A Novel Affine Arithmetic Method to Solve Optimal Power Flow Problems with Uncertainties

Mehrdad Pirnia, *Student Member, IEEE*, Claudio A. Cañizares, *Fellow, IEEE*, Kankar Bhattacharya, *Senior Member, IEEE*, and Alfredo Vaccaro, *Senior Member, IEEE*

***Abstract*— An affine arithmetic (AA) method is proposed in this paper to solve the optimal power flow (OPF) problem with uncertain generation sources. In the AA-based OPF problem, all the state and control variables are treated in affine form, comprising a center value and the corresponding noise magnitudes, to represent forecast, model error, and other sources of uncertainty without the need to assume a probability density function (pdf). The proposed AA-based OPF problem is used to determine the operating margins of the thermal generators in systems with uncertain wind and solar generation dispatch. The AA-based approach is benchmarked against Monte-Carlo Simulation (MCS) intervals in order to determine its effectiveness. The proposed technique is tested and demonstrated on the IEEE 30-bus system and also a real 1211-bus European system.**

***Index Terms*— Optimal power flow, generation uncertainty, affine arithmetic, interval analysis.**

I. INTRODUCTION

The increased focus on renewable generation has brought forth several concerns pertaining to planning and operation of modern power systems, because of their inherent characteristic of intermittency. In order to fulfill the requirements of the evolving smart power grid, the intermittency need be taken into account in traditional unit commitment (UC), economic load dispatch (ELD), Optimal Power Flow (OPF) models, as well as in long term planning models of transmission and generation systems.

Obtaining the margins of operations for thermal generators in the presence of uncertain generation (e.g., wind and solar) and load [1] helps significantly in determining the required reserve capacities, so that the system operates reliably and economically. System operators provide such ancillary services to maintain system reliability [2], [3]. The Electric Reliability Council of Texas (ERCOT) has faced resource adequacy issues, since less thermal capacity has been added as compared to wind, since 2003, and the additional wind capacity has not contributed to the peak load carrying capability, as wind blows more during off-peak periods [4].

OPF has been used for determining adequate generation reserves in power systems dispatch [5]. Methods such as the Monte-Carlo Simulation (MCS) and Interval Arithmetic (IA) have been proposed in the literature to model and analyze uncertainties arising from load and generation, in OPF problems. Although the MCS method is popular for its simple implementation, it is computationally expensive; on the other hand, analytical methods suffer from lack of accuracy and mathematical complexity [6]. Therefore, other methods have been suggested in the literature as alternatives to analytical and numerical approaches.

Probabilistic approaches are suggested to solve the OPF problems considering uncertainties in power system operations. One of the first attempts in solving the probabilistic OPF (P-OPF) problem is reported in [7], where the multivariate Gram-Charlier method is employed to model the probability density function (pdf) of uncertain variables. In [8], the error between the forecasted and the actual demand is assumed to have a Gaussian probability function. The model is then solved using MCS, and the statistical characteristics (e.g., mean and variance) of the system variables, i.e., active power generation and system losses are calculated, so that the dispatcher is able to allocate enough spinning reserve capacity for a given time interval. A different approach based on a sensitivity analysis technique is proposed in [9], wherein operating constraint violations and their probabilities are determined for the whole planning horizon, and then an iterative approach is used to adjust the control variables while satisfying their limits.

In order to improve the computational efficiency of OPF solution methods, analytical methods have been proposed. For instance, in [10] the Cumulant method is suggested to simplify the convolution of independent random variables, and then Gram-Charlier/Edgeworth Expansion theory is used to reconstruct the pdfs from the cumulants. Furthermore, in [11] the Cumulant method is used to solve the P-OPF problem using the logarithmic barrier Interior Point method and the results for variable load are compared using Gaussian distribution and Gamma distribution. Since some of these approaches require derivatives of nonlinear functions, the functions are assumed to be differentiable. Another method to solve the P-OPF problem based on the two-point estimate method is proposed in [12] which alleviates the calculation of derivatives and thus improves the computational efficiency of the solution process. Finally, in [13], a Chance Constrained

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M. Pirnia, C.A. Cañizares and K. Bhattacharya are with the Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, ON, N2L 3G1, Canada (e-mail: mpirnia@uwaterloo.ca; ccanizares@uwaterloo.ca; kankar@uwaterloo.ca).

A. Vaccaro is with the Department of Engineering, University of Sannio, 82100 Benevento, Italy (e-mail: vaccaro@unisannio.it).

Programming (CPP) method is used to solve an OPF problem with uncertain demand variables; in order to enhance the execution time, the OPF problem is linearized and a back-mapping approach is implemented to compute the probabilities of satisfying the inequality constraints.

An interval arithmetic (IA) method is used in [14] to determine strict bounds to the solution of the power flow problem for uncertain system parameters, where the interval linear power flow equations are solved using iterative methods or employing explicit inverse of matrices to obtain the hull of the solution set. The IA method lacks accuracy because of expansive intervals, with increasing sources of uncertainties and system size. Self-validated computation approaches, based on Affine Arithmetic (AA) are proposed in [15], [16] and [17] for power flow analysis, treating uncertain variables in affine form. The AA-based method is shown to provide better bounds than IA, since it considers the correlation between the input variables.

The present paper proposes a novel application of the AA method to the OPF problem to compute intervals to capture the internal and external uncertainties, and subsequently determine the spinning reserve requirement for a system with variable generation resources without the need for pdfs. The AA-based method has significant advantages over exiting methods to study the uncertainties in OPF problems, which can be summarized as follows:

- The method does not rely on the pdf approximation of the uncertain variables, such as demand and generation.
- In the AA approach, the correlation among the variables is considered and, therefore, the output intervals are not as conservative as in IA.
- The proposed method takes into account the internal errors caused by truncations and approximations.
- The method is accurate, as it yields results close to the "exact" intervals.
- The proposed approach is efficient, as it does not require many iterations to converge, and hence it is much faster than other existing methods such as MCS.

The proposed method is tested and validated using the IEEE 30-bus system and a real 1211-bus European system, demonstrating the efficiency and accuracy of the technique in realistic applications, using MCS as the reference for comparison purposes.

The rest of the paper is organized as follows: Section II presents a brief background on the AA method, and Section III proposes an AA-based OPF solution methodology. In Section IV, the proposed method is tested and validated on the IEEE 30-bus system and a real 1211-bus European system. Finally, in Section V the main conclusions and contributions of the paper are highlighted.

II. BACKGROUND

Numerical calculations are prone to errors in obtaining the exact values of mathematical computations, because of the presence of uncertainties. Most of the uncertainty analysis techniques, such as the MCS method, try to capture external uncertainties affecting the input data, and neglect the impact of internal errors, caused by approximation and truncation. To resolve this issue, "self-validated computation" (SVC) or "automatic result verification" such as the AA methods are proposed, which inherently keep track of the internal errors [18].

