Sensitivity-Indices Based Risk Assessment of Large Scale Solar PV Investment Projects

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***Abstract*—Large scale solar photovoltaic (PV) generation is now a viable, economically feasible and clean energy supply option. Incentive schemes, such as the Feed-in-Tariff (FIT) in Ontario, have attracted large-scale investments in solar PV generation. In a previous work, the authors presented an investor-oriented planning model for optimum selection of solar PV investment decisions. In this paper, a method for determining the sensitivity indices, based on the application of duality theory on the Karush-Kuhn-Tucker (KKT) optimality conditions, pertaining to the solar PV investment model is presented. The sensitivity of the investors' profit to various parameters, for a case study in Ontario-Canada are presented and discussed and these are found to be very close to those obtained using the Monte Carlo simulation and finite-difference (individual parameter perturbation) based approaches. Furthermore, a novel relationship is proposed between the sensitivity indices and the investor's profit for a given confidence level to evaluate the risk for an investor in solar PV projects.**

***Index Terms*—Solar photovoltaic, investor planning, sensitivity indices, duality theory, risk assessment**

I. Nomenclature

The main notations used throughout the paper are stated below for quick reference. In the paper, matrices are in boldface, vectors are in bold and italics, while scalar quantities are in italics.

Subscripts and superscripts

$base$	Base case optimization output
DT	Duality Theory
FD	Finite Difference
MC	Monte Carlo
MIN	Minimum value
MAX	Maximum value
new	Re-optimization output after parameter perturbation
o	Profit at a given Confidence Level
$\%$	Quantity in percentage
$\$$	Quantity in dollars

Indices

b	Index for binary variables
i, j	Index for transmission zones
J	Set of indexes of active inequality constraints
k	Index of years of study horizon
m	Index of all inequality constraints
p	Index of solar PV plan model parameters
$\boldsymbol{p}$	Set of parameters of the solar PV plan model
$\boldsymbol{p}_o$	Set of parameter base values of the solar PV plan model

Functions

$f(\cdot)$	Objective Function
$F(\cdot)$	Cumulative Distribution Function
$\boldsymbol{g}(\cdot)$	Inequality constraints
$\boldsymbol{h}(\cdot)$	Equality constraints

Parameters

$\mathbf{B}_{k,i,j}$	Elements of zonal transmission matrix, p.u.
$\mathbf{Cap}_{k,i}^{Conv}$	Capacity of conventional generation, MW
$\boldsymbol{CF}_i^{Conv}$	Capacity Factor of conventional generation, %
$\boldsymbol{CF}_i^{PV}$	Capacity Factor of PV unit, %
$\boldsymbol{d}_k$	Discount rate, %
DB	Initial number of years of zero investment
$\boldsymbol{FIT}_k$	Feed-in-Tariff, \$/kWh
L	Number of equality constraints
$\mathbf{LbC}_{k,i}^{PV}$	Solar PV installation labour cost, \$/kW
$\mathbf{LdC}_{k,i}^{PV}$	Solar PV installation land cost, \$/kW
M	Number of inequality constraints
m_J	Number of non-zero active inequality constraints
N	Number of years of solar PV plan horizon
n	Number of variables
$\mathbf{OM}_{k,i}^{PV}$	Solar PV operation and maintenance cost, \$/kWh
$\mathbf{P}_{k,i,j}^{MAX}$	Maximum power transfer across zones, MW
$\mathbf{PD}_{k,i}$	Power demand, MW
$\mathbf{TC}_{k,i}^{PV}$	Solar PV unit transportation cost, \$/kW
$\mathbf{UC}_{k,i}^{PV}$	Cost of solar PV unit, \$/kW
$\boldsymbol{\beta}_k$	Annual budget of solar PV investor, \$
β_T	Total budget of solar PV investor, \$
δ_{MIN}	Minimum bus angle, rad
δ_{MAX}	Maximum bus angle, rad
μ	Mean value of a data set
σ	Standard deviation of a data set

Variables

$\mathbf{Cap}_{k,i}^{PV}$	Capacity of solar PV generation, MW
$\mathbf{E}_{k,i}^{PV}$	Annual energy generation from solar PV, MWh
$\mathbf{E}_{k,i}^{Conv}$	Annual energy generation from conventional, MWh
$\mathbf{NC}_{k,i}^{PV}$	New capacity of solar PV generation, MW
$\mathbf{P}_{k,i,j}$	Power transfer across zones, MW

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$\mathbf{P}_{k,i}^{Conv}$ Power generated from conventional sources, MW
$\mathbf{P}_{k,i}^{PV}$ Power generated from solar PV, MW
VaR Value at risk, \$
W_b Binary variables
γ Lagrange multiplier for inequality constraints
$\boldsymbol{\delta}_{k,i}$ Bus angle, rad
λ Lagrange multiplier for equality constraints
ξ Sensitivity index
ρ Confidence level of a portfolio of profit, %
Ω Net present value (NPV) of profit, \$

II. Introduction

The growing costs of fossil fuels coupled with concerns for environmental emissions are strong motivations for increased penetration of renewable sources of energy. Electricity generated from renewable resources, particularly solar photovoltaic (PV) has a large potential in the global energy market. Since PV modules designed for both commercial and residential applications suitable for grid-connected and stand-alone systems have become readily available in the market, investments in solar PV projects have been growing. Favourable government policies, incentives and support mechanisms have fuelled its growth to such an extent that it is now one of the fastest growing alternative energy sources in the world [1]. Minimal running costs, zero emissions and steadily declining module and inverter costs of solar PV units render these attractive to investors for large-scale solar PV generation projects.

In [2], the economic planning of solar PV integration with the power grid presents an assessment of the economic viability of a grid-connected solar PV plant; a relationship for the breakeven capital cost for PV generation as a function of different levels of PV penetration is developed, demonstrating that with higher penetration percentages, the breakeven cost decreases. In [3], the distributed utility concept is explored and a market entry strategy is proposed for the introduction and growth of solar PV in utility applications; a diffusion model strategy is developed that bridges the gap between economic stand-alone special applications and bulk power production. Articles [4], [5], [6], [7], [8] published recently showcase the present scenario of grid integration of solar PV generation in the USA.

An optimization framework, presented in [9], facilitates a prospective investor to arrive at an optimal investment plan in large-scale solar PV generation projects. Optimal decisions on location, sizing and time of investment that yields the maximum profit to the investor are determined. The set of investment decision variables is discrete in nature and thus incorporates binary variables in the optimization model. The objective of the developed mixed integer linear programming (MILP) optimization model is to maximize the Net Present Value (NPV) of investors profit. The data that are used to estimate the parameters of this model are subject to errors, lack of precision, etc.; therefore, conclusions drawn from simulations are sensitive to the choice of input data. Since small changes in input data can have significant effects on the results, it is essential to assess the sensitivity of the results to various model parameters and input data, which is the main objective of the present paper.

