An Affine Arithmetic Based Method for Voltage Stability Assessment of Power Systems with Intermittent Generation Sources

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***Abstract*—This paper presents a novel method based on Affine Arithmetic (AA) for voltage stability assessment of power systems considering uncertainties associated with operating conditions, which may be attributed to intermittent generation sources, such as wind and solar. The proposed AA based method reduces the computational burden as compared to Monte Carlo (MC) simulations, and also improves the accuracy as compared to some analytical approaches. The proposed method is tested using two study cases, first, a 5-bus test system is used to illustrate the proposed technique in detail, and thereafter a 2383-bus test system to demonstrate its practical application. The results are compared with those obtained using MC simulations to verify the accuracy and computational burden of the proposed AA-based method, and also with respect to a previously proposed technique to estimate parameter sensitivities in voltage stability assessment.**

***Index Terms*—Voltage stability, PV curves, uncertain systems, affine arithmetic**

I. Nomenclature

A. Indices

- i, k Bus indices.
- h Index for noise symbols introduced by approximations of non-affine operations.
- max Index for maximum limit.
- min Index for minimum limit.
- m Index for buses with intermittent sources of power.
- n Index for buses with voltage control settings.
- o Index indicating initial values or central values in case of affine forms.
- s Iteration number.
- sa Index for saddle node bifurcation.

B. Variables

- A Matrix of coefficients for affine forms used in AA-based power flow analysis.
- A_{vs} Matrix of coefficients for affine forms used in AA-based voltage stability assessment (p.u.).
- b_r Floating-point series coefficients of the Chebyshev polynomials.
- C Auxiliary vector for linear programming formulation in the AA-based power flow analysis (p.u.).
- C_{max} Maximum elements of C (p.u.).
- C_{min} Minimum elements of C (p.u.).
- C_{vs} Auxiliary vector for the linear programming formulation in the AA-based voltage stability assessment (p.u.).
- ΔL Load changes (MW).
- $\Delta\lambda$ Variation of bifurcation parameter (p.u.).
- $\Delta\theta$ Variation of bus voltage angle (p.u.).
- ΔV Variation of bus voltage magnitude (p.u.).
- $D_p f$ Jacobian of f with respect to the system parameters (p.u.).
- $D_x f$ Jacobian of f with respect to the power flow variables (p.u.).
- $D_\lambda f$ Jacobian of f with respect to the bifurcation parameter (p.u.).
- e Error measured with respect to MC simulations (%).
- ε Vector of noise symbols associated with AA-based power flow analysis.
- ε^A Vector of noise symbols associated with approximation errors.
- ε_{IG} Noise symbols associated with intermittent sources of power.
- $\varepsilon_{PVa,b}$ Noise symbols associated with voltage control settings.
- ε_{VS} Vector of noise symbols associated with AA-based voltage stability assessment.
- f Set of non-linear power flow equations.
- Hc Chebyshev polynomial approximation of the sinusoidal functions.
- J_{aug} Augmented Jacobian matrix.
- λ Bifurcation parameter.
- L Vector of active and reactive power injections for the AA-based power flow analysis (p.u.).
- L_{vs} Vector of active and reactive power injections for the AA-based voltage stability assessment (p.u.).
- N_h Set of noise symbols introduced by approximations of non-affine operations.
- P Injected active power (p.u.).
- P_G Active power generated (p.u.).
- P^{IG} Computed partial deviation of the injected active power related to intermittent sources of power

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(p.u.).

P^{PV} Computed partial deviation of the injected active power related to PV buses (p.u.).

P^{A} Computed partial deviation of the injected active power related to approximation errors (p.u.).

Q Injected reactive power (p.u.).

q Matrix of errors introduced by approximations of non-affine operations in AA-based power flow analysis.

q_{vs} Matrix of errors introduced by approximations of non-affine operations in AA-based voltage stability assessment.

Q_G Reactive power generated (p.u.).

Q^{IG} Computed partial deviation of the injected reactive power related to intermittent sources of power (p.u.).

Q^{PV} Computed partial deviation of the injected reactive power related to PV buses (p.u.).

Q^{A} Computed partial deviation of the injected reactive power related to approximation errors (p.u.).

R_o Vector of central values for active and reactive power in AA-based power flow analysis (p.u.).

R_{vs_o} Vector of central values for active and reactive power in AA-based voltage stability assessment (p.u.).

s Iteration number for the parametrization approach.

θ Bus voltage angles (rad).

$\widehat{\theta}$ Affine form for bus voltage angles (rad).

V Bus voltage magnitudes (p.u.).

$\widehat{V}$ Affine form for bus voltage magnitudes (p.u.).

$V_{a,b}$ Auxiliary variables (p.u.).

V_{PQ} Variable associated with the voltage of the PQ-bus used as parameter (p.u.).

$\widehat{V}_{PQ}$ Affine form of the variable associated with the voltage of the PQ-bus used as parameter (p.u.).

ω Left eigenvector associated with the zero eigenvalue of $D_x f$.

x Bus voltage magnitudes, angles and other relevant unknown variables such as generator reactive powers.

C. Parameters

B Susceptance matrix (p.u.).

ΔP_G Direction of generator dispatch (p.u.).

ΔP_{G_A} Variation of generated power (p.u.).

ΔP_L Direction of the active power variations of load (p.u.).

ΔQ_L Direction of reactive power variations of load (p.u.).

ΔV_A Variation of voltage settings (p.u.).

ΔV_p Step size used in the parametrization technique (p.u.).

ε_A Noise symbols introduced by the approximation of non-affine operations.

G Conductance matrix (p.u.).

K_G Variable for representing power share in the distributed slack bus approach.

N Number of buses.

N_{IG} Set of buses with intermittent sources of power.

N_{PV} Set of PV buses.

Np number of polynomials of the Chebyshev approximations.

p Vector representing specified active and reactive powers injected at each node, as well as terminal generator voltage set points (p.u.).

P_L Active power demand (p.u.).

P_{max} Maximum injected active power (p.u.).

P_{min} Minimum injected active power (p.u.).

$P_{G_{max}}$ Generator maximum active power generated (p.u.).

$P_{G_{min}}$ Generator minimum active power generated (p.u.).

Q_L Reactive power demand (p.u.).

Q_{max} Generator maximum injected reactive power (p.u.).

Q_{min} Generator minimum injected reactive power (p.u.).

$Q_{G_{max}}$ Generator maximum reactive power limit (p.u.).

$Q_{G_{min}}$ Generator minimum reactive power limit (p.u.).

V_p Magnitude of the voltage used as a parameter (p.u.).

II. Introduction

Large-scale renewable energy projects are being developed in several countries to harness the potential of renewable energy sources and reducing CO_2 emissions, with the aim of attaining predefined penetration levels of renewables. However, some renewable energy sources such as wind and solar generation exhibit intermittency in their output power, which presents a challenge to power system planning, operation, and control because of the uncertainties associated with these non-dispatchable sources [1]. Depending on the penetration level of intermittent sources of power, the electric grid may experience considerable and uncertain changes in power flows. These changes may lead to system limit violations which affect system security [1]. Adequate analytical tools are thus required to properly analyze power systems with intermittent generation sources.

A variety of methods are available in the literature to perform voltage stability studies considering uncertainties associated with electrical parameters. These methods, which are briefly discussed in Section III-B, are mostly focused on computing voltage stability indices based on maximum loadabilities while reducing the computational burden associated with Monte Carlo (MC) simulations. Studies dealing with the computation of PV curves for uncertain power injections are not available in the literature; these PV curves offer important information regarding the voltage profiles that are useful for power system operators to avoid insecure operation of the power system.