The IA method, is the simplest self-validated range analysis technique, providing conservative bounds for interval values of $\hat{x}$, with an upper bound $\bar{X}$ and a lower bound $\underline{X}$. Basic IA operations between two intervals $\hat{x}$ and $\hat{y}$ assume two conditions: (1) none of the intervals are empty, and (2) the upper and lower bounds of each interval are finite real numbers. These assumptions prevent certain operations such as $0\times\infty$ and $(-\infty)\times(+\infty)$ [18]. One of the main disadvantages of IA is error explosion, resulting in very conservative final bounds, which are too wide to be useful. Error explosion usually occurs in long chains of computation, where the input of one operation is the output of another and is typical in power system analysis problems. The main reason for the wider bounds in IA calculations is the independency problem, in which the correlation between IA values is neglected. The AA method, on the other hand is a range analysis technique, which not only handles external uncertainties such as forecast errors, but also internal errors such as arithmetic roundoff, series truncation and function approximation [18]. Although the AA method is computationally more expensive than the IA method, it provides narrower intervals, thus justifying the extra cost.

An affine representation $\tilde{x}$ of a value x (e.g., variable or parameter) is represented in the following form:

$$\tilde{x} = x_0 + x_1\varepsilon_1 + x_2\varepsilon_2 + \cdots + x_n\varepsilon_n \tag{1}$$

Each noise variable ε_i, which lies between -1 and 1, represents an independent source of uncertainty, and each coefficient x_i models the magnitude of that uncertainty. Each affine form can be converted to an interval form by adding or subtracting the summation of the absolute values of all noise magnitudes to or from the central value x_0 in order to find the upper bound $\bar{X}$ and lower bound $\underline{X}$, as follows:

$$[\tilde{x}] = [\underline{X}\,, \bar{X}] = \left[x_0 - \sum_i |x_i|\ , x_0 + \sum_i |x_i|\right] \tag{2}$$

where $\sum_i |x_i|$ is called the total deviation of the affine form $\tilde{x}$.

A. *Affine Operations* [18]

Each affine operation $\tilde{f}(.)$ computes an affine form $\tilde{z}$, which is consistent with the affine input values as follows:

$$\tilde{f}(\tilde{x},\tilde{y}) \rightarrow \tilde{z} \tag{3}$$

Thus, consider $\tilde{x}$ and $\tilde{y}$:

$$\tilde{x} = x_0 + x_1\varepsilon_1 + x_2\varepsilon_2 + \cdots + x_n\varepsilon_n = x_0 + \sum_{i=1}^{n} x_i\varepsilon_i \tag{4}$$

$$\tilde{y} = y_0 + y_1\varepsilon_1 + y_2\varepsilon_2 + \cdots + y_n\varepsilon_n = y_0 + \sum_{i=1}^{n} y_i\varepsilon_i \tag{5}$$

Then, the following elementary affine operations can be defined:

$$\tilde{z}_1 = \tilde{x} \pm \tilde{y} = (x_0 \pm y_0) + (x_1 \pm y_1)\varepsilon_1 + \cdots + (x_n \pm y_n)\varepsilon_n \tag{6}$$

$$\tilde{z}_2 = \varphi \hat{x} = (\varphi x_0) + (\varphi x_1)\varepsilon_1 + \cdots + (\varphi x_n)\varepsilon_n \tag{7}$$

$$\tilde{z}_3 = \tilde{x} \pm \varphi = (x_0 \pm \varphi) + x_1\varepsilon_1 + x_2\varepsilon_2 + \cdots + x_n\varepsilon_n \tag{8}$$

where φ is a constant. In (4)-(5), observe that each noise variables ε_i shows a dependency between the two values $\tilde{x}$ and $\tilde{y}$. Therefore, in (6), ε_i contributes to the same uncertainty in the resulting affine form $\tilde{z}_1$, with the noise magnitudes corresponding to the coefficients of ε_i in the affine values $\tilde{x}$ and $\tilde{y}$.

B. Non-Affine Operations

Non-affine operations require an affine approximation and an extra term, called approximation error, to represent internal errors. For instance, a non-affine multiplication operation between the two affine values $\tilde{x}$ and $\tilde{y}$ can be written as:

$$\tilde{z} = \tilde{x}\,\tilde{y} = x_0\,y_0 + \sum_{i=1}^{n}(x_0\,y_i + y_0\,x_i)\,\varepsilon_i + z_k\,\varepsilon_k \tag{9}$$

The result of this non-affine function is an affine value containing the information provided by $\tilde{x}$ and $\tilde{y}$, and the approximation error represented by $z_k\varepsilon_k$, where the lower bound of $|z_k|$ can be obtained as follows:

$$|z_k| \geq \left(\sum_{i=1}^{n} x_i\,\varepsilon_i\right)\left(\sum_{i=1}^{n} y_i\,\varepsilon_i\right) \tag{10}$$

The simplest and most conservative affine approximation (e.g., trivial affine approximation) can be calculated as follows:

$$z_k = \sum_{i=1}^{n}|x_i|\sum_{i=1}^{n}|y_i| \tag{11}$$

Therefore, after substituting (11) in (9), the product function can be represented as:

$$\tilde{x}\,\tilde{y} = x_0\,y_0 + \sum_{i=1}^{n}(x_0\,y_i + y_0\,x_i)\,\varepsilon_i + \left(\sum_{i=1}^{n}|x_i|\sum_{i=1}^{n}|y_i|\right)\varepsilon_k \tag{12}$$

This affine approximation is computationally efficient, but not the most accurate because the error of this approximation is at most four times the error reported by the most accurate method, i.e., the Chebyshev approximation. Furthermore, ε_k is assumed to be 1 throughout this paper, i.e.,

$$\tilde{x}\,\tilde{y} = x_0\,y_0 + \sum_{i=1}^{n}(x_0\,y_i + y_0\,x_i)\,\varepsilon_i + \sum_{i=1}^{n}|x_i|\sum_{i=1}^{n}|y_i| \tag{13}$$

Even though this simplification may result in wider intervals, it reduces the number of noise symbols for each non-affine operation, and since there are many non-affine operations in OPF models, this would significantly reduce the number of noise symbols in realistic applications. In order to overcome the large intervals, and provide more accurate bounds the contraction method described in [19] is used in this paper, as demonstrated in Section III.

III. AA-based Optimal Power Flow

An AA-based method is proposed in this paper to solve OPF problems with uncertainties. Thus, all the variables associated with the proposed AA-based OPF problem (e.g., the bus voltage magnitude and the voltage angle, or real and imaginary components of bus voltages in rectangular form) can be stated in affine form with a center value and noise magnitudes. Center values represent the value of a variable under deterministic assumptions, i.e., when uncertainties are neglected, and noise magnitudes represent deviations of the OPF variables due to the uncertainties.

