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Comparison of Performance Indices for Detection of Proximity to Voltage Collapse

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Abstract—**The paper proposes a new test function to be used in an existent performance index for detection of proximity to a static voltage collapse point. This test function is based on a reduction of the load flow Jacobian with respect to the critical bus of a system. The test function is compared with known singular values and eigenvalues indices, and with other previously proposed test functions. A thorough analysis of the similarities, advantages, and disadvantages of all these indices and test functions is presented. The techniques are tested and compared on the IEEE 300 bus test system, showing the effect of system characteristics and limits in these indices and functions.**

I. Introduction

Voltage collapse problems have been a subject of great concern during planning and operation of power systems due to several voltage collapse events around the world [1, 2]. For certain power system models, some voltage collapse occurrences have been associated to "bifurcation" problems, particularly to saddle-node bifurcations [3, 4, 5].

Saddle-node bifurcations in power systems can be simply explained in the following way: For a load condition, in addition to the normal load-flow solution, which is typically the actual operating point or stable equilibrium point (s.e.p.), several solutions may be found for the load-flow equations. The "closest" one to the s.e.p. is the unstable equilibrium point (u.e.p.) of interest for voltage collapse studies [6, 7]. These equilibrium points approach each other as the system is loaded, up to the point when only one solution exists. If the system is loaded further, all system equilibria disappear. (This phenomenon of changing number of load-flow solutions with system loading is thoroughly discussed from a point of view other than bifurcation theory in [8].) The "last" equilibrium has been identified as the steady-state voltage collapse point, since it can be mathematically associated to a saddle-node bifurcation point of some particular underlying dynamic equations. At this bifurcation point, a real eigenvalue of the load-flow Jacobian becomes zero, i.e., the Jacobian becomes singular.

Identification of this bifurcation point plays an important role on voltage collapse analyses, and a fair amount of research effort has been concentrated on this issue (e.g., [9, 10, 11, 12, 13, 14, 15], to name a few). In this particular approach to the problem, the voltage collapse point may be identified through a static analysis, even though the final consequence is the loss of dynamic stability characterized by a system wide voltage collapse. References [13] and [14] show how the voltage collapse point can be identified by using singular value and eigenvalue decomposition, respectively. Particularly, in [13] the authors demonstrate that a singular value analysis of a reduced load-flow Jacobian with respect to the reactive power equations, provides better results than the analysis of the full load-flow Jacobian. Such reduction is also employed in reference [14] for eigenvalue analyses. The minimum singular value and the minimum magnitude of the eigenvalues of such reduced matrix are then proposed as performance indices to detect proximity to this voltage collapse or "singular" point. On the other hand, the authors in [16] proposed a novel approach to detect proximity to voltage collapse. A test function, as defined by Seydel in [17], is used in this case to define a proximity index that seems to present an advantage over previously proposed indices, due to its apparent "linear" behavior as the system approaches the voltage collapse point. However, the latter is true only if the "right" bus is chosen and no limits are considered in the system, as it is demonstrated on this paper.

The authors in [9, 18, 19, 20] introduced the use of a reduced load-flow Jacobian determinant with respect to the critical bus(es) of a system as an indicator of voltage security. These critical buses are determined based on the maximum entries on the system tangent vector when close to the voltage collapse point [18], as described in detail below. Based on this technique, this paper proposes the use of these particular determinants as test functions for indices of proximity to voltage collapse.

The paper shows that very similar results are obtained using the techniques proposed in references [13, 14], and explains why the reduction of the load-flow Jacobian to a smaller proper set of equations provides better results. It is important to mention that all of the indices discussed in this paper can be considered to be of the same "linear" nature, i.e., they are based on a linearization of the system at different operating points.

The paper is organized as follows. Section II reviews the techniques proposed in [13], [14], and [16]. Section III introduces the reduced load-flow Jacobian determinant technique. Finally, section IV depicts some simulation results, and discusses the shortcomings of the presented methods.

II. Existent Performance Indices

Performance indices to predict proximity to voltage collapse problems have been a permanent concern of researchers and technical staff in power systems operation, as these indices could be used on-line or off-line to help operators determine how close the system is to a possible collapse [21]. Voltages profiles, also known as QV, PV or nose curves, are currently in use at Tokyo Electric Power Co. for determining proximity to collapse, so that operators can take timely preventive measures to avoid losing the system [22]. The problem with these curves is that, although reliable, they are rather expensive to compute. Hence, cheaper indices that do not require of exhaustive calculations have been sought and proposed; however, some of these indices are highly nonlinear, as depicted in this paper, making them unreliable. Low-cost indices that have a known shape, so that operators can rely on them to predict proximity to collapse, are certainly of interest to industry and researchers alike.

This section describes several known inexpensive, "linear" proximity indices, namely, singular value and eigenvalue decomposition and test functions. The first two display highly nonlinear behavior, whereas the latter could be classified as of predictable shape.

A. Singular Value Decomposition

Singular values have been employed in power systems because of the useful orthonormal decomposition of the Jacobian matrices.

For a real square matrix A of dimension $n \times n$, one has that

$$A = U\Sigma V^T = \sum_{i=1}^{n} u_i \sigma_i v_i^T$$

where the singular vectors u_i and v_i are the i^{th} columns of the unitary matrices U and V, and Σ is a diagonal matrix of positive real singular values σ_i, such that $\sigma_1 \geq \sigma_2 \geq .... \geq \sigma_n$. The diagonal entries of Σ^2 correspond to the eigenvalues of matrix AA^T.

