Smart Charging of PEVs Penetrating into Residential Distribution Systems

Isha Sharma, Student Member, IEEE, Claudio Cañizares, Fellow, IEEE, and Kankar Bhattacharya, Senior Member, IEEE,

Abstract—This paper presents a novel modeling framework for the analysis of Plug-in Electric Vehicle (PEV) charging in unbalanced, residential, distribution systems. A Smart Distribution Power Flow (SDPF) framework is proposed to determine the controlled or smart charging schedules and hence address the shortcomings of uncontrolled charging. The effect of peak-demand constraint imposed by the Local Distribution Company (LDC) is also studied within the SDPF framework for the smart charging scenarios. Uncontrolled versus smart charging schemes are compared for various scenarios, from both the customer’s and the LDC’s perspective. Various objective functions, such as energy drawn by the LDC, total feeder losses, total cost of energy drawn by LDC and total cost of PEV charging are considered. Studies are carried out considering two sample systems i.e., the IEEE 13-node test feeder and a real distribution feeder. Analyses are also presented considering a probabilistic representation of the initial state of charge (SOC) and start time of charging for various scenarios to take into account the difference in customers’ driving patterns. The results show that uncontrolled charging of PEVs results in increased peak demand, low node voltage levels, and increased feeder current magnitudes. On the other hand, the SDPF framework provides very satisfactory operating schedules for the overall system including smart PEV charging.

Index Terms—Plug-in electric vehicles, Uncontrolled charging, smart charging, unbalanced distribution system, optimal feeder operation.

I. NOMENCLATURE

Indices

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<tr>
<td>C</td>
<td>Controllable capacitor banks, $C \in n$.</td>
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<td>$C_n$</td>
<td>Controllable capacitor banks at node $n$.</td>
</tr>
<tr>
<td>$EV$</td>
<td>PEV loads, $EV \in L$.</td>
</tr>
<tr>
<td>$EV_n$</td>
<td>PEV load at node $n$.</td>
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<tr>
<td>$k$</td>
<td>Hours, $k = 1,2,...,24$.</td>
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<td>$l$</td>
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<td>$L$</td>
<td>Loads, $L \in n$.</td>
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<td>$L_n$</td>
<td>Loads at node $n$.</td>
</tr>
<tr>
<td>$n$</td>
<td>Nodes.</td>
</tr>
<tr>
<td>$N_o$</td>
<td>Set of nodes, $n \in N_o$.</td>
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<tr>
<td>$p$</td>
<td>Phases, $p = a, b, c$.</td>
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<tr>
<td>$r$</td>
<td>Receiving-end.</td>
</tr>
<tr>
<td>$r_n$</td>
<td>Receiving-ends connected at node $n$.</td>
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<td>$S$</td>
<td>Sending-end.</td>
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<tr>
<td>$s_n$</td>
<td>Sending-ends connected at node $n$.</td>
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<td>$SS$</td>
<td>Substation node, $SS \in n$.</td>
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Parameters

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<tr>
<th>Symbol</th>
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<tr>
<td>$\Delta Q$</td>
<td>Size of each capacitor block in capacitor banks [Var].</td>
</tr>
<tr>
<td>$\Delta S_n$</td>
<td>Percentage voltage change for each LTC tap.</td>
</tr>
<tr>
<td>$\eta$</td>
<td>PEV charging efficiency.</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Average hourly load of a residence [W].</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Hourly forecast market price [$/MWh$].</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Time interval in hours [h].</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Load power factor angle [rad].</td>
</tr>
<tr>
<td>$ABC$</td>
<td>Three-phase ABCD matrices; $A$ unitless, $B$ in $\Omega$, $C$ in Siemens, $D$ unitless.</td>
</tr>
<tr>
<td>$C_{\text{max}}$</td>
<td>PEV battery capacity [Wh].</td>
</tr>
<tr>
<td>$I_{\text{c}}$</td>
<td>Load current at specified power and nominal voltage [A].</td>
</tr>
<tr>
<td>$I_{\text{d}}$</td>
<td>Load current at nominal voltage [A].</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of PEVs owned by a residence.</td>
</tr>
<tr>
<td>$N'$</td>
<td>Number of PEVs connected in each phase and node.</td>
</tr>
<tr>
<td>$N_{\text{cap}}$</td>
<td>Maximum number of capacitor blocks available in capacitor bank.</td>
</tr>
<tr>
<td>$P_{\text{D}}$</td>
<td>Maximum power drawn by PEV battery [W].</td>
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<tr>
<td>$P_{\text{c}}$</td>
<td>Active power of load [W].</td>
</tr>
<tr>
<td>$P_{\text{D}}$</td>
<td>Maximum allowable peak demand [W].</td>
</tr>
<tr>
<td>$Q_{\text{c}}$</td>
<td>Reactive power of load [Var].</td>
</tr>
<tr>
<td>$SOCT$</td>
<td>Final state of charge of the PEV battery.</td>
</tr>
<tr>
<td>$SOCT^*$</td>
<td>Initial state of charge of the PEV battery.</td>
</tr>
<tr>
<td>$V_{\text{o}}$</td>
<td>Specified nominal voltage [V].</td>
</tr>
<tr>
<td>$V_{\text{max}}$</td>
<td>Maximum voltage limit [V].</td>
</tr>
<tr>
<td>$V_{\text{min}}$</td>
<td>Minimum voltage limit [V].</td>
</tr>
<tr>
<td>$X$</td>
<td>Reactance of capacitor [\Omega].</td>
</tr>
<tr>
<td>$Z$</td>
<td>Load impedance at specified power and nominal voltage [\Omega].</td>
</tr>
</tbody>
</table>

Variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$\text{cap}$</td>
<td>Number of blocks of switched capacitor banks.</td>
</tr>
<tr>
<td>$I$</td>
<td>Current phasor [A].</td>
</tr>
<tr>
<td>$\mathbf{I}$</td>
<td>Vector of three-phase line current phasors [A].</td>
</tr>
<tr>
<td>$I_{\text{c}}^*$</td>
<td>Load current at nominal voltage [A].</td>
</tr>
<tr>
<td>$J_1 - J_4$</td>
<td>Objective functions.</td>
</tr>
<tr>
<td>$P$</td>
<td>Power drawn by PEV [W].</td>
</tr>
<tr>
<td>$Q_{\text{c}}$</td>
<td>Reactive power of capacitor banks [Var].</td>
</tr>
<tr>
<td>$\text{tap}$</td>
<td>Tap position.</td>
</tr>
<tr>
<td>$V$</td>
<td>Voltage phasor [V].</td>
</tr>
<tr>
<td>$\mathbf{V}$</td>
<td>Vector of three-phase line voltages [V].</td>
</tr>
</tbody>
</table>

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II. Introduction

Plug-in Electric Vehicles (PEVs) will play a significant role in reducing greenhouse gas emissions from the transport sector and the sector’s dependence on imported fossil fuels [1]. A PEV draws some, or all, of its power from the power grid. Recharging the PEV battery is typically carried out in residential garages equipped with standard outlets, and takes several hours. However, a specialized high voltage/high current electrical outlet can also be used for fast charging; using a higher level charging brings about significant reduction in charging time [2].

In Canada, the province of Ontario has an aggressive plan for the adoption of smart grid technologies in the distribution system, starting with the installation of smart meters. Time-of-use (TOU) tariffs have been introduced with the intention of reducing electricity use during peak hours. Technologies for convenient charging of PEVs are expected to be available by 2020 [1]. The Electric Vehicle Incentive Program provides incentives to customers from $5,000 - $8,500 (depending on battery capacity) towards the purchase or lease of a new PEV or battery electric vehicle [3]. It is estimated that the number of PEVs in the US and Canada would increase significantly by 2017-2018, resulting in considerable increase in charging demand [4], [5].

With increasing penetration of PEVs into distribution systems, there is a need to examine their effect on distribution feeders. In order to study the effect of PEV charging schedules on local distribution company’s (LDC) system load profiles, a detailed analysis is needed considering PEV penetration levels and local concentrations.

The impact of coordinated charging, vis-à-vis uncontrolled charging has been reported in various papers considering objective functions such as minimizing power losses and maximizing grid load factor [6], minimizing cost of charging and energy losses [7], and minimizing losses, and load factor maximization [8]. It is noted in these papers that uncoordinated charging leads to increased peak, decrease in distribution system efficiency and threatens grid performance and asset’s life. These works are based on balanced distribution systems; thus, random placement of PEVs are studied, and balanced power flows are solved using traditional methods such as backward forward sweep ([6], [8]) or modified Newton-based load flow ([7]) to calculate node voltages and feeder currents.

A demand response strategy is proposed in [9] where customers can control their own loads based on their preferences and comfort levels; the PEV charging schedule is created based on a stochastic method for modeling driving patterns and home arrival times. A scheduling algorithm is proposed in [10] to determine the optimum power purchase schedule for an aggregate PEV fleet from a day-ahead electricity market. An algorithm is proposed to dispatch the purchased energy to PEV loads on the operating day. It is noted that there is a need to revise current market regulations for PEV loads to improve system reliability and operation. In [11], controlled charging of PEVs based on the charging behavior of a number of vehicles, users, travel data and LDC system data, is proposed. It is observed that the impact of PEV charging varies across areas and can be reduced by shifting some charging load to a commercial distribution system.

The effect of fast-charging PEVs in a distribution system is examined in [12] using power-flow, short-circuit and protection studies. It is observed that the location of charging station can limit the maximum number of vehicles being simultaneously charged within the operating limits. In [13], probabilistic power flow analysis is used to study the impact of uncontrolled PEV charging on the grid; a methodology is proposed to model PEV charging loads at a charging station and in a local residential community. PEV loads are modeled in [14] considering the stochastic nature of the start-time of battery charging and the initial State-of-Charge (SOC).

This paper proposes a smart distribution system operation framework including PEV smart charging in LDC operations. The proposed framework is depicted in Fig. 1, where forecasted inputs of the LDC’s load profile for the next day, real-time price (RTP), number of PEVs to be charged at a node and phase, and their initial SOC are available. Executing a Smart Distribution Power Flow (SDPF), which is based on a distribution optimal power flow model [15], the PEV smart charging schedules and operating decisions for taps, capacitors and switches for the next day can be determined. The SDPF model could be used within a Model Predictive Control (MPC) framework to revise its operating decisions when there is a change in any of the inputs, the SDPF model could be rerun to obtain revised decisions decisions. Subsequently, the time horizon would move forward for the next optimization stage. However, the proposed framework cannot be implemented within the existing communication infrastructure now available in distribution feeders; more sophisticated two-way communication devices would be necessary for both LDCs and customers to realize this approach.

This paper builds upon the preliminary work reported in [17], with several additional model improvements and case studies that allow to draw more relevant and important conclusions. Thus, the main differences and contributions of this paper over and above previous works in the subject including [17], are as follows:

- The SDPF mathematical model proposed in this paper is a comprehensive representation of the LDC’s operating criteria in Smart Grids, since it represents customer’s perspective in the LDC’s decision making.
- The presented model includes a peak-demand constraint imposed by the LDC as a control mechanism to define customers their PEV charging schedules in real-time.
- This paper presents a thorough examination of uncontrolled PEV charging and its impact on LDC operation, and determines smart charging schedules for various relevant scenarios.
- The proposed SDPF model is also tested on a realistic distribution feeder presented in [15].
Two different load models are considered, i.e., constant impedance and ZIP loads, for the analysis. A probabilistic representation of the initial SOC and starting time of charging is considered, to take into account different customers’ driving patterns.

The rest of the paper is structured as follows: Section III presents the mathematical model of the proposed SDPF and discusses various objective functions, considered in this paper, from LDC’s and customers’ perspective. Section IV presents the main assumptions made in this work and description of the scenarios constructed for the analyses. Section V discusses the results obtained from various case studies and scenario analyses carried out using the IEEE 13-node test feeder and a real feeder. Section VI highlights the main conclusions from this research.