A. AA-based Mathematical Model

The proposed AA-based OPF model uses a cost minimization objective function, with all the uncertain variables, such as real and reactive power generation $\tilde{P}_i^G$ and $\tilde{Q}_i^G$, real and reactive power demand $\tilde{P}_i^D$ and $\tilde{Q}_i^D$, bus voltage magnitude $|\tilde{V}_i|$, real and imaginary components of bus voltages $\tilde{e}_i$ and $\tilde{f}_i$, real and imaginary components of bus currents $\tilde{I}_{r_i}$ and $\tilde{I}_{im_i}$, and line currents $\tilde{I}_{ij}$ in affine form. The following equations correspond to the rectangular form of the AA-based OPF problem:

$$\min\ F(\tilde{P}^G) = \sum_{i\in Th}\alpha_i {\tilde{P}_i^G}^2 + \beta_i\tilde{P}_i^G + c_i \tag{14}$$

$$\text{s.t.:}\quad \Delta\tilde{P}_i\left(\tilde{e}_i, \tilde{f}_i, \tilde{I}_{r_i}, \tilde{I}_{im_i}, \tilde{P}_i^G, \tilde{P}_i^D\right) = 0 \qquad \forall i \in N \tag{15}$$

$$\Delta\tilde{Q}_i\left(\tilde{e}_i, \tilde{f}_i, \tilde{I}_{r_i}, \tilde{I}_{im_i}, \tilde{Q}_i^G, \tilde{Q}_i^D\right) = 0 \qquad \forall i \in N \tag{16}$$

$$|\tilde{V}_i|^2 = \tilde{e}_i^2 + \tilde{f}_i^2 \qquad \forall i \in N \tag{17}$$

$$P_i^{min} \leq \tilde{P}_i^G \leq P_i^{max} \qquad \forall i \in NPG \tag{18}$$

$$Q_i^{min} \leq \tilde{Q}_i^G \leq Q_i^{max} \qquad \forall i \in NPG \tag{19}$$

$$I_{ij}^{min} \leq \tilde{I}_{ij} \leq I_{ij}^{max} \qquad \forall ij \in L \tag{20}$$

$${V_i^{min}}^2 \leq \left|\tilde{V}_i\right|^2 \leq {V_i^{max}}^2 \qquad \forall i \in N \tag{21}$$

where N is the set of all buses, NPG is the set of all generator buses, Th is the set of all thermal generates, and L is the set of all lines. Furthermore, $\Delta\tilde{P}_i(.)$ and $\Delta\tilde{Q}_i(.)$ are affine real and reactive power mismatch functions, and P_i^{min}, P_i^{max}, Q_i^{min}, Q_i^{max}, I_{ij}^{min}, I_{ij}^{max}, V_i^{min} and V_i^{max} are minimum and maximum limits for real power, reactive power, line currents, and bus voltage magnitudes, respectively, at bus i. This model cannot be solved directly, as it needs affine representations of the uncertain variables. The objective function (14) is the cost interval for the affine function $F(\tilde{P}^G)$, which is dependent on the affine real power generation of thermal generators. The objective is neither to minimize the cost of the worst/best case nor the expected value, but the cost interval.

All the state and control variables in (14)-(21) are in affine form, comprising a center value and the corresponding noise magnitudes. The center values are obtained by solving a deterministic OPF, in which the median of the given intervals for upper and lower bounds of real and reactive power demand

$\overline{P}_i^D$ and $\underline{P}_i^D$, and $\overline{Q}_i^D$ and $\underline{Q}_i^D$ are considered deterministic demands as follows:

$$P_i^D = \frac{\overline{P}_i^D + \underline{P}_i^D}{2} \qquad \forall i \in ND \tag{22}$$

$$Q_i^D = \frac{\overline{Q}_i^D + \underline{Q}_i^D}{2} \qquad \forall i \in ND \tag{23}$$

where ND is the set of all buses with uncertain demand. In this paper, uncertain wind and solar generation sources are treated as interval negative loads with a constant power factor, and are hence represented using (22) and (23) in the deterministic OPF.

To obtain the noise magnitudes of real and imaginary components of bus voltages, a sensitivity analysis is carried out, where the generation and demand are perturbed by a small magnitude (e.g., ±1%) at each node. A deterministic OPF is solved for each of these variations, thus solving as many OPFs as the number of uncertain inputs in the system, which corresponds to the number of renewable generators being studied. Therefore, the noise magnitudes can be obtained as follows [15]:

$$e_{i,j}^P = \left.\frac{\partial e_i}{\partial P_j^D}\right|_0 \approx \frac{e_i^N - e_i^0}{\Delta P_j^D} \qquad \forall i,j \in N \tag{24}$$

$$e_{i,j}^Q = \left.\frac{\partial e_i}{\partial Q_j^D}\right|_0 \approx \frac{e_i^N - e_i^0}{\Delta Q_j^D} \qquad \forall i,j \in N \tag{25}$$

$$f_{i,j}^P = \left.\frac{\partial f_i}{\partial P_j^D}\right|_0 \approx \frac{f_i^N - f_i^0}{\Delta P_j^D} \qquad \forall i,j \in N \tag{26}$$

$$f_{i,j}^Q = \left.\frac{\partial f_i}{\partial Q_j^D}\right|_0 \approx \frac{f_i^N - f_i^0}{\Delta Q_j^D} \qquad \forall i,j \in N \tag{27}$$

where $e_{i,j}^P$ and $f_{i,j}^P$ are partial deviations of real and imaginary components of bus voltages due to changes in real power injection, and $e_{i,j}^Q$ and $f_{i,j}^Q$ are partial deviations of real and imaginary components of bus voltages at bus i due to changes in reactive power injection at bus j. The parameters e_i^N and f_i^N are the new real and imaginary components of the bus voltages when the real and reactive power injections are perturbed; e_i^0 and f_i^0 are the initial values of real and imaginary components of bus voltages obtained from the deterministic model; and ΔP_j^D and ΔQ_j^D are the amount of perturbation in real and reactive power injections at bus j.

Note that since the power flow equations are nonlinear, the suggested calculations in (24)-(27) may result in the underestimation of the solution and, therefore, not include the “exact” solutions. In order to avoid underestimation of the solution, an “affine extension” technique suggested in [15] and [20] is used here to increase each noise magnitude using an independent magnification coefficient. Also, it is assumed that an OPF solution for the center value can be obtained as the starting point; hence, the derivatives (24)-(27), which can be obtained by inverting Jacobians of the center-value OPF solution, exist. In the case that a center-value OPF solution cannot be obtained, the proposed technique cannot be applied; this may be resolved by redefining the intervals. It should be noted that, since the classical OPF problem is highly non-convex, using NLP solution methods such as the Interior Point (IP), do not guarantee a global optimal solution. To prevent widely different solutions during the perturbation method used in the sensitivity analysis, at each run, the initial values for each variable are set at the values obtained from solving the center-value OPF. Since these perturbations are small, this procedure guarantees in practice obtaining local maxima close to the center-value OPF solutions.

The affine forms of the real and imaginary components of the bus voltage magnitude $\tilde{e}_i$ and $\tilde{f}_i$, which are linear functions of noise variables $\varepsilon_{P_j^D}$ and $\varepsilon_{Q_j^D}$ representing the uncertainties of active power and reactive power injections at bus j, can then be presented as follows:

$$\tilde{e}_i = e_{i0} + \sum_{j\epsilon N} e_{i,j}^P\, \varepsilon_{P_j^D} + \sum_{j\epsilon N} e_{i,j}^Q\, \varepsilon_{Q_j^D} \qquad \forall i \in N \tag{28}$$

$$\tilde{f}_i = f_{i0} + \sum_{j\epsilon N} f_{i,j}^P\, \varepsilon_{P_j^D} + \sum_{j\epsilon N} f_{i,j}^Q\, \varepsilon_{Q_j^D} \qquad \forall i \in N \tag{29}$$

where e_{i0} and f_{i0} are the center values for real and imaginary components of bus voltages at bus i, respectively, and $e_{i,j}^P$, $f_{i,j}^P$, $e_{i,j}^Q$, and $f_{i,j}^Q$ are defined in (24)-(27).