Thus, in this paper, a novel approach based on Affine Arithmetic (AA) for PV curve computation is proposed, aimed at reducing the computational effort associated with MC simulations, while reducing the complexities and improving the accuracy of some analytical approaches. Specifically, the proposed method accurately computes the upper and lower bounds of PV curves considering uncertainties in the operating conditions based on a computing paradigm that is an alternative to sampling-based approaches, and does not require to model the uncertainties using Probability Density Functions (PDFs) or uniform random number generators.

The rest of this paper is organized as follows: Section III discusses some relevant voltage stability definitions and analysis techniques, as well as a brief description of available methods for stochastic voltage stability assessment. In Section IV, the AA-based power flow method is discussed and a new approach to account for generator reactive power limits is presented. In Section V, the proposed AA based voltage stability method is formulated and discussed. Section VI presents the results of applying the proposed method to the two test systems, using MC simulations as the benchmark for comparison purposes, since this is an accurate and reliable method to compute system responses when multi-dimensional parameter uncertainties are considered [2]. Furthermore, a well-known and relatively simple technique [3] to compute parameter sensitivities of loadability margins is also used for comparison purposes. Finally, in Section VII, the main conclusions and contributions of the present work are highlighted.

III. Background

A. Voltage Stability

Voltage stability deals with the ability of the power system to maintain acceptable voltage levels under normal operating conditions and after the system is perturbed by small or large disturbances. Short-term voltage stability is studied considering the system dynamics and using time domain simulation tools, whereas long-term voltage stability is commonly studied by means of steady-state analysis techniques. For instance, voltage collapse, which is usually associated with long-term phenomena, is commonly analyzed using power flow techniques and linearization of the system equations [4]. Even though voltage stability is a dynamic process, steady-state analysis techniques are used to identify the absence of a long-term equilibrium in the post-contingency configuration which leads to voltage instability [5]. This paper is focused on this particular voltage stability phenomena.

The maximum loadability, before the system experiences a voltage collapse, is commonly used to compute voltage stability indices. This maximum loadability is characterized by the existence of a saddle-node bifurcation or a limit-induced bifurcation [4]. Saddle-node bifurcations refer to the operating point where the system state matrix becomes singular; which usually coincides with the singularity of the power flow Jacobian, thus resulting in no power flow solutions. On the other hand, limit-induced bifurcations arise when generators reach their reactive power limits and lose voltage control, thus leading in some cases to no power flow solutions.

This maximum loadability can be computed using direct methods [6], [7], a series of power flows [8], or the continuation power flow technique [9], [10], [11]. The general idea behind continuation power flow methods is the solution of the power flow equations, given by:

$$f(x, p, \lambda) = 0 \tag{1}$$

The parameter λ is used to generate different scenarios for loads and generator outputs according to the following equations:

- For generator powers excluding the slack bus:

$$P_{Gi}(\lambda) = P_{Goi} + \lambda\Delta P_{Gi} \qquad \forall i = 1, ..., N-1 \tag{2}$$

- For generator powers including the slack bus (distributed slack bus):

$$P_{Gi}(\lambda) = P_{Goi} + (\lambda + K_G)\Delta P_{Gi} \qquad \forall i = 1, ..., N \tag{3}$$

- For load powers:

$$P_{Li}(\lambda) = P_{Loi} + \lambda\Delta P_{Li} \qquad \forall i = 1, ..., N \tag{4}$$

$$Q_{Li}(\lambda) = Q_{Loi} + \lambda\Delta Q_{Li} \qquad \forall i = 1, ..., N \tag{5}$$

The solutions of (1) for different values of λ are obtained using a continuation approach based on two basic steps: predictor and corrector. In the predictor step an approximate power flow solution, for an increase in the loading level, is obtained from an initial operating point. In the corrector step, the approximate solution obtained from the predictor step is used as an initial guess to solve the power flow equations; and an additional equation which is used to obtain power flow solutions near the maximum loading point. Solutions obtained from the continuation power flow yield PV curves and the maximum system loadability, commonly used to compute voltage stability margins.

The AA-based method for voltage stability assessment proposed in this paper is designed to compute these PV curves and associated maximum loadability points, based on an AA power flow approach, similar to the one described in [12].

The maximum loadability obtained using these PV curves is used in this paper to compute the system Available Transfer Capability (ATC), which is defined as [13]:

$$ATC = TTC - TRM - ETC - CBM \tag{6}$$

where:

- TTC or Total Transfer Capability is the maximum loadability of the system, defined as the minimum of the thermal, voltage or stability limits considering the worst N-1 contingency. This paper is focused on the computation of the TTC based on voltage stability limits. Thus, the TTC used in (6) corresponds to the maximum loadability associated with the worst N-1 contingency criteria.
- TRM or Transmission Reliability Margin takes into account uncertainties associated with other contingencies and is usually assigned a fixed value, as a percentage of the TTC, or computed using probabilistic methods. This paper is based on the computation of maximum loadabilities considering uncertainties associated with power injections and uses the N-1 contingency criterion. Uncertainties attributed to load forecasting errors, as well as other contingencies that may affect the system security, are not considered in this paper. Thus, to account for these additional uncertainties, a 5% of the TTC is used to compute the TRM, as recommended in [14].
- ETC or Existing Transmission Commitments which denotes the current power transferred between the concerned areas.

- CBM or Capacity Benefit Margin is the reserve required by load-serving entities to meet their generation reliability requirements. This is assumed to be zero, in this paper.

The ATC is computed in this paper as an interval which denotes the bounds of the possible ATCs due to the assumed intermittent sources of power. The power flow base case is used to compute the central values of the affine forms of the corresponding power flow variables.

B. Existing Stochastic Voltage Stability Assessment Methods

MC simulations has been used to model a variety of system complexities for voltage stability assessment [15], [16]. However, the high computational cost of MC simulations renders it unsuitable for certain applications. Thus, most of the effort in the literature has concentrated on reducing the computational cost of MC simulations while achieving acceptable accuracy. For instance, the two-point estimate method used in [17] computes the transfer capability associated with uncertainties of transmission line parameters and bus injections. This method computes approximations for the moments of the output variable, using only two probability concentrations for each uncertain input variable. However, in certain cases, for a relatively large number of uncertain input variables and/or large statistical dispersion of these variables, the two-point estimate method is not sufficiently accurate as compared to MC simulations [18]. Moreover, a higher order point estimate method is required for the cases where the PDFs of the input variables are not normal, which adversely affects the computational efficiency of the method. In this sense, one of the advantages of the proposed AA based technique is its independence with respect to the PDFs associated with the uncertain variables, which are modeled as intervals with no assumptions regarding their probabilities.

Truncated Taylor series expansion methods are used in [19] to efficiently account for variations in the transfer capability due to variations in system parameters. A formula to compute reliability margins is derived, assuming conditions associated with the central limit theorem. According to this theorem, the uncertain parameters must be independent and their number be sufficiently large, in order to approximate the output variable as normally distributed. In the proposed AA based method for voltage stability assessment, the uncertain input variables are not necessarily assumed to be statistically independent; in fact, correlations among uncertain variables are explicitly represented in the model.

In [3], [20] the effect of system control parameters and system data on voltage stability margins is accounted for by means of linearization techniques and direct formulas. Moreover, in [21], these formulas are used to efficiently estimate changes in saddle node bifurcations and limit induced bifurcations, due to changes in power system transactions in a market environment. These methods, based on sensitivity analysis and formulas, are able to efficiently estimate voltage stability indices due to system parameter variations, as demonstrated in the example in Section VI, but within certain limits in which the assumed linearizations are valid. Moreover, they are not intended to compute full PV curves under uncertainties, which is the main contribution of the proposed AA based method.