III. OPERATIONS MODEL OF A SMART DISTRIBUTION SYSTEM

A. Three-phase Distribution System

A generic distribution system comprises series and shunt components. The series components include conductors / cables, transformers, transformer load tap changers (LTCs) and switches which are modeled using ABCD parameters, computed from relationships between sending and receiving end voltages and currents [18]:

\[
\begin{bmatrix}
V_{s,p,k} \\
T_{s,p,k}
\end{bmatrix}
= \begin{bmatrix}
A_{(l \times p)} & B_{(l \times p)} \\
C_{(l \times p)} & D_{(l \times p)}
\end{bmatrix}
\begin{bmatrix}
V_{r,p,k} \\
T_{r,p,k}
\end{bmatrix}
\quad \forall l, \forall p, \forall k
\]

(1)

The ABCD parameters for conductors, cables, transformers and switches are constants. Switches are represented as zero impedances, conductors and cables as \(\pi\)-equivalent circuits, and three-phase transformers are modeled based on the type of connection (wye or delta). Voltage regulating transformers in a distribution system are equipped with LTCs, whose ABCD parameters cannot be considered constant since they depend on the tap position at a given time \(k\). Thus, B and C are null matrices while the A and D matrices of the LTCs are modeled using the following equations:

\[
A_{l,k} = \begin{bmatrix}
1 + \Delta S \text{tap}_{l,a,k} & 0 & 0 \\
0 & 1 + \Delta S \text{tap}_{l,b,k} & 0 \\
0 & 0 & 1 + \Delta S \text{tap}_{l,c,k}
\end{bmatrix}
\]

\[
D_{(l \times k)} = A_{(l \times k)}^{-1} \quad \forall l, \forall p, \forall k
\]

(2)

where \(\text{tap}_{l,a,k}\), \(\text{tap}_{l,b,k}\) and \(\text{tap}_{l,c,k}\) are tap controls in the respective phases for the LTC and are considered continuous variables, which can take any value between -16 and +16, for a 32-step LTC. For a three-phase tap changer, the tap operations are considered identical in the three phases:

\[
\text{tap}_{l,a,k} = \text{tap}_{l,b,k} = \text{tap}_{l,c,k} \quad \forall t, \forall k
\]

(4)

Shunt components comprise loads and capacitor banks and are modeled separately to represent unbalanced three-phase loads. In distribution systems, the electrical loads are typically modeled as voltage dependent to better represent the types of loads encountered in distribution feeders [19]. In this work, at first, the loads are modeled as constant impedance, with the following equations representing wye-connected impedance loads on a per-phase basis:

\[
V_{L,p,k} = Z_{L,p,k} I_{L,p,k} \quad \forall L, \forall p, \forall k
\]

(5)

Subsequently, a mix of types of loads such as constant impedance (Z), constant current (I), and constant power (P), i.e. ZIP loads, are considered. Capacitor banks are modeled as multiple capacitor blocks with switching options, on a per-phase basis, as follows:

\[
V_{C,p,k} = X_{C,p,k} I_{C,p,k} \quad \forall C, \forall p, \forall k
\]

\[
j X_{C,p,k} = \frac{V_{C,p,k}^2}{Q_{C,p,k}} \quad \forall C, \forall p, \forall k
\]

\[
Q_{C,p,k} = \text{cap}_{C,p,k} \Delta Q_{C,p,k} \quad \forall C, \forall p, \forall k
\]

(8)

where the variable, \(\text{cap}_{C,p,k}\) is a continuous variable, and can take any positive value from 0 to \(N_{C,p}\). The PEV is modeled as a controlled current load, on a per-phase basis, as follows:

\[
|I_{EV,p,k}|(\angle V_{EV,p,k} - \angle I_{EV,p,k}) = |I_{EV,p,k}| \angle \theta_{EV,p,k} \quad \forall EV, \forall p, \forall k
\]

\[
\text{Re}(V_{EV,p,k} I_{EV,p,k}^*) = P_{EV,p,k} \quad \forall k, \forall p, \forall EV
\]

(10)

It is assumed that no significant reactive power is drawn by the PEV loads, thus treating them as unity power factor loads. Furthermore, the total energy drawn by the PEV battery over
the charging period, taking into account its efficiency $\eta_{EV}$, is equal to the battery charging capacity; thus:

$$\tau \sum_k \eta_{EV} P_{EV,p,k} = (SOC_{EV,p}^f \leftarrow SOC_{EV,p}) C_{EV,p}^{\max} N_{EV,p}^\prime \forall EV, \forall p$$ (11)

The maximum power that can be drawn by the battery is constrained by $P_{EV}^\prime$, the socket capacity of a standard electrical outlet, which depends on the level of charging, as follows:

$$P_{EV,p,k} \leq P_{EV}^\prime N_{EV,p}^\prime \forall EV, \forall p, \forall k$$ (12)

Equations (1)-(12) represent the various components of the three-phase distribution system including the PEV loads. To model the distribution system in its totality, these elements are required to satisfy the current balance at each node and phase:

$$\sum_l I_{l,p,k}(\forall r_n) = \sum_l I_{l,p,k}(\forall s_n) + \sum_l I_{L,n,p,k} + \sum_l I_{C,n,p,k}$$

$$+ \sum_{EV} I_{EV,n,p,k} \forall n, \forall p, \forall k$$ (13)

And the voltage at the node and phase, at which a given set of components are connected, are the same as the corresponding nodal voltages:

$$V_{l,p,k}(\forall r_n) = V_{l,p,k}(\forall s_n) = V_{L,n,p,k} = V_{C,n,p,k} = V_{EV,n,p,k} \forall p, \forall n, \forall k$$ (14)

B. Smart Distribution System Operation

The three-phase SDPF model determines the PEV charging schedules for various objective functions, considering the specified range of charging periods and grid operational constraints. The decision variables in the proposed SDPF model are the power drawn by the aggregate PEV loads at each node and phase, and the taps and capacitor switching decisions while the nodal voltages and feeder currents are the state variables. The different objective functions considered in this work represent the perspectives of the LDC and the customers, as follows:

- Minimize the total energy drawn by the LDC from the external grid, over a day:

$$J_3 = \sum_k \left( \sum_p \left( \sum_{EV} P_{EV,p,k} \right) \rho(k) \right)$$ (17)

This objective (17) represents a situation where the LDCs are stipulated by regulatory agencies, as in Ontario, Canada, to bring about a reduction in their energy consumption levels.

