Unit Commitment for Isolated Microgrids Considering Frequency Control

Mostafa Farrokhabadi, *Student Member, IEEE,* Claudio Cañizares, *Fellow, IEEE*
and Kankar Bhattacharya, *Senior Member, IEEE*

***Abstract*—This paper presents a mathematical model of frequency control in isolated microgrids, which is integrated into the Unit Commitment (UC) problem. In conventional UC formulations, power outputs are considered fixed between two periods, yielding a staircase pattern with respect to the energy balance of the generation and demand for a typical dispatch time horizon (e.g., 24 h). However, in practice generation units that participate in frequency control may see a change in their output within a single dispatch time interval (e.g., 5 min), depending on the changes in the demand and/or renewable generation. The proposed approach considers these changes in the generation output using a linear model, and based on that, a novel UC Mixed Integer Quadratic Programming (MIQP), with linear constraints and quadratic objective function, is developed which yields a more cost efficient solution for isolated microgrids. The proposed UC is formulated based on a day-ahead with Model Predictive Control (MPC) approach. To test and validate the proposed UC, a modified version of a CIGRE benchmark test system is used. The results demonstrate that the proposed UC would reduce the operational costs of isolated microgrids compared to conventional UC methods, at similar complexity levels and computational costs.**

***Index Terms*—Isolated microgrids, Unit Commitment, generation dispatch, frequency control.**

NOMENCLATURE

Indices and Superscripts

g	Generation units
i, j	Microgrid asset
k	Time step
r	Renewable generation units
s	ESS units

Sets

$\mathcal{F}$	Dispatchable units that participate in frequency control
$\mathcal{G}$	Generation units
$\mathcal{P}$	Dispatchable units that do not participate in frequency control
$\mathcal{R}$	Renewable generation units
$\mathcal{S}$	ESS units
$\mathcal{T}$	Time steps
$\mathcal{T}_1, \mathcal{T}_2$	Subsets of $\mathcal{T}$
$\mathcal{T}^*$	Time steps excluding the first step

Parameters

a_i	Quadratic term of cost function of diesel engine i (\$/kW^{2}h)
b_i	Linear term of cost function of diesel engine i (\$/kWh)
c_i	Constant term of cost function of diesel engine i (\$/h)
Csh_i^g	Shut-down cost of diesel engine i (\$)
Cst_i^g	Start-up cost of diesel engine i (\$)
D_k	Net demand at time step k (kW)
E_k	Required energy for dispatch time interval k (kWh)
ID_i	Inverse of droop of dispatchable unit i (kW/Hz)
MD_i^g	Minimum down-time of dispatchable unit i (h)
MU_i^g	Minimum up-time of dispatchable unit i (h)
$P_{i,k}^r$	Forecasted power output of renewable unit i at time step k (kW)
$P_{L,k}$	Loading at time step k (kW)
$\bar{P}_i^g$	Maximum output power of generation unit i (kW)
$\underline{P}_i^g$	Minimum output power of generation unit i (kW)
$\bar{P}_i^s$	Maximum charging/discharging power of storage unit i (kW)
$\bar{R}_i^g$	Maximum ramp-rate of dispatchable unit i (kW/5-min)
$\underline{R}_i^g$	Minimum ramp-rate of dispatchable unit i (kW/5-min)
R_i	Droop of dispatchable unit i (Hz/kW)
$\underline{RES}_k$	Spinning-up reserve limit at time step time k (kW)
$\overline{SOC}_i$	Maximum state of charge of ESS i (kWh)
$\underline{SOC}_i$	Minimum state of charge of ESS i (kWh)
$\Delta\tau$	Dispatch interval (5 min.)
ΔP_L	Load change (kW)
ΔP_i^r	Power output change of renewable generation unit i (kW)
ΔP_{ref}	Reference power change of the dispatchable unit i
η_i	Charging/discharging efficiency of ESS i

Variables

$\alpha_{i,j,k}^g$	Auxiliary variable for diesel engines i and j at time step k (kW)
$\omega_{i,k}^g$	Binary variable for unit commitment decision of dispatchable unit i at time step k
$d_{i,k}^s$	Binary variable for ESS i representing the discharging(1)/charging(0) status at time step k
$u_{i,k}^g$	Start-up decision binary variable for diesel engine i at time step k
$v_{i,k}^g$	Shut-down decision binary variable for diesel engine i at time step k
C_i^g	Cost function of dispatchable unit i (\$/h)

This work was supported by NSERC Smart Microgrid Network (NSMG-Net).