In [10] and [11], a perturbation approach to compute sensitivities in optimization based models is discussed. This method is used in [12] to derive general sensitivity formulas for maximum loading conditions in power systems. It provides generalized sensitivity expressions based on the solution of a voltage stability constrained optimal power flow. These sensitivity expressions use the dual variables (Lagrangian multipliers) at the optimal solution and the properties of the Karush-Kuhn-Tucker (KKT) optimality conditions. Reference [13] presents the application of the method [11] to find locational marginal price sensitivities with respect to changes in nodal demands.

In the current paper the concept of local sensitivity analysis based on duality theory (DT), [10] and [11], is applied to determine the plan sensitivity indices pertaining to the solar PV investment model [9]. The sensitivity of the NPV of profit to changes in model input parameters is determined. This approach is computationally less expensive than the Monte Carlo simulation based approach [14] and is also more advantageous than the Finite Difference (FD) approach, [15] and [16], as it determines all the parameter sensitivities simultaneously. Additionally, this method allows determining the sensitivities of the decision variables as well as the Lagrange multipliers with respect to all the model parameters, at once. However, since this analysis is based on a linearization approach, it may not yield accurate results for large variations in the input data for a nonlinear system model; therefore a comparison of the sensitivities obtained using Monte Carlo simulation and FD approaches are presented. Furthermore, the solar PV investment risk is assessed with respect to various parameters based on Value at Risk (VaR) and Confidence Level (CL) indices, which are determined from the DT based sensitivity indices.

The rest of this paper is organized as follows: Section III provides a review of the solar PV investment model developed in [9]. Section IV presents the methodology and formulation of calculation of sensitivity indices using DT, FD and Monte Carlo approaches. Section V presents the development of the relationship of investment risk indices with the proposed sensitivity indices. In Section VI, the sensitivity indices are computed and validated based on MC simulations and FD calculations, and the derived relationships in Section V are used to asses the risk levels with respect to various parameters. Finally, Section VII highlights the conclusions and contributions from this work.

III. The Solar PV Investment Model

A solar PV investment model, from the perspective of an investor, reported in [9], is briefly discussed next for the sake of completeness. The model is a MILP problem comprising continuous and binary variables. The objective of the model is to maximize Ω and hence determine the optimal set of solar PV investment decisions, where:

$$\Omega = \sum_{k} \frac{\sum_{i} (Revenue_{k,i} - Cost_{k,i})}{(1 + \boldsymbol{d}_k)^k} \tag{1}$$

and

$$Revenue_{k,i} = \mathbf{FIT_k}\ \mathbf{E}^{PV}_{k,i} \tag{2}$$

$$Cost_{k,i} = \mathbf{CC}^{PV}_{k,i}\ \mathbf{NC}^{PV}_{k,i} + \mathbf{OM}^{PV}_{k,i}\ \mathbf{E}^{PV}_{k,i} \tag{3}$$

Here $\mathbf{NC}^{PV}_{k,i} = \sum_b 5W_b$ is an integer variable, constructed using the auxiliary binary variables W_b. Note that all variables, parameters and indices in these and following equations are defined in Section I. In (3), the Capital Cost (CC) is described as:

$$\mathbf{CC}^{PV}_{k,i} = \mathbf{UC}^{PV}_{k,i} + \mathbf{LbC}^{PV}_{k,i} + \mathbf{TC}^{PV}_{k,i} + \mathbf{LdC}^{PV}_{k,i} \tag{4}$$

The various operational, planning and financial constraints of the MILP model are:

- Supply demand balance:

$$\mathbf{P}^{Conv}_{k,i} + \mathbf{P}^{PV}_{k,i} - \mathbf{PD}_{k,i} = \sum_j \mathbf{B}_{k,i,j}\ \left(\boldsymbol{\delta}_{k,i} - \boldsymbol{\delta}_{k,j}\right) \tag{5}$$

- Conventional Energy Generation Limit:

$$\mathbf{E}^{Conv}_{k,i} \leq\ 8760\ \mathbf{Cap}^{Conv}_{k,i}\ \boldsymbol{CF}^{Conv}_i \tag{6}$$

- Solar PV Energy Generation Limit:

$$\mathbf{E}^{PV}_{k,i} \leq\ 8760\ \mathbf{Cap}^{PV}_{k,i}\ \boldsymbol{CF}^{PV}_i \tag{7}$$

- Transmission Line Flow Limits:

$$\mathbf{B}_{k,i,j}\ \left(\boldsymbol{\delta}_{k,i} - \boldsymbol{\delta}_{k,j}\right) \leq\ \mathbf{P}^{MAX}_{k,i,j} \tag{8}$$

- Power angle limits:

$$\delta_{MIN} \leq \boldsymbol{\delta}_{k,i} \leq \delta_{MAX} \tag{9}$$

- Annual budget limit:

$$\sum_i \mathbf{CC}^{PV}_{k,i}\ \mathbf{NC}^{PV}_{k,i} \leq\ \boldsymbol{\beta}_k \tag{10}$$

- Total budget limit:

$$\sum_k \sum_i \left(\mathbf{CC}^{PV}_{k,i}\ \mathbf{NC}^{PV}_{k,i}\ +\ \mathbf{OM}^{PV}_{k,i}\ \mathbf{E}^{PV}_{k,i}\right) \leq \beta_T \tag{11}$$

- Dynamic Constraint on Solar PV Capacity Addition:

$$\mathbf{Cap}^{PV}_{k+1,i}\ =\ \mathbf{Cap}^{PV}_{k,i}\ +\ \mathbf{NC}^{PV}_{k,i}\quad \forall\ k = 1, 2, \cdots, (N-1) \tag{12}$$

- Initial Year Investment Constraint:

$$\mathbf{Cap}^{PV}_{k+1,i}\ =\ 0\ \ \forall\ k = 1, 2, \cdots, DB \tag{13}$$

- Terminal Year Investment Constraint:

$$\mathbf{Cap}^{PV}_{k+1,i}\ \leq\ \mathbf{Cap}^{PV}_{k,i}\quad \forall\ k\ \geq\ N \tag{14}$$

- Solar PV Lifetime Constraint:

$$\mathbf{Cap}^{PV}_{k+Z+1,i} = \mathbf{Cap}^{PV}_{k+Z,i} - \mathbf{NC}^{PV}_{k,i}\quad \forall\ k = 1, 2, \cdots, (N-1) \tag{15}$$

The interested reader is referred to [9] for more detailed information on the selection of the model parameters.

