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Pricing System Security in Electricity Markets

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Abstract— **This paper proposes and describes in detail two distinct and novel methodologies to try to address the issue of a proper representation of system security in the operation of decentralized and hybrid electricity markets, so that adequate price signals can be given to all market participants. Both methodologies are based on the use of voltage stability theory to better account for system security. For each of the presented techniques, a methodology to determine "nodal" marginal prices is proposed and described in detail. A 6-bus system with both supply and demand-side bidding is described and used to test and compare the proposed techniques.**

Keywords— **Electricity markets, locational marginal prices (LMP), security, available transfer capability (ATC).**

I. Introduction

THE current rush to deregulate electricity markets throughout the world has led to the proposal of several market structures that could be categorized into three main groups, namely, pool or centralized markets (e.g., the "old" U.K. market, Chile, and PJM), standard auction or decentralized markets (e.g., Spain and the former California markets), and spot pricing or hybrid markets (e.g., New Zealand, Ontario, and the current U.K. and New England markets) [1], [2], [3].

A. Centralized or Pool Markets

Centralized markets can be basically viewed as unit commitment problems where a "central" broker/operator takes care of "dispatching" the market participants based on their bids, while accounting somewhat for the transmission system and network security. Because of the complexity of the problem, which is in itself a disadvantage since it is not "transparent" to the market participants, the network is usually modeled using only a dc power flow model. Limits on the transmission line power flows used to try to represent system security tend to be somewhat arbitrary, as these limits are predetermined based on off-line studies that do not necessarily correspond to the actual system operating conditions.

This model has the perceived advantage of producing an "optimal" solution, taking into consideration temporal restrictions such as minimum up and down times, but there is no guarantee that a unique solution, or even a solution may be found for this problem [4]. The dc model used to represent the network leads to solutions that most likely do not reflect the actual security level of the system, especially if conservatively low power flow limits are used in the transmission system, which is usually the case. The latter might lead to high prices as a result of unrealistic constraints becoming active.

Another disadvantage of this methodology is that it forces the unbundling of ancillary services, such as reactive power, requiring iterative techniques to obtain adequate solutions for energy and ancillary service markets [2].

Although some of these problems could be resolved in theory by including the ac power flow equations, this is not considered practical due to the complexity of the problem. Thus, this problem, together with the "centralized" nature of this type of market, gives participants a sense of lack of "transparency" in the handling of transactions. This has led to the use of other market architectures in the implementation of most new electricity markets. For all of these reasons, the issue of proper consideration and pricing of security in this type of market structure is not pursued further in this paper.

B. Decentralized or Simple Auction Markets

Decentralized markets are basically "transparent" markets run by a central broker where only the participants' bids are used to determine a market clearing price using a simple auction mechanism, without considering system constraints. The results of this auction are then passed on to a market operator who may approve, modify and/or reject the transactions, depending on the market rules.

In these types of markets, ancillary services are basically unbundled and determined in sequential markets (e.g., reactive power and reserve markets). This approach has the problem of not considering the actual system conditions during the biding process, particularly system security, which may lead to inadequate price signals that would make it difficult for the participants to develop appropriate biding strategies.

The present paper proposes a methodology to properly introduce system security in this type of market structure, so that prices somewhat reflect the "level" of security of the real system. The proposed methodology is based on an iterative computation of the Available Transfer Capability (ATC) and the "redispatch" of generators and loads by determining the impact of the different transactions through sensitivity analysis. The ATC and required sensitivities are all determined based on voltage stability criteria. These sensitivities are also used to determine a marginal price for each of the market participants, so that their effect on system security is accounted for in the final prices.

C. Hybrid or OPF-based Markets

Hybrid markets are based on classical spot pricing theory [1], [2], and allow for an adequate bundling of energy and ancillary service markets, so that proper price signals can be given to all market participants from a global system operation perspective through the use of locational or "nodal" marginal prices [5].

These markets can be viewed basically as Optimal Power Flow (OPF) "security constrained" problems, with the objective of maximizing social welfare as opposed to minimizing costs, i.e., the objective is to minimize the differences between supply and demand bids. The unit commitment problem is assumed to be resolved by "preprocessing" of the bids; for example, temporal restrictions such as ramp rates may be included as limits on the power bids (e.g., [6]).

An advantage of these types of markets is that there is extensive experience with the use of OPF in power system operation, with mature solution methodologies that have been proven to work in practice and that can be readily applied to electricity markets. For these reasons, many electricity markets are being implemented or modified to operate based on these techniques. A problem with these types of markets, as with the classical OPF problem, is that system security is somewhat arbitrarily represented through limits in power transfers that most likely do not reflect the actual security levels associated with a given operating condition, thus yielding inappropriate price signals [7].

In this paper, a technique to better account for system security than just using somewhat "arbitrary" limits on transmission line power flows is proposed for hybrid markets. The proposed methodology is based on a voltage stability constrained OPF proposed in [8]. Although in this method it is not possible to directly represent the (N-1) contingency criteria used in the computation of the ATC, the effects of contingencies can be indirectly represented through an "adequate" choice of the value of the stability margin in the optimization procedure, as explained below.

D. Content

The methodologies presented in this paper try to address the issue of a proper representation of system security in the operation of a decentralized and a hybrid market, so that adequate price signals can be given to all market participants. Both methodologies are based on the use of voltage stability theory to better account for system security [9]; although this approach does not fully account for the actual dynamical security of the system, it does represent system security in a much more accurate way than the simple use of predetermined limits on power flows.

The paper is thus structured as follows: Section II describes in detail the proposed auction system with redispatch methodology and the associated congestion pricing mechanisms. Section III discusses the introduction of voltage stability constraints in an OPF-based electricity market, including a discussion on the effect of these constraints on the marginal prices. Section IV comments on the results obtained for a 6 bus test system with demand-side bidding, concentrating especially in comparing the different proposed methodologies. Finally, Section V summarizes the main ideas and results obtained in this paper, as well as the advantages and disadvantages of the proposed techniques.

II. Simple Auction System with Redispatch

The redispatch and pricing methodology proposed here is basically an implementation of a simple auction system where ATC computations and sensitivity analyses, based on voltage stability criteria, are carried out for the given bidding conditions. This method is based on sensitivity formulas developed for the stability limit conditions defined by the ATC; the redispatch costs are then distributed among the market participants based on the sensitivity factors obtained from these formulas.

