Optimal Energy Management of Greenhouses in Smart Grids

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Abstract—This work presents a novel hierarchical control approach and new mathematical optimization models of greenhouses, which can be readily incorporated into Energy Hub Management Systems (EHMSs) in the context of smart grids, to optimize the operation of their energy systems. In greenhouses, artificial lighting, CO₂ production, and climate control systems consume considerable energy; thus, a mathematical model of greenhouses appropriate for their optimal operation is proposed, so that it can be implemented as a supervisory control in existing greenhouse control systems. The objective is to minimize total energy costs and demand charges while considering important parameters of greenhouses; in particular, inside temperature and humidity, CO₂ concentration, and lighting levels should be kept within acceptable ranges. Therefore, the proposed model incorporates weather forecasts, electricity price information, and the end-user preferences to optimally operate existing control systems in greenhouses. Effects of uncertainty in electricity price and weather forecast on optimal operation of the storage facilities are studied through Monte Carlo simulations. The presented simulation results show the effectiveness of the proposed model to reduce total energy costs while maintaining required operational constraints.

Index Terms—Smart grids, energy management systems, energy hubs, greenhouses, mathematical modeling, optimization.

I. NOMENCLATURE

The sets, indices, subscripts, variables, and parameters used in the equations throughout the paper are presented here. Where applicable, the parameter values used for the simulations are provided as well.

Sets

A Set of devices; $A = \{cf, chl, chl_v, co_2, dh, f_v, fog, hu, ht, hv, nv\}$

T Set of time intervals in scheduling horizon;

Z Set of zones;

Indices

i Index of devices; $z$ Index of zones

t Index of time intervals

Functions

$J$ Objective function of the optimization model

Subscripts

a Air; $cf$ Circulation fans

chl Chiller; $chl_p$ Chilling pipe

chl_v Valve of chilling pipe; $co_2$ CO₂ generator

dh Dehumidifier; $eu$ Evaporation

$f_v$ Forced ventilation; $fog$ Fogging system

$gh$ Greenhouse; $ht$ Heating system

$ht_p$ Heating pipe; $ht_v$ Heating system valve

$nv$ Natural ventilation; $out$ Outdoor

$p$ Plants; $phot$ Photosynthesis

$sl$ Soil; $sr$ Solar radiation

$w$ Water; $wl$ Wall

Variables

$\bar{p}$ Peak demand variable (kW)

$p_{sat}$ Saturated water vapour pressure (Pa)

$p_{par}$ Partial vapour pressure (Pa)

$\phi_z(t)$ Relative humidity of zone $z$ at time $t$ (%)

$q_i(t)$ Thermal effect of component $i$ on temperature in zone $z$ (kJ/h)

$\theta_z(t)$ Temperature of zone $z$ at time $t$ (°C)

$s_i,z(t)$ Operation state of device $i$ of zone $z$ at time $t$; $0 \leq s_i,z(t) \leq 1$

$w_z(t)$ Water content of air in zone $z$ at time $t$ ($g_{w}/kg_{a}$)

$c(t)$ CO₂ concentration in zone $z$ at time $t$ ($g_{CO_2}/kg_{a}$)

$\mu_{s,x}$ Axillary variable to represent $s \cdot x$

Parameters

$A_{gh,z}$ Area of greenhouse in zone $z$ (m²); 20000

$A_{wl,z}$ Area of greenhouse walls in zone $z$ (m²); 22400

$A_{nv,z}$ Ventilation window area in zone $z$ (m²); 450

$A_{htp,z}$ Area of heating pipe in zone $z$ (m²); 1625

$A_{chl_p,z}$ Area of cooling pipe in zone $z$ (m²); 1625

$C_1$ Constant (no dim.); 100

$C_2$ Constant ($Pa$); 1.7001

$C_3$ Constant ($Pa$); 7.7835

$C_4$ Constant ($K^{-1}$); 1/17.0789

$C_5$ Constant ($kg_{w}/kg_{a}$); 0.6228

$C_6$ Conversion factor from (1/s) to (1/h); 3600

$C_7$ Coefficients associated with the respiration rate of the crop (°C); -0.27

$C_8$ Coefficients associated with the respiration rate of the crop (no dim.); 0.05

$C_{a}$ Specific heat of air (kJ/(kg K)); 1.006

$C_{e}(t)$ Cost of energy at time $t$ ($$/kWh$$)