IV. Calculation of Sensitivity Indices

A. Duality Theory

A description of the duality theory based method for obtaining local sensitivities of all the parameters is discussed in [10] and [11], and is briefly explained next, in the context of the aforementioned model. Thus, let consider a primal Non-Linear Programming (NLP) problem as follows:

$$\begin{aligned} Min.\quad & z = f(\boldsymbol{x}, \boldsymbol{a}) \\ s.t.\quad & \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{a}) = \boldsymbol{b} \\ & \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{a}) \leq \boldsymbol{c} \end{aligned} \tag{16}$$

where $\boldsymbol{h}(\boldsymbol{x}, \boldsymbol{a}) = [h_1(\boldsymbol{x}, \boldsymbol{a}), \cdots, h_L(\boldsymbol{x}, \boldsymbol{a})]^T$ and $\boldsymbol{g}(\boldsymbol{x}, \boldsymbol{a}) = [g_1(\boldsymbol{x}, \boldsymbol{a}), \cdots, g_M(\boldsymbol{x}, \boldsymbol{a})]^T$. In order to simplify the mathematical derivations, the parameters $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are assumed to be subsets of a set $\boldsymbol{p}$, i.e., $\boldsymbol{p} = [\boldsymbol{a}\ \ \boldsymbol{b}\ \ \boldsymbol{c}]^T$. The sensitivities of the optimal solution $(\boldsymbol{x}^*, \boldsymbol{\lambda}^*, \boldsymbol{\gamma}^*, z^*)$ with respect to local changes in the parameter $\boldsymbol{p}$ can be obtained by differentiating the objective function and the KKT conditions of optimality of the NLP model (16). Thus, the matrix with all derivatives, i.e., the sensitivity indices, is given by:

$$\left[\frac{d\boldsymbol{x}}{d\boldsymbol{p}}\quad \frac{d\boldsymbol{\lambda}}{d\boldsymbol{p}}\quad \frac{d\boldsymbol{\gamma}}{d\boldsymbol{p}}\quad \frac{dz}{d\boldsymbol{p}}\right]^T = \mathbf{U}^{-1}\mathbf{S} \tag{17}$$

where

$$\mathbf{U} = \begin{bmatrix} \boldsymbol{F_x} & \mathbf{0} & -1 \\ \boldsymbol{F_{xx}} & \boldsymbol{H_x^T} & 0 \\ \boldsymbol{H_x} & \mathbf{0} & 0 \end{bmatrix} \tag{18}$$

$$\mathbf{S} = -\left[\boldsymbol{F_p}\ \boldsymbol{F_{xp}}\ \boldsymbol{H_p}\right]^{\mathbf{T}} \tag{19}$$

and

$$\boldsymbol{F}_{\boldsymbol{x}(1\times n)} = [\nabla_{\boldsymbol{x}} f(\boldsymbol{x}^*, \boldsymbol{p})]^T \tag{20}$$

$$\boldsymbol{F}_{\boldsymbol{p}(1\times p)} = \left[\nabla_{\boldsymbol{p}} f(\boldsymbol{x}^*, \boldsymbol{p})\right]^T \tag{21}$$

$$\boldsymbol{H}_{\boldsymbol{x}(L\times n)} = [\nabla_{\boldsymbol{x}} \boldsymbol{h}(\boldsymbol{x}^*, \boldsymbol{p})]^T \tag{22}$$

$$\boldsymbol{H}_{\boldsymbol{p}(L\times p)} = \left[\nabla_{\boldsymbol{p}} \boldsymbol{h}(\boldsymbol{x}^*, \boldsymbol{p})\right]^T \tag{23}$$

$$\begin{aligned} \boldsymbol{F}_{\boldsymbol{xx}(n\times n)} = \nabla_{\boldsymbol{xx}} f(\boldsymbol{x}^*, \boldsymbol{p}) &+ \sum_{l=1}^{L} \lambda_l^* \nabla_{\boldsymbol{xx}} h_l(\boldsymbol{x}^*, \boldsymbol{p}) \\ &+ \sum_{m=1}^{m_J} \gamma_m^* \nabla_{\boldsymbol{xx}} g_m(\boldsymbol{x}^*, \boldsymbol{p}) \end{aligned} \tag{24}$$

$$\begin{aligned} \boldsymbol{F}_{\boldsymbol{xp}(n\times n)} = \nabla_{\boldsymbol{xp}} f(\boldsymbol{x}^*, \boldsymbol{p}) &+ \sum_{l=1}^{L} \lambda_l^* \nabla_{\boldsymbol{xp}} h_l(\boldsymbol{x}^*, \boldsymbol{p}) \\ &+ \sum_{m=1}^{m_J} \gamma_m^* \nabla_{\boldsymbol{xp}} g_m(\boldsymbol{x}^*, \boldsymbol{p}) \end{aligned} \tag{25}$$

The matrix $\mathbf{U}$ in (17) is generally invertible, as the solution to the optimization problem $(\boldsymbol{x}^*, \boldsymbol{\lambda}^*, \boldsymbol{\gamma}^*, z^*)$ is a regular non-degenerate point. If any degenerate constraint, i.e., a zero-valued Lagrange multiplier of an active constraint, is present, then the corresponding rows and columns in the matrices $\mathbf{U}$ and $\mathbf{S}$ are removed [12], making $\mathbf{U}$ invertible. Additionally, the non-degenerate inequality constraints are converted to equality constraints [10].

This method provides, in one shot, the sensitivity indices for the objective function $\xi_{DT} = \frac{dz}{d\boldsymbol{p}}$, as well as the primal $\frac{d\boldsymbol{x}}{d\boldsymbol{p}}$ and dual variables, $\frac{d\boldsymbol{\lambda}}{d\boldsymbol{p}}$ and $\frac{d\boldsymbol{\gamma}}{d\boldsymbol{p}}$, sensitivities with respect to the model parameters. The units of ξ_{DT} are \$/(unit of the parameter p_0), which can be appropriately normalized to represent in terms of \$/(1% change in p_0). This way, the sensitivity indices are scaled appropriately in order to compare and rank them with respect to their severity. Additionally, the dollar value of ξ_{DT} can be represented as a percentage of Ω_{base}, i.e.,

$$\xi^{\%}_{DT} = 100 \left(\frac{\xi^{\$}_{DT}}{\Omega_{base}} \right) \tag{26}$$

where Ω_{base} is the NPV of the profit obtained from solving the solar PV investment model in Section III.