Some research work is reported on the computation of loadability margins considering uncertainties associated with intermittent sources of generation. For instance, in [22], a stochastic response surface is used to estimate the PDF of the loadability margins considering stochastic generation variations attributed to renewable energy sources and assuming non-conforming loads. This method is able to reduce the computational burden of MC simulations by using a polynomial chaos expansion of a relatively low order. As the number of uncertain variables increase, the order of this polynomial chaos expansion may increase and hence improve the accuracy of the method. Alternatively, various papers are focused on finding maximum loadabilities in terms of load uncertainties. For example, in [23], the stochastic nature of loads is modeled using a hyper-cone model whose thickness represents the uncertainty of future loading and the vertex is the current operating point. This approach is focused on finding the worst case scenario for maximum loadabilities in terms of load uncertainties, while the proposed AA based method computes the bounds of PV curves and associated maximum loadabilities, considering uncertainties due to power injections. In [24], a cumulant method is used for assessment of saddle node bifurcations considering uncertainties in load forecast. The method is used to solve the resultant stochastic non-linear programming formulation that leads to the estimation of the PDF for maximum loadability. The cumulant method requires information on the cumulants of the uncertain variables which may not be accurately available for uncertainties associated with intermittent sources of generation, where there is no general agreement on the PDF to best represent the forecasting errors [25], [26].

In order to overcome some of the limitations of sampling- and statistical-based methods, the use of soft-computing-based methodologies for uncertainty representation in voltage stability assessment has been proposed in the literature. In particular, the application of fuzzy set theory to represent uncertainties has been proposed in [27]- [28]. In this paradigm, the input data are modeled by fuzzy numbers, which are special types of fuzzy sets. Defining the connection between interval analysis and fuzzy set theory is not a trivial task [29]. In [30], the ideas of fuzzy sets and interval analysis are both connected to a general topological theory. Similarly, in [31], it is argued that the theory of fuzzy information granulation, the rough set theory and interval analysis can all be considered as subsets of a conceptual and computing paradigm of information processing called Granular Computing. An essential aspect of this paradigm is that its constituent methodologies are complementary and symbiotic, rather than competitive and exclusive. Based on these principles, the main contributions of the present paper lie on the application of advanced interval based solution approaches to voltage stability analysis with data uncertainties.

More recently, computational intelligence based techniques have been proposed in both state selection (as an alternative to MC Simulations) and in state evaluation (as an aid to MC Simulations). The application of these techniques allows deploying directed intelligent search paradigms which are an alternative to the proportionate sampling approach characterizing standard MC simulations. Amongst these techniques, the most promising for the problem under study, is the Population-based Intelligent Search (PIS) [32]. This algorithm tries to generate the dominant failure states and minimize the generation of success states. Thanks to this feature, fewer states need to be evaluated compared to standard MC simulations (where the majority of the states sampled are success states). The proposed AA approach is intrinsically different from this computing paradigm since it does not integrate either sampling based methods or evolutionary computation processes. By using the proposed methodology, a reliable estimation of the problem solution hull can be directly computed taking into account the parameter uncertainty interdependencies as well as the diversity of uncertainty sources. The main advantage of this solution strategy is that it neither requires derivative computations nor interval systems, and hence are suitable, in principle, for large scale problems, where robust and computationally efficient solution algorithms are required.

Another general class of statistical methods aimed at empirically assessing the parametric confidence intervals of a sampling distribution are based on resampling theory. The most common technique in this class is the so called bootstrap method, a technique for using the data collected from a single experiment to simulate what the results might be, if the experiment was repeated by sampling (with replacement) data from the original dataset [33]. This paradigm has been recognized as a powerful tool for testing or avoiding parametric assumptions when computing confidence intervals in many engineering problems [34]-[35]. Although the adoption of the bootstrap method is expected to be more effective, compared to a standard MC simulations, especially in terms of computational efforts, it still requires repetitive simulations. In this paper, a computing paradigm is proposed that is an alternative to sampling based approaches; thus, the employment of AA to represent the uncertainties of the power system state variables is proposed. This allows expressing the system equations in a more convenient formalism compared to the traditional and widely used linearization, frequently adopted in interval Newton methods. By using the proposed methodology, a reliable estimation of the problem solution hull can be computed taking into account the parameter uncertainty interdependencies, as well as the diversity of uncertainty sources, without requiring repetitive simulations.

The main scope of this paper is to conceptualize a new AA based computing paradigm for PV curve computation as an alternative to sampling based approaches. The main contributions of the paper are as follows:

- Assumption of a PDF of the uncertain variables are not required, contrary to some analytical methods where particular PDFs need to be assumed. This characteristic is important when the uncertainties are due to intermittent renewable sources, since the PDF of the forecasting error depends on the time horizon and the prediction method, among other factors.
- The method is reasonably accurate and computationally more efficient as compared to MC simulations for real-sized systems.
- Finally, the technique is able to compute the hull of PV curves and associated maximum loadabilities based on a self-validated technique which tracks the correlations among variables to avoid error explosion, commonly attributed to other self-validated techniques.

The results obtained in this paper for several test systems using the proposed AA formulation, are compared with those obtained using Crude Monte Carlo sampling (CMC) to verify the accuracy of the proposed approach. CMC sampling offers the following desirable characteristics which makes it suitable for comparison purposes:

- By definition, it does not produce any spurious trajectories.
- Results are based on the hypothesis that the number of CMC trials is large enough to assume that the union of the uncertainty region described by Monte Carlo is a very close approximation of the correct problem solution.

It is expected that the application of a more sophisticated data sampling based paradigm, based on cross-entropy methods and variance reduction, will reduce the required computational resources. Nevertheless, the main scope of this paper is to conceptualize a new AA based computing paradigm as an alternative to sampling based approaches. A formal analysis aimed at characterizing the algorithm complexities (i.e. number of floating points operations) compared to advanced sampling paradigms is outside the scope of this paper.

In addition, the sensitivity formula described next is used for comparison purposes.

C. Sensitivity Formula

The sensitivity formula proposed in [3] is used in this paper to compute the maximum loadability intervals; this formula is derived as follows: given the power system equilibria equation (1), at a saddle node bifurcation point $(x_{sa}, p_{sa}, \lambda_{sa})$, $D_x f\big|_{sa}$ exhibits a zero eigenvalue which left eigenvector ω satisfies the following condition:

$$\omega^T D_x f\big|_{sa} = 0 \tag{7}$$

assuming a small perturbation $(\Delta x, \Delta p, \Delta\lambda)$ around the saddle node bifurcation point, the linearization of f can be written as follows, premultiplied by ω^T:

$$\omega^T D_x f\big|_{sa}\Delta x + \omega^T D_p f\big|_{sa}\Delta p + \omega^T D_\lambda f\big|_{sa}\Delta\lambda = 0 \tag{8}$$

thus, the third term $\omega^T D_\lambda f\big|_{sa}\Delta\lambda$ becomes zero. The sensitivity formula is thus obtained as:

$$\frac{\Delta\lambda}{\Delta p} = -\frac{\omega^T D_p f\big|_{sa}}{\omega^T D_\lambda f\big|_{sa}} \approx \frac{d\lambda}{dp}\bigg|_{sa} \tag{9}$$

In case of multiple parameter variation, as in this paper, $\omega^T D_p f\big|_{sa}$ becomes a vector. This formula is used at the nominal bifurcation point to compute the sensitivity of the loading margins with respect to the assumed parameters, which is then used for the linear computation of the interval loading margins. One of the main drawbacks of this approach is that it neglects the violation of reactive power limits that may occur when power system parameters change.