M. Farrokhabadi, C. A. Cañizares, and K. Bhattacharya are with the Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, ON, N2L 3G1, Canada (e-mail: m5farrok@uwaterloo.ca; ccanizares@uwaterloo.ca; kankar@uwaterloo.ca).

$C\tau_i^g$ Cost of energy delivered by dispatchable unit i during dispatch time interval k ($)
$P_{i,k}^g$ Power output of diesel engine i at the beginning of time step k (kW)
$P_{i,k}^{s,chg}$ Charging power of ESS i at time step k (kW)
$P_{i,k}^{s,dch}$ Discharging power of ESS i at time step k (kW)
$\boldsymbol{Pa_{i,k}^g}$ Auxiliary variable for power output of diesel engine i at time step k (kW)
$PE_{i,k}^g$ Power output of diesel engine i at the end of the time step k (kW)
$P_i^g(t)$ Time-domain function of power output of diesel engine i over a certain dispatch interval (kW)
$OC_{i,k}^g$ Total operating cost of dispatchable unit i during dispatch time interval k ($)
$SOC_{i,k}$ SOC of ESS i at time step k (kWh)
Δf_k Frequency change during dispatch time interval k (Hz)
$\Delta P_{i,k}^g$ Power output change of diesel engine i at the end of dispatch time interval k due to changes in D_k (kW)

I. Introduction

THe Unit Commitment (UC) problem determines the optimal generation schedule to supply the demand, while ensuring that the system operates within certain technical constraints [1]. In isolated microgrids, the UC problem functions as a secondary control to ensure its reliable and economical operation [2]. The generation scheduling of dispatchable units obtained from a conventional UC are considered fixed between two dispatch time intervals, yielding a staircase generation profile over the UC time horizon. This approach is reasonable in large interconnected systems, where UC and frequency regulation are treated separately; however, the staircase schedule of generation outputs is shown in [3] to create large frequency deviations at the beginning and end of each dispatch interval. On the other hand, in isolated microgrids, all dispatched Distributed Generation (DG) units participate in frequency regulation, especially if renewable generation is present, given their high output power variability, and thus DG units would not remain fixed between two time intervals.

Several papers have proposed UC models for microgrids with different configurations and constraints. Thus, in [4], [5], and [6], a UC model that includes operational constraints pertaining to Distributed Energy Resources (DERs) and Energy Storage Systems (ESS) such as ramp-up, ramp-down, and minimum up/down-time constraints is proposed. In [7], the UC problem is formulated based on a Model Predictive Approach (MPC) to account for errors in renewable generation forecast. In [8], a combined UC and OPF problem that utilizes smart loads in microgrids to obtain the optimal dispatch decisions of generation units is proposed. However, none of the above mentioned works account for the impact of frequency regulation on generation, assuming that the generation outputs are fixed between two dispatch intervals. In practice, the output of the units participating in microgrid frequency regulation would continuously change, to balance the power mismatch due to renewable generation and load changes.

There are some works that consider constraints related to frequency control within the framework of UC for microgrids, with the majority focusing on reserve-related constraints. For example, in [9], the reserve required for frequency regulation is modelled as a decrease in the minimum limit and an increase in the maximum limit of the largest generator involved in the control process. In [10], a new constraint is introduced to control the frequency levels, which determines the minimum frequency reached if the system loses the largest generator; reserve levels are then adjusted through an iterative process until the frequency constraint is satisfied. In [11], a frequency-regulating reserve constraint and a load-frequency sensitivity index are introduced to calculate the proper amount of reserves required to keep the system frequency higher than the minimum acceptable value. In [12], the isochronous mode of generation is modelled and integrated into the UC problem, with a particular emphasis on the microgrid reserve requirements. None of these references actually model or consider the impact of the frequency control mechanism on the generation output, and hence on the UC objective function; the primary assumption of these works remains that the generation power outputs are fixed between two dispatch time intervals.