$C_{dc}$ Demand charge ($$/kW$$); 8

$C_{p}$ Specific heat of the plants (woods and leaves) (kJ/(kg K)); 3

$C_{w}$ Specific heat of water (kJ/(kg K)); 4.1855

$C_{r}$ Respiration coefficient of crops in $z$ (g/(m² h K)); 1.224e-3

This work was supported by the Ontario Centres of Excellence, Hydro One Networks Inc., Milton Hydro Distribution Inc., Emergent Inc., and Ontario Power Authority. A PCT patent application has been submitted on this work. The authors are with the Department of Electrical and Computer Engineering, University of Waterloo, Ontario, Canada N2L 3G1 (e-mail: {mchehreg, ccanizar, kankar}@uwaterloo.ca).
\( C_{\text{phot}} \) Photosynthesis coefficient of crops in zone \( z \) (g/J); 46.03e-3
\( C_{\text{max}}^{\text{inj}} \) Max. carbon injected by CO\(_2\) generator in zone \( z \) (g/m\(_2\)h); 0.8
\( C_{\text{max}}^{\text{CO}_2} \) Max. \( \text{CO}_2 \) concentration in zone \( z \) (g/m\(_2\)h); 1.3
\( C_{\text{min}}^{\text{CO}_2} \) Min. \( \text{CO}_2 \) concentration in zone \( z \) (g/m\(_2\)h); 0.7
\( \varepsilon \) Volumetric ratio of air to crops in the greenhouse (m\(^3\)/m\(^3\)h); 0.85
\( \eta_{\text{ch}} \) Performance coefficient of the chilling system; 1
\( \eta_{\text{fog}} \) Fog to vapour conversion factor of the fogging system (no dim.); 0.05
\( H_{\text{gh}} \) Average height of the greenhouse (m); 4
\( I(t) \) Solar irradiation at time \( t \) (W/m\(^2\))
\( L \) Large positive number; 10
\( \lambda \) Percentage of wind speed which enters into the greenhouse (no dim.); 0.0075
\( N_T \) Number of intervals in scheduling horizon \( T \); 24
\( P_a \) Actual water vapour pressure (Pa); 0.65e5
\( P_i \) Rated power of device \( i \) (kW); \( \{c.f.: 30.5, \text{chl: 175}\}
\( Q_z \) Volumetric air flow rate of ventilation fans in zone \( z \) (m\(^3\)/(h m\(^2\)h)); 18.3
\( R_w \) Heat transfer coefficient of greenhouse walls (kJ/(h K m\(^2\))); 180
\( R_d \) Heat transfer coefficient of greenhouse soil (kJ/(h K m\(^2\))); 20.7
\( R_{\text{pipe}} \) Heat transfer coefficient of pipes (kJ/(h K m\(^2\))); 1200
\( R_{\text{soil}} \) Heat transfer coefficient of greenhouse cover (kJ/(h K m\(^2\))); 0.7
\( R_{\text{pipe,soil}} \) Heat transfer coefficient between pipes and soil (kJ/(h K m\(^2\))); 3.6
\( \rho_a \) Density of air (kg/m\(^3\)); 1.27
\( \rho_w \) Density of water (kg/m\(^3\)); 1000
\( \rho_p \) Density of plants (kg/m\(^3\)); 1010.2
\( R_{\text{gh}} \) Rate of \( \text{CO}_2 \) emissions from natural gas consumption (tonne/MWh); 0.5148
\( R_i \) Required operation time of device \( i \) (h); 12
\( \Theta_{sl} \) Soil temperature (°C); 8
\( \Theta_{\text{min, out}} \) Min. acceptable outdoor weather temperature to allow outdoor air ventilation (°C); -3
\( \Theta_{\text{set}} \) Inside temperature set point in zone \( z \) (°C); 17
\( \Theta_{z}^{\text{in}} \) Min. inside temperature in zone \( z \) (°C); 15
\( \Theta_{z}^{\text{max}} \) Max. inside temperature in zone \( z \) (°C); 19
\( \Theta_{z}^{\text{ave}} \) Min. average temperature in zone \( z \) (°C); 16
\( \Theta_{z}^{\text{ave, max}} \) Max. average temperature in zone \( z \) (°C); 18
\( \Theta_{\text{min,ht}} \) Min. hot water temperature (°C); 60
\( \Theta_{\text{max,ht}} \) Max. hot temperature (°C); 95
\( \Theta_{\text{min,chl}} \) Min. chilled water temperature (°C); 4
\( \Theta_{\text{max,chl}} \) Max. chilled water temperature (°C); 10
\( \tau \) Length of time interval (h); 1
\( V_{\text{gh,z}} \) Volume of greenhouse zone \( z \) (m\(^3\)); 80000
\( V_{\text{ht},z} \) Volume of water in heating pipes and tank in zone \( z \) (m\(^3\)); 100
\( V_{\text{chl},z} \) Volume of water in chilling pipes and tank in zone \( z \) (m\(^3\)); 50
\( W_{\text{evp}}(z) \) Crop evaporation at each hour in zone \( z \) (g/(h m\(^2\)h)); 125.8
\( W_{\text{Max}}^{\text{fog}} \) Max. water rate of fogging systems (g/(w m\(^2\)h)); 9.6
\( W_{\text{max}, dh} \) Max. rate of dehumidifier (g/(w m\(^2\)h)); 145
\( W_{\text{out}}(t) \) Absolute water content of outdoor air at time \( t \) (g/(w kg\(_a\)));
\( \xi_z \) Effect of the fans operation on \( \text{CO}_2 \) concentration in zone \( z \) (h\(^{-1}\)); 366000