This singular value decomposition is typically used to determine the rank of a matrix, which is equal to the number of non zero singular values of A [23, 24]. Hence, its application to static voltage collapse analysis focuses on monitoring the smallest singular value up to the point when it becomes zero [13].

In power systems, one is usually interested in determining the singularity of the Jacobian associated to the system dynamic equations, as this singularity corresponds to a bifurcation point, an undesirable stability condition of the system [3, 4, 5]. Different models of the system elements, particularly generators and loads, certainly affect the location of these bifurcation points [25, 26]. Furthermore, changing various parameters in the system can produce different types of bifurcating phenomena [27]. Nevertheless, for the purpose of this paper and given the limited space available, the popular load-flow model is used, where the variations of constant active and reactive powers are assumed to be the parameter that drive the system to a singularity, which can be associated to a saddle-node bifurcation of a dynamic model of the system as thoroughly demonstrated in [5], Notice that without loss of generality, the methodologies described in this paper can be directly applied to any dynamic system model.

Based on the power flow model, the matrix A above is assumed to represent the load-flow Jacobian J, which typically contains the first derivatives of active and reactive power mismatch equations, $\Delta P = \Delta P(\theta, E)$ and $\Delta Q = \Delta Q(\theta, E)$, with respect to the voltage magnitude E and angles θ, i.e., the linearization of these equations yields

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = J \begin{bmatrix} \Delta\theta \\ \Delta E \end{bmatrix} \tag{1}$$

Thus, one can rewrite (1), at equilibrium points other than the bifurcation point, as

$$\begin{bmatrix} \Delta\theta \\ \Delta E \end{bmatrix} = V\,\Sigma^{-1}\,U^T \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \sum_{i=1}^{n} \sigma_i^{-1} v_i u_i^T \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \tag{2}$$

Notice that the minimum singular value is a relative measure of how close the system is to the voltage collapse or singular point. Furthermore, near this bifurcation point, since σ_n is close to zero, equation (2) can be rewritten as

$$\begin{bmatrix} \Delta\theta \\ \Delta E \end{bmatrix} \approx \sigma_n^{-1} v_n u_n^T \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

Thus, the associated left and right singular vectors u_n and v_n, respectively, can be interpreted as follows [13]:

1. The maximum entries in v_n indicate the most sensitive voltage magnitudes and angles (critical buses).
2. The maximum entries in u_n correspond to the most sensitive direction for changes of active and reactive power injections.

In [10], the authors proposed to reduce the load-flow Jacobian to the first derivative of reactive power equations in relation to voltage magnitude, by assuming that the generator and load buses present no active power variation, i.e., $\Delta P = 0$. Thus,

$$\begin{aligned} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} &= \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta E \end{bmatrix} \\ \Rightarrow \quad \Delta Q &= (J_4 - J_3 J_1^{-1} J_2)\Delta E = J_{QV}\Delta E \end{aligned} \tag{3}$$

where $J_1 = [\partial P/\partial\theta]$, $J_2 = [\partial P/\partial E]$, $J_3 = [\partial Q/\partial\theta]$, and $J_4 = [\partial Q/\partial E]$. In general, at the saddle-node bifurcation point one has that J_1 is not singular, even though J is singular. There is not definite proof of the latter statement, but in practice that seems to be the case in all examples discussed in the literature and is also the experience of the authors. Thus, J_{QV} is assumed well defined in (3), becoming singular at the bifurcation point since

$$\det J_{QV} = \frac{\det J}{\det J_1}$$

The singular values of this reduced matrix can then be used to determine proximity to voltage collapse. Furthermore, these singular values show "better" behavior than the ones of J, as demonstrated in [13] and in section IV of this paper.

It is interesting to highlight the fact that sub-matrix J_3 is quasi-symmetric, for small values of transmission system resistances. Therefore, one expects a similar attribute for J_{QV}, making the singular values and eigenvalues practically identical, since symmetric matrices have similar singular value and eigenvalue decomposition [23, 24], as demonstrated for the IEEE 300 bus test system below.

B. *Eigenvalue Decomposition*

The authors in [14] apply an eigenvalue decomposition to the reduced load-flow Jacobian J_{QV}. This decomposition for a real square diagonalizable (semi-simple) matrix A can be written as [23]:

$$A = X\Lambda Y^T = \sum_{i=1}^{n} x_i \mu_i y_i^T$$

where X represents a complex matrix of right eigenvectors x_i, Y corresponds to the complex matrix of left eigenvectors y_i, and Λ is a diagonal matrix of complex eigenvalues μ_i of A.

For the J_{QV} matrix defined in (3), this decomposition may be applied directly, since this matrix is quasi-symmetric and, therefore, diagonalizable. Furthermore, due to its quasi-symmetric structure, one expects to obtain a set of only real eigenvalues and eigenvectors, very similar in value to the corresponding singular values and singular vectors. Thus, near the bifurcation or voltage collapse point the eigenvectors associated to the eigenvalue closest to zero have the same interpretation as the singular vectors, i.e., the maximum entries in the right eigenvector correspond to the critical buses (most sensitive voltages) in the system, and the maximum entries in the left eigenvector pinpoints the most sensitive direction for changes of power injections [28, 14]. The results of the singular value and eigenvalue decomposition for the IEEE 300 bus test system below clearly show how these two methods basically yield the same information. It is the observation of the authors that as the size of the system increases, the singular values and absolute eigenvalues become closer.