This section concentrates on describing the techniques used for the ATC and sensitivity calculations, and their use in the proposed redispatch and security pricing algorithm.

A. ATC Calculations and Sensitivity Formulas

The ATC, as defined by NERC, is a measure of the transfer capability remaining in the physical transmission network for further commercial activity over and above already committed uses [10]. Thus, mathematically it is defined as

$$\mathrm{ATC} = \mathrm{TTC} - \mathrm{ETC} - \mathrm{TRM} \tag{1}$$

where

$$\mathrm{TTC} = \min(P_{\max_{I_{lim}}}, P_{\max_{V_{lim}}}, P_{\max_{S_{lim}}})$$

represents the Total Transfer Capability, i.e., the maximum power that the system can deliver given the security constraints defined by thermal limits (I_{lim}), voltage limits (V_{lim}) and stability limits (S_{lim}), and an (N-1) contingency criterion (worst single contingency of the transmission system). ETC stands for the Existing Transmission Commitments, and TRM corresponds to the Transmission Reliability Margin, which includes a Capacity Benefit Margin (CBM) and is meant to account for uncertainties in system operation.

From the ATC definition (1), it is clear that its value changes with the system operating conditions, and hence has to be computed for every bidding condition. However, in most of the current implementations of electricity markets, the ATC is determined in off-line studies and represented in the bidding process as rather conservative limits on the power flowing through the transmission lines or main transmission "corridors". This is obviously unrealistic and could easily lead to either fictitious congestion problems, with the corresponding pricing implications, or unsecured operating conditions, which may lead to collapse problems.

The problem in the computation of the ATC is the actual determination of the stability limits, which require costly time domain simulations. Hence, in the present paper, the stability limits are approximately represented using voltage stability margins, which can be readily and quickly computed, giving a good idea of the "relative" stability of the network [9]. In this case, a continuation power flow approach is used to determine the maximum system loading or TTC, as defined in (1) [11].

Since voltage stability criteria are used to determine the ATC value, one can also readily determine the sensitivities of the ATC with respect to various systems parameters, especially with respect to the participants' bids. The required sensitivity formulas can be obtained from the definition of the TTC and the use of basic voltage stability concepts [9].

Thus, consider that the system can be represented in steady state with the following set of nonlinear equations:

$$f(x, \lambda, p) = 0 \tag{2}$$

where $x \in \Re^n$ stands for the state and algebraic system variables, such as bus voltage magnitudes and angles; $\lambda \in \Re$ is the "bifurcation" or loading parameter used to represent the system loading level, as the load powers are modeled as $P_L = P_{L_o} + \lambda P_D$, with P_{L_o} representing the load power levels at the initial loading conditions and P_D corresponding to the demand power bids (all loads are assumed to have constant power factors); and $p \in \Re^m$ corresponds to "controllable" system parameters, such as the supply and demand power bids P_S and P_D, respectively. In this analysis, generator powers are modeled as $P_G = P_{G_o} + (\lambda + k_G)P_S$, where P_{G_o} is the initial generation loading conditions and k_G is a variable used to represent a distributed slack bus. Equations (2) typically correspond to a set of "modified" power flow equations, which basically result from modeling system controls and limits in greater detail than in the typical power flow equations.

The voltage stability limits for system (2) are basically associated with saddle-node and limit-induced bifurcations of the corresponding set of nonlinear equations [9]; at these bifurcation points, the system collapses. Thermal and voltage limits, on the other hand, can be treated mathematically, for the purpose of sensitivity analyses, in a similar way as limit-induced bifurcations, although the system does not collapse when these limits are reached. Hence, for (2), the ATC can then be defined as

$$\text{ATC} = \lambda_c \tag{3}$$

if the TRM is neglected, and where λ_c is the "critical" ("maximum") loading level at which the system is at a limit condition, as per definition (1).

In this paper, all ATC and required sensitivity values are computed based on the results generated by UWPFLOW [12], which is a continuation power flow program capable of representing various power system elements using "detailed" steady state models.

A.1 Saddle-Node Bifurcations

Saddle-node bifurcations (SNB) are characterized by a pair of equilibrium points coalescing and disappearing as the bifurcation parameter λ "slowly" changes. Mathematically, the SNB point is an equilibrium point (x_c, λ_c, p_c) with a singular Jacobian $D_x f|_c$ and associated unique right and left "singular" eigenvectors v and w, respectively, i.e., $D_x f|_c v = D_x^T f|_c w = 0$.

By taking the derivatives of (2), one has at the SNB point that

$$\begin{aligned} D_x f|_c \, \mathrm{d}x + D_\lambda f|_c \, \mathrm{d}\lambda + D_p f|_c \, \mathrm{d}p &= 0 \\ \Rightarrow \quad w^T D_x f|_c \, \mathrm{d}x + w^T D_\lambda f|_c \, \mathrm{d}\lambda + w^T D_p f|_c \, \mathrm{d}p &= 0 \end{aligned}$$

Hence, from these equations and as proposed in [13], one has that the sensitivities of the system loading λ with respect to changes in the parameters p at the SNB point can be determined by using

$$\left.\frac{\mathrm{d}\lambda}{\mathrm{d}p}\right|_c = -\frac{1}{\omega^T D_\lambda f|_c} \, \omega^T D_p f|_c \tag{4}$$

A.2 Limits

Limit-induced bifurcations (LIB) are equilibrium points where a system control limit is reached, which in some cases may lead to a system collapse characterized by a pair equilibrium points coalescing and disappearing for slow changes of the bifurcation parameter λ. At a LIB, as opposed to a SNB, the system Jacobian is not singular at the bifurcation point (x_c, λ_c, p_c); hence, equation (4) does not apply at this point.