- Minimize total feeder losses over a day:

$$J_2 = \sum_k \sum_p \sum_n \left( \sum_{EV} Re(V_{L,n,p,k} \leftarrow V_{r,n,p,k}) I_{r,n,p,k}^* \right)$$ (16)

In the analysis of distribution systems, where loads are modeled to be voltage dependent, this objective seeks to improve the voltage profile across distribution nodes.

- Minimize the total cost of energy drawn by the LDC from the external grid, over a day:

$$J_1 = \sum_k \left( \sum_p \sum_{EV} P_{EV,p,k} \right) \rho_1(k)$$ (18)

This objective can be used by the LDC to study the system impact of PEV charging, expecting rational behavior of customers, i.e., customers seeking to minimize their charging costs. It is assumed that all customer homes are equipped with smart meters and are subject to RTP or TOU tariffs $\rho_1(k)$, about which the customers are assumed to have sufficient information, so that they schedule their PEV charging accordingly. In real life, the two prices $\rho(k)$ and $\rho_1(k)$, are different from each other, because of the LDC’s network costs, global adjustments, etc., included in the latter. However, in this paper, it is assumed that $\rho(k)$ is equal to $\rho_1(k)$ without loss of generality. In such a situation, the LDC needs to understand the system impact of such a charging approach, which is carried out using $J_4$.

The 3-phase distribution feeder and its components, modeled by (1)-(12), and the network equations (13)-(14), are the constraints of the SDPF model. Other constraints of SDPF model comprise the feeder operating limits that include the limits on node voltages, feeder currents, taps, and capacitors, as follows:

$$V_{n,p,k}^{\min} \leq V_{n,p,k} \leq V_{n,p,k}^{\max} \forall n, \forall p, \forall k$$ (19)

$$|I_{i,j,p,k}| \leq I_{i,j,p,k}^{\max} \forall i \in s_n, \forall j \in r_n, \forall p, \forall k$$ (20)

$$\text{tap}_{i,p,k} \leq \text{tap}_{i,p,k} \leq \text{tap}_{i,p,k} \forall i, \forall p, \forall k$$ (21)

$$0 \leq \text{cap}_{C,p,k} \leq C_{p,k} \forall C, \forall p, \forall k$$ (22)

The LDC may also impose a peak demand constraint, based on its substation capacity, as follows:

$$\sum_{L_n} \sum_{p} \left( \sum_{EV} \left( V_{L,n,p,k} \leftarrow V_{r,n,p,k}) I_{r,n,p,k}^* \right) + P_{EV,p,k} \right) \leq P_{D_k}$$ (23)

The proposed SDPF model given by (1)-(23) is a non-linear programming (NLP) problem, which is modeled in GAMS and solved using the SNOPT solver [20]. In this paper, the taps and capacitors are modeled as continuous variables, as mentioned earlier, to alleviate the introduction of integer variables and hence retain the model as an NLP problem, thereby keeping the computational burden reasonable, considering that the main purpose of the paper is to study the impact of PEV
charging on feeders, and not the optimal voltage control of these feeders.

The output of the SDPF pertaining to the PEVs is the aggregated power drawn by PEV loads, at an hour, node and phase \( (P_{EV,p,k}) \). The PEV charging current \( (I_{EV,p,k}) \) can then be obtained from the model. In order to determine the number of PEVs to be charged, \( P_{EV,p,k} \) can be used, after the optimization solution is obtained, as follows:

\[
N_{EV,p,k}^{chg} = \frac{P_{EV,p,k}}{P_{EV}}
\]  

(24)

For example, from the above equation, if a \( P_{EV,p,k} = 17 \) kW is obtained, and knowing that \( P_{EV} = 4.8 \) kW, using (24), \( N_{EV,p,k}^{chg} \) is calculated to be 3.54. From this, the LDC can infer that there should be three PEVs allowed to charge at this node, phase and hour, drawing 4.8 kW each, while a fourth PEV should be allowed to only draw 2.6 kW. Note that the determination of individual PEV charging levels is not considered here, since the objective is to study the charging problem from the feeder perspective; in this context, the simple method outlined above is one reasonable way to determine the number of PEVs charging at a node and phase.

IV. ASSUMPTIONS AND SCENARIOS

A. Assumptions

In order to evaluate the system impact of smart charging of PEVs vis-à-vis their uncontrolled charging, the following assumptions are made:

- A 24 h time horizon is assumed, with time interval of \( \tau = 1 \)h. However, the proposed SDPF can accommodate smaller time steps if required, but at the cost of increased computational costs.

- Mid-size sedans, PHEV30 km with 9.76 kWh [21] battery capacities are considered. PHEV30 km implies that 60% of the vehicle kilometers are driven on battery and the rest on gasoline [22].

- PEVs are the only dispatchable loads at a node and are not capable of delivering power back to the grid. Note that it has been assumed that all the PEVs at a node and phase are aggregated, which is reasonable in the context of the studies presented here that concentrate on the feeder.

- Since all PEVs are assumed to be residential loads, these are not available for charging between 7 AM to 5 PM, when people are at work.

- The charging efficiency \( \eta_{EV} \) is assumed to be 85%, and only Level 2 charging (208-240V/40-100A) is considered.

- For the deterministic studies, the SOC of the PEV battery at the start of charging is 20% and it is charged to 90% of its full capacity at every node. In probabilistic studies, lognormal distribution of initial SOC is considered for each node.

- In deterministic studies, all the PEVs are assumed to have a minimum charging time of 2 hours [23], since the value of \( P_{EV} \) considered is 4.8 kW [1]. For probabilistic studies, the minimum charging time of PEVs depends on the initial SOC.

- The charging current of the PEV is assumed to remain constant and not vary with the SOC of the battery.