The idea that dispatchable units' power outputs would not be fixed between two dipatch intervals has been investigated in [13]–[15]. In [13], it is demonstrated that considering generation levels in UC problems as hourly energy blocks may not be realizable in practice. To address this problem, the UC problem is reformulated in [14] to incorporate energy delivery constraints based on a sub-hourly energy demand profile. In [15], a UC-based market clearing model is proposed, considering the difference between power and energy, and accounting for start-up and shut-down power trajectories and ramping constraints; in this case, demand and energy are modelled as piecewise-linear functions representing their power trajectories. The methods proposed in these works have not been applied to microgrids with various DERs; in addition, none of these works investigate the impact of frequency control on power trajectories of dispatchable units.

Based on the aforementioned literature review, the current paper presents a novel UC model for isolated microgrids that integrates the impact of frequency control on generation outputs, thus reducing the operation cost of isolated microgrids. The problem is fomulated as a mixed-integer quadratic program (MIQP), with linear constraints and quadratic objective function. Therefore, the main contributions of this work are the following:

- Development of a novel mathematical formulation of the frequency control mechanism integrated within a UC framework for isolated microgrids and its impact on generation scheduling.
- Comparison of frequency control mechanisms based on single unit control, droop control, or isochronous load sharing (ILS) control mode.
- Introduction of a new reserve power constraint to represent the corresponding frequency control mechanism, resulting in a more economic and realistic dispatch solution.

Various comparisons are carried out in a large and complex isolated microgrid, demonstrating the practical feasibility of the proposed approach, and that adopting it would reduce the operational costs.

The rest of the paper is organized as follows: Section II reviews various frequency control methods in isolated microgrids in the context of UC. Section III describes the mathematical modelling of the proposed UC, and discusses the integration of the presented frequency control formulation into the UC problem. Section IV presents the test system considered, and discusses the results obtained with the proposed UC, demonstrating its practical feasibility and benefits. Finally, Section V highlights the main conclusions and contributions of the paper.

II. Isolated Microgrid Frequency Control

A. Single Unit Control

In this control mode, a single generation unit is in charge of restoring the active power balance in the system, while the rest of the generation units' outputs remain fixed at the dispatch level. This type of frequency control is usually suitable for small isolated microgrids with low penetration of renewable resources, where a single DG unit provides for a significant share of the active power demand of the system and changes in active power mismatch are not significant. For larger isolated microgrids with higher penetration of renewable resources, the changes in the active power mismatch could be substantial, and hence one single controllable unit may not be able to properly regulate the system frequency; this may result in the system frequency deviating from its acceptable range of operation [16]. In this case, frequency control should be divided among multiple generators.

B. Droop Load Sharing Control

Large isolated microgrids with more than one generation unit participating in frequency control require droop control to avoid interference among the multiple generators controlling frequency. In this case, the steady-state frequency changes as the system demand changes, allowing for proper load sharing between the units. In this approach, the slope of the power-frequency relationship is unique for each generation unit, and determines the level of its contribution in frequency control; this is referred to as droop (R). According to the droop operation principle [17], generators with the higher R participate less in compensating for the active power mismatch, as per:

$$\Delta P_i^g = \Delta P_{ref,i} - \frac{1}{R_i}\Delta f \tag{1}$$

where all variables and parameters in this and other equations are defined in the Nomenclature section. Therefore, under the droop control paradigm, and given that ΔP_{ref} is equal to zero for all generation units during a dispatch interval, the changes in the generation output could be mathematically modelled as follows:

$$\sum_{i\in\mathcal{F}} \Delta P_i^g = \Delta P_L - \sum_{i\in\mathcal{R}} \Delta P_i^r \tag{2}$$

$$\Delta P_i^g\, R_i = \Delta P_j^g\, R_j \quad \forall i,j \in \mathcal{F} \tag{3}$$