In general, a system reaching a limit at an equilibrium point (x_c, λ_c, p_c) can be characterized by two different sets of equations, i.e.,

$$\begin{aligned} f_1(x_c, \lambda_c, p_c) &= 0 \\ f_2(x_c, \lambda_c, p_c) &= 0 \end{aligned} \tag{5}$$

where the first set $f_1(\cdot)$ corresponds to the "original" system equations, whereas the second set $f_2(\cdot)$ corresponds to a modified set of equations where the limit is active. For example, when a reactive power generator limit is reached at a bus i, a generator voltage control equation, say $V_i - V_{i_c} = 0$, may be replaced by $Q_{G_i} - Q_{G_{lim}} = 0$ at the limit condition. Hence, taking the derivatives of (5) at the equilibrium point where the limit becomes active,

$$\begin{aligned} D_x f_1|_c \, \mathrm{d}x + D_\lambda f_1|_c \, \mathrm{d}\lambda + D_p f_1|_c \, \mathrm{d}p &= 0 \\ D_x f_2|_c \, \mathrm{d}x + D_\lambda f_2|_c \, \mathrm{d}\lambda + D_p f_2|_c \, \mathrm{d}p &= 0 \end{aligned}$$

Eliminating $\mathrm{d}x$ from these equations, leads to

$$\left.\frac{\mathrm{d}\lambda}{\mathrm{d}p}\right|_c = \frac{1}{\mu^T \mu} \, \mu^T \left(D_x f_2|_c \, D_x f_1|_c^{-1} \, D_p f_1|_c - D_p f_2|_c \right) \tag{6}$$

where

$$\mu = D_\lambda f_2|_c - D_x f_2|_c \, D_x f_1|_c^{-1} \, D_\lambda f_1|_c$$

Observe that the sensitivity formula (6) applies to any limit condition, independent of whether it corresponds to a LIB or a thermal or voltage limit. Hence, this equation together with (4) are used to determine the sensitivity of the ATC with respect to the system parameters p, which for the purpose of this paper correspond to the supply and demand bids P_S and P_D, respectively.

B. Redispatch and Security Pricing Algorithm

The redispatch technique proposed in this paper follows similar general criteria as the one currently used in the New England electricity market [14], where redispatch is used to address the issue of a simple auction mechanism, which neglects system losses, yielding a market clearing condition that violates certain congestion criteria, as defined by power flow limits on the transmission system. The costs resulting from dispatching a more expensive unit than the market clearing price (MCP) to solve the congestion problem are then "distributed" among the different participants. The main improvements of the technique proposed here with respect to known methodologies are:

- The ATC is computed "on-line", i.e., taking into consideration the actual system conditions, as opposed to using power flow limits determined off-line that do not necessarily represent the system security level.
- The congestion costs are distributed among the market participants based on the actual impact that each one of them has on the ATC value.

The suggested redispatch technique to address the problem of market clearing conditions that do not meet the actual ATC requirements, as defined in the previous section, is summarized in the flow-diagram shown in Fig. 1. This methodology determines the transaction costs for the different market participants as follows:

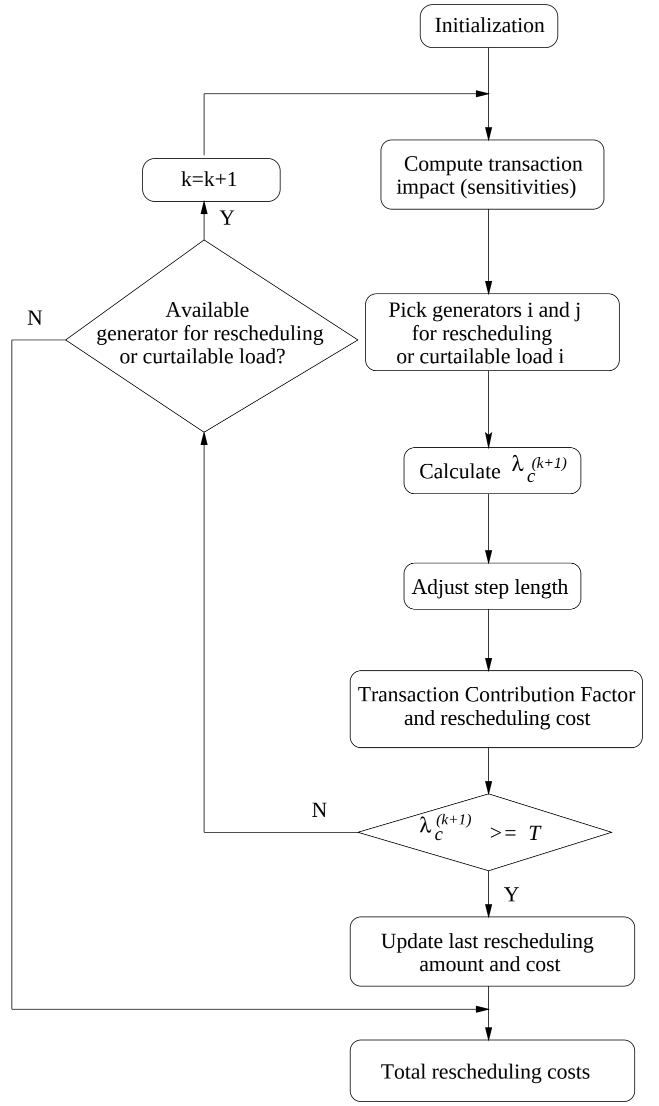

Fig. 1. Redispatch technique for a simple auction system. The ATC at each iteration is represented by $\lambda_c^{(k+1)}$, whereas T represents the transaction level that clears the market.

B.1 Step 1

Using a simple auction mechanism, the MCP and associated total transaction power level T are determined, together with the load and generator power levels that clear the market, namely, P_D and P_S and for all loads and generators, respectively. The values of P_D and P_S are used in the determination of the ATC, as these define the load and generator direction used for the computation of λ_c in UWPFLOW [12].

B.2 Step 2

If the ATC is violated, i.e., if $\lambda_c < T$, the impact of each possible system transaction is determined using (4) or (6), depending on the limiting factor that defines the ATC. Thus, the generator i with the most positive impact on the ATC that has not been fully dispatched in the bidding process, and the generator j dispatched in step 1 with the most negative or least positive impact in the ATC are chosen for rescheduling. Thus, the corresponding increase and decrease in generation is defined as

$$\Delta P_{S_i}^{(k)} = -\Delta P_{S_j}^{(k)} = \Delta P_S^{(k)}$$

where k is the number of iteration in the redispatch process, and $\Delta P_S^{(k)}$ is chosen depending on the value that one wants for the ATC, since the new value of the ATC may be approximated by

$$\lambda_c^{(k+1)} \approx \lambda_c^{(k)} + \left.\frac{\mathrm{d}\lambda}{\mathrm{d}P_{S_i}}\right|_c^{(k)} \Delta P_{S_i}^{(k)} + \left.\frac{\mathrm{d}\lambda}{\mathrm{d}P_{S_j}}\right|_c^{(k)} \Delta P_{S_j}^{(k)} \qquad (7)$$

It is assumed here that $\mathrm{d}\lambda/\mathrm{d}P_{S_i}|_c^{(k)} > \mathrm{d}\lambda/\mathrm{d}P_{S_j}|_c^{(k)}$, otherwise no ATC improvements can be attained by redispatching generation.