- The entire load at a node on the feeder section, denoted by \( PD_{p,L} \), are residential loads. Furthermore, it is assumed that if a house owns a PEV, it owns exactly \( N_{p,L} = 1 \) number of vehicles.

- The average monthly electricity consumption of a residence is 1500 kWh [24]. The average hourly load of the residence (\( \gamma \)) is calculated to be 2.08 kW.

- As mentioned in Section III.B, \( \rho(k) \) is assumed equal to \( \rho_1(k) \). In this paper, these prices are assumed to be the Hourly Ontario Electricity Price (HOEP) which applies to participants in the wholesale electricity market of Ontario ( [25], [26]) and is used for all the analysis reported here. This HOEP is an hourly price, uniform across all of Ontario, computed at the end of every hour of real-time operation of the Ontario electricity market. The HOEP profile used is depicted in Fig. 2, wherein it can be noted that peak prices occur from hours 13-18.

Using the aforementioned assumptions, the number of PEVs connected in each phase and node, \( N_{n,p} \), can be realistically estimated for an \( x \) p.u. penetration of PEV by:

\[
N_{n,p} = \text{floor} \left( x \ \text{floor} \left( \frac{PD_{n,p}}{\gamma} \right) \right) N_{p,L} \ \forall n, \forall p
\]  

(25)

A penetration of \( x = 1.0 \) means that every residence has one PEV, while \( x = 0 \) is the base case with no PEVs in the system. All analysis presented in this paper are carried out for \( x = 0.5 \) (i.e., a PEV penetration of 50%, which means every second house has a PEV), the PEV load being added to the existing distribution system, spread over the 13-hour charging period.

B. Scenarios

1) Business as Usual: Uncontrolled Charging (Case 1):

This case assumes that customers charge their PEVs as and when they want to, without any regard for system constraints. The LDC has no control on the PEV charging schedules and hence an uncontrolled mode charging takes place. The following two scenarios are considered in this case, as presented in Table I.

- \( S1 \): This scenario assumes that the customers charge their PEVs in the shortest possible time (two hours), after plugging in. Although, they are not interested in minimizing the charging cost, they are aware of the prevailing tariff rates, for example, that the off-peak TOU price begins at 7 PM, as in Ontario [27], or that the off-peak RTP commences from 7 PM as shown in Fig. 2. Accordingly, PEV charging is carried out between 8 to 10 PM. Also since node voltages or feeder...
current limits are not of concern, this scenario is simulated using a distribution load flow without any limits on node voltages or feeder currents. This scenario effectively represents the worst case, as it results in concentrated buildup of charging loads, which coincides with the period of peak demand on the distribution feeder.

$S_2$: This scenario assumes that customers receive fairly precise forecast of $p_1(k)$. And they are equipped with smart home energy management systems (e.g. [28]) that determines optimal operational schedules for various home appliances such as air conditioner or heating, washer, dryer, and other appliances, as well as PEV charging, so as to reduce their overall electricity cost. In this scenario, the LDC has no control on the PEV charging schedules, and hence limits on node voltages or feeder currents are not considered.

2) Smart Charging (Case 2): In smart charging, the LDC is envisaged to send control signals to PEVs for charging purpose, considering grid constraints such as node voltage limits and feeder current limits. The LDC may also impose the system peak-demand constraint (23) while determining the PEV charging schedules. The following four different smart charging scenarios are considered, as summarized in Table II:

$S_3$: In this scenario, the LDC minimizes the total energy drawn from the substation over a day, as per (15).

$S_4$: In this scenario, the LDC seeks to minimize the total feeder losses in the system over a day, as per (16).

$S_5$: In this scenario, the LDC minimizes the total cost of energy drawn from the external grid over a day, as per (17).

$S_6$: In this scenario, the LDC determines the charging schedules assuming rational behavior of customers, while at the same time respecting system constraints to prevent feeder problems. Node voltage limits and feeder current limits are considered while the LDC seeks to minimize the total cost of PEV charging over a day, as per (18). It should be noted that this scenario is the “controlled” or “smart” version of Scenario $S_2$.

V. RESULTS AND ANALYSIS

A. IEEE 13-Node Test Feeder

The IEEE 13-node test feeder (Fig. 3) [29] is used in this work to examine the proposed SDPF model and smart charging aspects of PEVs in distribution systems. As mentioned earlier, the capacitors are modeled as multiple capacitor banks with switching options; i.e., the capacitor at Bus 675 is assumed to comprise five capacitor blocks of 100 kVar in each phase, and that at Bus 611 comprising 5 capacitor blocks of 50 kVar in phase $c$. A 24-hour load profile is used in this work [17], for the analytical studies. The number of PEVs at each node and phase for a 50% penetration is given in Table III.

1) Uncontrolled Charging

Figure 4 shows a comparison of the total system demand over a 24-hour period for the uncontrolled charging scenarios $S_1$ and $S_2$. Note that for $S_1$, a new peak is created at 9 PM and the system demand increases by 100% and 103% at 8 PM and 9 PM, respectively. On the other hand, in $S_2$ the charging occurs at 4 AM and at midnight, when the energy price is low, also creating new peaks in the system. Figure 5 shows how the main feeder (650-632) current is impacted in scenarios $S_1$ and $S_2$. Observe that the main feeder current exceeds the maximum limit in all three phases, thereby leading to possible feeder problems. Figure 6 shows that the node voltages drop significantly at 8 PM, especially in phases $a$ and $c$, because of uncontrolled charging in $S_1$.

A summary comparison of the two scenarios of uncontrolled charging $S_1$ and $S_2$ is presented in Table IV. Note that for uncontrolled charging of PEVs without regard to charging costs or system conditions ($S_1$), the system impact is much more severe than when the PEV owners seek to minimize their charging costs ($S_2$). For instance, the energy drawn is

![Fig. 3. IEEE 13-node test feeder [29].](image-url)
the system close to the lower acceptable voltage limit, (0.95 p.u.), the total energy drawn is minimized. Figure 7 shows the system base demand (without PEV loads) and the system demand profile including PEV charging loads, with and without the peak-demand constraint (23). The system demand exceeds \( PD \) at hour 21 because the PEV charging load coincides with the system peak at this hour. Note that when peak demand is limited, the PEV charging schedule is adjusted appropriately within the allowable charging hours, and the system demand is restricted. The base demands corresponding to scenarios \( S_3-S_5 \) are obtained by minimizing the respective objective function without considering PEVs, and the base demand in \( S_6 \) is obtained by subtracting the PEV charging load from the corresponding total demand.