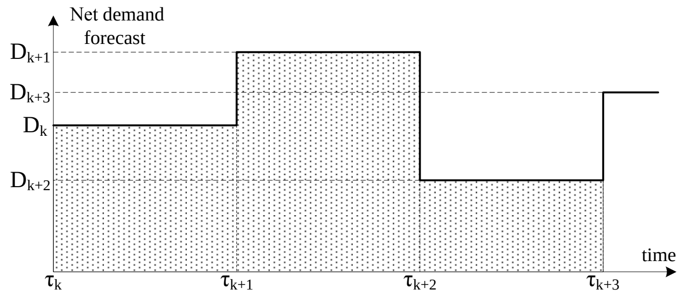

Fig. 1. Energy provision in the conventional UC.

C. ILS Control

Under the ILS control paradigm, each unit operates based on the single unit control principle described previously; however, the units communicate their loading level to each other through load sharing communication lines to guarantee that each unit is operating at the same percentage of its full-load rating. Hence, the steady-state frequency of the isolated microgrid is maintained at its nominal point. In this case, the changes in the generation units' outputs can be mathematically modelled as follows [12]:

$$\sum_{i\in\mathcal{F}} \Delta P_i^g = \Delta P_L - \sum_{i\in\mathcal{R}} \Delta P_i^r \tag{4}$$

$$\frac{\Delta P_i^g}{\bar{P}_i^g} = \frac{\Delta P_j^g}{\bar{P}_j^g} \quad \forall i,j \in \mathcal{F} \tag{5}$$

It is crucial for the generation units participating in the ILS control mode to establish and maintain reliable communication among themselves, otherwise the units oppose each other when regulating frequency. Thus, to ensure reliable operation, the units should be physically close to each other [12].

III. Proposed UC Mathematical Modelling

A. Objective Function

Generally in UC problems, the objective function is formulated based on the cost of providing a certain amount of energy between two dispatch time intervals; this energy is determined by the net demand of the system D, which is defined as follows:

$$D_k = P_{L,k} - \sum_{i\in\mathcal{R}} P_{i,k}^r \tag{6}$$

In conventional UC, the basic assumption is that output power levels are fixed over a dispatch interval and jump to another value at the next interval, forming a staircase profile, as seen in Fig. 1. Hence, the amount of energy provided by the generation units during each interval, is calculated as follows:

$$E_k = D_k\, \Delta\tau \tag{7}$$

However, in practice, the net demand does not jump from one value to another every dispatch interval, but it gradually changes until it reaches another value. Given that dispatch intervals are short, these changes can be modelled linearly, as shown in Fig. 2, and is shown in Section IV to be a valid assumption based on realistic measurements. In Fig. 2, the

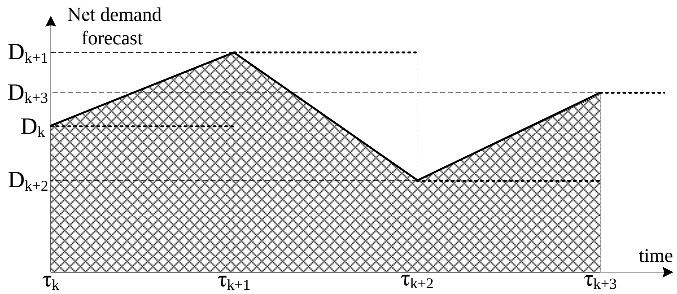

Fig. 2. Energy provision in the proposed UC.

forecasted net demand for each dispatch time is the same as in Fig. 1; however, in this case, the power output levels are not fixed during the each dispatch time interval, yielding a different energy profile. Thus, the energy required during each dispatch time interval can be calculated as follows:

$$E_k = \frac{D_k + D_{k+1}}{2}\,\Delta\tau \tag{8}$$

which is a generalized form of (7), and is equal to it only when D_k is equal to D_{k+1}, i.e. when the net demand does not change for two consecutive dispatch times.