Since the whole process is based on a linearization, one cannot make large changes in generated power, otherwise this might have a large effect on the actual ATC value, which changes nonlinearly as the parameters change; hence the need for an iterative process. The amount of generation chosen for redispatch $\Delta P_S^{(k)}$ may be readjusted when determining the actual value of $\lambda_c^{(k+1)}$ using the full nonlinear system, as explained below.

Observe that system losses are not considered here, as these are just assigned to a "slack" bus. This basically means that other market mechanisms are used to account for losses in the process, which is typically the case in decentralized market structures.

Only if there are no adequate generators available for redispatch, is the load considered for curtailment. This approach is to be expected when the load is inelastic, as these types of loads require that the forecasted load be dispatched, given the high "costs" of load curtailment. In the case of demand-side bidding, however, the load could be considered for redispatching in the same way as the generators, i.e., the load with the most negative impact on the ATC, say i, may be reduced by an amount that has a "significant" impact on the ATC value, as per approximation

$$\lambda_c^{(k+1)} \approx \lambda_c^{(k)} - \left.\frac{\mathrm{d}\lambda}{\mathrm{d}P_{D_i}}\right|_c^{(k)} \Delta P_{D_i}^{(k)} \tag{8}$$

This equation can be used for rescheduling inelastic loads as well. In this case, there is no security cost for load curtailment, since one assumes that the loads are intrinsically running a risk of not being dispatched by participating in the market. However, this means that overall system revenues are lost, as loads that could be served by redispatching available generation would not be dispatched, even if these loads are willing to pay the higher costs of rescheduling generated by the proposed market process. Hence, the load redispatch option, whether the loads are inelastic or not, is only considered if no generation bids are available to solve the ATC violation. Observe that in this case, the transaction level T is affected by the load reduction; thus,

$$T^{(k+1)} = T^{(k)} - \Delta P_{D_i}^{(k)}$$

This is not the case for generation redispatch; in this case, $T^{(k+1)} = T^{(k)}$.

B.3 Step 3

The step changes in generation or load are readjusted by computing the actual value of $\lambda_c^{(k+1)}$ and comparing it to the approximated value computed using (7) or (8). If the difference is greater than a chosen tolerance, the previous step and this one are repeated with smaller changes in the supply and demand until the desired tolerance is met.

B.4 Step 4

The redispatching costs for the given iteration k are then determined based on a Transaction Contribution Factor (TCF) as defined by

$$TCF_i^{(k)} = \frac{\mathrm{d}\lambda/\mathrm{d}p_i|_c^{(k)}\ p_i^{(k)}}{\sum_j \mathrm{d}\lambda/\mathrm{d}p_j|_c^{(k)}\ p_j^{(k)}} \tag{9}$$

where i stands for the bus number, and $p_i^{(k)}$ corresponds to the value of the corresponding parameter, i.e., the value of $P_{S_i}^{(k)}$ or $P_{D_i}^{(k)}$. Only buses with negative impact on the ATC, i.e., buses with $\mathrm{d}\lambda/\mathrm{d}p_i|_c^{(k)} < 0$, are considered in this computation; buses with positive impact are given a zero TCF value, so that market participants that do not create the security problem are not charged for the cost of keeping the system secure. The parameter values $p_i^{(k)}$ are included in this "normalization" process to account for the "size" of the corresponding transactions in the security cost.

The total generator rescheduling security cost of the k^{th} iteration may be defined as

$$SC_k = (C_{S_i} - MCP)\ \Delta P_{S_i}^{(k)} \tag{10}$$

where i is the generator chosen in Step 2, with a marginal cost or bid C_{S_i}, and MCP is the market clearing price obtained from the simple bidding process in Step 1. In the case of inelastic loads, one can argue that there should be a cost associated with curtailing the load that should be considered as part of the cost of keeping the system secure; thus, one would have in this case that

$$SC_k = A_{D_i}\ \Delta P_{D_i}^{(k)} \tag{11}$$

where A_{D_i} is the "cost" of curtailing the load at the chosen bus i.

B.5 Step 5

If the ATC requirements are met, i.e., if $\lambda_c^{(k+1)} < T^{(k+1)}$, then the iterative process stops, say at $k = m$. At this point, the final generator or load reschedules are adjusted based on (7) or (8), respectively, so that the ATC and final transaction level are the same, i.e., $\lambda_c^{(m+1)} \approx T^{(m)}$ Thus,

$$\Delta P_{S_i}^{(m)} = -\Delta P_{S_j}^{(m)} = \frac{T^{(m)} - \lambda_c^{(m)}}{\mathrm{d}\lambda/\mathrm{d}P_{S_i}|_c^{(m)} - \mathrm{d}\lambda/\mathrm{d}P_{S_j}|_c^{(m)}}$$

or

$$\Delta P_{D_i}^{(m)} = \frac{T^{(m)} - \lambda_c^{(m)}}{\mathrm{d}\lambda/\mathrm{d}P_{D_i}|_c^{(m)}}$$

B.6 Step 6

The final transaction levels and marginal costs for each i node are readily determined as follows:

- Generators:

$$\begin{aligned} P_{S_i} &= P_{S_i}^{(0)} + \sum_{k=1}^{m} \Delta P_{S_i}^{(k)} \\ \rho_{S_i} &= MCP - \frac{1}{P_{S_i}} \sum_{k=1}^{m} TCF_i^{(k)}\ SC_k \end{aligned} \tag{12}$$

- Loads

$$\begin{aligned} P_{D_i} &= P_{D_i}^{(0)} - \sum_{k=1}^{m} \Delta P_{D_i}^{(k)} \\ \rho_{D_i} &= MCP + \frac{1}{P_{D_i}} \sum_{k=1}^{m} TCF_i^{(k)}\ SC_k \end{aligned}$$

B.7 Observations

It is important to highlight the fact that the proposed technique to compute "nodal" marginal prices assumes that all the costs of redispatching are fully distributed among the market participants, as is the case of the current New England electricity markets [14]. Hence, since a "zero sum" approach is used in this case, there are no "leftovers" to be used by the ISO for possible upgrades of the transmission system. Furthermore, costs associated with losses and reactive power dispatch, as well as reserves for frequency control, are assumed to be resolved in other independent auction markets or by other market mechanisms, which may in turn yield system operating conditions that could have a significant effect on the marginal costs generated by each one of the different markets. Although the latter might be resolved by using other iterative processes, somewhat similar to the one proposed here for the integration of the ATC into the auction system, this is certainly a disadvantage of these types of markets.