II. INTRODUCTION

SMART GRIDS are envisioned to support large penetrations of distributed demand-side resources coupled with system-wide Demand Response (DR) driven by economic and reliability signals. In this context, utilities are offering Demand Side Management (DSM) and DR services to better manage their networks [1, 2]. These DR programs incentivize customers with payments or economic penalties to reduce consumption during periods of critical grid conditions or periods of high energy costs. With the integration of information technology and Advanced Metering Infrastructure (AMI) into smart grids, both utilities and customers can have access to two-way communication infrastructures, control devices, and visual interfaces that allow them to send, retrieve, visualize, process, and/or control their energy needs [3]. These developments make automated operational decisions feasible in energy systems, presenting a significant potential to improve performance and effectiveness of DSM and DR programs, allowing customer direct involvement in these programs to better manage energy and power consumption.

To date, large industrial and commercial customers have been the most active participants in DSM and DR programs because of their potential to achieve large peak-load and energy consumption reductions [1]. Other sectors such as residential, small commercial, and agricultural customers have traditionally participated less in DSM and DR activities mainly because of their individually smaller contributions to the system peak-load and energy consumption, as well as the technical difficulties of integrating these customers due to the nature of their activities [4]. Therefore, in the new era of smart grids, these small customers also could be important resources for DR and DSM programs, and thus exploring new opportunities to better manage energy requirements in these sectors to reduce their demand is relevant and timely.

In the USA, poultry farms, dairy farms, and greenhouses are some of the major energy consuming customers in the agricultural sector, being about 16% of the total energy consumption [5]. Most of the existing DSM programs in this sector are focused on energy efficiency programs in farms to reduce total energy consumption by installing more energy efficient technologies and the reduction of energy losses [6–8]. The potential for DSM and DR participation in greenhouses is much higher than farms because of the nature of activities that take place in these places. In this context, climate control systems are the main mechanism that regulate energy and power consumption in greenhouses [9]. Therefore, these
systems could be the principal means by which these loads could participate in DSM and DR programs.

In the literature, the main types of climate control for greenhouses are: feedback controllers [10], [11]; optimal control [12]; Neural Networks (NN) [13], [14]; Fuzzy Logic Controllers (FLC) [15], [16]; Model Predictive Control (MPC) [17]–[19] and hierarchical control [12], [20]. Most of the work reported in the literature only focus on improving climate control of greenhouses without taking into account energy costs. However, there is some reported work on greenhouse climate control with energy cost minimization, mostly focusing on minimization of CO₂ production and heating costs (e.g., [15], [19]). Nevertheless, in general, existing methods for energy management in greenhouses fail to fully optimize total energy utilization mainly due to a lack of a general optimization framework based on comprehensive internal and external information such as weather and energy price forecast, and other associated variables.

In view of the above discussions, this paper focuses on the optimal operation of energy systems of greenhouses in the context of smart grids, making the following novel contributions:

- A hierarchical operation strategy is proposed and demonstrated.
- A detailed mathematical model of greenhouses is developed and presented, considering their operational requirements and appropriate for the optimal and real-time scheduling of these hubs’ electricity, gas, and heat equipment.
- Tests and validation of the proposed optimization approach are presented and discussed for a realistic greenhouse, demonstrating the benefits and feasibility of the proposed approach.

The proposed models can be readily incorporated into Energy Management Systems (EMS) and implemented as a supervisory real-time control in existing greenhouse controllers, thus empowering greenhouses to effectively manage their overall energy demand, production, and storage in real-time. The objective is to minimize total energy costs while important parameters of greenhouses, i.e., inside temperature and humidity, CO₂ concentration, and lighting levels, are kept within acceptable ranges. The proposed supervisory control in conjunction with current existing climate controllers would allow coordinated optimal operation of greenhouse while considering the user preferences, thus facilitating the integration of these agricultural customers into smart grids.

The remainder of the paper is organized as follows: The EMS of greenhouses is discussed in Section III, together with the proposed supervisory operation strategy. The developed mathematical models are presented and discussed in Section IV. Numerical results of the application of the proposed models to a realistic greenhouse are presented and discussed in Section V. Finally, the main conclusions and contributions of the paper are highlighted in Section VI.