IV. AA-based Power Flow Analysis

AA is a numerical analysis technique where the variables of interest are represented as affine combinations of data uncertainties and/or approximation errors. AA is intended to reduce the excessively conservative width of the output intervals that arise in some cases by Interval Arithmetic (IA) [36]. This interval width reduction is achieved in AA by tracking the correlation between the computed quantities, which is not possible in IA. However, AA requires more complex operations and larger computational cost as compared to IA.

An AA-based technique was proposed in [12] to solve the power flow problem considering uncertainties in load and generation. According to this approach, the power flow equations are initially solved without limits. In case of a limit violation, the associated bus or buses are treated as PQ buses as in standard power flow methods, and the procedure to compute the affine forms for voltages and angles is applied again. This is repeated until no more limit violations are observed.

A different approach is proposed in this paper to account for generator reactive power limits, based on the following formulation of voltages and angles in affine form, $\forall i = 1, .., N$:

$$\widehat{V_i} = V_{o_i} + \sum_{m \in N_{IG}} \frac{\partial V_i}{\partial P_{G_m}}\bigg|_o \Delta P_{G_A m} \varepsilon_{IGm} + \sum_{n \in N_{PV}} \frac{\partial V_i}{\partial V_n}\bigg|_o \Delta V_{A_n} (\varepsilon_{PVan} - \varepsilon_{PVbn}) \tag{10}$$

$$\widehat{\theta_i} = \theta_{o_i} + \sum_{m \in N_{IG}} \frac{\partial \theta_i}{\partial P_{G_m}}\bigg|_o \Delta P_{G_A m} \varepsilon_{IGm} + \sum_{n \in N_{PV}} \frac{\partial \theta_i}{\partial V_n}\bigg|_o \Delta V_{A_n} (\varepsilon_{PVan} - \varepsilon_{PVbn}) \tag{11}$$

where, the noise symbols ε_{PVa} and ε_{PVb} are introduced to account for the reactive power limits of generators, and thus avoid the need for PV-PQ bus switching of the power flow AA formulation used in [12]. This approach is based on the representation of PV buses discussed in [7], where the voltage at these buses is defined as:

$$V_n = V_{o_n} + V_{a_n} - V_{b_n} \qquad \forall n \in N_{PV} \tag{12}$$

where:

$$\begin{aligned} &V_{a_n} > 0, V_{b_n} = 0 \quad \text{for} \quad Q_{G_n} < Q_{G_{min_n}} \\ &V_{a_n} = 0, V_{b_n} > 0 \quad \text{for} \quad Q_{G_n} > Q_{G_{max_n}} \\ &V_{a_n} = V_{b_n} = 0 \quad \text{for} \quad Q_{G_{max_n}} \le Q_{G_n} \le Q_{G_{min_n}} \\ &\qquad \forall n \in N_{PV} \end{aligned} \tag{13}$$

the first condition in (13) is a strict complementarity condition and guarantees that the minimum reactive power constraint of the generator is not violated. This is achieved by increasing the magnitude of the reference voltage by means of the auxiliary variable V_{a_n} according to (12). Similarly, the second condition avoids violation of the upper reactive power limit by adjusting the magnitude of the auxiliary variable V_{b_n}, which reduces the reference voltage of the generator. When no reactive power limits are violated, the auxiliary variables in (12) are zero as in the third condition, thus keeping the voltage reference at its pre-established value. This model yields the following Mixed Complementarity Problem (MCP):

$$\begin{aligned} \min \quad & \tfrac{1}{2}\|f(x, p, \lambda)\|_2^2 \\ \text{s.t.} \quad & (Q_{G_n} - Q_{G_{min_n}})V_{a_n} = 0 \\ & (Q_{G_n} - Q_{G_{max_n}})V_{b_n} = 0 \\ & Q_{G_{max_n}} \le Q_{G_n} \le Q_{G_{min_n}} \\ & V_n = V_{o_n} + V_{a_n} - V_{b_n} \\ & V_{a_n}, V_{b_n} \ge 0 \\ & \forall n \in N_{PV} \end{aligned} \tag{14}$$

Using a similar approach, the formulation in AA of the voltage at PV buses may be written as follows:

$$V_{pv_n} = V_{o_{pv_n}} + \Delta V_{A_n} \varepsilon_{PVan} - \Delta V_{A_n} \varepsilon_{PVbn} \qquad \forall n \in N_{PV} \tag{15}$$

where:

$$\begin{aligned} &0 < \varepsilon_{PVan} < 1, \varepsilon_{PVbn} = 0 \quad \text{for} \quad Q_G < Q_{G_{min}} \\ &\varepsilon_{PVan} = 0, 0 < \varepsilon_{PVbn} < 1 \quad \text{for} \quad Q_G > Q_{G_{max}} \qquad \forall n \in N_{PV} \\ &\varepsilon_{PVan} = 0, \varepsilon_{PVbn} = 0 \quad \text{for} \quad Q_{G_{max}} \le Q_G \le Q_{G_{min}} \end{aligned} \tag{16}$$

the noise symbols ε_{PVan} and ε_{PVbn} in (15) act as the auxiliary variables V_{a_n} and V_{b_n} in (12) to keep the reactive power within limits. The effect of the variation of the voltage specified at PV nodes due to these noise symbols is accounted for in (10) and (11) by means of the partial derivatives of the bus voltages and angles, with respect to the voltage reference set points at PV buses. This approach is implemented using the Linear Programming (LP) formulations described in detail at the end of this section.

The coefficients of the noise symbols in (10) and (11), given by the partial derivatives of voltage magnitudes and angles with respect to the injected active power and reference voltages at voltage controlled buses, can be calculated from a "base" power flow solution and its associated Jacobian matrix. These derivatives are evaluated at the central values of the active powers injected at buses where intermittent generators are considered. The central values are defined as follows:

$$P_{G_0 m} = \frac{P_{G_{max} m} + P_{G_{min} m}}{2} \qquad \forall m \in N_{IG} \tag{17}$$

The active and reactive power affine forms $\widehat{P_i}$ and $\widehat{Q_i}$ are obtained by replacing the voltage and angle affine forms given in (10) and (11), in the following equations $\forall i = 1, .., N$:

$$\widehat{P_i} = \sum_{j=1}^{N} \widehat{V_i}\widehat{V_j}[G_{ij}\cos(\widehat{\theta_i} - \widehat{\theta_j}) + B_{ij}\sin(\widehat{\theta_i} - \widehat{\theta_j})] \tag{18}$$

$$\widehat{Q_i} = \sum_{j=1}^{N} \widehat{V_i}\widehat{V_j}[G_{ij}\sin(\widehat{\theta_i} - \widehat{\theta_j}) - B_{ij}\cos(\widehat{\theta_i} - \widehat{\theta_j})] \quad (19)$$

Since the sinusoidal functions are non-affine operations, they are expanded in this paper in terms of a series of Chebyshev polynomials for five digits accuracy. This approximation is written as follows:

$$Hc(z) = \sum_{r=0}^{Np} b_r T_r(z) \qquad r = 0, 1, 2... \quad (20)$$

where the Chebyshev polynomials $T_r(z)$ are computed using the following recurrence relation [37]:

$$\begin{aligned} T_o(z) &= 1 \\ T_1(z) &= z \\ T_{r+1}(z) &= 2zT_r(z) - T_{r-1}(z) \end{aligned} \quad (21)$$