To apply the duality theory based sensitivity method to the solar PV investment model, the variables $\boldsymbol{x}$ and parameters $\boldsymbol{p}$ are defined next. Thus, from the model discussed in Section III, the vector $\boldsymbol{x}$ of model variables are identified as follows:

$$\boldsymbol{x} = \left[\mathbf{E}^{PV}_{k,i} \; \mathbf{NC}^{PV}_{k,i} \; \mathbf{Cap}^{PV}_{k,i} \; \mathbf{E}^{Conv}_{k,i} \; \boldsymbol{\delta}_{k,i} \right]^T \tag{27}$$

and the vector of model parameters are defined as:

$$\begin{aligned} \boldsymbol{p} = \Big[& \mathbf{B}_{k,i,j} \; \mathbf{Cap}^{Conv}_{k,i} \; \boldsymbol{CF}^{Conv}_{i} \; \boldsymbol{CF}^{PV}_{i} \; \boldsymbol{d}_k \; \boldsymbol{FIT}_k \\ & \mathbf{LbC}^{PV}_{k,i} \; \mathbf{LdC}^{PV}_{k,i} \; \mathbf{OM}^{PV}_{k,i} \; \mathbf{P}^{MAX}_{k,i,j} \; \mathbf{PD}_{k,i} \\ & \mathbf{TC}^{PV}_{k,i} \; \mathbf{UC}^{PV}_{k,i} \; \boldsymbol{\beta}_k \; \beta_k \; \delta_{MIN} \; \delta_{MAX} \Big]^T \end{aligned} \tag{28}$$

B. Monte Carlo and Finite Difference

These sensitivities, computed based on a linearized approach, are validated here using the well-known Monte Carlo simulation approach and a FD approach. This allows to determine their range of application.

The Monte Carlo simulation is applied to the optimization model on an OAT (One-factor-at-A-Time) basis. The parameters need to be perturbed symmetrically around their base values in order to obtain unskewed and unbiased sensitivity indices. In a normally distributed probability density function (p.d.f.) of a parameter, the expected value (mean) is the same as the median of the distribution; this allows for a symmetrical variation of the parameter for various standard deviations. Additionally, for normally distributed p.d.f., the mode of the variation is also equal to its median, which enables equal perturbation of a parameter around its deterministic value. Therefore, all the parameters in this paper, are considered to be normally distributed for probabilistic studies with the standard deviation being 1% of its mean value, i.e.,

$$\sigma^{\%}_{p} = 100 \left(\frac{\sigma_p}{\mu_p} \right) \tag{29}$$

where $\mu_p = p_0$.

The solar PV investment model optimization program is run for 2000 iterations with the normally distributed parameter keeping other parameters unaltered. The standard deviation of the resulting Ω is computed as a percentage of its mean value, as follows:

$$\sigma^{\%}_{\Omega} = 100 \left(\frac{\sigma^{\$}_{\Omega}}{\mu^{\$}_{\Omega}} \right) \tag{30}$$

The ratio of the standard deviations (in percentage) of output (Ω) and input (a parameter) is indicative of the sensitivity index computed by the Monte-Carlo method:

$$\xi^{\%}_{MC} = \frac{\sigma^{\%}_{\Omega}}{\sigma^{\%}_{p}} \tag{31}$$

i.e., for 1% standard deviation in input parameter $\xi^{\%}_{MC} = \sigma^{\%}_{\Omega}$.

The sensitivity index computed using Monte Carlo simulation based approach, can also be represented as follows:

$$\xi^{\$}_{MC} = \frac{\sigma^{\$}_{\Omega}}{\sigma^{\%}_{p}} \tag{32}$$

Thus, from (31) and (32):

$$\xi^{\%}_{MC} = 100 \left(\frac{\xi^{\$}_{MC}}{\mu^{\$}_{\Omega}} \right) \tag{33}$$

For the FD approach, each parameter is increased by 1% of its base value (p_0) and the Ω is computed again from the solar PV investment model (Ω_{new}). The difference between Ω_{new} and Ω_{base} denotes the change in Ω for a 1% increase in a given parameter while other parameters remain unaltered. This change in Ω is, in essence, the true sensitivity index for the said parameter with respect to Ω, i.e.,

$$\Omega_{new} - \Omega_{base} = \xi^{\$}_{FD} \tag{34}$$

The sensitivity index from (34) can also be represented as a percentage of Ω_{base} using the following expression:

$$\xi^{\%}_{FD} = \frac{\Omega_{new} - \Omega_{base}}{\Omega_{base}} \tag{35}$$

V. Risk Analysis using Sensitivity Indices

In risk analysis, VaR is a measure that estimates how much a portfolio could lose because of market movements for a given probability of occurrence of that portfolio variable [17], and it is referred to as CL. VaR and CL are computed from the cumulative distribution function (c.d.f.) constructed from the p.d.f. of the output quantity. In the context of this paper, the output quantity is Ω and the portfolio is the normally distributed p.d.f. of Ω. Thus, from the c.d.f. of Ω, a given profit Ω_0 with a corresponding cumulative probability $F(\Omega_0)$, indicates confidence level $\rho = 1 - F(\Omega_0)$, which means that there is a ρ% likelihood that $\Omega \geq \Omega_0$ and the VaR from the expected profit $\left(\mu_{\Omega}{}^{\$}\right)$ is given by:

$$VaR = \mu^{\$}_{\Omega} - \Omega_0 \tag{36}$$

Monte Carlo simulations are typically used to compute the VaR and CL of the investment portfolio. However, this involves significant computational burden arising from the large number of simulation cases required and from establishing the c.d.f. of the investment portfolio. It is demonstrated next, through mathematical derivations, that the proposed sensitivity index $\xi^{\$}_{DT}$ can be directly utilized to compute the risk parameters VaR and CL, and hence reduce the computational effort required to determine these values.

If a linear relationship exists between an input parameter and the output variable, the p.d.f. of the output is expected to remain similar to the input p.d.f., and the mean of the input distribution is expected to generate the mean of the output distribution. In this paper, the mean of the normally distributed p.d.f. of a parameter is considered to be the base value of the parameter (p_0) being perturbed. Hence, the mean of Ω from every Monte-Carlo simulation output will generally be equal to Ω_{base}, i.e.,

$$\mu^{\$}_{\Omega} = \Omega_{base} \tag{37}$$

Then, from (26) and (33), the following can be obtained:

$$\frac{\xi^{\%}_{MC}}{\xi^{\%}_{DT}} = \frac{\xi^{\$}_{MC}}{\xi^{\$}_{DT}}\left(\frac{\Omega_{base}}{\mu^{\$}_{\Omega}}\right) \tag{38}$$

Thus, using (37) in (38), the following relations are proposed:

$$\xi^{\$}_{MC} \cong \left|\xi^{\$}_{DT}\right| \tag{39}$$

and

$$\xi^{\%}_{MC} \cong \left|\xi^{\%}_{DT}\right| \tag{40}$$

From (26) and (31) in (40), the following relation holds:

$$\frac{\sigma^{\%}_{\Omega}}{\sigma^{\%}_{p}} = \xi^{\%}_{MC} \cong \left|\xi^{\%}_{DT}\right| = 100\left(\frac{\left|\xi^{\$}_{DT}\right|}{\Omega_{base}}\right) \tag{41}$$