Hybrid markets, on the other hand, produce solutions that account for several of the market costs through an "integrated" solution approach, thus resolving most of these problems. However, these market structures present several other disadvantages associated with a "black box" approach, such as lack of "transparency" and convergence difficulties, as discussed in the next section.

III. Spot Price of Security

Hybrid market structures are based on the determination of a spot price of electricity obtained from an OPF solution and its associated Lagrangian multipliers [1], [2], [4]. This technique basically consists on solving the following OPF problem [5]:

$$\begin{aligned}
\text{Min.} \quad & C_S^T P_S - C_D^T P_D && (13)\\
\text{s.t.} \quad & f(\delta, V, Q_G, P_S, P_D) = 0 \quad \text{(PF eqs.)}\\
& 0 \leq P_S \leq P_{S\max} \quad \text{(supply bid blocks)}\\
& 0 \leq P_D \leq P_{D\max} \quad \text{(demand bid blocks)}\\
& |P_{ij}(\delta, V)| \leq P_{ij\max} \quad \text{("security" limits)}\\
& Q_{G\min} \leq Q_G \leq Q_{G\max} \quad \text{(gen. Q limits)}\\
& V_{\min} \leq V \leq V_{\max} \quad \text{(bus V limits)}
\end{aligned}$$

where C_S and C_D are vectors of supply and demand bids in \$/MWh, respectively; Q_G stands for the generator reactive powers; V and δ represent the bus phasor voltages; and P_{ij} represent the powers flowing through the lines, which are typically used to "indirectly" represent system security. P_S and P_D are the vectors of supply and demand bids of power in MW, provided in blocks of $P_{S\max}$ and $P_{D\max}$, and are associated with generator and load powers, since $P_G = P_{G_o} + P_S$, and $P_L = P_{L_o} + P_D$, where P_{G_o} and P_{L_o} basically define the initial system operating conditions (loads are assumed to have constant power factors).

The OPF problem (13) can be transformed into the following optimization problem based on the Lagrangian that results from a logarithmic barrier Interior Point approach:

$$\begin{aligned}
\text{Min.}\ \mathcal{L} \;=\;& C_S^T P_S - C_D^T P_D && (14)\\
& -\rho^T\ f(\delta, V, Q_G, P_S, P_D)\\
& -\mu_{P_{S\max}}^T\ (P_{S\max} - P_S - s_{P_{S\max}})\\
& -\mu_{P_{D\max}}^T\ (P_{D\max} - P_D - s_{P_{D\max}})\\
& -\mu_{P_{ij\max}}^T\ (P_{ij\max} - |P_{ij}(\delta, V)| - s_{P_{ij\max}})\\
& -\mu_{Q_{G\min}}^T\ (Q_G - Q_{G\min} - s_{Q_{G\min}})\\
& -\mu_{Q_{G\max}}^T\ (Q_{G\max} - Q_G - s_{Q_{G\max}})\\
& -\mu_{V_{\min}}^T\ (V - V_{\min} - s_{V_{\min}})\\
& -\mu_{V_{\max}}^T\ (V_{\max} - V - s_{V_{\max}})\\
& -\mu_s \left(\sum_i \ln s_i \right)
\end{aligned}$$

where $\rho \in \Re^n$ and

$$\mu = \left[\mu_{P_{S\max}}\ \mu_{P_{D\max}}\ \mu_{P_{ij\max}}\ \mu_{Q_{G\min}}\ \mu_{Q_{G\max}}\ \mu_{V_{\min}}\ \mu_{V_{\max}}\ \mu_s\right]^T$$

$\mu > 0$, are the Lagrangian multipliers, and

$$s = \left[s_{P_{S\max}}\ s_{P_{D\max}}\ s_{P_{ij\max}}\ s_{Q_{G\min}}\ s_{Q_{G\max}}\ s_{V_{\min}}\ s_{V_{\max}}\right]^T$$

$s \geq 0$, are the slack variables ($s = [s_i]$).

The marginal costs for the supply and demand can be defined from (14), since

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial P_{S_i}} &= C_{S_i} - \rho_{S_i} + \mu_{P_{S\max_i}} = 0 && (15)\\
\frac{\partial \mathcal{L}}{\partial P_{D_i}} &= -C_{D_i} + \rho_{D_i} + \mu_{P_{D\max_i}} = 0
\end{aligned}$$

as $\partial f_i / \partial P_{S_i} = 1$ and $\partial f_i / \partial P_{D_i} = -1$, from the power flow equations. Thus, from (15), the marginal or shadow price for each market participant, i.e., the Locational Marginal Price (LMP), is given by the corresponding Lagrangian multiplier [5], i.e.,

$$LMP_i = \rho_i \qquad (16)$$

In (13), the limits on the line power flows are used to somehow represent the system security level, i.e., indirectly represent the system's ATC. These limits are determined off-line, assuming certain "typical" system conditions and contingencies, and hence do not represent the actual operating conditions and the effect of the bidding process on system security. This is obviously one of the major disadvantages of this method, and hence is resolved here by including additional constraints that define a minimum voltage stability margin, as originally proposed for the standard OPF problem in [8]. Thus, the following modifications are proposed to better account for system security:

$$\begin{aligned}
\text{Min.} \quad & C_S^T P_S - C_D^T P_D \qquad (17)\\
\text{s.t.} \quad & f(\delta, V, Q_G, P_S, P_D) = 0 \quad \text{(PF base case)}\\
& f(\delta_c, V_c, Q_{G_c}, \lambda_c, P_S, P_D) = 0 \quad \text{(PF max. load)}\\
& \lambda_c \geq \lambda_{c_o} \quad \text{(min. stab. margin)}\\
& 0 \leq P_S \leq P_{S\max} \quad \text{(supply bid blocks)}\\
& 0 \leq P_D \leq P_{D\max} \quad \text{(demand bid blocks)}\\
& |P_{ij}(\delta, V)| \leq P_{ij_{\max}} \quad \text{(thermal limits)}\\
& |P_{ij}(\delta_c, V_c)| \leq P_{ij_{\max}}\\
& Q_{G\min} \leq Q_G \leq Q_{G\max} \quad \text{(gen. Q limits)}\\
& Q_{G\min} \leq Q_{G_c} \leq Q_{G\max}\\
& V_{\min} \leq V \leq V_{\max} \quad \text{(bus V limits)}\\
& V_{\min} \leq V_c \leq V_{\max}
\end{aligned}$$

where λ_c is a scalar that represents a loading parameter that drives the system to a "critical" point, and thus defines the generation and load powers at the critical point as follows: $P_{G_c} = P_{G_o} + (1 + \lambda_c + k_{G_c})\ P_S$, and $P_{L_c} = P_{L_o} + (1 + \lambda_c)\ P_D$, with k_{G_c} representing a variable used to distribute the system losses among the generators in proportion to the generator bids (distributed slack bus model). The loading parameter λ_c is used to guarantee a minimum voltage stability margin λ_{c_o}, and thus indirectly represents the system ATC; however, contingencies cannot be directly included in this problem as opposed to the previously proposed technique. The latter can somewhat be resolved with the choice of the value of λ_{c_o}, as the larger this value, the larger the contingency that the system can withstand, given that this indirectly represents the system stability margin. However, one has to be careful when choosing this margin, as a " large" value might lead to convergence difficulties or a lack of solution all together, which is one of the main problems of this approach. Notice that, in this case, the limits in the transmission line flows are only used to represent actual thermal limits as opposed to the "security" constraints in (13).

The LMPs can also be determined using the corresponding Lagrangian multipliers for (17). Thus, it can be readily shown that

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial P_{S_i}} &= C_{S_i} - \rho_{S_i} - \rho_{c_{S_i}}(1 + \lambda_c + k_{G_c}) \qquad (18)\\
&\quad + \mu_{P_{S\max_i}} = 0\\
\frac{\partial \mathcal{L}}{\partial P_{D_i}} &= -C_{D_i} + \rho_{D_i} + \rho_{c_{D_i}}(1 + \lambda_c)\\
&\quad + \mu_{P_{D\max_i}} = 0
\end{aligned}$$

where ρc corresponds to the Lagrangian multiplier of the power flow equations at the critical point. Hence, the LMP for each market participant is also given by equation (16), i.e., $LMP_i = \rho_i$. Observe that in (18), the LMP value accounts for the actual bid (C_i), the bid limits ($\mu_{P_{\max_i}}$), as well as the stability margin (ρ_{c_i} and λ_c).

As with any spot pricing methodology, the binding constraints have a direct effect on the solution of optimization problem (17), and hence are indirectly represented in the price through the value of the corresponding Lagrangian multipliers [5]. For example, generator Q limits do have an effect on the optimal solution and thus affect the LMPs, even though reactive power pricing is not directly considered in the optimization problem.

For inelastic loads, all the previous equations should be changed to assume a fixed demand value. In this case, the load is not treated as a variable parameter within the optimization process. However, one may still use the Lagrangian multipliers at each load bus as the LMPs for the load, since an argument can be made that these represent the marginal costs of supplying power at the load buses [5]. Observe that lack of solution or convergence difficulties may be encountered in this case, depending on the value of the inelastic demand and other system constraints; in other words, the system might not be able to supply the required demand under the given constraints.

The possible convergence difficulties or lack of solution, as well as the integrated approach that somewhat "hides" the effect of the different market components and constraints into a final overall solution, are certainly disadvantages of OPF-based approaches when compared to the use of simple auctions. Furthermore, as opposed to the auction mechanism proposed in this paper, an adequate representation of the effect of contingencies on system security, and the inclusion of a mechanism to allow and account for inelastic load curtailment due to security reasons are not feasible in the OPF methodology proposed in this paper. However, OPF-based techniques yield price values that directly or indirectly account for the effect that the different system constraints have in the market. Furthermore, these methodologies yield a relatively simple mechanism to generate revenues for the ISO to be used in possible system improvements, as demonstrated on the example discussed in the next section.

IV. Example

The two techniques proposed in the previous section are implemented and compared to standard simple auction and spot pricing techniques based on the results obtained for the test system of Fig. 2 [2].

This is a simple 6 bus test system with 3 generation companies (GENCOs) that provide supply bids, and three energy supply companies (ESCOs) that provide demand bids, as shown in Table I. The data for this system is extracted from [2] and is shown in Tables II and III. The maximum transmission line power flows were determined

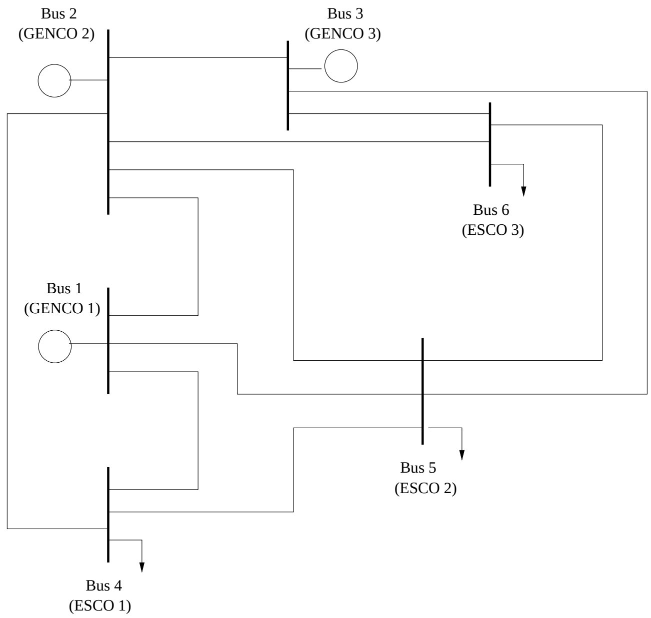

Fig. 2. Test system.