III. EXISTING AND PROPOSED GREENHOUSE EMS

A. Existing EMS

Figure 1 shows an overview of a greenhouse energy system. All growing phases of crops can be modified by the control of temperature, humidity, light, and CO₂ in a greenhouse, making climate control in greenhouses a multi-variable problem.

Operational constraints of physical devices such as maximum window opening, flow rate of fans, rate of fogging systems, and temperature of hot water tubes are limiting features which need to be considered in these control systems. Also, predefined ranges for controlled variables should be chosen properly considering the physical limits of devices and related physical and thermodynamic laws (e.g., saturation bounds enforced by saturation law [10]).

In greenhouses, transpiration of a crop can be controlled by manipulating the temperature and ventilation rate of the greenhouse [21], and photosynthesis is a function of irradiance, temperature and CO₂ concentration [22]. CO₂ enrichment is usually used to decrease the amount of supplemental lighting, as this is a much less expensive alternative; however, it is expensive to maintain elevated CO₂ concentrations inside a greenhouse during periods of high ventilation rates. It should also be noted that the greenhouse layout and available equipment, as well as the type of plants grown in the greenhouse affect the climate control strategy and model.

Automated Control Systems (ACS) in most greenhouses consist of central computers, sensors and a data acquisition system connected through communication protocols such as RS-232 and ModBus [23]. These ACS coordinate and integrate the control of greenhouse equipment and systems such as heaters, coolers, motors for windows opening and closing, pumps and irrigation systems in real time.

A typical existing climate control system in a greenhouses is depicted in Fig. 2, together with the proposed supervisory control described below. Currently, most control algorithms...
The proposed optimization model looks ahead in time and updates the actual control actions such as turning devices on and off. The supervisory control generates set points for the existing controllers, which per-
in Section IV, reside at the supervisory level to generate set points. Greenhouses usually have weather stations that provide external and internal information on temperature, relative humidity, radiation, and wind speed to be used for their real-time climate control.

B. Proposed Supervisory Operation Strategy

As just discussed, common existing ACSs coordinate and integrate real-time control of greenhouse equipment and systems using logical On-Off and PID based controllers, and typically do not optimize the energy utilization in such multi-carrier facility. Therefore, a hierarchical supervisory framework is proposed and discussed here based on a mathematical model of greenhouses, using internal and external information such as weather and energy price forecasts for optimal operation of greenhouses. The proposed framework maintains the greenhouse climate within proper conditions to achieve the best plant growth, and controls important parameters such as greenhouse temperature, relative humidity, lighting levels, and CO₂ concentrations while reducing total energy costs.

1) Hierarchical Operation Scheme: Nowadays, appropriate day-ahead electricity price forecasts and weather forecasts for the next few days, updated every few hours, are available. These forecasts are used here to design a hierarchical operation strategy to improve the operation of greenhouse climate control systems to reduce total energy costs and demand. The architecture of the proposed hierarchical scheme for the optimal operation of greenhouses is presented in Fig. 2. The existing feedback control systems remain at the lower hierarchical level, while at the higher level the proposed supervisory control generates set points for the existing controllers, considering appropriate set-point ranges and user preferences to optimize the operation of the climate control system.

The proposed optimization greenhouses model, presented in Section IV, reside at the supervisory level to generate set points for the existing feedback control systems, which perform the actual control actions such as turning devices on and off. The optimization model looks ahead in time and updates the optimal outputs every hour, while the feedback controller continuously monitors the parameters under control and tracks the target set points in real time. The supervisory control also monitors the system, and in case of large discrepancies between the calculated and measured parameters, re-runs the model and updates the set points, using an MPC approach.

2) External Information: External information used in the proposed supervisory control of Fig. 2 is weather forecasts such as average hourly outdoor temperature, humidity, wind speed, and solar irradiations. Day-ahead forecasts of the electricity and gas prices and peak-demand charges are also used to calculate the expected energy costs of greenhouses. This external information is assumed to be exogenous inputs to the proposed optimization model.

3) Scheduling Horizon: The scheduling horizon in the optimization model can vary from a few hours to days, with the selection depending on the type of the activities which take place within the greenhouse and the accuracy of weather and energy price forecasts. A daily scheduling horizon with time intervals of one hour is used in this work for optimal energy management of greenhouses.

IV. MATHEMATICAL MODELING OF GREENHOUSES

In a typical greenhouse, the following categories of energy consuming components can be identified: supplementary lighting; climate controls of temperature, humidity, and CO₂ levels through heating and cooling systems; and natural and forced air ventilation and circulation. The mathematical models that represent the components of the system considering their operational constraints are described next.