C. *Test Functions*

Seydel in [17] discusses a family of scalar test functions t_{lk} defined as

$$t_{lk} \triangleq e_l^T J\, J_{lk}^{-1} e_l \tag{4}$$

where J corresponds to the system Jacobian, and e_l is the l^{th} unit vector, i.e., a vector with all zero entries except for an entry of 1 in row l. The matrix J_{lk} is defined as follows:

$$J_{lk} \triangleq (I - e_l e_l^T)J + e_l e_k^T \tag{5}$$

where I represents the identity matrix. Equation (5) can be simply interpreted as an operation on the Jacobian matrix J where the l^{th} row is removed and replaced by the row e_k^T. Notice that for the load-flow equations at the voltage collapse or bifurcation point J is singular, but matrix J_{lk} is guaranteed not singular if the l^{th} and k^{th} are chosen so that they correspond to non zero entries in the "zero" eigenvectors, say vectors x_n and y_n associated to the zero eigenvalue of J. Furthermore, if $l = k = c$, where c corresponds to the maximum or critical entry in x_n, the test function becomes the "critical" test function

$$t_{cc} = e_c^T J\, J_{cc}^{-1} e_c \tag{6}$$

The authors in [16] use the test function of equation (4) to define an index of proximity to collapse. Thus, lets assume that the load-flow vector nonlinear equations are defined as the active and reactive power mismatches at the system buses

$$\begin{aligned} \Delta P(\theta, E, \lambda) &= 0 \\ \Delta Q(\theta, E, \lambda) &= 0 \end{aligned} \tag{7}$$

where E and θ represent the phasor bus voltages, as stated above, and λ represents a scalar parameter or loading factor used to simulate the system load changes that drive the system to collapse in the following way:

$$\begin{aligned} P_L &= P_{L_P}(1+k_P\lambda)+P_{L_I}\left(\frac{E}{E_0}\right)(1+k_{V_P}\lambda) \\ &\quad +P_{L_Z}\left(\frac{E}{E_0}\right)^2(1+k_{Z_P}\lambda) \\ Q_L &= Q_{L_Q}(1+k_Q\lambda)+Q_{L_I}\left(\frac{E}{E_0}\right)(1+k_{V_Q}\lambda) \\ &\quad +Q_{L_Z}\left(\frac{E}{E_0}\right)^2(1+k_{Z_Q}\lambda) \end{aligned} \tag{8}$$

Here P_L and Q_L represent the load at bus L, and P_{L_P}, P_{L_I}, P_{L_Z}, k_P, k_{V_P}, k_{Z_P}, Q_{L_Q}, Q_{L_I}, Q_{L_Z}, k_Q, k_{V_Q}, k_{Z_Q}, and E_0 are all pre-defined constants that determine the composition of the ZIP load (constant impedance-current-power load) [26]. Hence, the Jacobian matrices and test function family become functions of the system variables and parameter, i.e., for $z \triangleq [\theta^T \; E^T]^T$, $J = J(z,\lambda)$, $J_{lk} = J_{lk}(z,\lambda)$, and $t_{lk} = t_{lk}(z,\lambda)$. As the parameter λ changes approaching bifurcation, the system variables change, with the critical test function t_{cc} displaying a quadratic or quartic shape [16], as observed in some of the plots of section IV, i.e.,

$$t_{cc}(z,\lambda) \approx a\Delta\lambda^{1/c}$$

where a is a scalar constant, c is either 2 (quadratic) or 4 (quartic), and $\Delta\lambda = \lambda - \lambda_0$ (λ_0 is the maximum loading factor, i.e., the bifurcation value of the system parameter). In general, test functions t_{lk} for buses other than the critical bus c do not display this particular shape. Based on this approximation of t_{cc}, the index of proximity to collapse is defined in [16] as

$$\tau \triangleq -\frac{1}{c}\frac{t_{cc}(z,\lambda)}{\frac{\mathrm{d}t_{cc}}{\mathrm{d}\lambda}(z,\lambda)} \tag{9}$$

This index τ presents in some cases a linear behavior with respect to changes in the loading factor λ, as illustrated in [16]. However, when limits are considered t_{cc} does not exhibit an overall quadratic or quartic shape, as demonstrated for the IEEE 300 bus system below, resulting in an ill-defined τ.

Another difficulty with τ is the problem of determining the critical bus c. This bus can be pinpointed from the maximum entries in the vector $\mathrm{d}z/\mathrm{d}\lambda$ at an operating point (z_1, λ_1) [12, 18], where

$$\frac{\mathrm{d}z}{\mathrm{d}\lambda} \triangleq J^{-1}\left[\begin{array}{c} \left.\frac{\partial \Delta P}{\partial \lambda}\right|_1 \\ \left.\frac{\partial \Delta Q}{\partial \lambda}\right|_1 \end{array}\right]$$

since this "tangent" vector converges to the "zero" right eigenvector x_n at the bifurcation point [11, 29]. However, in general, one needs to be rather close to the bifurcation point to accurately estimate this bus. This problem is depicted below for the IEEE 300 bus test system.

For the reasons stated above, this paper will not compute τ in the examples shown in section IV, and it will only concentrate on studying the different shapes of the family of test functions t_{lk} to compare them to the other indices and test functions presented in the paper.