TABLE I
GENCO and ESCO Bids

Bus i	Participant	C [\$/MWh]	$P_{\max}$ [MW]
1	GENCO 1 (S_1)	9.7	20
2	GENCO 2 (S_2)	8.8	25
3	GENCO 3 (S_3)	7	20
4	ESCO 1 (D_1)	12	25
5	ESCO 2 (D_2)	10.5	10
6	ESCO 3 (D_3)	9.5	20

TABLE II
Bus Data at Base Loading

Bus i	P_{L_o} [MW]	Q_{L_o} [MVar]	P_{G_o} [MW]	$Q_{G_{\max}}$ [MVar]	$Q_{G_{\min}}$ [MVar]
1	0	0	90	150	-150
2	0	0	140	150	-150
3	0	0	60	150	-150
4	90	60	0	0	0
5	100	70	0	0	0
6	90	60	0	0	0

Bus i	V [p.u.]	$V_{\max}$ [p.u.]	$V_{\min}$ [p.u.]
1	1.05	1.1	0.9
2	1.05	1.1	0.9
3	1.05	1.1	0.9
4	0.9754	1.1	0.9
5	0.9677	1.1	0.9
6	0.9930	1.1	0.9

TABLE III
Line Data

Line i-j	R_{ij} [p.u.]	X_{ij} [p.u.]	$B_i/2$ [p.u.]	$P_{ij_{\max}}$ [MW]
1-2	0.1	0.2	0.02	-29.88
1-4	0.05	0.2	0.02	90.62
1-5	0.08	0.3	0.03	29.14
2-3	0.05	0.25	0.03	22.62
2-4	0.05	0.1	0.01	81.67
2-5	0.1	0.3	0.02	51.43
2-6	0.07	0.2	0.025	56.3
3-5	0.12	0.26	0.025	42.13
3-6	0.02	0.1	0.01	57.27
4-5	0.2	0.4	0.04	-21.83
5-6	0.1	0.3	0.03	-13.67

"off-line" through a series of maximum loading studies using UWPFLOW [12], and based on (N-1) contingency criteria; these limits represent the maximum power flow on the different lines for a maximum loading condition calculated using the load and generation pattern defined by the bids, for a worst contingency scenario. Thermal limits on the lines were assumed to be large, and hence are neglected in this case.

Results for inelastic demand were not obtained for this system, to simplify the analysis of the results as well as to shorten the length of the paper. Nevertheless, as discussed in the previous sections, this problem can be viewed as a special case of demand-side biding.

A. Simple Auctions

A simple auction mechanism, neglecting losses, yields the results depicted in Fig. 3 and Table IV, which included the voltages, total transaction level (T), losses, total cost, and the leftover payment for the ISO for comparison purposes. Observe that the results consider that one of the generators (GENCO 2), picks up the losses, thus leaving a cost that has to be covered by the ISO using another auction or market mechanism. The ATC computations, however, yield a value of $\lambda_c = 38.23$ MW corresponding to a minimum load bus voltage limit for a worst contingency on Line 2-4. Hence, since $\lambda_c < T$, generator redispatch is necessary to make the transaction feasible.

The generation redispatch procedure of Fig. 1 yields the results depicted on Tables V and VI. This procedure is based on the sensitivity vector $\mathrm{d}\lambda/\mathrm{d}p|_c^{(k)}$, as previously explained; for example, at the initial iteration $k = 1$ is com-

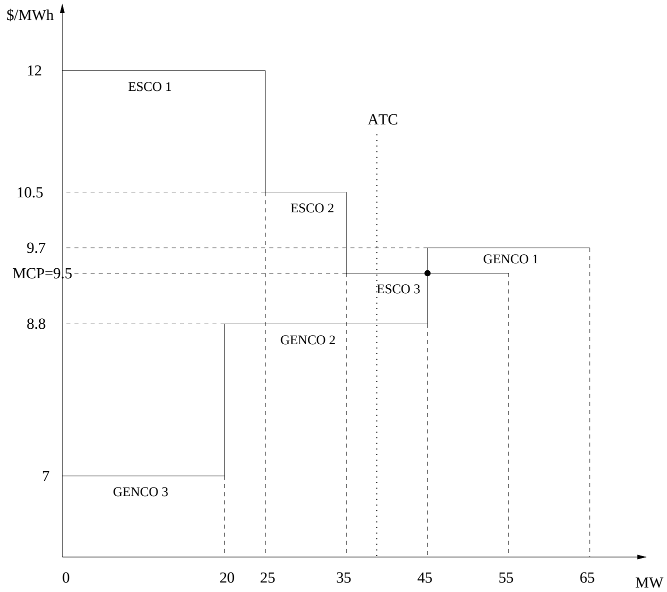

Fig. 3. Solution of simple auction for 6-bus test system.

TABLE IV
SIMPLE AUCTION RESULTS

Bus i	Part.	V [p.u.]	$\rho = MCP$ [\$/MWh]	P [MW]	Pay [\$/h]
1	S_1	1.050	9.5	0.00	0.00
2	S_2	1.050	9.5	28.55	-271.23
3	S_3	1.050	9.5	20.00	-190.00
4	D_1	0.967	9.5	25.00	237.50
5	D_2	0.956	9.5	10.00	95.00
6	D_3	0.983	9.5	10.00	95.00
TOTALS		T=45 MW Losses=3.55 MW ATC=38 MW		Cost=427.50 \$/h ISO=-34.23 \$/h	

puted using (6) and is equal to

$$\left.\frac{\mathrm{d}\lambda}{\mathrm{d}p}\right|_c^{(1)} = \begin{bmatrix} \mathrm{d}\lambda/\mathrm{d}P_{S_1}|_c^{(1)} \\ \mathrm{d}\lambda/\mathrm{d}P_{S_2}|_c^{(1)} \\ \mathrm{d}\lambda/\mathrm{d}P_{S_3}|_c^{(1)} \\ \mathrm{d}\lambda/\mathrm{d}P_{D_1}|_c^{(1)} \\ \mathrm{d}\lambda/\mathrm{d}P_{D_2}|_c^{(1)} \\ \mathrm{d}\lambda/\mathrm{d}P_{D_3}|_c^{(1)} \end{bmatrix} = \begin{bmatrix} 0.95 \\ -0.03 \\ 0.03 \\ -1.55 \\ -0.49 \\ -0.09 \end{bmatrix}$$

Table VII shows the final results of the proposed redispatch procedure. Observe the difference between these results and the original simple auction results of Table IV; although the total transaction level is the same, the power levels, marginal prices, and/or transaction costs for each market participant are different.