A. Objective Functions

1) Minimization of costs of energy consumption: The following objective function corresponds to the minimization of the customer’s energy costs over the scheduling horizon:

\[ J_1 = \sum_{i\in A} \sum_{t\in T} \tau C_e(t) P_i s_{i,z}(t) \quad (1) \]

2) Minimization of peak demand charges: The following objective seeks to minimize the customers’ demand charges:

\[ J_2 = C_{dc} \cdot \hat{\rho} \quad (2) \]

where \( \hat{\rho} \) is a non-negative variable used along with the following constraint to represent the peak demand during the scheduling horizon:

\[ \hat{\rho} \geq \sum_{i\in A} \sum_{t\in T} P_i s_{i,z}(t) \quad \forall t \in T \quad (3) \]

3) Minimization of total energy costs: Total energy costs include costs of energy consumption plus peak demand charges as follows:

\[ J_3 = J_1 + J_2 \quad (4) \]
B. Model Constraints

1) Indoor Humidity: Humidity inside a greenhouse needs to be controlled to provide a suitable environment for plant growth and to prevent fungal diseases. In the case of high humidity, which usually happens in winter nights, the plants stop transpiration, and condensation from the roof and plant leaves may cause fungal diseases. In the case of low humidity the plants stop absorbing CO₂ and the photosynthesis process, resulting in slow plant growth. Therefore, controlling relative humidity in greenhouses should be modeled properly in the mathematical model.

Relative humidity of greenhouses is defined as [24]:

\[
\phi = \frac{p_{par}}{p_{sat}} \times 100\%
\]

(5)

where the saturated vapor pressure and the partial pressure can be approximated by:

\[
p_{sat} = C_1 \left( -C_2 + C_3 e^{C_4 \theta} \right)
\]

(6)

\[
p_{par} = \frac{w P_{atm}}{C_5}
\]

(7)

The saturated vapor pressure equation is linearized as follows, based on a Taylor series expansion:

\[
p_{sat} = C_1 \left\{ -C_2 + C_3 e^{C_4 \left( (\Theta^l + \Theta^u)/2 \right)} \right\} [1 + C_4 (\theta - (\Theta^l + \Theta^u)/2)]
\]

(8)

(9)

The linearization of this and other equations is done to make the optimization model a Mixed Integer Linear Programming (MILP) problem, for practical application purposes. To model the humidity inside the greenhouses, the water content of air inside the greenhouse needs to be modeled based on the mass balance principle, as follows [9]:

\[
w_z(t) = w_z(t - 1) + \frac{\tau}{\rho_a V_z} W_{exp,z} A_z + Q_z \rho_a A_z s_{fw,z}(t) (W_{out}(t) - w_z(t)) + \theta(t) \lambda_p A_z s_{nv,z}(t) (W_{out}(t) - w_z(t)) + s_{fo,g,z}(t) W_{max,fo,g,z} A_z - s_{dh,z}(t) W_{max,dp,z} A_z
\]

(10)

This equation states that the water content of the greenhouse air at time \( t \) is a function of its water content at time \( t - 1 \); moisture ventilated by the forced and natural air ventilation system; and the operation of the fogging and dehumidification systems.

Using (5) and substituting the associated terms from (7) and (9), the following constraints guarantee that the relative humidity of inside air is kept within the desired limits:

\[
w_z(t) \leq \phi_{max} \frac{p_{sat} C_5}{P_{atm}} \quad \forall t \in T \quad (11a)
\]

\[
w_z(t) \geq \phi_{min} \frac{p_{sat} C_5}{P_{atm}} \quad \forall t \in T \quad (11b)
\]

2) Indoor Temperature: Thermal dynamics of the greenhouse are modeled based on the energy balance principle, as follows [9]:

\[
\theta_z(t) = \theta_z(t - 1) + \frac{\tau}{C_w P_{atm}} \left[ q_{\text{sat}}^{\text{evp},z}(t) - q_{\text{sat}}^{\text{ch},z}(t) - q_{\text{sat}}^{\text{atm},z}(t) \right] - q_{\text{atm},z}(t)
\]

(12)

This constraint states that the temperature of the greenhouse space at time \( t \) is a function of its temperature at time \( t - 1 \); absorbed heat from sunshine; heat transfer through heating and chilling pipes; heat loss through walls, soil, air leakage and ventilation; heat produced by the lighting, CO₂ generation and dehumidification systems; and evaporation heats of the fogging system.