The affine forms of real and imaginary components of bus voltages, $\tilde{e}_i$ and $\tilde{f}_i$ can be used to calculate, from (13), the square of bus voltage magnitude, using the following relationship:

$$|\tilde{V}_i|^2 = \tilde{e}_i^2 + \tilde{f}_i^2 \qquad \forall i \in N \tag{30}$$

Thus, $|\tilde{V}_i|^2$ has the following form:

$$\begin{aligned} |\tilde{V}_i|^2 = (e_{i0}^2 + f_{i0}^2) &+ 2\sum_{j\epsilon N}\left(e_{i0}e_{i,j}^P + f_{i0}f_{i,j}^P\right)\varepsilon_{P_j^D} \\ &+ 2\sum_{j\epsilon N}\left(e_{i0}e_{i,j}^Q + f_{i0}f_{i,j}^Q\right)\varepsilon_{Q_j^D} \\ &+ (e_i^T + f_i^T) \qquad \forall i \in N \end{aligned} \tag{31}$$

Here e_i^T and f_i^T are truncation errors due to non-affine operations. Furthermore, knowing the affine forms of the real and imaginary components of the bus voltage magnitude, the real and reactive power can be calculated using the following affine operations:

$$\tilde{I}_i = \sum_{j\epsilon N}\left(\mathrm{G}_{ij} + \hat{\jmath}\mathrm{B}_{ij}\right)(\tilde{e}_j + \hat{\jmath}\tilde{f}_j) \qquad \forall i \in N \tag{32}$$

where $\hat{\jmath} = \sqrt{-1}$, and G_{ij} and B_{ij} are the real and imaginary components of the Y-bus matrix, respectively. Therefore, the linear affine forms of real and imaginary bus currents $\tilde{I}_{r_i}$ and $\tilde{I}_{im_i}$ can be calculated as follows:

$$\tilde{I}_{r_i} = \sum_{j\epsilon N}\left\{\mathrm{G}_{ij}\left(\tilde{e}_i - \tilde{e}_j\right) - \mathrm{B}_{ij}\left(\tilde{f}_i - \tilde{f}_j\right)\right\} \qquad \forall i \in N \tag{33}$$

$$\tilde{I}_{im_i} = \sum_{j\epsilon N}\{\mathrm{G}_{ij}(\tilde{f}_i - \tilde{f}_j) + \mathrm{B}_{ij}(\tilde{e}_i - \tilde{e}_j)\} \qquad \forall i \in N \quad (34)$$

where $\tilde{I}_{r_i}$ and $\tilde{I}_{im_i}$ have the following general forms after affine operations:

$$\tilde{I}_{r_i} = I_{r_{i0}} + \sum_{j\epsilon N} I_{r_{i,j}}^{P}\, \varepsilon_{P_j^D} + \sum_{j\epsilon N} I_{r_{i,j}}^{Q}\, \varepsilon_{Q_j^D} \qquad \forall i \in N \quad (35)$$

$$\tilde{I}_{im_i} = I_{im_{i0}} + \sum_{j\epsilon N} I_{im_{i,j}}^{P}\, \varepsilon_{P_j^D} + \sum_{j\epsilon N} I_{im_{i,j}}^{Q}\, \varepsilon_{Q_j^D} \qquad \forall i \in N \quad (36)$$

Here, $I_{r_{i0}}$ and $I_{im_{i0}}$ are the center values for real and imaginary components of current magnitudes. $I_{r_{i,j}}^{P}$ and $I_{r_{i,j}}^{Q}$ are partial deviations of the real component of current at bus i for deviation in real and reactive power injection at a bus j, respectively; and $I_{im_{i,j}}^{P}$ and $I_{im_{i,j}}^{Q}$ are partial deviations of imaginary component of current at bus i for deviation in real and reactive power injection at bus j, respectively. Note that the real and imaginary components of the current share the same sources of uncertainties, i.e., real and reactive power injections $\varepsilon_{P_j^D}$ and $\varepsilon_{Q_j^D}$. Furthermore, using affine and non-affine operations and $\tilde{e}_i$, $\tilde{f}_i$, $\tilde{I}_{r_i}$, and $\tilde{I}_{im_i}$, the real and reactive power mismatch $\Delta\tilde{P}_i(.)$and $\Delta\tilde{Q}_i(.)$ in (15) and (16), respectively, can be calculated as follows:

$$\Delta\tilde{P}_i(\tilde{e}_i, \tilde{f}_i, \tilde{I}_{r_i}, \tilde{I}_{im_i}, \tilde{P}_i^G, \tilde{P}_i^D) = \tilde{P}_i - \tilde{e}_i\, \tilde{I}_{r_i} - \tilde{f}_i\, \tilde{I}_{im_i} = 0 \qquad \forall i \in N \quad (37)$$

$$\Delta\tilde{Q}_i(\tilde{e}_i, \tilde{f}_i, \tilde{I}_{r_i}, \tilde{I}_{im_i}, \tilde{Q}_i^G, \tilde{Q}_i^D) = \tilde{Q}_i - \tilde{f}_i \tilde{I}_{r_i} + \tilde{e}_i\, \tilde{I}_{im_i} = 0 \qquad \forall i \in N \quad (38)$$

Where $\tilde{P}_i$ and $\tilde{Q}_i$ represent the affine real and reactive power injections, and have the following affine forms, with center value and associated partial deviations:

$$\tilde{P}_i = P_{i0} + \sum_{j\epsilon N} P_{i,j}^{P}\, \varepsilon_{P_j^D} + \sum_{j\epsilon N} P_{i,j}^{Q} \varepsilon_{Q_j^D} + P_i^T \qquad \forall i \in N \quad (39)$$

$$\tilde{Q}_i = Q_{i0} + \sum_{j\epsilon N} Q_{i,j}^{P} \varepsilon_{P_j^D} + \sum_{j\epsilon N} Q_{i,j}^{Q}\, \varepsilon_{Q_j^D} + Q_i^T \qquad \forall i \in N \quad (40)$$

Where P_{i0} and Q_{i0} are the center values of affine real and reactive power injections; $P_{i,j}^P$ and $Q_{i,j}^P$ are the partial deviations of real and reactive power injections due to changes in real power injections at a bus j, respectively; $P_{i,j}^Q$ and $Q_{i,j}^Q$ are the partial deviations of real and reactive power injections due to changes in reactive power injections at bus j, respectively; and $P_i^T$and Q_i^T are real and reactive power injection truncation errors, based on (13). The above affine forms can be calculated as follows:

$$P_{i0} = e_{i0}\, I_{r_{i0}} + f_{i0}\, I_{im_{i0}} \qquad \forall i \in N \quad (41)$$

$$Q_{i0} = f_{i0} I_{r_{i0}} - e_{i0}\, I_{im_{i0}} \qquad \forall i \in N \quad (42)$$

$$P_{i,j}^{P} = e_{i0}\, I_{r_{i,j}}^{P} + I_{r_{i0}}\, e_{i,j}^{P} + f_{i,0}\, I_{im_{i,j}}^{P} + I_{im_{i0}}\, f_{i,j}^{P} \forall i,j \in N \quad (43)$$

$$Q_{i,j}^{P} = f_{i0}\, I_{r_{i,j}}^{P} + I_{r_{i0}}\, f_{i,j}^{P} - e_{i,0}\, I_{im_{i,j}}^{P} - I_{im_{i0}}\, e_{i,j}^{P} \forall i,j \in N \quad (44)$$