TABLE V
RESCHEDULING AND SECURITY COSTS

k	$\Delta P_{G_1}^{(k)}$ [MW]	$\Delta P_{G_2}^{(k)}$ [MW]	$\lambda_c^{(k)}/T$	SC_k [\$/h]
1	5	-5	0.90094	1
2	5	-5	0.95647	1
3	3	-3	0.992	0.6
4	1	-1	1.0042	0.2

TABLE VI
TRANSACTION CONTRIBUTION FACTORS $TCF_i^{(k)}$

k	1	2	3	4
$TCF_1^{(k)}$	0	0	0	0
$TCF_2^{(k)}$	0.013	0.0462	0.065	0.075
$TCF_3^{(k)}$	0	0.0329	0.0785	0.1084
$TCF_4^{(k)}$	0.858	0.8325	0.789	0.7606
$TCF_5^{(k)}$	0.109	0.0884	0.0673	0.056
$TCF_6^{(k)}$	0.02	0	0	0

B. OPF-based Pricing

The results of applying the basic OPF-based procedure (13), which does not properly account for system security, to determine the participants' marginal costs and power levels are depicted in Table VIII. These results were obtained in MATLAB based on an Interior Point optimization approach. In this case, only the upper voltage limits on generator buses play a role, as these voltages are forced to their maximum values by the optimization procedure, which is to be expected, since this would reduce losses and in general improve system security. The latter is demonstrated by the higher value of ATC obtained in this case, which was calculated "off-line" to determine the feasibility of the given transaction. However, observe that the transaction level T is smaller, even though the ATC is larger; there is no clear reason for this reduction, which highlights

TABLE VII
SIMPLE AUCTION WITH REDISPATCH RESULTS

Bus i	Part.	V [p.u.]	ρ [\$/MWh]	P [MW]	Pay [\$/h]
1	S_1	1.050	9.7000	14.00	-135.80
2	S_2	1.050	9.4897	14.56	-138.17
3	S_3	1.050	9.4949	20.00	-189.90
4	D_1	0.966	9.5926	25.00	239.80
5	D_2	0.956	9.5249	10.00	95.20
6	D_3	0.984	9.5020	10.00	95.00
TOTALS		T=45 MW Losses=3.56 MW ATC=45 MW		Cost=430.00 \$/h ISO=-33.87 \$/h	

TABLE VIII

OPF-based Results

Bus i	Part.	V [p.u.]	ρ [\$/MWh]	P [MW]	Pay [\$/h]
1	S_1	1.100	8.8415	0.0	0.00
2	S_2	1.100	8.8000	24.0	-211.31
3	S_3	1.100	8.9583	20.0	-179.17
4	D_1	1.026	9.3651	25.0	234.13
5	D_2	1.015	9.4560	10.0	94.56
6	D_3	1.038	9.2361	7.4	68.32
TOTALS		T=42.4 MW Losses=1.6 MW ATC=69.37 MW		Cost=397.01 \$/h ISO=6.54 \$/h	

TABLE IX

OPF-based Results Including Security ($\lambda_{c_o} = 5.5$ p.u.)

Bus i	Part.	V [p.u.]	ρ [\$/MWh]	P [MW]	Pay [\$/h]
1	S_1	1.100	8.8306	0.0	-0.00
2	S_2	1.100	8.7911	21.9	-192.62
3	S_3	1.100	8.9422	20.0	-178.84
4	D_1	1.026	9.3541	25.0	233.85
5	D_2	1.015	9.4413	10.0	94.41
6	D_3	1.039	9.2154	5.5	50.22
TOTALS		T=40.5 MW Losses=1.4 MW ATC=66.59 MW		Cost=378.48 \$/h ISO=7.02 \$/h	

the fact that, as previously mentioned, it is not easy to clearly pinpoint all the factors that influence the final solution in an integrated OPF-based technique. Notice that there is some money left from the transaction, which the ISO can use to cover for the transmission system losses.

When system security is directly included in the OPF-based approach, the solution obtained from solving (17) for a minimum "stability" margin value of $\lambda_{c_o} = 0.1$ p.u. is exactly the same as the one obtained without considering system security. The reason for this is that the required minimum margin is much smaller than the actual system ATC, and hence the corresponding system constraints have no influence in the solution of the optimization problem. However, when this value is significantly increased to $\lambda_{c_o} = 5.5$ p.u., the solution changes to that shown in Table IX, as the stability margin becomes a binding constraint. Observe that the marginal prices are smaller for all market participants in this case, as one would expect from equations (18), and that the transaction level has decreased slightly, which is also to be expected, since the security constraints now play a role on the solution process, reducing the amount of power that can be transacted through the transmission system. On the other hand, the amount of money left over has increased, which basically means that the ISO can use these funds to take care of losses as well as congestion problems in the system.

V. Conclusions

Two distinct techniques to manage and price system security in different electricity market structures are proposed, explained in detail, and briefly tested in this paper. Both techniques are based on basic concepts of voltage stability in power systems, and can be viewed as improvements of methodologies currently used on-line to determine electricity prices in actual markets.

From the tests and analyses, the following main conclusions can be drawn:

- The system ATC can be adequately accounted for in simple auction markets through the iterative technique proposed here. In the case of OPF-based markets, although the current system stability level can be accounted for in the solution process, the issue of contingencies cannot be directly included.
- In both techniques, a value for the price of security is generated. However, the OPF-based approach has the advantage of directly generating additional funds for the ISO to take care of system upgrades associated with its losses and security; other mechanisms must be devised and used in simple auctions for this specific purpose.
- The auction-based mechanism is robust and does not present major numerical difficulties. Furthermore, costs of load curtailment for inelastic loads can be readily included in the process. This is not the case for the OPF-based approach, which may present convergence problems and, in some cases, even a lack of solution, especially for inelastic loads; the inclusion of load curtailment costs does not appear to be a simple matter in this case.
- In the OPF-based technique, it is not simple to readily distinguished the different factors that influence the final results, as these are somewhat hidden in the solution. In the case of the auction-based technique, this is relatively simple to do, as the different parts and transactions that make up the market are handled independently of each other. However, iterative procedures are required in this case to be able to account for all the different factors that affect the final solution.

The authors are currently working on testing further the proposed techniques to better understand their benefits and drawbacks, and to determine possible improvements.

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