Temperatures of hot and chilled water inside pipes are calculated based on the energy balance principle, as follows [9]:

\[
\theta_{ht_{p,z}}(t) = \theta_{ht_{p,z}}(t - 1)
\]

\[
+ \frac{\tau}{C_w P_{atm}} \left[ 3.6 \times P_{ht_{max}}^{\text{sat}} s_{ht,z}(t) - R_{\text{pipe,ht_{p,z}}} s_{ht,z}(t) (\theta_{ht_{p,z}}(t) - \theta_z(t)) - R_{\text{sl,ht_{p,z}}} (\theta_{ht_{p,z}}(t) - \Theta_{sl}(t)) \right] \quad \forall t \in T
\]

(13)

\[
\theta_{ch_{p,z}}(t) = \theta_{ch_{p,z}}(t - 1)
\]

\[
+ \frac{\tau}{C_w P_{atm}} \left[ 3.6 \times P_{ch_{max}}^{\text{sat}} s_{ch,z}(t) - R_{\text{pipe,ch_{p,z}}} s_{ch,z}(t) (\theta_{ch_{p,z}}(t) - \theta_z(t)) - R_{\text{sl,ch_{p,z}}} (\theta_{ch_{p,z}}(t) - \Theta_{sl}(t)) \right] \quad \forall t \in T
\]

(14)

These constraints state that the average temperature inside the pipes at time \( t \) is a function of: its temperature at time \( t - 1 \); absorbed heat (cold) from operation of heating (chilling) system; heat transfer through pipes to the greenhouse space; and heat loss through soil.

The calculated inside temperature of the greenhouses must be kept within a range specified by minimum and maximum limits:

\[
\Theta_{z}^l \leq \theta_z(t) \leq \Theta_{z}^u \quad \forall t \in T
\]

(15)

And the average inside temperature over the scheduling horizon must be within a tighter predefined temperature range:

\[
\Theta_{z}^{lu} \leq \frac{1}{T} \sum_{t \in T} \theta_z(t) / N_T \leq \Theta_{z}^{lu}
\]

(16)

3) Indoor CO₂ Level: Plants need sunlight and CO₂ for photosynthesis. When there is sunlight, plants consume CO₂ inside the greenhouse and thus CO₂ concentration drops; thus, to keep a high level of photosynthesis and plant growth, it is essential to supply CO₂ into the greenhouse and maintain CO₂ concentration within a desired range. CO₂ concentration within the greenhouse is modeled as follows, based on the
mass balance principle [9]:
\[
c_z(t) = c_z(t - 1) + \frac{\tau}{\rho_q V_{gh,z}} \left[ C^{\text{max}}_{s,x} \left( s_{co_2,z}(t) \right) A_{gh,z} \right.
+ C_c v_c A_{mv,z} s_{mv,z}(t) \left( C_{out}(t) - c_z(t) \right)
+ \xi_{s,x} f_{rv,z}(t) \left( C_{out}(t) - c_z(t) \right)
+ C_{res,z} A_{gh,z} \left( C_7 + C_8 \theta_2(t) \right)
- C_c C_{\text{phot},z} I(t) \eta_{\text{sr}} A_{gh,z} \biggr) \forall t \in T \quad (17)
\]

This constraint states that CO\textsubscript{2} balance within the greenhouse is determined by the CO\textsubscript{2} supply, the plants consumption of CO\textsubscript{2} and the air exchange by ventilation.

Concentration of CO\textsubscript{2} inside the greenhouses must be kept within a range specified by minimum and maximum values, as follows:
\[
C_{z}^\text{min} \leq c_z(t) \leq C_{z}^\text{max} \forall t \in T \quad (18)
\]

4) Lighting: Supplemental lighting for greenhouses is required to increase photosynthesis and plants growth especially in areas that receive few hours average daily sunshine. High Intensity Discharge (HID) lamps such as metal halide and high pressure sodium lamps are commonly used for the purpose of supplying artificial lighting in greenhouses. Operational requirements of these supplementary lighting systems in the proposed model are formulated and included in the model. Since HID lamps are not designed for cyclical On/Off operation, minimum up time and minimum down time requirements of these lighting systems need to be modeled as well.

Minimum and maximum aggregated lighting requirements of the plants from sunshine and supplementary lighting systems installed in each zone are also modeled. Constraints that enforce maximum successive On time of the lighting system, and the minimum duration and the minimum lighting of cloudy weather to turn on the supplementary lighting systems are included to ensure that the plants use the artificial lighting more efficiently. The required equations to model these systems are similar to those used in conventional unit commitment problems in power systems, and are not provided here due to space limitation, but can be found in [25].

5) Air Circulation: Air circulation is needed in greenhouses to maintain a uniform temperature and CO\textsubscript{2} concentration throughout the greenhouse. The circulation fans should operate for at least a user-defined required operation time (\(R_{cf}\)), which can be modeled as follows:
\[
\sum_{k=1}^{N_{R}} s_{cf,z}(t) \geq R_{cf} \quad (19)
\]
The circulation fans should also operate whenever the CO\textsubscript{2} generation unit is On to distribute CO\textsubscript{2} uniformly; this is modeled as follows:
\[
s_{cf,z}(t) \geq s_{co_2,z}(t) - L(1 - s_{co_2,z}(t)) \quad (20)
\]

6) Other Devices’ Operational Constraints: As previously mentioned, the operation of the ventilation fans are controlled in the proposed model based on their effects on greenhouse temperature, humidity, and CO\textsubscript{2} concentration. However, when the outdoor temperature is less than a pre-specified value \(\theta_{out}^{\text{min}}\), forced ventilation and natural ventilation should not operate and circulate very cold air into the greenhouse. Also, fogging and dehumidification systems should not operate simultaneously, and the valves of the heating and chilling pipes do not open simultaneously to inject heat and cold into the greenhouse at the same time. Notice that the heating system (boiler) and the cooling system (chiller) may operate simultaneously to take advantage of storing heat and cold during low electricity prices.