And from (30) in (41) and rearranging terms, one has that:

$$100\left(\frac{\sigma^{\$}_{\Omega}}{\mu^{\$}_{\Omega}}\right) = 100\ \sigma^{\%}_{p}\left(\frac{\left|\xi^{\$}_{DT}\right|}{\Omega_{base}}\right) \tag{42}$$

From (37) in (42), a relationship between the standard deviation of the output variable with the sensitivity index of the perturbed parameter is obtained, as follows:

$$\sigma^{\$}_{\Omega} = \sigma^{\%}_{p}\ \left|\xi^{\$}_{DT}\right| \tag{43}$$

If more than one input parameters are perturbed, the first term of the product on the right hand side of the equality in (43), i.e., the standard deviation of the input parameter perturbation in percentage, can be represented by the equivalent standard deviation of the parameters perturbed based on a well-known expression from multivariate normal distributions [18]. The second term of the product can then be replaced by the weighted average of the sensitivity indices of the parameters that are perturbed. Hence, the standard deviation of the output can be represented in generic form as follows:

$$\sigma^{\$}_{\Omega} = \sqrt{\sum_{p}\left(\sigma^{\%}_{p}\right)^2}\left(\frac{\sum_{p}\sigma^{\%}_{p}\ \left|\xi^{\$}_{DT_p}\right|}{\sum_{p}\sigma^{\%}_{p}}\right) \tag{44}$$

Note that (43) is a specific case of (44) when $p = 1$, i.e., when only one input parameter is perturbed. The relationship described in (44) is very important as it allows computing the standard deviation of a normally distributed output variable resulting from the input of one or more normally distributed parameter(s) without the need to carryout the computationally expensive Monte Carlo simulations.

A c.d.f. of a normally distributed p.d.f. of Ω is shown in Fig. 1. Although the c.d.f. extends to the left asymptotically, it is assumed that $F(\Omega_{MIN}) = 0$, neglecting the asymptotic nature of the curve in that region. A linear approximation of the c.d.f.

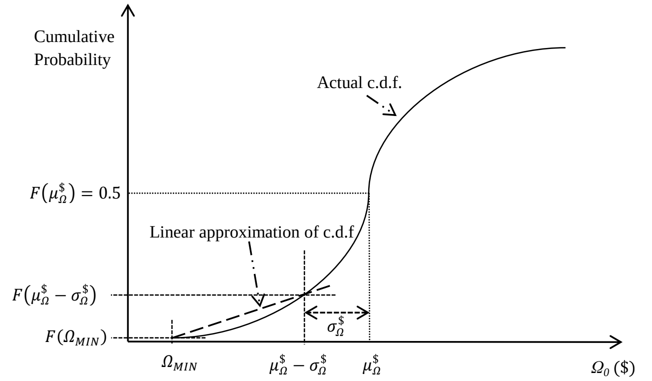


Fig. 1. Typical c.d.f. plot of depicting the linear approximation segment.

in the region $\Omega \in \left[\Omega_{MIN}, \left(\mu^{\$}_{\Omega} - \sigma^{\$}_{\Omega}\right)\right]$ can be represented by:

$$\Omega_0 = \Omega_{MIN} + \frac{\left(\mu^{\$}_{\Omega} - \ \sigma^{\$}_{\Omega}\right) - \Omega_{MIN}}{F\left(\mu^{\$}_{\Omega} - \sigma^{\$}_{\Omega}\right)}\left(1 - \frac{\rho}{100}\right) \tag{45}$$

A standard normal p.d.f. has $\mu = 0$ and $\sigma = 1$ and assuming a variable range $[-\tau, \tau]$, where $F(-\tau) \approx 0$, a relationship between $\mu^{\$}_{\Omega}$ and Ω_{MIN} can then be obtained, as follows:

$$\Omega_{MIN} = \mu^{\$}_{\Omega} - \tau\ \sigma^{\$}_{\Omega} \tag{46}$$

From the Standard Normal Cumulative Distribution Function Table [18], one has for $\tau = 4$, $F(-4) \approx 0$. The value of $\mu^{\$}_{\Omega} - \sigma^{\$}_{\Omega}$ corresponds to one standard deviation below the mean and thus, the value of $F\left(\mu^{\$}_{\Omega} - \sigma^{\$}_{\Omega}\right)$ is as follows:

$$F\left(\mu^{\$}_{\Omega} - \sigma^{\$}_{\Omega}\right) = F(-1) = \alpha \tag{47}$$

Using the same table given in [18], it is obtained that $\alpha =\ 0.1587$. Thus, finally, the relationship of Ω_0 for a CL of $\rho\%$ with $\xi^{\$}_{DT}$ proposed here, replacing, (37), (46) and (47) in (45), is as follows:

$$\Omega_0 = \Omega_{base} - \tau\ \sigma^{\$}_{\Omega} + \sigma^{\$}_{\Omega}\left(\frac{\tau - 1}{\alpha}\right)\left(1 - \frac{\rho}{100}\right) \tag{48}$$

For $\tau = 4$ and $\alpha = 0.1587$:

$$\Omega_0 = \Omega_{base} - 4\ \sigma^{\$}_{\Omega} + 18.904\ \sigma^{\$}_{\Omega}\left(1 - \frac{\rho}{100}\right) \tag{49}$$

The proposed equations (36), (44) and (49) show that for a linear optimization problem, the VaR for a given CL can be closely estimated using the DT-based sensitivity analysis without actually running the computationally expensive Monte Carlo simulations.

VI. Results and Discussions

A. Sensitivity Indices

The optimization model described in [9] is solved using the CPLEX solver [19] in the GAMS [20] environment with relative optimality tolerance of 0.1%. The input parametric data is taken for the case of Ontario from [9] and extrapolated for a study period of 35 years (2009 - 2043). The model determines that all solar PV generation are to be installed in the Bruce zone, 35 MW of installed solar PV

TABLE I
SENSITIVITY INDICES OF PARAMETERS IN DOLLARS

Parameters	$\xi_{DT}^{\$}$ \$/1\%p_0	$\xi_{MC}^{\$}$ \$/1\%p_0	$\xi_{FD}^{\$}$ \$/1\%p_0
FIT	9,733,465	9,706,000	9,733,464
d	-8,946,481	8,943,838	-8,882,920
CF_{Bruce}^{PV}	7,351,319	7,442,591	7,351,315
β_T	6,290,000	6,215,678	6,114,413
UC^{PV}	-5,543,933	5,546,823	-5,543,933
OM^{PV}	-2,382,150	2,382,665	-2,360,953
LbC^{PV}	-729,100	729,150	-729,100
LdC^{PV}	-280,302	280,497	-280,302
TC^{PV}	-22,421	22,115	-22,421