III. Reduced Load-Flow Jacobian Determinant

To define this index for a given load bus, say l, one starts by assuming that active and reactive power variations occur only at the system bus of interest. Hence, re-ordering the load-flow Jacobian in such a way that the P and Q equations for bus l are the last ones, yields

$$\left[\begin{array}{c} 0 \\ 0 \\ \hline \Delta P_l \\ \Delta Q_l \end{array}\right] = \left[\begin{array}{cc} A & B \\ C & D \end{array}\right]\left[\begin{array}{c} \Delta\theta \\ \Delta E \\ \hline \Delta\theta_l \\ \Delta E_l \end{array}\right] \tag{10}$$

where matrices A, B, C, and D represent the corresponding blocks of the Jacobian matrix J. Observe that D is a 2×2 matrix. Thus, equation (10) can be reduced to

$$\left[\begin{array}{c} \Delta P_l \\ \Delta Q_l \end{array}\right] = D'_{ll}\left[\begin{array}{c} \Delta\theta_l \\ \Delta E_l \end{array}\right] \tag{11}$$

where

$$D'_{ll} \triangleq D - CA^{-1}B \tag{12}$$

Matrix D'_{ll} in (12) can be obtained by a partial factorization of the corresponding load-flow Jacobian J [30], which suggests slightly less computational costs than determining the test function t_{lk} in (4), since it does not require of repeat solutions and matrix and vector products, as shown below. Thus, the difference in computational burden can be approximately quantified by identifying the different steps of the typical procedure one would use to find t_{lk}, i.e.,

1. Order and factor J_{lk} (same cost as that of finding D'_{ll}).
2. Apply a forward and backward substitution (repeat solution) to obtain $a = J_{lk}^{-1}e_l$.
3. Calculate by a matrix-vector product $b = J^T e_l$.
4. Finally, compute by a vector product $t_{lk} = b^T a$.

Based on this procedure, the computational cost of calculating t_{lk} can be estimated as approximately 10% more than what is required for determining D'_{ll}, if efficient ordering and factorization processes are used [30]. Although the improvement is not significant, if several values of the test function or matrix are required, which is typically the case, the difference could amount to a sensible CPU time difference, particularly for large systems.

Observe that the matrix D'_{ll} is well defined at all operating points, since A is guaranteed nonsingular even at the collapse point, as long as bus l has non zero entries in the "zero" eigenvectors of J at the bifurcation point; this is particularly true for $l = c$ (the critical bus). Thus, the determinant of D'_{ll},

$$\det D'_{ll} = \frac{\det J}{\det A} \tag{13}$$

becomes zero only at the bifurcation or collapse point. Monitoring $\det D'_{ll}$ at different operating points for changes in the loading factor λ provides identical information and behavior as the test function t_{ll}, especially for $l = c$, as illustrated for the test system in the next section. Hence, the same performance index τ of equation (9) can be defined using $\det D'_{cc}$ as the test function instead of t_{cc} .

IV. Results and Discussion

In this section, the indices and test functions discussed above are evaluated and compared for the IEEE 300 bus test system, with and without generator limits. Only constant active and reactive power load models were considered in this case for simplicity and comparison purposes. One can easily repeat the same studies for different type of static load models, with similar results.

A. No Limits

First, the test system is studied with no generator limits considered. A load increase direction is chosen, i.e., k_P and k_Q are defined in (8) according to the initial load at each bus. The system is then driven from an initial operating point up to the bifurcation point by changing the loading factor λ, with the help of a continuation power flow [12, 18].

For each operating point, the smallest singular value and absolute eigenvalue are calculated for the load-flow Jacobian J and for the reduced load-flow Jacobian J_{QV}, and the results are depicted in Fig. 1. Observe that the minimum magnitude of the eigenvalues and the minimum singular value of the reduced matrix J_{QV}, exhibit "better" behavior than the corresponding values for the full Jacobian J, i.e., they are more sensitive to changes in λ, which agrees with the results and arguments presented in [13]. However, none of these values can be used as an index of proximity to voltage collapse, due to their sharp drop when close to the bifurcation or collapse point λ_0.

To study the symmetry of the reduced load-flow Jacobian J_{QV} its singular values (Σ) and corresponding eigenvalues (Λ) were studied. The first observation is that all eigenvalues of J_{QV} are real. Furthermore, these eigenvalues are rather close in value to the corresponding singular values of the matrix. To illustrate the latter

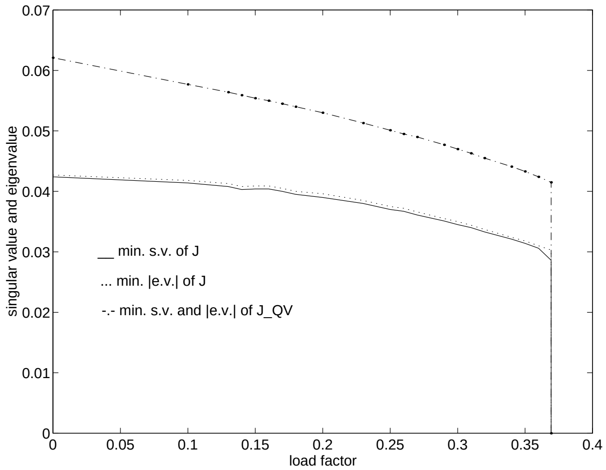
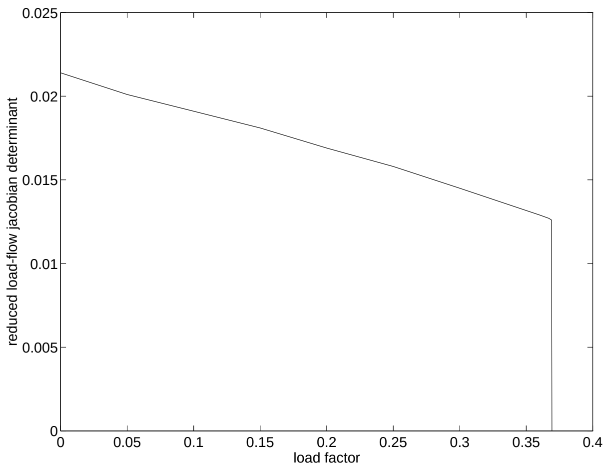


Fig. 1. Minimum singular value and absolute eigenvalue for the test system with no limits.