C. Exact Linear Equivalent of Bi-linear Terms

In the problem formulation presented in the previous sections, there are products of binary and continuous variables that make the model nonlinear. Thus, all these bi-linear terms in the developed model are linearized to obtain an MILP problem which is more suitable for real-time applications. For that, assume that \(s\) is a binary variable and \(x\) is a positive continuous variable bounded by \(x \leq x \leq \pi\). Hence a new variable \(\mu_{s,x}\) can be defined to obtain the exact equivalent of the product \(s x\) as follows:
\[
\mu_{s,x} \geq x - (1 - s) \pi \quad (21a)
\]
\[
\mu_{s,x} \leq x \quad (21b)
\]
\[
s \leq \mu_{s,x} \quad (21c)
\]
\[
\mu_{s,x} \leq x \quad (21d)
\]

Therefore, all the bi-linear variable terms in the model, e.g., \(s_{ht}_{z}(t) \theta_{ht}(t)\) and \(s_{ht}_{z}(t) \theta_{z}(t)\) in (13), are replaced with the corresponding \(\mu_{s,x} = s x\) variables and constraints, resulting in an MILP mathematical optimization model.

V. NUMERICAL RESULTS OF GREENHOUSE MODEL

Several case studies have been conducted to examine the performance of the developed mathematical model for optimal operation of greenhouses, of which the most relevant ones are presented here. In case studies shown here, the mathematical model is run for a typical greenhouse, for which parameters and device ratings are suitably chosen (see Section I), and realistic data inputs for outside temperatures, humidity, wind speed, solar irradiation, electricity price and demand charges are used [25]. Figures 3 and 4 show the electricity price and outdoor temperature, respectively, used in the simulations for summer and winter days; other input data can be found in [25]. Real-Time Pricing (RTP) and demand charges for electricity costs, and Flat Rate Price (FRP) for natural gas in Ontario, Canada are used to calculate total energy costs. AMPL [26], a modeling language for mathematical programming, is used to implement the developed mathematical models of the greenhouse, and CPLEX [27], a popular solver for LP and MILP problems, is used to solve these models.

A. Simulations

The case studies presented in Table I are considered here to examine various operation paradigms of greenhouses using the developed model. The solution presented for Case 0 is just a feasible solution of the model, thus representing the actual operation of a greenhouse for typical existing climate conditions.
control systems, and hence is considered here to correspond to a realistic “base case” for comparison purposes. Case 1, Case 2, and Case 3 represent different applications of the proposed models for the optimal operation of greenhouses. The constraints of the model for all cases are the same, with only the objective functions changing.

Optimal operational decisions and resulting trajectories generated by the proposed model for a winter day are presented in Fig. 5, showing the decision variables for all devices and the resulting inside temperatures, relative humidities, and \( CO_2 \) concentrations for Case 3. Observe that the model operates the heater, dehumidifier, chiller, \( CO_2 \) generator, natural ventilation, and circulation fans to maintain greenhouse climate conditions within predefined ranges. Observe that the model reduces total costs by operating the devices during lower energy price periods, as shown in Fig. 5.

The resulting electric power demands of the greenhouse for each case on a winter day are shown in Fig. 6. For a winter day, the peak demand cannot be significantly reduced due to the need to operate the supplementary lighting system; however, the model reduces total costs by operating the devices during lower energy price periods, as shown in Fig. 5.

A comparison of energy costs and demand charges for optimal operation of the greenhouse in all cases for summer and winter days are presented in Tables II and III, respectively. In Case 1, the energy costs are reduced significantly as compared to Case 0 and are the least among all cases, while the demand charges remain the same as the base case for both summer and winter days. In Case 2, the demand

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**Fig. 3.** Electricity price for a summer day and a winter day used in the simulations.