$$P_{i,j}^{Q} = e_{i0}\, I_{r_{i,j}}^{Q} + I_{r_{i0}}\, e_{i,j}^{Q} + f_{i,0}\, I_{im_{i,j}}^{Q} + I_{im_{i0}}\, f_{i,j}^{Q} \forall i,j \in N \quad (45)$$

$$Q_{i,j}^{Q} = f_{i0}\, I_{r_{i,j}}^{Q} + I_{r_{i0}}\, f_{i,j}^{Q} - e_{i,0}\, I_{im_{i,j}}^{Q} - I_{im_{i0}}\, e_{i,j}^{Q} \forall i,j \in N \quad (46)$$

$$P_i^T = \sum_{j\epsilon N}\{|e_{i,j}^P| + |e_{i,j}^Q|\} \sum_{j\epsilon N}\left\{\left|I_{r_{i,j}}^{P}\right| + \left|I_{r_{i,j}}^{Q}\right|\right\} + \sum_{j\epsilon N}\{|f_{i,j}^P| + |f_{i,j}^Q|\} \sum_{j\epsilon N}\left\{\left|I_{im_{i,j}}^{P}\right| + \left|I_{im_{i,j}}^{Q}\right|\right\} \qquad \forall i \in N \quad (47)$$

$$Q_i^T = \sum_{j\epsilon N}\{|f_{i,j}^P| + |f_{i,j}^Q|\} \sum_{j\epsilon N}\left\{\left|I_{r_{i,j}}^{P}\right| + \left|I_{r_{i,j}}^{Q}\right|\right\} - \sum_{j\epsilon N}\{|e_{i,j}^P| + |e_{i,j}^Q|\} \sum_{j\epsilon N}\left\{\left|I_{im_{i,j}}^{P}\right| + \left|I_{im_{i,j}}^{Q}\right|\right\} \qquad \forall i \in N \quad (48)$$

This formulation, as previously mentioned, is the most conservative but computationally efficient for calculating the magnitude of the internal errors P_i^T and Q_i^T. The affine forms (39) and (40) for real and reactive powers can be represented in interval forms $[\underline{P}_i, \overline{P}_i]$ and $[\underline{Q}_i, \overline{Q}_i]$, where:

$$\overline{P}_i = P_{i0} + radP_i\left(\varepsilon_{P_j^D}, \varepsilon_{Q_j^D}\right) \qquad \forall i \in N \quad (49)$$

$$\underline{P}_i = P_{i0} - radP_i\left(\varepsilon_{P_j^D}, \varepsilon_{Q_j^D}\right) \qquad \forall i \in N \quad (50)$$

$$\overline{Q}_i = Q_{i0} + radQ_i\left(\varepsilon_{P_j^D}, \varepsilon_{Q_j^D}\right) \qquad \forall i \in N \quad (51)$$

$$\underline{Q}_i = Q_{i0} - radQ_i\left(\varepsilon_{P_j^D}, \varepsilon_{Q_j^D}\right) \qquad \forall i \in N \quad (52)$$

Here, $radP_i\left(\varepsilon_{P_j^D}, \varepsilon_{Q_j^D}\right)$ and $radQ_i\left(\varepsilon_{P_j^D}, \varepsilon_{Q_j^D}\right)$ are the following functions of noise variables $\varepsilon_{P_j^D}$ and $\varepsilon_{Q_j^D}$, with the most conservative values when they are equal to 1, and represent the total amount of deviation from the center value:

$$radP_i\left(\varepsilon_{P_j^D}, \varepsilon_{Q_j^D}\right) = \sum_{j\epsilon N} P_{i,j}^P \varepsilon_{P_j^D} + \sum_{j\epsilon N} P_{i,j}^Q \varepsilon_{Q_j^D} + P_i^T \qquad \forall i \in N \quad (53)$$

$$radQ_i\left(\varepsilon_{P_j^D}, \varepsilon_{Q_j^D}\right) = \sum_{j\epsilon N} Q_{i,j}^P \varepsilon_{P_j^D} + \sum_{j\epsilon N} Q_{i,j}^Q\, \varepsilon_{Q_j^D} + Q_i^T \qquad \forall i \in N \quad (54)$$

These intervals can be used to determine the operational range of generation for dispatchable generators (e.g., thermal, hydro), when there are sources of uncertainty such as renewable generation in the system.

B. Contraction Method

The intervals obtained in (49)-(52) are dependent on the value of the noise variables $\varepsilon_{P_j^D}$ and $\varepsilon_{Q_j^D}$, which are between -1 and 1. The most conservative intervals are obtained by fixing these variables at their maximum values; however, this results in large intervals. Hence, based on the method presented in [19], [21] and [22], a contraction method is used to reduce the bounds. In order to reasonably contract the intervals, while respecting the physical characteristics of the system (e.g. voltage and generation limits), the discussed next LP is solved to obtain the minimum values of noise variables; the suggested optimization problem is linear since all the affine variables are constructed using linear affine operations (6)-(13). Thus, by replacing in the AA-based OPF model (14)-(21), all uncertain variables with their AA forms defined in (28)-(40), one obtains the following representation of the OPF model:

$$\min \ \tilde{F}\left(\varepsilon_{P_j^D}, \varepsilon_{Q_j^D}\right) \quad (55)$$

$$\text{s.t.:} \ \Delta\tilde{P}_i\left(\varepsilon_{P_j^D}, \varepsilon_{Q_j^D}\right) = 0 \qquad \forall i \in N \quad (56)$$

$$\Delta\tilde{Q}_i\left(\varepsilon_{P_j^D}, \varepsilon_{Q_j^D}\right) = 0 \qquad \forall i \in N \quad (57)$$

$$P_i^{min} \leq P_i\left(\varepsilon_{P_j^D}, \varepsilon_{Q_j^D}\right) \leq P_i^{max} \qquad \forall i \in NPG \quad (58)$$

$$Q_i^{min} \leq Q_i\left(\varepsilon_{P_j^D}, \varepsilon_{Q_j^D}\right) \leq Q_i^{max} \qquad \forall i \in NPG \quad (59)$$

$$I_{ij}^{min^2} \leq \tilde{I}_{r_{ij}}^{\,2}\left(\varepsilon_{P_j^D}, \varepsilon_{Q_j^D}\right) + \tilde{I}_{im_{ij}}^{\,2}\left(\varepsilon_{P_j^D}, \varepsilon_{Q_j^D}\right) \leq I_{ij}^{max^2} \qquad \forall ij \in L \quad (60)$$

$$V_i^{min^2} \leq \tilde{V}_i^2\left(\varepsilon_{P_j^D}, \varepsilon_{Q_j^D}\right) \leq V_i^{max^2} \qquad \forall i \in N \quad (61)$$

$$-1 \leq \varepsilon_{P_j^D} \leq 1 \qquad \forall j \in N \quad (62)$$

$$-1 \leq \varepsilon_{Q_j^D} \leq 1 \qquad \forall j \in N \quad (63)$$

The above linear noise contraction model can be easily solved using available LP solvers, such as CPLEX [23]. Then the obtained values of $\varepsilon_{P_j^D}$ and $\varepsilon_{Q_j^D}$ are used in (49)-(52).