The affine form for the active power equations (18) and reactive power equations (19) exhibit the following form, after all affine function operations and approximations given in [36]:

$$\widehat{P_i} = P_{i_o} + \sum_{m\in N_{IG}} P^{IG}_{i,m}\varepsilon_{IG_m} + \sum_{n\in N_{PV}} P^{PV}_{i,n}\varepsilon_{PV_{an}} - \sum_{n\in N_{PV}} P^{PV}_{i,n}\varepsilon_{PV_{bn}} + \sum_{h\in N_h} P^{A}_{i,h}\varepsilon^{A}_{h} \quad (22)$$

$$\widehat{Q_i} = Q_{i_o} + \sum_{m\in N_{IG}} Q^{IG}_{i,m}\varepsilon_{IG_m} + \sum_{n\in N_{PV}} Q^{PV}_{i,n}\varepsilon_{PV_{an}} - \sum_{n\in N_{PV}} Q^{PV}_{i,n}\varepsilon_{PV_{bn}} + \sum_{h\in N_h} Q^{A}_{i,h}\varepsilon^{A}_{h} \quad (23)$$

These equations may be written in compact form as follows:

$$A\varepsilon = C \quad (24)$$

$$C = L - (R_o + q) \quad (25)$$

where:

$$A = \begin{bmatrix} P^{IG}_{1,1} & \cdots & P^{IG}_{1,N_{IG}} & P^{PV}_{1,1} & \cdots & P^{PV}_{1,N_{PV}} & -P^{PV}_{1,1} & \cdots & -P^{PV}_{1,N_{PV}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ P^{IG}_{N-1,1} & \cdots & P^{IG}_{N-1,N_{IG}} & P^{PV}_{N-1,1} & \cdots & P^{PV}_{N-1,N_{PV}} & -P^{PV}_{N-1,1} & \cdots & -P^{PV}_{N-1,N_{PV}} \\ Q^{IG}_{1,1} & \cdots & Q^{IG}_{1,N_{IG}} & Q^{PV}_{1,1} & \cdots & Q^{PV}_{1,N_{PV}} & -Q^{PV}_{1,1} & \cdots & -Q^{PV}_{1,N_{PV}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Q^{IG}_{N-1,1} & \cdots & P^{IG}_{N-1,N_{IG}} & Q^{PV}_{N-1,1} & \cdots & P^{PV}_{N-1,N_{PV}} & -Q^{PV}_{N-1,1} & \cdots & -Q^{PV}_{N-1,N_{PV}} \end{bmatrix}$$

$$\varepsilon = \begin{bmatrix} \varepsilon_{IG_1} \\ \vdots \\ \varepsilon_{IG_{N_{IG}}} \\ \varepsilon_{PV_{a,1}} \\ \vdots \\ \varepsilon_{PV_{a,N_{PV}}} \\ \varepsilon_{PV_{b,1}} \\ \vdots \\ \varepsilon_{PV_{b,N_{PV}}} \end{bmatrix} \qquad L = \begin{bmatrix} [P_{G_{min_1}} - P_{L_1}], [P_{G_{max_1}} - P_{L_1}] \\ \vdots \\ [P_{G_{min_{N-1}}} - P_{L_{N-1}}], [P_{G_{max_{N-1}}} - P_{L_{N-1}}] \\ [Q_{G_{min_1}} - Q_{L_1}], [Q_{G_{max_1}} - Q_{L_1}] \\ \vdots \\ [Q_{G_{min_{N-1}}} - Q_{L_{N-1}}], [Q_{G_{max_{N-1}}} - Q_{L_{N-1}}] \end{bmatrix}$$

$$R_o = \begin{bmatrix} P_{1_o} \\ \vdots \\ P_{N_o} \\ Q_{1_o} \\ \vdots \\ Q_{N_o} \end{bmatrix} \qquad q = \begin{bmatrix} P^{A}_{1,1} & \cdots & P^{A}_{1,Nh} \\ \vdots & \vdots & \vdots \\ P^{A}_{N-1,1} & \cdots & P^{A}_{N-1,Nh} \\ Q^{A}_{1,1} & \cdots & Q^{A}_{1,Nh} \\ \vdots & \vdots & \vdots \\ Q^{A}_{N-1,1} & \cdots & Q^{A}_{N-1,Nh} \end{bmatrix} \begin{bmatrix} \varepsilon^{A}_{1} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \varepsilon^{A}_{N_h} \end{bmatrix}$$

To obtain the maximum and minimum values of the noise vector ε in (24), the following LP problems need to be solved:

$$\begin{aligned} \min \quad & \textstyle\sum \varepsilon_{IG_{min}m} + \sum \varepsilon_{PVa_{min}n} + \sum \varepsilon_{PVb_{min}n} \\ \text{s.t.} \quad & -1 \le \varepsilon_{IG_{min}m} \le 1 \\ & 0 \le \varepsilon_{PVa_{min}n}, \varepsilon_{PVb_{min}n} \le 1 \\ & C_{min} \le A\varepsilon_{min} \le C_{max} \\ & \forall m \in N_{IG}, \forall n \in N_{PV} \end{aligned} \quad (26)$$

$$\begin{aligned} \max \quad & \textstyle\sum \varepsilon_{IG_{max}m} + \sum \varepsilon_{PVa_{max}n} + \sum \varepsilon_{PVb_{max}n} \\ \text{s.t.} \quad & -1 \le \varepsilon_{IG_{max}m} \le 1 \\ & -1 \le \varepsilon_{PVa_{max}n}, \varepsilon_{PVb_{max}n} \le 0 \\ & C_{min} \le A\varepsilon_{max} \le C_{max} \\ & \forall m \in N_{IG}, \forall n \in N_{PV} \end{aligned} \quad (27)$$

The values of $\varepsilon_{IG_{min}}$, $\varepsilon_{PVa_{min}}$, $\varepsilon_{PVb_{min}}$, and $\varepsilon_{IG_{max}}$, $\varepsilon_{PVa_{max}}$, $\varepsilon_{PVb_{max}}$ obtained by solving these simple LP problems are used in (10) and (11) to compute the maximum and minimum bus voltages and angles. Notice that the objective functions in (26) and (27) are mathematically formulated to contract the vector ε according to the physical restrictions imposed by the power flow equations and the bounds of the noise symbols. Thus, there is no particular physical meaning attributed to the objective functions. Since (26) corresponds to a minimization problem, and the bounds of $\varepsilon_{PVa_{min}n}$ and $\varepsilon_{PVb_{min}n}$ are restricted to the interval [0,1], these variables will tend to zero unless there is any reactive power limit violation. Similarly, because in (27) the objective function is formulated as a maximization problem, and the bounds of $\varepsilon_{PVa_{max}n}$ and $\varepsilon_{PVb_{max}n}$ are restricted to the interval [-1,0], these noise symbols tend to zero, unless a reactive power limit is violated. On the other hand, the noise symbols $\varepsilon_{IG_{min}m}$ and $\varepsilon_{IG_{max}m}$, which are related to the intermittent sources of power, are allowed to vary within the interval [-1,1], which is the hull assigned to affine arithmetic variables.