TABLE II
SENSITIVITY INDICES OF PARAMETERS IN PERCENTAGE

Parameters	$\xi_{DT}^{\%}$	$\xi_{MC}^{\%}$	$\xi_{FD}^{\%}$
	$\%of\,\Omega_{base}/1\%p_0$		
FIT	1.3776	1.3736	1.3776
d	-1.2662	1.2659	-1.2572
CF_{Bruce}^{PV}	1.0405	1.0533	1.0405
β_T	0.8903	0.8799	0.8654
UC^{PV}	-0.7847	0.7851	-0.7847
OM^{PV}	-0.3372	0.3372	-0.3342
LbC^{PV}	-0.1032	0.1032	-0.1032
LdC^{PV}	-0.0397	0.0397	-0.0397
TC^{PV}	-0.0032	0.00313	-0.0032

capacity during the fourth to sixth year, and 25 MW in the seventh year of the plan horizon. The resulting (Ω_{base}) is \$706.54 Million. The optimal solution so obtained, identifies both zero and non-zero Lagrange multipliers associated with both equality and inequality constraints. The constraints pertaining to non-zero Lagrange multipliers are used for computing the sensitivities based on the DT method, as explained in Section IV-A. The variables $\mathbf{E}_{k,i}^{Conv}$ and $\boldsymbol{\delta}_{k,i}$ prove to be non-basic and have no significant impact on the PV investment, as it is evident from the zero valued Lagrangian multipliers associated with appropriate constraints.

A MATLAB code is developed using Symbolic Math Toolbox [21] and the functions pertaining to $f(\boldsymbol{x},\boldsymbol{p})$ and $\boldsymbol{h}(\boldsymbol{x},\boldsymbol{p})$ are fed into it. The matrices $\mathbf{U}$ and $\mathbf{S}$, given by (18) and (19), are computed symbolically in MATLAB, and then the numerical values of parametric data and active Lagrange multipliers are substituted to compute the sensitivity indices. The sensitivity indices are ranked as per their severity in Table I. The table shows the dollar amount by which Ω_{base} changes for a 1% increase in the base value (p_0) of a parameter. This table also presents a comparison of the sensitivity indices, in dollars, computed using the Monte Carlo simulation approach and the FD method. Similar results are shown in Table II for the sensitivity indices computed using the three methods as a percentage of Ω_{base} for a 1% increase in the base value of a parameter. From Tables I and II, it is found that the profit of an investor is most sensitive to the Feed-in-Tariff rate. Observe that both the FD method and the DT based method yield the proper signs depicting the increment or decrement of the Ω_{base} value with respect to an increase in the parameter value; on the other hand, the Monte Carlo simulation approach failed to provide this information due to the fact that the sensitivity is computed from Monte Carlo simulations as a ratio of the output and input standard deviations, and standard deviations always have positive values. Note as well in Table I and II that the sensitivity indices computed using the DT based method are numerically very close to the true sensitivity indices computed using the FD method as well as those obtained from the Monte Carlo simulation approach. This validates the relationships of (39) and (40) for the given model.

TABLE III
COMPARISON OF CALCULATED AND ACTUAL STANDARD DEVIATIONS

Input Perturbation $\sigma_p^{\%}$				Actual $\sigma_{\Omega}^{\$}$	Calculated $\sigma_{\Omega}^{\$}$
FIT	d	CF_{Bruce}^{PV}	β_T	(Million \$)	(Million \$)
1	0	0	0	9.640	9.734
2	0	0	0	19.271	19.466
3	0	0	0	28.907	29.200
0	1	0	0	8.854	8.946
0	2	0	0	17.711	17.893
0	3	0	0	26.578	26.839
0	0	1	0	7.273	7.351
0	0	2	0	13.753	14.703
0	0	3	0	19.007	22.054
1	1	0	0	12.902	13.209
1	1	1	0	14.728	15.029
1	1	1	1	16.033	16.161
3	2	1	0	34.497	33.952
4	3	2	1	48.630	46.942

B. *Risk Analysis*

The standard deviation of the output variable Ω when any number of input parameter(s) are perturbed with $\sigma_p^{\%}$ can be computed using (44). A comparison between the calculated standard deviation of versus the actual value obtained from Monte Carlo simulation is presented in Table III. Note that these results validate the proposed expression (44) for various combinations of input parameter perturbations, thus justifying the use of (49) to compute the VaR for various CLs.

Table IV presents a comparison of calculated Ω_0 using (49) versus the actual Ω_0 obtained from Monte Carlo simulations Ω_0^{MC} for a selected set of parameters being perturbed and for various values of CL. Observe that the relation proposed in (49) yields a fairly close value to the actual Ω_0. It is to be noted that the relation in (49) provides a more conservative estimate of Ω_0 for each confidence level. These conservative estimates of profit are particularly beneficial to solar PV investors to evaluate their risk strategies, i.e., their VaR and the expected rate of return, and thus arrive at an appropriate investment decision.

The percentage errors between Ω_0 computed from (49) and the actual values Ω_0^{MC} obtained from the c.d.f. plot resulting from the Monte Carlo simulations with respect to ρ is shown in Fig. 2 for four parameters, namely, FIT, d, and

TABLE IV
COMPARISON OF NPV OF PROFIT FOR VARIOUS CLS

Parameter Perturbed by 1%	ρ	Ω_0^{MC} $Million\$$	Ω_0 $Million\$$
FIT	99%	684.1	674.01
	95%	690.6	680.14
	90%	694.4	687.81
	85%	696.9	695.47
d	99%	686.3	676.64
	95%	691.8	682.28
	90%	695.3	689.32
	85%	697.3	696.37
CF_{Bruce}^{PV}	99%	689.4	681.97
	95%	694.5	686.6
	90%	697.4	692.39
	85%	699.3	698.18
β_T	99%	692.1	685.52
	95%	695.5	689.48
	90%	698.3	694.43
	85%	700.1	699.39

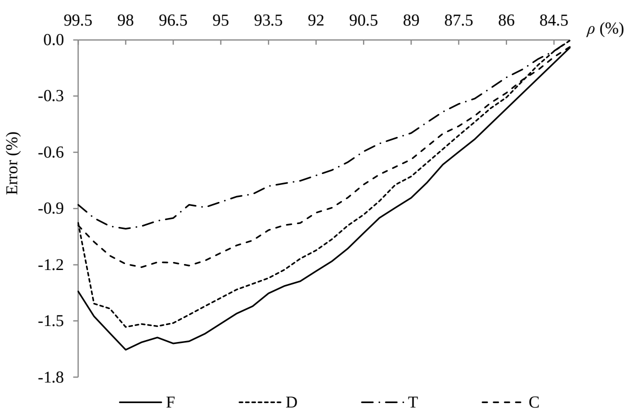


Fig. 2. Percentage error versus confidence level when parameters are perturbed by 1% standard deviation (F: FIT; D: Discount Rate; C: CF^{PV} (Bruce); T: Total Budget).