In the above LP problem, the objective function $\tilde{F}(.)$ is the affine linear expansion of (14), as follows:

$$\tilde{F}\left(\varepsilon_{P_j^D}, \varepsilon_{Q_j^D}\right) = \sum_{i\epsilon N}\left[\alpha_i\left\{P_{i0}^{\ 2} + \sum_{j\epsilon N} 2P_{i0}\, P_{i,j}^P \varepsilon_{P_j^D} + \sum_{j\epsilon N} 2P_{i0} P_{i,j}^Q \varepsilon_{Q_j^D} + 2P_{i0}P_i^T + P_i^{T'}\right\} + \beta_i\left\{\sum_{j\epsilon N} P_{i0} + P_{i,j}^P\, \varepsilon_{P_j^D} + \sum_{j\epsilon N} P_{i,j}^Q \varepsilon_{Q_j^D} + P_i^T\right\} + c_i\right] \quad (64)$$

Note that in the above equation, due to an affine product, a new error magnitude $P_i^{T'}$ is introduced. The line currents $\tilde{I}_{r_{ij}}$ and $\tilde{I}_{im_{ij}}$ in (60) are calculated as follows:

$$\tilde{I}_{r_{ij}} = \mathrm{G}_{ij}\left(\tilde{e}_i - \tilde{e}_j\right) - \mathrm{B}_{ij}\left(\tilde{f}_i - \tilde{f}_j\right) \qquad \forall ij \in L \quad (65)$$

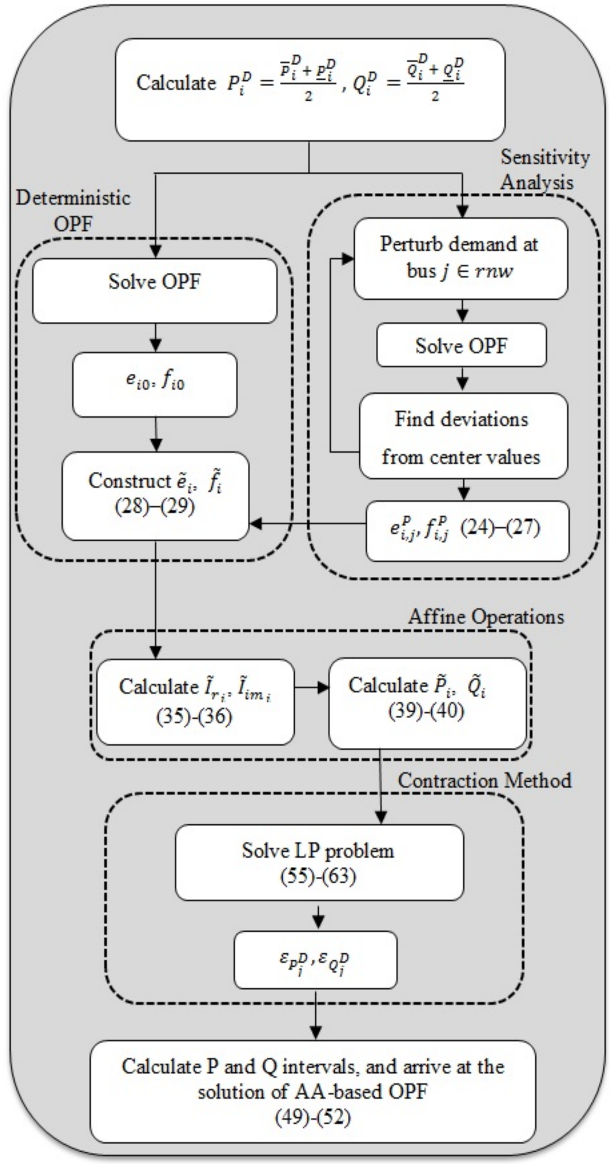

Fig. 1. Proposed AA-based OPF.

$$\tilde{I}_{im_{ij}} = \mathrm{G}_{ij}\left(\tilde{f}_i - \tilde{f}_j\right) + \mathrm{B}_{ij}\left(\tilde{e}_i - \tilde{e}_j\right) \qquad \forall ij \in L \quad (66)$$

Fig. 1 depicts the procedure used to calculate the intervals for the affine variables and hence arrive at the solution intervals to the OPF with uncertainties. Note that the demand intervals at each bus are the input of the model. These intervals consider the uncertainties associated with both demand and renewable generation by considering the latter as negative loads. Then affine variables $\tilde{e}_i$ and $\tilde{f}_i$ are constructed using both a deterministic OPF model to obtain center values, and a sensitivity analysis based on several perturbed OPFs to obtain the noise magnitudes. Affine operations are then applied on these affine variables in order to calculate other affine variables, such as $\tilde{P}_i$ and $\tilde{Q}_i$. To minimize the size of the intervals, a contraction method is used to minimize the noise magnitudes associated with each of these variables.

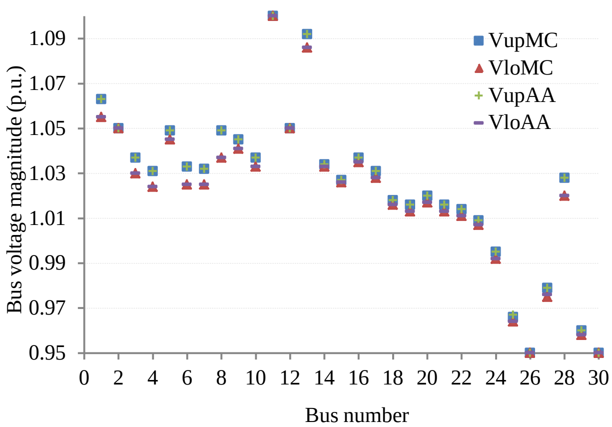

Fig.2. Bus voltage magnitude intervals.

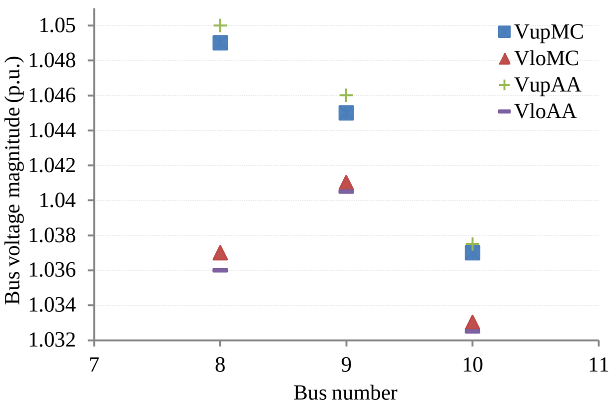

Fig.3. Bus voltage magnitude intervals, selected buses.

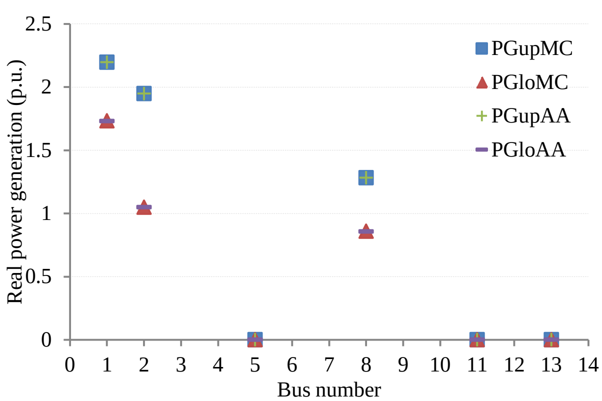

Fig. 4. Real power generation intervals.