V. AA-based Computation of PV Curves

Equation (2) can be reformulated for dispatchable generators, excluding the slack bus, as follows:

$$\widehat{P_{G_i}}(\lambda) = P_{G_{oi}} + \widehat{\lambda}\Delta P_{Gi} \qquad \forall i = 1, .., N-1 \quad (28)$$

Similarly, the load equations can be written as:

$$\widehat{P_{L_i}}(\lambda) = P_{L_{oi}} + \widehat{\lambda}\Delta P_{Li} \qquad \forall i = 1, .., N \quad (29)$$

$$\widehat{Q_{L_i}}(\lambda) = Q_{L_{oi}} + \widehat{\lambda}\Delta Q_{Li} \qquad \forall i = 1, .., N \quad (30)$$

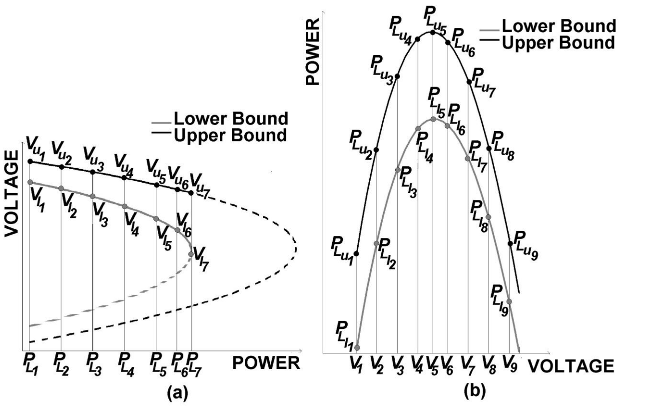

Fig. 1. PV curves: (a) using the load power as the parameter; (b) using a PQ-bus voltage magnitude as the parameter.

where:

$$\widehat{\lambda} = \lambda_o + \sum_{m \in N_{IG}} \frac{\partial \lambda}{\partial P_{G_m}}\bigg|_o \Delta P_{G_{A}m} \varepsilon_{IGm} + \sum_{n \in N_{PV}} \frac{\partial \lambda}{\partial V_n}\bigg|_o \Delta V_{A_n} (\varepsilon_{PVan} - \varepsilon_{PVbn}) \tag{31}$$

Notice that the loading parameter λ is now in affine form, and thus is written as a function of the variability of the output power of intermittent generators. This variability is modeled as intervals that represent the maximum and minimum power injections associated with the simulation time period as follows:

$$P_{G_m} = [P_{G_om} - \Delta P_{G_Am}, P_{G_om} + \Delta P_{G_Am}] \qquad \forall m \in N_{IG} \tag{32}$$

or equivalently in AA:

$$\widehat{P}_{G_m} = P_{G_om} + \Delta P_{G_Am} \varepsilon_{IGm} \qquad \forall m \in N_{IG} \tag{33}$$

For a different period of time, a different interval for the uncertain power injections may be needed; for instance, ΔP_{G_A} may be chosen according to the forecasting error associated with the concerned planning horizon. Thus, the temporal dependency of the uncertainties can be reflected on the size of the interval, i.e. the larger the time window, the larger the interval, and vice versa.

The value of ΔV_{A_n} in (31) may be decided so that there is a sufficient margin for the variation of the reference voltage setting of PV buses in order to keep generator's reactive power within limits. In this paper, ΔV_{A_n} has been assumed to be a 10% of the pre-established reference voltage control settings for all study cases.

The affine forms for voltages and angles can be written as in (10) and (11).

For the computation of the central values and the upper and lower bounds of the PV curves, a parametrization technique is used in this paper. Figure 1(a) illustrates the problems when the load power is used as a parameter to compute the lower and upper parts of the PV curves. It is noted that varying the load by varying the power from P_{L1} to P_{L7}, and computing the respective upper voltages V_{u1} to V_{u7} and lower voltages V_{l1} to V_{l7} based on AA or MC simulations can only be done until the lower bound of the PV curve reaches the maximum loading point given by P_{L7}. Beyond this point, the dotted parts of the PV curves cannot be readily computed. To overcome this problem, the voltage magnitude at a PQ bus is chosen as the parameter, as in Fig. 1(b), based on a parametrization approach [4], [9], [10], [11]. Notice that the voltage magnitude at the chosen PQ bus varies from V_1 to V_9, and for each of these voltages, the associated lower power loads P_{Ll1} to P_{Ll9} and upper power loads P_{Lu1} to P_{Lu9} can be readily computed. The PQ-bus voltage of the bus that exhibits the largest variation in voltage magnitude with respect to load variations is used as the parameter. The affine form for the voltage of this bus can be written as:

$$\widehat{V}_{PQ} = V_p \tag{34}$$

where V_p is updated for the computation of the PV curves as follows:

$$V_p = V_p - \Delta V_p \tag{35}$$

The step size ΔV_p varies depending on the proximity to the maximum loadability; thus, the closer to this maximum, the smaller the step size. The proximity to maximum loadability is detected by computing the difference between the former and the actual loading factor λ; the lower this difference, the closer the system is to a saddle node or limit-induced bifurcation. Notice that for computations beyond the maximum loadability, the loading factor starts decreasing, and thus, it is straightforward to detect when the maximum loadability has been reached.

The parametrization approach is implemented by augmenting the power flow equations by one equation, which refers to the value assigned to the voltage magnitude chosen as the parameter. Thus, the set of equations to be solved is:

$$\begin{bmatrix} f(\theta, V, \lambda) \\ V_{PQ} - V_p \end{bmatrix} = 0 \tag{36}$$

which is solved using a Newton approach, as follows:

$$\begin{bmatrix} f(\theta^s, V^s, \lambda^s) \\ V_{PQ} - V_p \end{bmatrix} = -J^s_{aug} \begin{bmatrix} \Delta\theta^s \\ \Delta V^s \\ \Delta\lambda^s \end{bmatrix} \tag{37}$$

$$\begin{bmatrix} \theta^{s+1} \\ V^{s+1} \\ \lambda^{s+1} \end{bmatrix} = \begin{bmatrix} \theta^s \\ V^s \\ \lambda^s \end{bmatrix} + \begin{bmatrix} \Delta\theta^s \\ \Delta V^s \\ \Delta\lambda^s \end{bmatrix} \tag{38}$$

where s refers to the iteration number. These equations are iteratively solved until convergence is attained.

The proposed PV-curve computation approach is based on the following AA form of power flow equations:

$$\widehat{P}_{G_i} - P_{L_{oi}} - \widehat{\lambda} \Delta P_{Li} - \widehat{P}_i = 0 \qquad \forall i = 1, ..., N \tag{39}$$

$$\widehat{Q}_{G_i} - Q_{L_{oi}} - \widehat{\lambda} \Delta Q_{Li} - \widehat{Q}_i = 0 \qquad \forall i = 1, ..., N \tag{40}$$

plus the additional equation corresponding to the parametrization approach:

$$\widehat{V}_{PQ} - V_p = 0 \tag{41}$$

where $\widehat{P}_i$ and $\widehat{Q}_i$ are written according to (22) and (23), and $\widehat{P}_{G_i}$ is given by (28) for dispatchable generators, excluding the slack bus. In case of non-dispatchable generators, $\widehat{P}_{G_i}$ is modeled as an interval which represents the uncertainties associated with the output power, as in (33).

Equations (39) and (40) exhibit the following compact form, after all affine function operations and approximations:

$$A_{vs}\varepsilon_{vs} = C_{vs} \tag{42}$$

$$C_{vs} = L_{vs} - (R_{vs_o} + q_{vs}) \tag{43}$$

where:

$$R_{vs_o} = \begin{bmatrix} P_{1_o} + \lambda_o(\Delta P_{L_1} - \Delta P_{G_1}) \\ \vdots \\ P_{N_o} + \lambda_o(\Delta P_{L_n} - \Delta P_{G_n}) \\ Q_{1_o} + \lambda_o(\Delta Q_{L_1} - \Delta Q_{G_1}) \\ \vdots \\ Q_{N_o} + \lambda_o(\Delta Q_{L_n} - \Delta Q_{G_n}) \end{bmatrix}$$

and A_{vs}, ε_{vs}, C_{vs}, L_{vs}, and q_{vs} have the form of the matrices used in (24) and (25).

Using a similar approach to the one described in Section IV, the LP formulation for the proposed AA based voltage stability assessment is obtained by substituting C by C_{vs} and A by A_{vs} in (26) and (27). The values for $\varepsilon_{IG_{min}}$, $\varepsilon_{PVa_{min}}$, $\varepsilon_{PVb_{min}}$, and $\varepsilon_{IG_{max}}$, $\varepsilon_{PVa_{max}}$, $\varepsilon_{PVb_{max}}$ obtained by solving the resultant LP formulations are used in (31) to compute the minimum and maximum values of λ. Figure 2 depicts the algorithm used to compute the PV-curve intervals. The stopping criteria for this algorithm is decided based on the number of solution points of the hull of the PV curves beyond the maximum loadabilities.