**Fig. 4.** Outdoor temperature for a summer day and a winter day used in the simulations.

**Fig. 5.** Optimal values of the variables in Case 3 using RTP for a winter day.

**Table I**

<table>
<thead>
<tr>
<th>Cases</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 0</td>
<td>The optimization model is solved with a constant value as the objective function, thus finding a feasible solution for the model with all constraints on operation of the devices, inside temperature, humidity, and ( CO_2 ) concentration being met.</td>
</tr>
<tr>
<td>Case 1</td>
<td>The objective is to minimize electricity demand charges.</td>
</tr>
<tr>
<td>Case 2</td>
<td>The objective is to minimize energy consumption costs.</td>
</tr>
<tr>
<td>Case 3</td>
<td>The objective is to minimize total energy costs, including energy consumption and demand charges.</td>
</tr>
</tbody>
</table>

**Table II**

<table>
<thead>
<tr>
<th>Item</th>
<th>Energy (kWh)</th>
<th>Peak demand (kW)</th>
<th>Energy costs ($)</th>
<th>Demand charges ($)</th>
<th>Energy costs savings w.r.t Case 0 (%)</th>
<th>Demand charges savings w.r.t Case 0 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 0</td>
<td>6629.47</td>
<td>206.00</td>
<td>161.47</td>
<td>1648.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Case 1</td>
<td>7535.11</td>
<td>206.00</td>
<td>168.34</td>
<td>1648.00</td>
<td>32.90</td>
<td>0.00</td>
</tr>
<tr>
<td>Case 2</td>
<td>8577.65</td>
<td>186.00</td>
<td>147.50</td>
<td>848.00</td>
<td>2.46</td>
<td>48.5</td>
</tr>
<tr>
<td>Case 3</td>
<td>4933.54</td>
<td>186.02</td>
<td>135.69</td>
<td>846.16</td>
<td>13.97</td>
<td>48.5</td>
</tr>
</tbody>
</table>
TABLE III
ENERGY COSTS AND DEMAND CHARGES FOR A WINTER DAY.

<table>
<thead>
<tr>
<th>Item</th>
<th>Energy (kW)</th>
<th>Peak demand (kW)</th>
<th>Energy charges ($)</th>
<th>Demand charges ($)</th>
<th>Energy costs savings w.r.t Case 0 (%)</th>
<th>Demand charges savings w.r.t Case 0 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 0</td>
<td>54169.50</td>
<td>4031.00</td>
<td>1167.39</td>
<td>2827.60</td>
<td>0.00</td>
<td>0.0</td>
</tr>
<tr>
<td>Case 1</td>
<td>55638.43</td>
<td>4031.00</td>
<td>948.21</td>
<td>2827.60</td>
<td>18.77</td>
<td>0.0</td>
</tr>
<tr>
<td>Case 2</td>
<td>54959.19</td>
<td>4000.00</td>
<td>1135.06</td>
<td>2800.00</td>
<td>2.77</td>
<td>0.8</td>
</tr>
<tr>
<td>Case 3</td>
<td>55133.69</td>
<td>4000.50</td>
<td>949.39</td>
<td>2800.03</td>
<td>18.67</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Random values of RTP (Hourly Ontario Electricity Price), temperature, and humidity for each hour are generated using a normal distribution with associated mean and standard deviations for each hour of a day obtained from actual data for each season for Ontario, Canada, thus representing worst case scenario since no correlation is considered among these input data. For wind speed, random values are generated using a Weibull distribution with the scale and shape parameters obtained from actual hourly data for each each season for Ontario. Random solar irradiation inputs are generated using uniform distribution with reasonable minimum and maximum values for each hour a day for each season for Ontario.

Energy costs and peak demand charges at each MCS iteration and their cumulative expected mean values obtained for a summer day in Case 3 are shown in Fig. 7; observe that the expected values converge in about 150 iterations. Expected average energy costs and peak demand charges in Case 0 are $269.8 and $8721.3, respectively, while these values for Case 3 are $159.6 and $5258.4, respectively. Hence, the expected monthly total costs, which are assumed to be 30 times the expected daily energy costs plus the expected peak demand charges, are $16816.8 and $10046.4 over a summer month for Case 0 and Case 3, respectively. Therefore, even when considering large variations in weather and energy price data, the model yields a significant total cost reduction of 40% for summer months. Similar MCS show that the model yields more than 19% and 2% reductions in expected energy costs and demand charges, respectively, for a winter day; these reductions yield a 13% expected monthly total cost savings for winter days.

VI. CONCLUSIONS

A hierarchical control approach was proposed for optimal operation of greenhouses in the context of smart grids, which includes novel mathematical models for the optimal operation scheduling of greenhouse’ electricity, gas, and heat systems. Thus, optimization models were formulated to optimally operate supplementary lighting, CO2 generation, air circulation and ventilation, and heating and cooling systems in existing greenhouses control systems. The developed models incorporated weather forecasts, electricity price information, and the end-user preferences to minimize total energy costs and peak
demand charges while considering important parameters of greenhouses climate control. The presented simulation results showed the effectiveness of the proposed model to reduce total energy costs while maintaining required operational constraints of a greenhouse, even in the presence of uncertainties.

REFERENCES