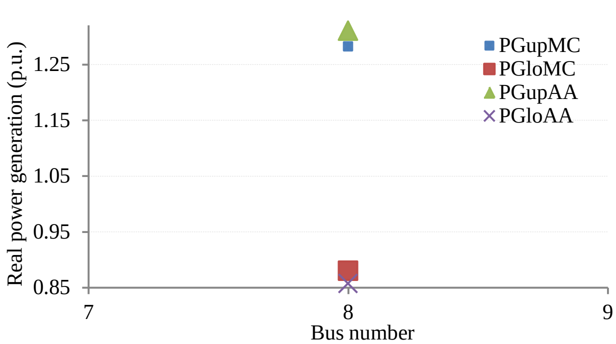

Fig.5. Real power generation interval at Bus 8.

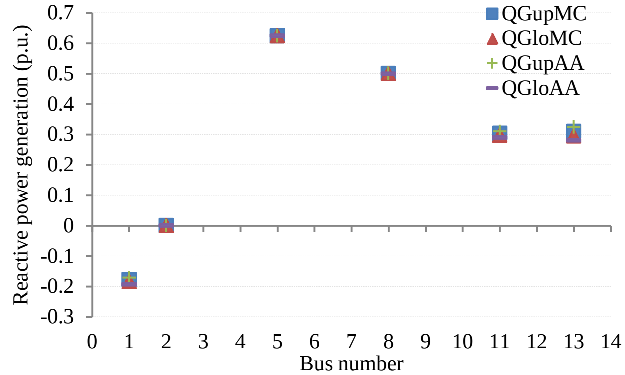

Fig. 6. Reactive power generation intervals

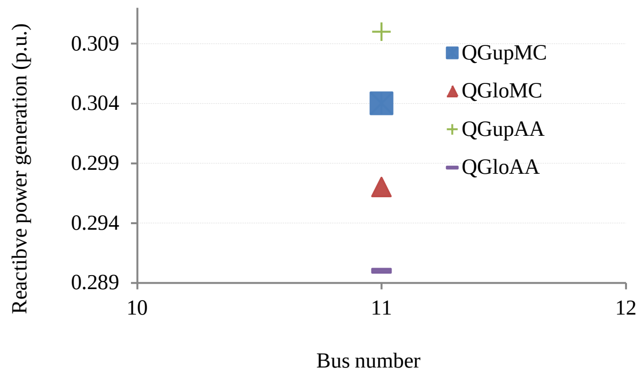

Fig. 7. Reactive power generation interval at Bus 11

IV. Results and Discussions

In this section, the proposed AA-based OPF is tested using the IEEE 30-bus system [24] and a real European 1211-bus system, and compared with the MCS solution. The IA method has not been used as a mean for comparison in this paper, as the bounds provided by the IA approach are too conservative or simply cannot be obtained and therefore it is not a feasible method. The IA wider bounds are simply because of not considering the correlation among the variables which leads to error explosion. In fact, one of the authors has studied the application of the IA method to the OPF, and observed that the IA-based OPF problem diverges for realistic systems with a reasonable number of uncertainties due to too wide intervals, which resulted in singularities of the Jacobian of the Lagrangian function, even after preconditioning, and hence it could not be used for the types of studies presented in the current paper.

In the 30-bus system, one of the generators is replaced with a wind turbine to study the effects of uncertainty arising from the renewable sources in the system. The 1211-bus real European system has 160 generators (thermal, hydro, wind, and solar), 58 are solar and 8 are wind generators, for a total renewable capacity of 11678 MW; considering that the total generation capacity of the system, including imports, is 183 GW, this corresponds to 6.4% of the generation capacity. The total system demand is 153 GW, including exports. The transmission system comprises 1567 lines and 122 fixed transformers. The intermittency in wind and solar generation is assumed to be compensated by thermal generation in both

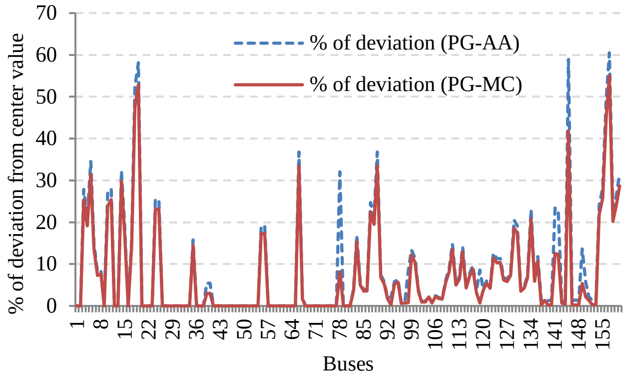

Fig. 8. Percentage of real power deviation for thermal generators.

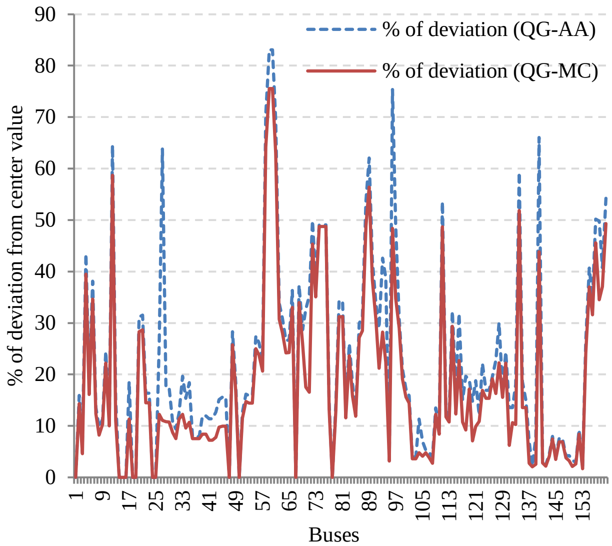

Fig. 9. Percentage of reactive power deviation for thermal generators.

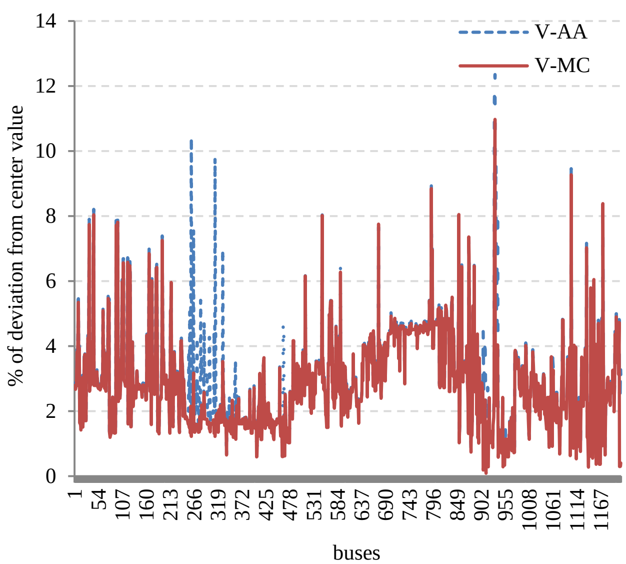

Fig. 10. Percentage of deviation for bus voltage magnitudes.

of the test systems, and thus the proposed AA-based method is used to determine the power output intervals of these generators, hence calculating the generation reserve needed to reliably and optimally supply the demand.