Generation units react to changes in the net demand D according to the frequency control mechanism used. Those units that do not participate in frequency control have a constant power output during each dispatch time interval. However, assuming that changes in D are linear, the output of units participating in frequency control would change linearly depending on their droop coefficient R in droop mode, or their nominal rating P^g_{rated} in ILS mode. For droop-based frequency regulation, the following equations can be used to determine the output of a generation unit:

$$\sum_{i\in\mathcal{F}} \Delta P^g_{i,k} = D_{k+1} - D_k \tag{9}$$

$$\frac{\Delta P^g_{i,k}}{ID_i} = \Delta f_k\,\omega_{i,k} \quad \forall i \in \mathcal{F} \tag{10}$$

Note that (10) is nonlinear; hence, to keep the problem within the linear framework, this equation can be decomposed into four linear constraints, as follows [18]:

$$\frac{\Delta P^g_{i,k}}{ID_i} - (1-\omega_{i,k})\,\frac{(D_k - D_{k+1})^2}{ID_i} \le \Delta f_k \quad \forall i \in \mathcal{F} \tag{11}$$

$$\frac{\Delta P^g_{i,k}}{ID_i} + (1-\omega_{i,k})\,\frac{(D_k - D_{k+1})^2}{ID_i} \ge \Delta f_k \quad \forall i \in \mathcal{F} \tag{12}$$

$$\omega_{i,k}\,\frac{-(D_k - D_{k+1})^2}{ID_i} \le \Delta P^g_{i,k} \quad \forall i \in \mathcal{F} \tag{13}$$

$$\omega_{i,k}\,\frac{(D_k - D_{k+1})^2}{ID_i} \ge \Delta P^g_{i,k} \quad \forall i \in \mathcal{F} \tag{14}$$

Observe in (11)-(14) that depending on whether $\omega_{i,k}$ is 0 or 1, $\Delta P^g_{i,k}/ID_i$ would be equal to 0 or Δf_k, as per (10); therefore, under the proposed UC paradigm, each dispatchable generation unit is dispatched at a certain level $P^g_{i,k}$ at dispatch time k. However, the output of units that participate in frequency control is expected to change by $\Delta P^g_{i,k}$ at the end of dispatch time interval k; hence, the power output of each dispatchable unit during the dispatch time interval k can be modelled as follows:

$$P^g_i(t) = P^g_{i,k} + \frac{\Delta P^g_{i,k}}{\Delta\tau}\,t \quad \forall i \in \mathcal{F} \wedge \tau_k \le t < \tau_{k+1} \tag{15}$$

The operation cost of dispatchable generation unit i is given by the quadratic cost function as follows:

$$C^g_i = a_i\,(P^g_i)^2 + b_i\,P^g_i + c_i \tag{16}$$

In conventional UC, where P^g_i is assumed fixed over the dispatch time interval, (16) can be multiplied by the duration of the time interval to calculate to total cost of energy delivery by unit i; however, in the proposed UC, $P^g_i(t)$ is a function of time, as per (15). Hence, cost of delivering energy can be derived as follows:

$$\begin{aligned} C\tau^g_{i,k} &= \int_0^{\Delta\tau} \left(a_i P^g_i(t)^2 + b_i P^g_i(t) + c_i\right) dt \\ &= \Bigg[a_i\left((P^g_{i,k})^2 + \frac{(\Delta P^g_{i,k})^2}{3} + P^g_{i,k}\Delta P^g_{i,k}\right) \\ &+ b_i\left(P^g_{i,k} + \frac{\Delta P^g_{i,k}}{2}\right) + c_i\Bigg]\,\Delta\tau \\ &\qquad \forall i \in \mathcal{F} \wedge \tau_k \le t < \tau_{k+1} \end{aligned} \tag{17}$$

Defining $\boldsymbol{Pa^g_{i,k} = P^g_{i,k} + \Delta P^g_{i,k}/2}$, (17) can be re-written as follows:

$$\begin{aligned} C\tau^g_{i,k} = &\Bigg[a_i\left((\boldsymbol{Pa^g_{i,k}})^2 + \frac{(\Delta P^g_{i,k})^2}{12}\right) + \\ &b_i\boldsymbol{Pa^g_{i,k}} + c_i\Bigg]\,\Delta\tau \quad \forall i \in \mathcal{F} \end{aligned} \tag{18}$$