Figure 8 presents the phase-wise demand with and without PEVs. Observe that the PEV charging mainly occurs during late evening and early morning hours. It should be mentioned that the feeder current magnitudes in this scenario are within limits, and the node voltages are observed to be close to 0.95 p.u.

2) Case 2: Smart Charging

a) Minimize Total Energy Drawn by LDC \( (S_3) \): Since the loads are modeled as voltage dependent, by operating the system close to the lower acceptable voltage limit, (0.95 p.u.), the total energy drawn is minimized. Figure 7 shows the system base demand (without PEV loads) and the system demand profile including PEV charging loads, with and without the peak-demand constraint (23). The system demand exceeds \( PD \) at hour 21 because the PEV charging load coincides with the system peak at this hour. Note that when peak demand is limited, the PEV charging schedule is adjusted appropriately within the allowable charging hours, and the system demand is restricted. The base demands corresponding to scenarios \( S_3-S_5 \) are obtained by minimizing the respective objective function without considering PEVs, and the base demand in \( S_6 \) is obtained by subtracting the PEV charging load from the corresponding total demand.

Figure 8 presents the phase-wise demand with and without PEVs. Observe that the PEV charging mainly occurs during late evening and early morning hours. It should be mentioned that the feeder current magnitudes in this scenario are within limits, and the node voltages are observed to be close to 0.95 p.u.

b) Minimize Total Feeder Losses \( (S_4) \): Figure 9 shows that the resulting system demand including PEV charging load is below \( PD \), and thus constraint (23) is not imposed in this scenario. The nodal voltages improve and feeder current magnitudes are below their respective maximum limits. Furthermore, most of the PEV charging occurs in all phases during early morning hours, from midnight to 6 AM, when the base load is low, as shown in Fig. 10. This scenario results in a fairly flat load profile, without any steep peaks at any hour, as compared to other scenarios, and the node voltages

### Table IV
Summary Results of Uncontrolled PEV Charging Scenarios; 13-node Feeder

<table>
<thead>
<tr>
<th></th>
<th>( S_1 )</th>
<th>( S_2 )</th>
<th>( S_2 ) vs ( S_1 ) change [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy drawn by LDC [kWh]</td>
<td>73,727</td>
<td>68,669</td>
<td>-6.8</td>
</tr>
<tr>
<td>Feeder losses [kWh]</td>
<td>2,232</td>
<td>1,950</td>
<td>-12.6</td>
</tr>
<tr>
<td>PEV charging cost of customers [$/day]</td>
<td>196</td>
<td>117</td>
<td>-40.3</td>
</tr>
<tr>
<td>Cost of energy drawn by LDC [$/day]</td>
<td>3,119</td>
<td>2,806</td>
<td>-10</td>
</tr>
</tbody>
</table>

reduced by 6.8% and the PEV charging cost is reduced by 40.3% in \( S_2 \) as compared to \( S_1 \). However, as noted in Figs. 5 and 6, both \( S_1 \) and \( S_2 \) are detrimental to the system because feeder current limits are exceeded and several bus voltages are below acceptable limits.

2) Case 2: Smart Charging

a) Minimize Total Energy Drawn by LDC \( (S_3) \): Since the loads are modeled as voltage dependent, by operating...
are significantly improved as compared to Scenario $S_3$.

c) Minimize Total Cost of Energy Drawn by LDC ($S_5$): Figure 11 shows the total system demand with and without peak-demand constraint (23), considering the PEV charging load. In this scenario, the LDC schedules the charging of PEV loads when electricity prices are low, i.e., at 4 AM, midnight and 3 AM. At midnight, the PEV charging load added to the system base load results in a system peak.

Because of the increase in demand when electricity prices are low, some feeder current magnitudes reach their maximum limit at midnight and 4 AM. By including constraint (23) in SDPF, the feeder current magnitudes are reduced below their maximum limit. Consequently, the PEV charging load is now distributed across more hours, as shown in Fig. 12. Voltages obtained in this scenario are close to their lower limit (0.95 p.u.), resulting in less energy drawn from the external grid.

d) Minimize Total Cost of PEV Charging ($S_6$): In this scenario, the LDC schedules the charging of PEV loads considering the system limits, and at the same time assumes that customers behave rationally and seek to minimize their charging cost. From Fig. 13, it is seen that when the peak-demand constraint (23) is not imposed, the PEV charging is concentrated at 4 AM and midnight when electricity prices are low. This results in a significant increase in system demand at these hours. On the other hand, by imposing constraint (23), the PEV charging load is distributed over the next available cheap price hours. The phase-wise PEV charging schedules are shown in Fig. 14.

Since the PEV charging load is concentrated at 4 AM and at midnight, some feeder current magnitudes reach their maximum limits. Imposition of constraint (23) does not reduce these feeder currents below their limits, since the system is operating at the limits, which is undesirable. Since minimization of energy drawn is not an objective for this scenario, the voltages are observed to be above their lower limits.

B. Comparison of Smart Charging Scenarios

A comparison of the total energy drawn by the LDC, total feeder losses, total cost of energy drawn by the LDC and the total PEV charging cost, for scenarios $S_3$, $S_4$, $S_5$ and $S_6$ are presented in Table V. As expected, minimum energy drawn by the LDC from the external grid is obtained in $S_3$, although the PEV charging cost for customers is maximum, because of the way their charging is scheduled by the LDC. On the other hand, $S_6$ results in the lowest PEV charging cost for the
customers, but requires the LDC to draw significantly larger amount of energy from the grid, thereby increasing cost and losses for the LDC.