The proposed model is simulated in the General Algebraic Modeling System (GAMS) [25]. The solvers used, i.e., COINIPOPT for the OPF solution [26], and CPLEX for the LP contraction problem, have their parameters set at their respective default, off-the-shelf settings, so as not to bias their "standard" performance. Major settings such as tolerance level or maximum number of iterations of the solver are by default the same for all solvers (e.g., feasibility tolerance is 10^{-6}). The implemented MCS method converges after 3000 iterations for both the IEEE 30-bus test systems, and the 1211-bus system, assuming that the uncertain parameters have uniform distribution within the bounds defined for the assumed variable generation.

A. 30-bus System

In the IEEE 30-bus test system, a wind turbine is assumed to be located at Bus 2. It is considered that this generator power output varies in a ±30% range of its forecasted center value, $P_{20}^{D} = -1.283\, p.u.$, since this is treated as a negative injection with unity power factor.

All the center values are obtained by executing the nonlinear OPF model with the deterministic data, and the noise magnitudes are obtained from the sensitivity analysis technique described in Section III. The intervals obtained by the MCS method are used as the benchmark to check the validity of the obtained AA-based intervals. Figures 2 and 3 compare the bus voltage magnitude bounds using both AA (VupAA and VloAA) and MCS (VupMC and VloMC) methods. Note that there is no significant difference between the two approaches. The bounds obtained by the MCS method lie slightly inside the AA-based ones, because the MCS method provides the "exact" intervals, whereas the AA-based approach provides more conservative margins, as expected. Observe that the difference between the AA-based and MCS-based methods is very small.

The real power intervals, shown in Figs 4 and 5, depict the interval output for thermal generators at Bus 1 and Bus 8, when the wind turbine output varies in the given interval at Bus 2; all other generators are synchronous condensers and hence have zero real power output. Figures 6 and 7 demonstrate the recative power generation inervals. As shown, the resulted AA-based margins of reactive power generation closely match the ones obtained from the MCS method. Note that the diffrence between the upper and lower bounds of the illustrated intervals in Fig. 6 is very small.

B. 1211-bus System

The AA method is also applied to a real European 1211-bus system. Figures 8 and 9 show the real and reactive power deviations of the thermal generators, and Fig. 10 shows the deviation of bus voltage magnitudes at all buses with respect to the center value, in percentage, obtained from both the AA-based approach and MCS. In this case, a ±10% deviation active power injection for wind and solar with unity power factor is assumed. The deviations are obtained as follows:

$$\%\ \text{deviation for } \tilde{x} = \frac{\overline{X} - \underline{X}}{X_0} * 100 \qquad (67)$$

Observe that the errors in this case are significantly large for some generators than those obtained for the smaller IEEE 30-bus system, as expected, since the number of uncertainties is much larger, i.e., 1 in the small system versus 66 in the real method in the maximum total thermal generation required to compensate for the wind and solar power generation uncertainties. Note that the 1.02% total error in Table I is only 1231 MW in the real European system.

TABLE I
THERMAL CAPACITIES, USING MCS AND AA-BASED APPROACHES

Total Thermal Reserve	AA-based Method (GW)	MCS Method (GW)	% of Error
Maximum	144.04	142.57	1.02%
Minimum	131.1	128.2	2.21

The AA method is faster than the MCS approach, as the MCS method needs 3000 iterations for convergence, while the AA method requires as many iterations as the number of uncertain variables, i.e., one iteration for the IEEE 30-bus test system and 66 iterations for the real 1211-bus system. Furthermore, the additional OPFs are relatively small perturbations of the center-value OPF, and hence convergence can be quickly attained by using the center-value OPF solution as starting point.

V. ACKNOWLEDGMENT

The authors would like to thank Professor Gregor Verbic for sharing his studies on the IA-based OPF.

VI. CONCLUSIONS

A novel AA-based model has been proposed to solve the OPF problem with intervals to represent system uncertainty, such as variable wind and solar power generation. The intervals obtained from the proposed technique can be used to approximate the margins of operation for dispatchable generators needed to properly account for system uncertainties. In order to test the efficiency and accuracy of the AA-based OPF, the method was tested on the IEEE 30-bus test system and a real 1211-bus European system, and benchmarked against the MCS method. The AA-based approach was shown to efficiently yield intervals close to the MCS bounds. This method can be used to also study the impact of demand variations in power systems, and other similar applications where intervals could be used to represent uncertainties, without the need to assume pdfs.

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Mehrdad Pirnia (M'08) received the B.Sc degree in Industrial Engineering from Bu-AliSina University, Iran, in 2001 and the M.A.Sc degree in Management Sciences in 2009 from the University of Waterloo. He finished his PhD studies in Electrical Engineering at the University of Waterloo in January 2014, and is currently an instructor at its Department of Management Science. His research interests are in electricity market modeling, renewable energy integration, and large scale optimization.

Claudio A. Cañizares (S'86-M'91-SM'00-F'07) received the Electrical Engineer degree from the Escuela Politecnica Nacional (EPN), Quito-Ecuador, in 1984, where he held different teaching and administrative positions from 1983 to 1993. His MSc (1988) and PhD (1991) degrees in Electrical Engineering are from the University of Wisconsin-Madison. He has been with the E&CE Department at the University of Waterloo since 1993, where he has held various academic and administrative positions and is currently a full Professor, the Hydro One Endowed Chair, and an Associate Director of the Waterloo Institute for Sustainable Energy (WISE). His research activities concentrate in the study of stability, modeling, simulation, control and computational issues in power systems within the context of competitive electricity markets and smart grids. He is a Fellow of the Royal Society of Canada and the Canadian Academy of Engineering, has been the recipient of various IEEE-PES Working Group awards, and holds and has held several leadership positions in IEEE-PES technical committees and subcommittees.

Kankar Bhattacharya (M'95-SM'01) received the Ph.D. degree in Electrical Engineering from the Indian Institute of Technology, New Delhi, in 1993. He was on the faculty of Indira Gandhi Institute of Development Research, Mumbai, India (1993-1998), and Chalmers University of Technology, Göteborg, Sweden (1998-2002). He has been with the Department of Electrical and Computer Engineering at the University of Waterloo since 2003, and is currently a full Professor an Associate Director of the Waterloo Institute for Sustainable Energy (WISE). His research interests are in power system operations and economics, electricity markets, deregulation and retail competition issues, distributed generation economics and planning, global energy models, scenario analysis, long-term planning, and energy-environment linkages and sustainable development.

Alfredo Vaccaro (M\`01-SM\`09) received the M.Sc. degree with honors in electronic engineering in 1998 from the University of Salerno, Salerno, Italy. From 1999 to 2002, he was an Assistant Researcher at the University of Salerno, Department of Electrical and Electronic Engineering. Since March 2002, he has been an Assistant Professor in electric power systems at the Department of Engineering of the University of Sannio, Benevento, Italy. His special fields of interest include soft computing and interval-based method applied to power system analysis and advanced control architectures for diagnostic and protection of distribution networks. He is an Associate Editor and member of the Editorial Boards of IET Proc. Renewable Power Generation, the International Journal of Electrical and Power Engineering, the International Journal of Reliability and Safety, the International Journal on Power System Optimization, and the International Journal of Soft Computing.