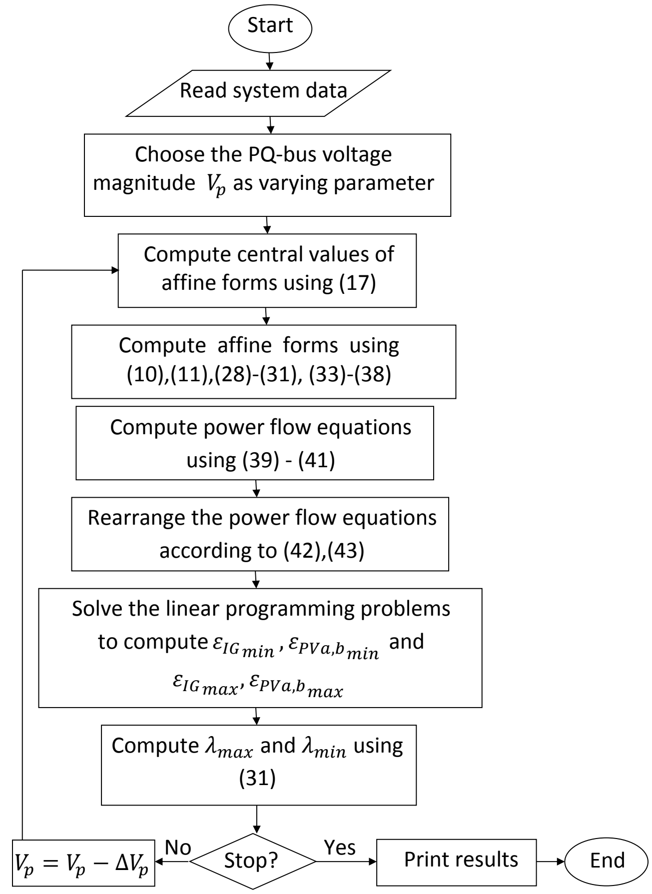

Fig. 2. Algorithm for the AA based PV curve computation method.

VI. Simulation Results and Discussions

To test the proposed AA-based methodology for voltage stability assessment, two test systems are considered. The first is a 5-bus test system [38], which allows a thorough test and demonstration of the proposed methodology without the complexities associated with system size. The second corresponds to a portion of the Polish system and comprises 2383 buses [39], allowing to test and demonstrate the proposed approach in a realistic system. To validate and compare the results obtained with the proposed AA technique, PV curves are computed using an MC simulation approach, assuming uniform distribution for the non-dispatchable generators intervals, and are thus treated as the benchmark for comparison purposes. The convergence of the MC simulations was determined by assuming a tolerance of 0.0001 in the change of the expected value. In the first iterations, the extreme values of the assumed intervals for the intermittent sources of power are evaluated to speed-up the convergence rate of the MC simulations. This strategy is based on the studies reported in [40], where it is shown that a useful approximation of the power flow solution bounds can be assessed by sampling a reduced set of deterministic power flow solutions. The idea is to develop a worst case analysis of the load/generation scenarios by introducing reasonable assumptions from operational experience, for example, low/high bus voltage magnitudes are expected when the power network is more/less loaded. The deterministic power flow solutions corresponding to these load/generation scenarios are then combined to determine a rough but fast approximation of the hull boundary of the power flow solutions. These solutions can then be refined by conventional MC simulations to obtain a more accurate estimation of the power flow solution bounds. In this case, an improvement of the convergence rate is expected since the algorithm starts with an initial solution which is very close to the expected one. The sensitivity formula described in Section III-C is also used to compute the maximum loadability intervals for comparison purposes.

A. 5-Bus Test System

This system comprises two generators which supply a base load of 440 MW. One of these generators is assumed to be an intermittent power source, while the second is assumed to be dispatchable, and is the system slack bus. For the computation of PV curves, the load directions in (39) and (40) are: $\Delta P_{L_i} = P_{L_{oi}}$, $\Delta Q_{L_i} = Q_{L_{oi}}$.

Table I shows the effect of increasing the size of the interval that models the uncertain variable on the computed maximum and minimum load changes. For comparison purposes, the MC simulations, and the sensitivity formula (SF) results are also depicted (the MC simulations required 50 samples to converge). Notice that as the margin of variation associated with the uncertain power injection increases, the error for the upper bound of the maximum

TABLE I
COMPARISON OF LOAD CHANGES USING DIFFERENT APPROACHES FOR 5-BUS SYSTEM

		Margin (%)			
		5	15	25	35
MC	$\Delta L_{max}(MW)$	335.46	336.67	337.52	338.05
AA	$\Delta L_{max}(MW)$	335.47	336.97	338.47	339.97
	$e(\%)$	0.003	0.090	0.280	0.566
SF	$\Delta L_{max}(MW)$	335.55	337.16	338.77	340.38
	$e(\%)$	0.026	0.146	0.369	0.688
MC	$\Delta L_{min}(MW)$	333.93	332.15	330.04	327.61
AA	$\Delta L_{min}(MW)$	333.67	331.57	329.51	327.45
	$e(\%)$	-0.082	-0.172	-0.162	-0.049
SF	$\Delta L_{min}(MW)$	333.94	332.33	330.72	329.11
	$e(\%)$	0.003	0.055	0.204	0.456

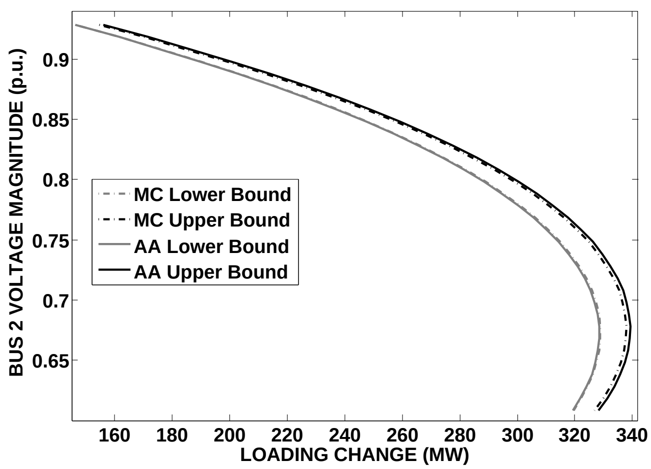

Fig. 3. PV curves for the IEEE 5-bus test system.

loadabilities using AA and SF also increases, with the error corresponding to the AA based method being slightly lower than the error computed using the SF approach. Also, notice that the lower bound for the maximum loadability is pessimistic for the AA approach and optimistic for the SF approach, which is an advantage of the proposed approach.

Figure 3 shows the PV curves obtained using MC simulations and the proposed AA methodology for a 30% margin variation of the uncertain variable. Notice that the AA approach allows efficiently computing the upper and lower bounds of the PV curves, providing information regarding the hull of voltage profiles. No other technique proposed in the literature is capable of quickly generating such curves, which could be used by power system operators, as standard PV curves are used now [4]. (i.e. to determine voltage profiles as the maximum system loadability is approached).