An interesting observation can be made for scenario $S_5$, which seems to be the optimal choice for both the LDC and the customer from their respective perspectives. This scenario results in almost close to minimum PEV charging cost for the customers, while at the same time ensures that the LDC’s energy drawn, feeder losses and cost of energy drawn are within acceptable values. Observe that scenario $S_3$ also yields reasonably balanced results for both the customers and the LDC.

Figure 15 shows the phase-wise voltage magnitudes for the various smart charging scenarios. Scenario $S_3$ shows voltages close to 0.95 p.u., which results in reduction of energy drawn by the LDC. Since phase $c$ is the most heavily loaded as compared to other phases, voltages in this phase are close to 0.95 p.u. for $S_3$-$S_5$. On the other hand, scenario $S_6$ results in somewhat higher voltages, closer to 1 p.u., in all phases, as compared to the other scenarios, since neither the system energy demand nor feeder losses are minimized in this case.

Figure 16 presents phase-wise currents of Feeder 692-675 for all smart charging scenarios. Observe that the feeder current in phase $a$ for $S_3$, $S_5$ and $S_6$ are at its maximum limit for some hours, while phases $b$ and $c$ (mostly) are below the maximum limit.

### C. Effect of ZIP Load Models on PEV Smart Charging

The analysis presented in this paper thus far, considered only constant impedance loads. However, in order to examine the charging impact for a broader-based representation of loads, a mix of constant impedance ($Z$), constant current ($I$), and constant power ($P$) loads, i.e., ZIP loads, are also considered, to account for other possible types of loads. While constant impedance loads are modeled using (5), constant power loads are modeled as follows:

\[ P_{L,p,k} + jQ_{L,p,k} = V_{L,p,k}I_{L,p,k}^* \quad \forall L, \forall p, \forall k \]

and constant current loads are modeled as:

\[ |I_{L,p,k}|(\angle V_{L,p,k} - \angle I_{L,p,k}) = |I_{L,p,k}^*|\angle \theta_{L,p,k} \]
Thus, in the proposed SDPF model, (26) and (27) are included along with other equations. To model voltage independent loads, and thus analyze the opposite case to the previous studies, the ZIP load mix is assumed to be dominated by constant power loads. Hence, loads at nodes 634, 645, 671, 675, and 632-671 are modeled as constant power loads; loads at nodes 646 and 652 are modeled as constant impedance loads; and finally loads at nodes 692 and 611 are modeled as constant current loads.

From the smart charging scenario studies it is noted that there is no significant change in the PEV charging schedules when compared to those with constant impedance loads. Accordingly, observe that there is very little effect of the load models on PEV charging cost. Table VI presents the summary results of the smart charging scenarios considering ZIP loads. All the scenarios depict an increase in energy drawn from the substation, feeder losses, and cost of energy drawn from the substation as compared to scenarios with constant impedance loads (Table V). As expected, since the ZIP loads are dominated by constant power, their energy consumption does not depend on the node voltages.

Figure 17 presents the phase voltage magnitudes at Node 675 for the smart charging scenarios with ZIP loads. Comparing these with those obtained with constant impedance loads (Fig. 15), it is noted that the voltage profiles are very similar for scenarios $S_3$, $S_5$ and $S_6$. In scenario $S_4$, the voltage profiles at Node 675 is significantly improved with ZIP loads. However, it is very difficult to draw general conclusions on the impact of loads models on node voltages by analyzing individual nodes. Therefore, an aggregated, phase-wise, voltage deviation index (VDI) is defined, as follows:

$$ VDI_p = \sum_k \sum_n \left[ V_{n,p,k} - V^{min} \right] $$

Thus, in the 13-node test feeder considered, when all the node voltages are at $V^{min}$ in a given phase $p$, the value of VDI will be zero, while when all node voltages are equal to 1 p.u., the VDI will take a value of 14.4. This implies that a low value of VDI indicate node voltages close to $V^{min}$, while VDIs close to or higher than 14.4 indicate a node voltage profile close to or above 1.0 p.u. Table VII presents phase-wise VDIs for constant impedance and ZIP load models, from which the following observations can be made:

- Constant impedance loads have low VDIs, in general, as compared to ZIP loads, implying that ZIP loads result in better voltage profiles.
- In all the scenarios, with both load models, the values of $VDI_L$ are the lowest, since phase $c$ is the most heavily loaded amongst the three phases.
- $S_6$ presents good voltage profiles, as evident from the high values of VDIs with both load models.
- $S_4$ with ZIP load models shows improved voltage profiles as compared to constant impedance loads.

Even though constant power loads are pre-dominant in the ZIP load models used, about 11% of the loads in phase a, 22% in phase b, and 27% in phase c are either constant impedance or constant current loads. This makes the behavior of the feeder different from a feeder with constant power loads only. Therefore, in $S_3$, voltages are close to their minimum allowable limits to draw minimum energy, while in $S_4$, voltages tend to improve so as to minimize the feeder losses, as shown in Table VII.

### D. Probabilistic Analysis

1) **Uncontrolled Charging Scenario ($S_1$):** As discussed in Section IV.B, $S_1$ assumes that all the PEVs start charging simultaneously at 8 PM and have a charging time of 2 hours, which leads to a concentrated charging load during 8-10 PM. Also, as per Section IV.A, $SOC^i$ is assumed to be 20%. These assumptions represent the worst case scenario for the feeder, since full battery charging takes place early, quickly, and simultaneously, coinciding with the system base
peak demand. However, in practice, the starting time of PEV charging and $SOC^i$ depend on the customers’ driving pattern, i.e., miles traveled by the PEV. To model these uncertainties, the starting time of charging and $SOC^i$ are now modeled as lognormally distributed random distributions, as suggested in [30] and shown in Fig. 18, for each load node separately. The above distributions are chosen such that the mean values of the starting charging time and $SOC^i$ are 8 PM and 0.35, respectively, since these can be reasonably considered to be the most likely plug-in time and battery levels for residential customers.