(a)



(b)

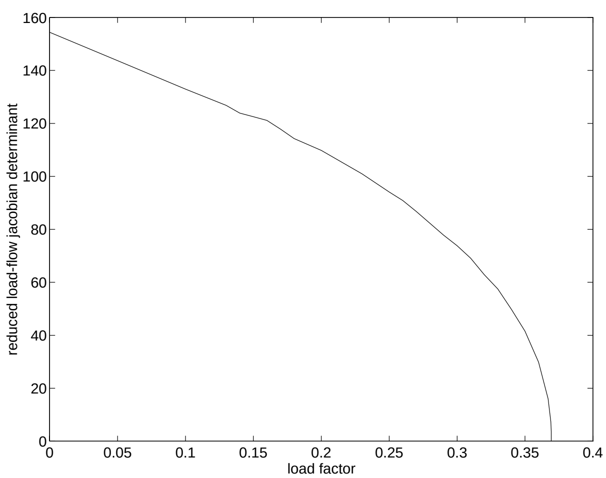


Fig. 2. Test function $\det D'_{ll}$ for the test system with no limits evaluated at (a) a non-critical bus $l = 9042$, and at (b) the critical bus $l = c = 192$.

TABLE I

J_{QV} SYMMETRY FOR SYSTEM WITH NO LIMITS

λ	$\epsilon \times 10^{-5}$
0	1.1792
0.2	1.8465
0.3	2.3538
0.36	2.7038
0.369 (λ_0)	2.7355

the following relative error (ϵ) was defined:

$$\epsilon \triangleq \frac{\| \Sigma + \Lambda \|_\infty}{\| \Sigma \|_\infty} = \frac{\max_i\{\sigma_i + \mu_i\}}{\max_i\{\sigma_i\}} \tag{14}$$

and evaluated for several operating points, up to the critical or maximum loading factor λ_0. The results of these computations are shown in Table I for the system without limits. The value of ϵ increases slightly as the minimum singular value and absolute eigenvalue approach zero; however, overall this value remains rather small, indicating that both singular value and eigenvalue decomposition are basically the same.

Figure 2 illustrates the shape of the test function $\det D'_{ll}$ for the actual critical bus ($c = 192$) and for a bus judged as critical at the initial operating point ($l = 9042$). Notice the striking similarity of the general shapes of this test function with the ones depicted in Fig. 3 for $|t_{ll}|$. In both cases, only the test functions evaluated at the critical bus have the quartic shape used in definition (9) of the performance index τ. Bus 9042 was determined as critical from the largest entries in the tangent vector at the initial operating point. However, bus 192 becomes critical when very close to the collapse point. The actual critical bus in this case could not be determined from the tangent vector information at operating points away from the bifurcation.

The PV curves, nose curves, or bifurcation diagrams of the test system without limits are shown in Fig. 4, to depict the voltage behavior for the buses under study.

B. *Limits*

The test system with all limits considered is employed here to show several of the shortcomings of the indices and test functions described in the paper, which were not originally addressed in [13] and [16].

The minimum singular values and absolute eigenvalues of J and J_{QV} are again monitored for changes in λ, as depicted in Fig. 5, with similar results as in the previous case, i.e., the indices show a sudden large drop when close to the bifurcation point, rendering them inadequate for detecting proximity to a collapse point. Also, the singular values and eigenvalues of J_{QV} are analyzed for symmetry, with similar results as in the case with no limits.

Figures 6 and 7 show the test functions $\det D'_{ll}$ and $|t_{ll}|$, respectively, for two different system buses. Bus 192 corresponds to the system critical bus when no system limits are considered, whereas bus 526 is the actual critical bus of the system when these limits are applied. First, notice the similarity once again of both test functions. Second, the critical bus test functions do not present a quartic or quadratic shape as required for the computation of τ in equation (9); however, they are better behaved than the test function of a bus other than the critical one. In this case, the performance index τ is not well defined due to the discontinuity in $\det D'_{cc}$ and t_{cc} depicted in Figs. 6 and 7. When limits are ignored, the test functions present the desired quartic shape as shown above, and τ is properly defined in that case. Once more, the critical bus can only be detected when the system is very close to the bifurcation point.

Figure 8 depicts the PV curves of the test system with limits to illustrate the voltage behavior for the buses under study. Notice the little change in bus voltage magnitude at bus 192 in this case, as compared to the changes shown in Fig. 4.