B. 2383-Bus Test System

The system comprises 323 generators and 4 synchronous condensers supplying a base load of 24558 MW. A total of 50 generators, with total capacity of 7677.2 MW (31% of the total system generation capacity) are assumed to be intermittent power sources, with 15% margin variations. The size and number of uncertainties of this system allows evaluating the performance of the proposed AA based

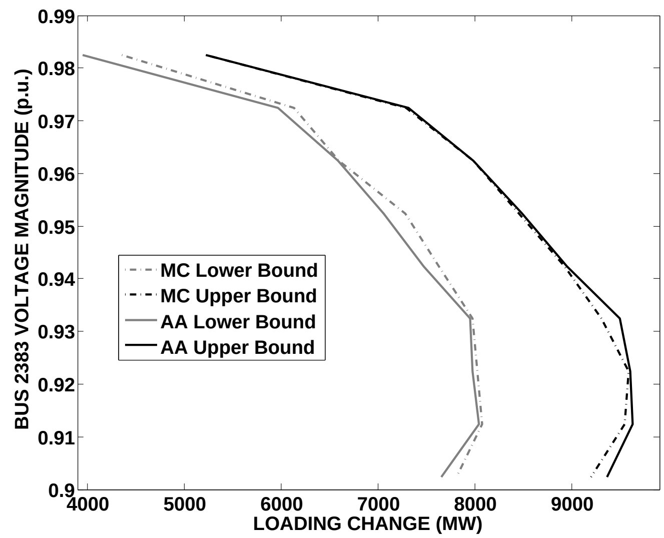

Fig. 4. PV curves for the 2383-bus test system.

voltage stability assessment for practical systems. As in the previous example, the following load and dispatch directions are assumed: $\Delta P_{L_i} = P_{L_{oi}}$, $\Delta Q_{L_i} = Q_{L_{oi}}$ and $\Delta P_{G_i} = P_{G_{oi}}$ $\forall i$.

The PV curves depicted in Fig. 4 are computed using the proposed AA based method and MC simulations for normal operating conditions. The probability distribution of the uncertain variables in the MC simulations are assumed to be uniform distributions, requiring 2000 samples to attain convergence. Observe again that the proposed AA-based approach yields slightly pessimistic results.

Table II shows the bounds for load changes obtained using MC simulations, the proposed AA based method, and the SF approach for the worst single line trip and single generator trip. Because of the difficulties encountered in the computation of the Jacobian matrices required in (9), the SF formula was approximated as per the following equation:

$$\left.\frac{d\lambda}{dp}\right|_o \approx \left.\frac{\Delta\lambda}{\Delta p}\right|_o \quad (44)$$

with Δp small enough to have a good approximation. It is observed that the worst contingency occurs when Line 67-138 is tripped. Observe that the errors in the values obtained using the proposed AA technique are lower than those obtained with the SF approach; furthermore, there are no underestimation of the margins by the AA method, as opposed to those obtained using the SF technique. In this case, the ATC for transmission line 31-32, which is the most loaded line in the system, computed using (6) and assuming CBM=0, varies within the intervals [31.83, 78.72] MW, [29.17, 82.56] MW, and [26.18, 85.31] MW, for MC simulations, the AA based method, and the SF approach, respectively; this results in a maximum error for the AA based approach of 8.37%, while the maximum error for the SF approach is 17.77%. Notice that the difference between the loading changes computed using MC simulations and the AA based approach tends to increase as the size of the system and number of uncertain variables increases. This result is to be expected, because, as the number of uncertain variables and operations increase, the approximation errors of the non-affine operations become larger.

TABLE II
COMPARISON OF LOAD CHANGES USING DIFFERENT APPROACHES FOR 2383-BUS SYSTEM

		Normal	Line 67-138 trip	Gen 131 trip
MC	$\Delta L_{max}(MW)$	9585	4160	6532
AA	$\Delta L_{max}(MW)$	9622	4300	6577
	$e(\%)$	0.39	3.37	0.69
SF	$\Delta L_{max}(MW)$	9702	4400	6410
	$e(\%)$	1.22	5.77	-1.87
MC	$\Delta L_{min}(MW)$	8072	2452	5120
AA	$\Delta L_{min}(MW)$	8038	2355	5111
	$e(\%)$	-0.42	-3.96	-0.18
SF	$\Delta L_{min}(MW)$	7958	2246	5176
	$e(\%)$	-1.41	-8.4	1.09

These simulations, based on MATLAB, were carried out on a computer with 8 GB of RAM and a processor of 3.40 GHz. The proposed AA approach was computationally more efficient than MC simulations, achieving 49.03% (0.889 s vs 1.744 s) and 90.90% (270.7 s vs 2975.7 s) savings in computational time for the 5-bus and the 2383-bus test system, respectively. These simulation times are referred to the system in normal operating conditions, considering the uncertainties associated with the intermittent sources of power.

It is important to point out that the proposed AA based methodology computes ATCs considering uncertainties attributed to intermittent sources of power given a system topology. Because of the N-1 contingency criterion was considered, the AA based method was repeatedly applied for each contingency. The uncertainty associated with equipment outages (i.e. lines and transformers) can also be modeled using the proposed AA based computing paradigm. According to a mathematical approach widely adopted in circuit analysis literature, the equipment outages can be described by proper variations of the admittance matrix elements, by defining the parameters of the equipment equivalent circuits using proper intervals (i.e. $R_i = [R_{min}, R_{max}] = [R, 1e6]$, where $R_i = 1e6$ corresponds to the i-th equipment out of service).

C. Sources of Error

The main sources of error of the proposed AA-based method for voltage stability assessment are as follows:

- Errors introduced by sinusoidal function approximations: In this paper, sinusoidal functions are written in terms of Chebyshev polynomial approximations for 5-digit accuracy as stated in Section IV. This approximation, given by (20), comprises a finite number of polynomials whose coefficients are floating-points. The number of polynomials Np are truncated depending on the number of digits accuracy. For 5-digit accuracy, as in this paper, the Chebyshev approximation error is 10^{-5} because of the truncation of the Chebyshev series approximation only. Since the polynomials of this approximation involves non-affine operations, additional truncation approximation errors are introduced [36]. Therefore, the final affine forms of the sine and cosine operations exhibit approximation errors greater than those attributed to the Chebyshev approximations only. Therefore, increasing the number of digits of the Chebyshev approximation beyond 5 digits does not improve the overall accuracy of the sine and cosine affine approximations.
- Errors due to the size of the intervals that model the uncertain variables: The greater the size of the interval that models the uncertain variables, the greater the error, as shown in Section VI-A. This is expected, since the initial coefficients of the affine forms are obtained from a "base" power flow solution and its associated Jacobian matrix. Thus, the greater the deviation of the central value of the affine forms with respect to their minimum and maximum values, the greater is the error of the initial coefficients of the affine forms.
- Errors due to the number of uncertain variables and size of the system: The greater the number of uncertain variables and size of the system, the greater the error, as shown in Section VI-B. This is also expected since with the number of equations and coefficients of the affine forms increasing, the truncation errors associated with non-affine operations also increase.

VII. CONCLUSIONS

A novel method for voltage stability assessment of power systems with intermittent sources of generation such as wind and solar power is presented in this paper. Specifically, the proposed methodology is able to compute the bounds of the PV curves and associated static load margins when the system presents uncertainties due to operating conditions, without any assumption on their PDFs. The results depict a reasonably good accuracy at significantly lower computational costs when compared to those obtained using simulation based techniques. Comparisons also show that the lower bound of the maximum loadability, obtained using the AA based approach is pessimistic, while it is optimistic in some cases for the SF approach, thus making the AA based approach more appropriate for practical applications.

The proposed AA based method takes into account the correlations among variables, which is not possible with other self-validated methods such as IA-based methods. The accuracy of the proposed method is mainly attributed to this characteristic. However, as the number of uncertain variables, size of the system, and size of the uncertainty intervals increase, the accuracy of the method decreases. This should not be a significant problem for the practical application of the proposed technique, since the number and sizes of uncertain generation sources in real systems is not expected to be more than 30% of the total system generation.

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