CF_{Bruce}^{PV}, individually perturbed by 1% from their base values $\left(\sigma_p^{\%} = 1\%\right)$. Note that the error values are very low in each case and lie in the range of -0.9% to -1.75% for 95% or more CL values, and improves as the CL reduces, i.e., as the risk averseness of an investor decreases. Figure 3 plots the error for various CLs for the same set of parameters, simultaneously perturbed in various combinations, by 1% from their base values. Observe that the error is still quite low and lies in the range of -2% to -3% for 95% or more CL values, and improves as the CL reduces. This demonstrates that the sensitivity indices, computed using the DT based method, very precisely calculates the risk parameters of the investor, for the given solar PV investment model, and provides significant computational advantages over the Monte Carlo simulation approach. Furthermore, (48) and (49) can also be used to compute the VaR for various CLs even when the output is not normally distributed. The latter is demonstrated based on computing the percentage errors when CF_{Bruce}^{PV} is perturbed with 1%, 2% and 3% standard deviations, resulting in ouput p.d.f.s that are not normally

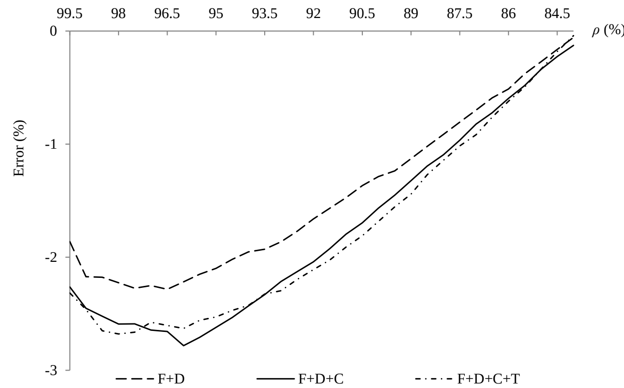
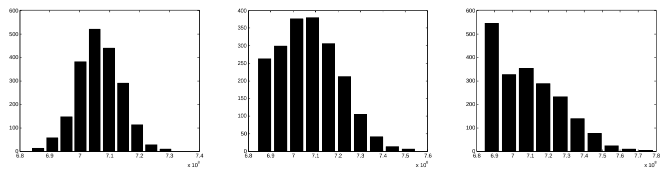


Fig. 3. Percentage error versus confidence level when all parameters are perturbed simultaneously by 1% standard deviation (F: FIT; D: Discount Rate; C: CF^{PV} (Bruce); T: Total Budget).



(a) $\sigma_{CF^{PV}}^{\%} = 1\%$ (b) $\sigma_{CF^{PV}}^{\%} = 2\%$ (c) $\sigma_{CF^{PV}}^{\%} = 3\%$

Fig. 4. Probability distributions of output Ω, with CF^{PV} perturbed by various standard deviation.

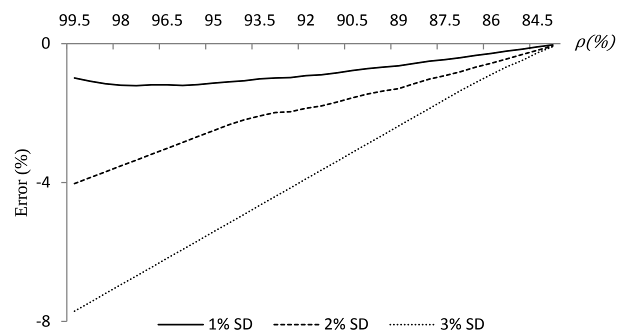


Fig. 5. Percentage error versus CL when CF^{PV} perturbed by various standard deviations.

distributed, as shown in Fig. 4. Observe that as that the histograms in this figure move away from normal distribution as the standard deviation increases, the percentage errors in Fig. 5 remain within 0 to -8% for these input perturbations. Furthermore, note also that the percentage errors are always negative, implying that a more conservative estimate of VaR is obtained while using (49). Thus, it can be argued, that (48) and (49), resulting from the linear approximation in Fig. 1, are also able to compute the VaR and CL for output portfolios not having normal distributions.

C. Range of Validity of DT Approach

In order to understand the range of perturbations for which the sensitivity indices computed using the DT based method are valid, parameters are individually perturbed in the range of +9% to -9% of their p_0 and Ω is computed for each perturbed value. Sensitivities are then computed using

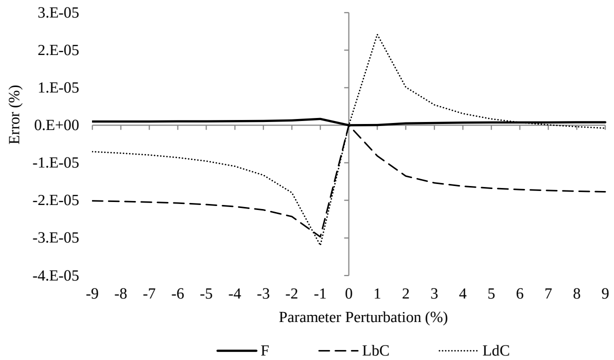


Fig. 6. Percentage error versus range of parameter perturbation (F: FIT; LbC: Labour cost; LdC: Land cost).

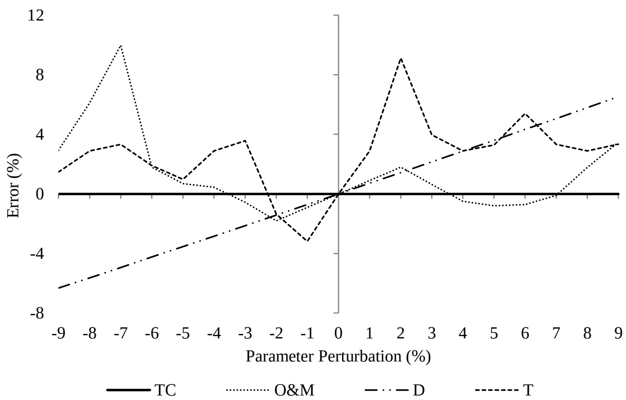


Fig. 7. Percentage error versus range of parameter perturbation (TC: Transportation cost; O&M: Operation & Maintenance cost; D: Discount rate; T: Total Budget).