(a)

(b)

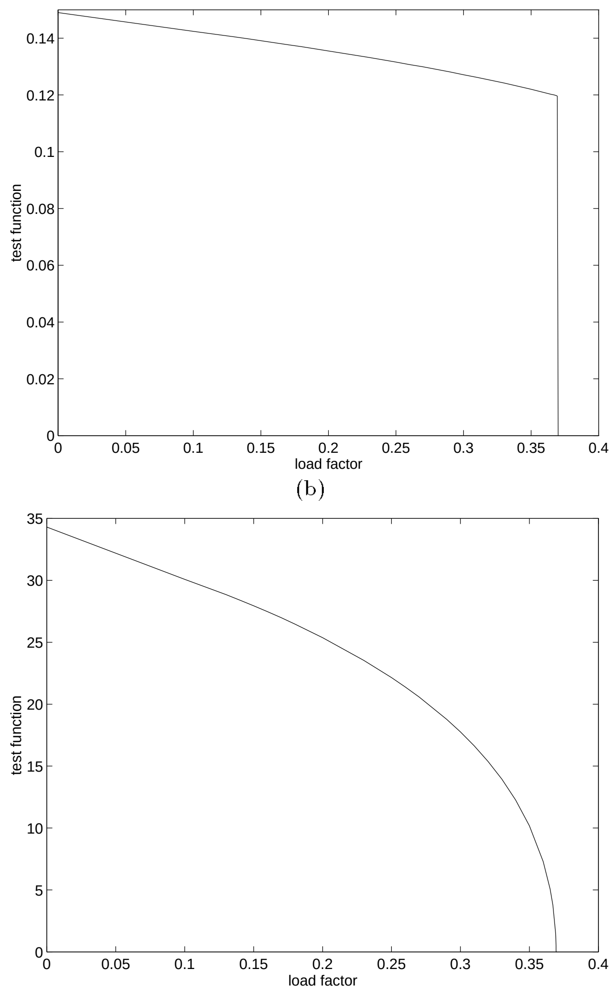


Fig. 3. Absolute value of the test function t_{ll} for the test system with no limits evaluated at (a) a non-critical bus $l = 9042$, and at (b) the critical bus $l = c = 192$.

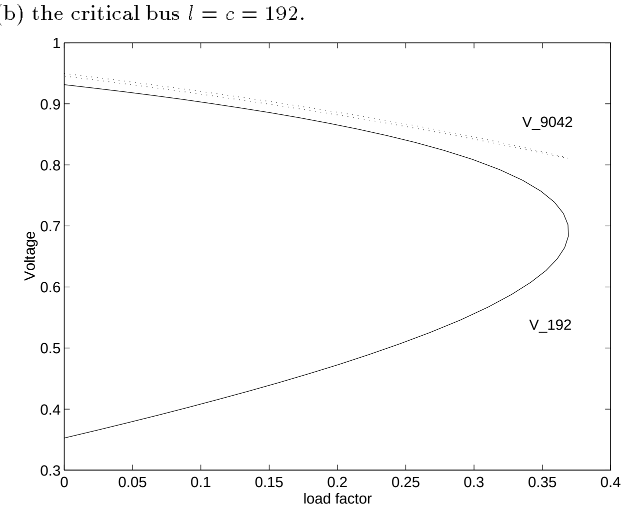


Fig. 4. PV curve or bifurcation diagram for the test system without limits.

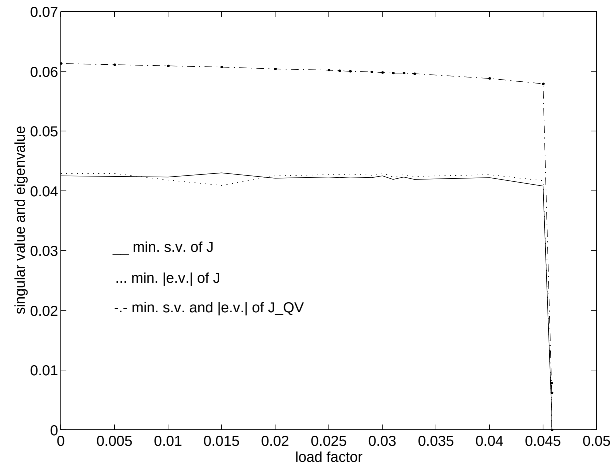


Fig. 5. Minimum singular value and absolute eigenvalue for the test system with limits.

(a)

(b)

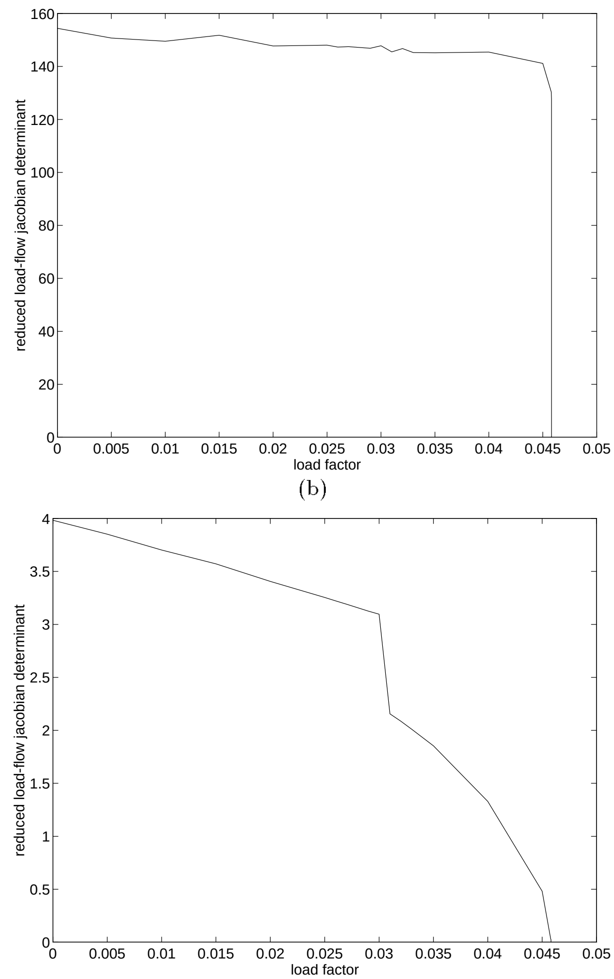


Fig. 6. Test function $\det D'_{ll}$ for the IEEE 300 bus test system evaluated at (a) a non-critical bus $l = 192$, and at (b) the critical bus $l = c = 526$.