The resulting plots of the probability distributions of the energy drawn by the LDC and the cost of PEV charging obtained from the model are shown in Fig. 19. The expected values of various decision variables are given in Table VIII, noting that these are somewhat lower than those obtained in the deterministic case presented in Section V.A.1., as expected. These results were obtained using a Monte Carlo Simulation (MCS) approach [31].

2) Smart Charging Scenarios ($S_3$–$S_6$): As discussed in Section IV.B.2, smart charging scenarios assume an initial SOC of 20%, and a charging window from 7 PM to 7 AM within which the optimal schedules are determined. Unlike $S_1$, in these scenarios, there is no specified fixed starting time for charging. Since $SOC^i$ depends on the miles driven in the PEVs, this can be modeled using a lognormally distributed probability distribution function (Fig. 18) at each individual load node. For this case, Table IX presents the expected values of various decision variables obtained using an MCS approach, i.e., energy drawn by LDC, feeder losses, PEV charging cost, and cost of energy drawn by the LDC. The expected values obtained are lower than those obtained in the deterministic case (Table V).

Figure 20 shows four different plots of probability distributions of the decision variables, corresponding to the scenarios in which these variables are being optimized. For example, Fig. 20(a) shows the probability distribution of the energy drawn by the LDC for scenario $S_5$, which has the objective of minimizing the energy drawn. Similarly, Fig. 20(b) shows the probability distribution of system losses when the LDC’s objective is loss minimization ($S_4$), and so on. These probability distributions depict the range of the values over which the system variables are expected to vary, for the assumed lognormally distributed initial SOCs.

The probabilistic analyses presented here show that the deterministic studies discussed earlier are reasonable, yielding results close to the expected trends for the various relevant variables under study.

E. Real Distribution Feeder

Simulations are also carried out considering a real unbalanced distribution feeder, presented in [32], and depicted in Figure 21. This feeder has three 3-phase transformers equipped with LTCs and a single phase transformer. There are 16 load nodes, with all loads being modeled as constant impedances. The feeder current limit information is not available in this

![Table VIII](image1)

**TABLE VIII**

<table>
<thead>
<tr>
<th>Probabilistic Studies for Uncontrolled Charging Scenario $S_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Expected energy drawn by LDC [kWh]</strong></td>
</tr>
<tr>
<td><strong>Expected feeder losses [kWh]</strong></td>
</tr>
<tr>
<td><strong>Expected PEV charging cost of customers [$/day</strong></td>
</tr>
<tr>
<td><strong>Expected cost of energy drawn by LDC [$/day</strong></td>
</tr>
</tbody>
</table>

![Figure 19](image2)

**Fig. 19.** Decision variable values for probabilistic uncontrolled charging.

![Figure 20](image3)

**Figure 20.** This feeder has three 3-phase transformers equipped with LTCs and a single phase transformer.

![Figure 21](image4)

**E. Real Distribution Feeder**
case; hence, constraint (20) is not included in the SDPF model. Table X presents the distribution of PEVs at all load nodes and each phase for a 50% PEV penetration.

As described in Section III-A and III-B, uncontrolled PEV charging ($S_1$ and $S_2$) and smart charging scenarios ($S_3$, $S_4$, $S_5$, and $S_6$) are also studied for this feeder. Table XI presents the summary results for the uncontrolled charging scenarios. The improvement in various parameters, such as energy drawn by the LDC, PEV charging costs, etc., in $S_2$ as compared to $S_1$ are very similar as for the IEEE 13-node test feeder.

Table XII presents the summary results for the smart charging scenarios. Observe that scenarios $S_5$ and $S_6$ result in the same PEV charging cost, since both the LDC and the customers are trying to minimize their respective costs.

It should be mentioned that in the uncontrolled scenario $S_2$ in Table I, node voltage limits are not considered and thus, the resultant node voltage profiles tend to be lower than the nominal limits. Therefore, since loads are modeled as voltage dependent, they draw less energy from the grid, and consequently incur lower feeder losses as compared to $S_3$ and $S_4$, respectively.

It should be noted from the analysis of the results for the two feeders that $S_5$ is an optimal scenario for both the LDC and the customers. Scenario $S_3$ results in a fairly uniform load profile, and thus does not require the peak-demand constraint (23) in the SDPF model. Scenarios $S_1$ and $S_6$ are extreme scenarios, with $S_1$ representing the perspective of the LDC only, while $S_6$ represents the customer interests without concern for the grid. Therefore, scenario $S_1$ results in minimum energy drawn by the LDC, but at maximum charging cost for the customers, and vice versa for $S_6$.

F. Computational Stats

The proposed SDPF model has been programmed and executed in the GAMS environment [20] on a Dell PowerEdge R810 server, Windows 64-bit operating system, with 4 Intel Xeon 1.87 GHz processors and 64 GB of RAM. The proposed SDPF model is an NLP problem which is solved using the SNOPT solver [20]. The model and solver statistics for the IEEE 13-node test feeder and the real distribution feeder are summarized in Table XIII.
VI. CONCLUSIONS

This paper presented a smart distribution power flow (SDPF) model which incorporates detailed representation of plug-in electric vehicles (PEVs) within a three-phase unbalanced distribution system. The proposed SDPF model is applied to study the effects on distribution feeders of uncontrolled PEV charging vis-à-vis smart charging schedules. These smart charging schedules were obtained from the LDC operator’s point of view based on various criteria, and considering system operational constraints, and customer’s rational behavior.

Studies on realistic test feeders were carried out considering uncontrolled and controlled (smart) charging. The effect of the LDC imposing a cap on system peak-demand was also studied for the smart charging scenarios. Probabilistic studies, to account for different and uncertain customers’ travel patterns, were carried out as well for uncontrolled charging and smart charging scenarios to determine the expected values of the various decision variables. The smart charging mode operation was determined to be effective in scheduling the PEV charging loads at appropriate hours, while meeting feeder constraints for the various objectives being considered.

It was observed from these studies that uncontrolled PEV charging can result in violation of grid constraints such as feeder current limits, bus voltages, and may also result in demand spikes. Therefore, controlled mode charging, through smart charging schedules is demonstrated to be the best approach for PEV charging in the context of smart grids.

REFERENCES


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