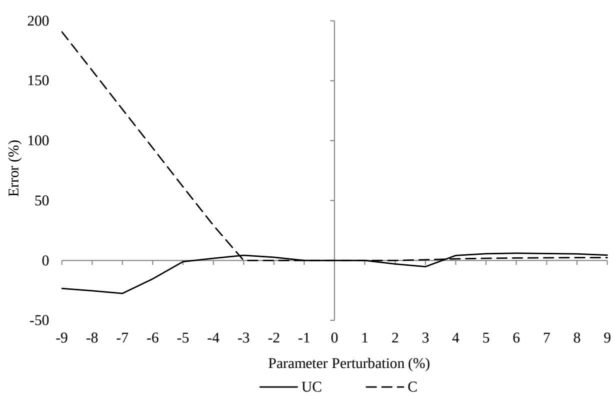


Fig. 8. Percentage error versus range of parameter perturbation (UC: Unit cost; C: CF^{PV} Bruce).

the FD approach for the parameter. Figures 6, 7 and 8 show the percentage errors between the computed DT and FD sensitivities. Observe that, for the parameters perturbed in Fig. 6, the error is close to zero; thus, one can conclude that the relation between these parameters and the NPV is linear in nature. In Fig. 7, the error lies between -7% and 10%, thus demonstrating the nonlinearity of the solar PV investment model with respect to these parameters. In Fig. 8, note that for the capacity factor of PV at Bruce, the error is quite low for positive perturbations, but for negative perturbations below -3%, the error increases; this is due to the fact that when the capacity factor of PV at Bruce is reduced below the PV capacity factor value at another zone, all the new PV installations shift to that other zone; in this case, the NPV is not affected by changes to the value of CF^{PV}at Bruce.

VII. Conclusion

This paper presented the application of the duality theory based method of computing sensitivity indices for a solar PV planning model from the perspective of an investor. The sensitivity indices were directly computed from the solar PV optimization model and only involved the computation of a set of Jacobian matrices and some matrix operations. In the context of this model, the sensitivity indices represent the change in the net present value (NPV) of investors profit when model parameters such as the Feed-in-Tariff, discount rate, total budget, etc., are varied from their respective base values. The presented results demonstrate that the sensitivity indices obtained using the DT based method are very close to those obtained using the Monte Carlo approach and the finite difference approach, which can be considered the true values.

Contrary to the Monte Carlo simulation based approach for determining the sensitivity of parameters, which involves a large number of simulations of the solar PV model in the order of thousands, this approach computes the indices directly in one simulation; thus, the computational burden is significantly reduced. Moreover, while the Monte Carlo simulation approach does not provide any information on the direction of change of the NPV of profit when a parameter is perturbed, the DT based method yields also the desired signs of the sensitivity values, which is valuable information for the investor.

A novel interpretation of the sensitivity indices is proposed thereafter, to evaluate the investors risk parameters pertaining to the solar PV projects selected by the model. Using an approximation of the cumulative distribution function of investors profit, a linear relation is developed between the sensitivity indices and investors profit for a certain confidence level. The proposed application of the sensitivity indices to determine the risk parameters provides valuable information on investment risks for an investor.

References

[1] "Global Market Outlook for Photovoltaics until 2015," European Photovoltaic Industry Association, EPIA Report, May 2011. [Online]. Available: www.epia.org

[2] W. Ku, N. Nour, T. Piascik, A. Firester, A. Stranix, and M. Zonis, "Economic Evaluation of Photovoltaic Generation Applications in a Large Electric Utility System," *IEEE Trans. Power App. Syst.*, vol. PAS-102, no. 8, pp. 2811 –2816, aug. 1983.

[3] J. Iannucci and D. Shugar, "Structural Evolution of Utility Systems and its Implications for Photovoltaic Applications," in *Record of the 22nd IEEE Photovoltaic Specialists Conference*, vol. 1, oct 1991, pp. 566 –573.

[4] G. Brinkman, P. Denholm, E. Drury, R. Margolis, and M. Mowers, "Toward a Solar-Powered Grid," *IEEE Power Energy Mag.*, vol. 9, no. 3, pp. 24 –32, may-june 2011.

[5] A. Mills *et al.*, “Dark Shadows,” *IEEE Power Energy Mag.*, vol. 9, no. 3, pp. 33 –41, may-june 2011.
[6] J. McCalley, W. Jewell, T. Mount, D. Osborn, and J. Fleeman, “A Wider Horizon,” *IEEE Power Energy Mag.*, vol. 9, no. 3, pp. 42 –54, may-june 2011.
[7] A. Ellis, M. Behnke, and J. Keller, “Model Makers,” *IEEE Power Energy Mag.*, vol. 9, no. 3, pp. 55 –61, may-june 2011.
[8] K. Katiraei and J. Aguero, “Solar PV Integration Challenges,” *IEEE Power Energy Mag.*, vol. 9, no. 3, pp. 62 –71, may-june 2011.
[9] W. Muneer, K. Bhattacharya, and C. A. Cañizares, “Large-Scale Solar PV Investment Models, Tools, and Analysis: The Ontario Case,” *IEEE Trans. Power Syst.*, vol. 26, no. 4, pp. 2547 –2555, nov. 2011.
[10] A. J. Conejo, E. Castillo, R. Minguez, and R. Garcia-Bertrand, *Decomposition Techniques in Mathematical Programming: Engineering and Science Applications*. Berlin: Springer-Verlag, 2006.
[11] E. Castillo, A. Conejo, C. Castillo, R. Mnguez, and D. Ortigosa, “Perturbation Approach to Sensitivity Analysis in Mathematical Programming,” *Journal of Optimization Theory and Applications*, vol. 128, pp. 49–74, 2006.
[12] F. Milano, A. Conejo, and R. Zarate-Minano, “General sensitivity formulas for maximum loading conditions in power systems,” *Generation, Transmission Distribution, IET*, vol. 1, no. 3, pp. 516 –526, may 2007.
[13] A. Conejo, E. Castillo, R. Minguez, and F. Milano, “Locational Marginal Price Sensitivities,” *IEEE Trans. Power Syst.*, vol. 20, no. 4, pp. 2026 – 2033, nov. 2005.
[14] A. Saltelli, *Global Sensitivity Analysis: The Primer*. Wiley, 2008.
[15] A. Saltelli, K. Chan, and E. Scott, *Sensitivity Analysis*. Wiley, 2000.
[16] J. Sobieszczanski-Sobieski, J. F. Barthelemyy, and K. M. Riley, “Sensitivity of Optimal Solutions of Problem Parameters,” *AIAA Journal*, vol. 20, pp. 1291 – 1299, 1982.
[17] M. Shahidehpour, H. Yamin, and Z. Li, *Market Operations in Electric Power Systems: Forecasting, Scheduling, and Risk Management*. IEEE, Wiley-Interscience, 2002.
[18] E. Suhir, *Applied Probability for Engineers and Scientists*. McGraw-Hill, 1997.
[19] *CPLEX 12*, Solver Manual. [Online]. Available: www.gams.com/dd/docs/solvers/cplex.pdf
[20] R. E. Rosenthal, *GAMS - A User’s Guide*. [Online]. Available: www.gams.com/dd/docs/bigdocs/GAMSUsersGuide.pdf
[21] *MATLAB Documentation*, The Mathworks Inc., 2011. [Online]. Available: www.mathworks.com/help/techdoc/

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