. 11, no. 3, August 1996, pp. 1473–1481.

[27] G. D. Irisarri, X. Wang, J. Tong, and S. Moktari, "Maximum loadability of power systems using interior point non-linear optimization method," *IEEE Trans. Power Systems*, vol. 12, no. 1, February 1997, pp. 162–172.

[28] R. Fletcher, *Practical Methods of Optimization*. John Wiley & Sons, New York, 1987.

[29] P. E. Gill, W. Murray, and M. H. Wright, *Practical Optimization*. Academic Press, London, 1981.

[30] J. Stoer and R. Bilirsch, *Introduction to Numerical Analysis*. Springer-Verlag, 1980.

[31] I. Dobson and L. Lu, "Voltage collapse precipitated by the immediate change in stability when generator reactive power limits are encountered," *IEEE Trans. Circuits and Systems—I*, vol. 39, no. 9, September 1992, pp. 762–766.

[32] P. A. Löf, G. Andersson, and D.J. Hill, "Voltage dependent reactive power limits for voltage stability studies," *IEEE Trans. Power Systems*, vol. 10, no. 1, February 1995, pp. 220–228.

[33] K. S. Chandrashekhar and D. J. Hill, "Dynamic security dispatch: Basic formulation," *IEEE Trans. Power Systems*, vol. 102, no. 7, July 1983, pp. 2145–2154.

[34] A. A. Fouad and T. Jianzhong, "Stability constrained optimal rescheduling of generation," *IEEE Trans. Power Systems*, vol. 8, no. 1, February 1993, pp. 105–112.

[35] T. J. Overbye and C. L. DeMarco, "Voltage security enhancement using energy based sensitivities," *IEEE Trans. Power Systems*, vol. 6, no. 3, August 1991, pp. 1196–1202.

Claudio A. Cañizares (S'87, M'92) received the Electrical Engineer diploma (1984) from the Escuela Politécnica Nacional (EPN), Quito-Ecuador, where he held different positions from 1983 to 1993. His MS (1988) and PhD (1991) degrees in Electrical Engineering are from the University of Wisconsin-Madison. Dr. Cañizares is currently an Associate Professor at the University of Waterloo and his research activities are mostly concentrated in the study of computational, modeling, and stability issues in ac/dc/FACTS systems.