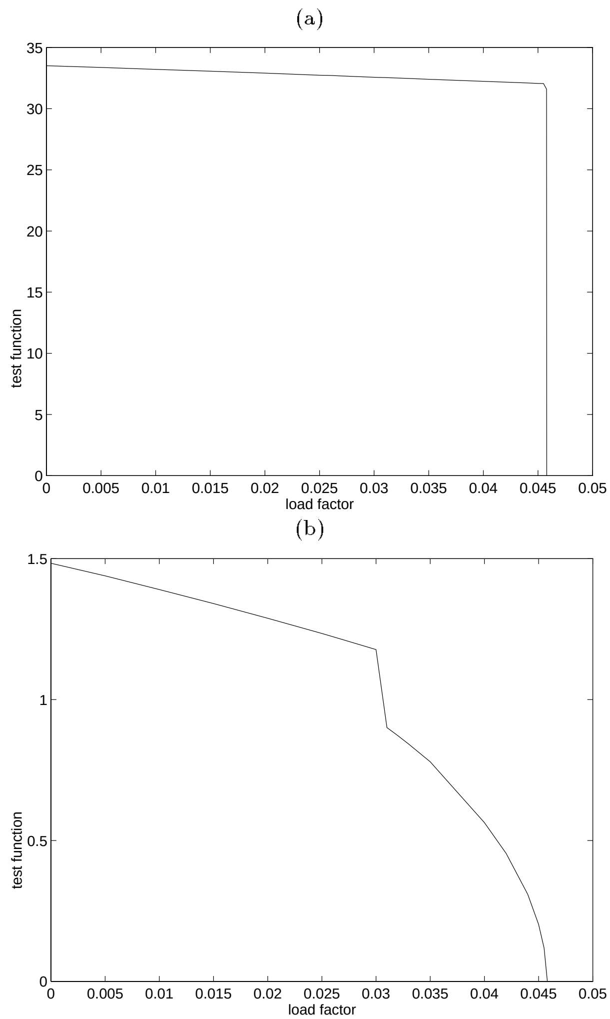


Fig. 7. Absolute value of the test function t_{ll} for the IEEE 300 bus test system evaluated at (a) a non-critical bus $l = 192$, and at (b) the critical bus $l = c = 526$.

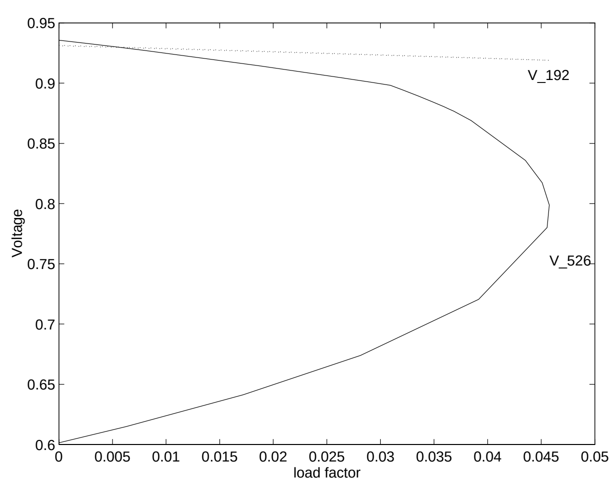


Fig. 8. PV curve or bifurcation diagram for the test system with limits.

V. Conclusions

The paper clearly demonstrates that singular values and eigenvalue decomposition of the reduced Jacobian J_{QV} basically provide the same information for static analysis of voltage collapse problems. This result is based on the quasi-symmetric structure of the matrix J_{QV}, and brings together the two seemingly dissimilar approaches proposed in references [13] and [14]. The paper also shows that the minimum singular value, or absolute eigenvalue for that matter, of J_{QV} is not a good indicator of proximity to the bifurcation or collapse point.

A new family of test functions is proposed. It is shown that these new test functions provide the same information, at slightly reduced computational costs, as the test function used in [16] to define a proximity index. One of the problems with these test functions and proximity index is the need of having to determine the critical bus of the system, since test functions associated to other system buses do not have the required quartic or quadratic shape, and this could represent a serious computational obstacle, due to the need in some cases of having to be close to the bifurcation point in order to pinpoint the true critical bus. Furthermore, discontinuities in the test functions due to system limits may also restrict the possible use of these functions in the study of voltage collapse.

References

[1] "Voltage stability of power systems: Concepts, analytical tools, and industry experience," technical report 90TH0358-2PWR, IEEE/PES, 1990.

[2] Y. Mansour, editor, "Suggested techniques for voltage stability analysis," technical report 93TH0620-5PWR, IEEE/PES, 1993.

[3] H. G. Kwatny, A. K. Pasrija, and L. Y. Bahar, "Static bifurcations in electric power networks: Loss of steady-state stability and voltage collapse," *IEEE Trans. Circuits and Systems*, vol. 33, no. 10, October 1986, pp. 981–991.

[4] I. Dobson and H. D. Chiang, "Towards a theory of voltage collapse in electric power systems," *Systems & Control Letters*, vol. 13, 1989, pp. 253–262.

[5] C. A. Cañizares, "Conditions for saddle-node bifurcations in ac/dc power systems," accepted for publication in the *Int. J. of Electric Power & Energy Systems*, July 1994.

[6] T. J. Overbye and C. L. DeMarco, "Voltage security enhancement using energy based sensitivities," *IEEE Trans. Power Systems*, vol. 6, no. 3, August 1991, pp. 1196–1202.

[7] T. Overbye, *Application of an Energy Based Security Method to Voltage Instability in Electrical Power Systems*, PhD thesis, University of Wisconsin–Madison, 1991.

[8] B. Berggren and G. Andersson, "On the nature of unstable equilibrium points in power systems," *IEEE Trans. Power Systems*, May 1993, pp. 738–745.

[9] A. C. Z. de Souza and V. H. Quintana, "A new technique of network partitioning for voltage collapse margin calculations," accepted for publication in *IEE Proc. Generation, Transmission and Distribution*, June 1994.

[10] T. Van Cutsem, "A method to compute reactive power margins with respect to voltage collapse," *IEEE Trans. Power Systems*, vol. 6, no. 1, February 1991, pp. 145–156.

[11] C. A. Cañizares, F. L. Alvarado, C. L. DeMarco, I. Dobson, and W. F. Long, "Point of collapse methods applied to ac/dc power systems," *IEEE Trans. Power Systems*, vol. 7, no. 2, May 1992, pp. 673–683.

[12] C. A. Cañizares and F. L. Alvarado, "Point of collapse and continuation methods for large ac/dc systems," *IEEE Trans. Power Systems*, vol. 8, no. 1, February 1993, pp. 1–8.

[13] P. A. Löf, T. Smed, G. Anderson, and D.J. Hill, "Fast calculation of a voltage stability index," *IEEE Trans. Power Systems*, vol. 7, no. 1, February 1992, pp. 54–64.

[14] B. Gao, G. K. Morrison, and P. Kundur, "Voltage stability evaluation using modal analysis," *IEEE Trans. Power Systems*, vol. 7, no. 4, November 1992, pp. 1529–1542.

[15] R. A. Schulueter, I. Hu, and M. W. Chang, "Methods for determining proximity to voltage collapse," technical report FEbruary 4-8, IEEE/PES Winter Meeting - Paper 90 WM 096-8 PWRS, 1990.

[16] H. D. Chiang and R. Jean-Jumeau, "Toward a practical performance index for predicting voltage collapse in electric power systems," IEEE/PES 93 SM 512-4 PWRS, Vancouver, BC, Canada, July 1993.

[17] R. Seydel, *From Equilibrium to Chaos—Practical Bifurcation and Stability Analysis.* Elsevier Science, North-Holland, 1988.

[18] C. A. Cañizares, A. Z. de Sousa, and V. H. Quintana, "Improving continuation methods for tracing bifurcation diagrams in power systems," in [31].

[19] A. C. Z. de Souza and V. H. Quintana, "Identification of voltage collapse margins in power systems," *CCECE/CCGEI Proceedings*, Vancouver, Canada, September 1993.

[20] A. C. Z. de Souza and V. H. Quintana, "Identification of voltage collapse using network partitioning," *Proc. NAPS*, Washington, DC, October 1993, pp. 191–196.

[21] N. D. Hatziargyriou and T. Van Cutsem, editors, "Indices predicting voltage collapse including dynamic phenomena," technical report TF 38-02-11, CIGRE, 1994.

[22] M. Suzuki, S. Wada, M. Sato, T. Asano, and Y. Kudo, "Newly developed voltage security monitoring system," *IEEE Trans. Power Systems*, vol. 7, no. 3, August 1992, pp. 965–973.

[23] G. Gollub and C. F. Van Loan, *Matrix Computations.* Johns Hopkins, London, fourth edition, 1993.

[24] B. Noble and J. W. Daniel, *Applied Linear Algebra.* Prentice-Hall, New York, third edition, 1988.

[25] P. W. Sauer, B. C. Lesieutre, and M. A. Pai, "Dynamic vs. static aspects of voltage problems," pp. 207–216 in [32].

[26] C. A. Cañizares, "On bifurcations, voltage collapse and load modeling," IEEE/PES 94 SM 512-4 PWRS, San Francisco, CA, July 1994.

[27] C. A. Cañizares and S. Hranilovic, "Transcritical and Hopf bifurcations in ac/dc systems," in [31].

[28] I. Dobson, "Observations on the geometry of saddle node bifurcations and voltage collapse in electrical power systems," *IEEE Trans. Circuits and Systems—I*, vol. 39, no. 3, March 1992, pp. 240–243.

[29] C. A. Cañizares, *Voltage Collapse and Transient Energy Function Analyses of AC/DC Systems*, PhD thesis, University of Wisconsin–Madison, 1991.

[30] I. S. Duff, A. M. Erisman, and J. K. Reid, *Direct Methods for Sparse Matrices.* Oxford Science Publications, New York, 1986.

[31] L. H. Fink, editor, *Proc. Bulk Power System Voltage Phenomena III—Voltage Stability and Security*, ECC Inc., Davos, Switzerland, August 1994.

[32] L. H. Fink, editor, *Proc. Bulk Power System Voltage Phenomena II—Voltage Stability and Security*, ECC Inc., Fairfax, VA, August 1991.

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