Therefore, considering the generation units start-up and shut-down costs, the final cost function for generation units that participate in frequency control can be stated as follows:

$$OC^g_{i,k} = C\tau^g_{i,k} + Cst^g_i.u^g_{i,k} + Csh^g_i.v^g_{i,k} \quad \forall i \in \mathcal{F} \tag{19}$$

The cost function for units that are not participating in frequency control can be derived by multiplying (16) by $\Delta\tau$, as follows:

$$C\tau^g_{j,k} = \left(a_j(P^g_{j,k})^2 + b_j P^g_{j,k} + c_j\right)\,\Delta\tau \quad \forall j \in \mathcal{P} \tag{20}$$

Thus, similar to (19), the final cost function for these units is:

$$OC^g_{j,k} = C\tau^g_{j,k} + Cst^g_j.u^g_{j,k} + Csh^g_j.v^g_{j,k} \quad \forall j \in \mathcal{P} \tag{21}$$

Therefore, the final objective function to be minimized can be defined as follows:

$$Z = \sum_{k\in\mathcal{T}}\left(\sum_{i\in\mathcal{F}} OC^g_{i,k} + \sum_{j\in\mathcal{P}} OC^g_{j,k}\right) \tag{22}$$

Note that (22) guarantees that the generation units dispatch level $P^g_{i,k}$ $\forall i$ are optimized based on the more realistic energy requirement (8) rather than (7). It is important to consider that (8) is not an actual constraint included in the UC problem; instead, the conventional power balance constraint is sufficient, since it is the frequency control responsibility to satisfy the power balance during the rest of the time interval, thus ensuring that the energy required is supplied by the generation units. Since the changes in the generation unit output due to frequency control is properly modelled in (22), the overall UC is guaranteed to consider and optimize for these changes.

Observe that the procedure used here to obtain the final objective function can be applied to any other linear or nonlinear heat-rate function. Furthermore, even though (11) is developed based on droop-based control, it can be modified to account for either single-unit or ILS control. For the former, the set $\mathcal{F}$ only contains a single generation unit index, and there is no need for (10)-(14); for the latter, To model the ILS control mode, ID_i should be replaced by $P^g_{rated,i}$ in (10)-(14).

B. Operating Constraints

1) Power Balance: Similar to conventional UC, the proposed UC requires that the generated power and demand be equal at each dispatch time, yielding the following constraints:

$$\sum_{i\in\mathcal{G}} P^g_{i,k} + \sum_{i\in\mathcal{R}} P^r_{i,k} + \sum_{i\in\mathcal{S}} \left(P^{s,dch}_{i,k} - P^{s,chg}_{i,k}\right) - P_{L,k} = 0 \quad \forall k \in \mathcal{T} \tag{23}$$

2) Dispatchable Units: There are certain constraints associated with dispatchable generation units such as acceptable power generation range, ramp-up and ramp-down limits, minimum-up and minimum-down time limits, and coordination constraints,, which are modelled as follows [19]:

$$\underline{P}^g_i \omega_{i,k} \le P^g_{i,k} \le \bar{P}^g_i \omega_{i,k} \quad \forall i \in \mathcal{G} \wedge k \in \mathcal{T} \tag{24}$$

$$P^g_{i,k+1} - P^g_{i,k} \le \bar{R}^g_i \Delta\tau + u^g_{i,k} \underline{P}^g_i \quad \forall i \in \mathcal{P} \wedge k \in \mathcal{T} \tag{25}$$

$$P^g_{i,k} - P^g_{i,k+1} \le \underline{R}^g_i \Delta\tau + v^g_{i,k} \underline{P}^g_i \quad \forall i \in \mathcal{P} \wedge k \in \mathcal{T} \tag{26}$$

$$u^g_{i,k} - u^g_{i,k-1} - u^g_{i,t} \le 0 \quad \forall i \in \mathcal{G} \wedge k \in \mathcal{T}^* \wedge t \in \mathcal{T}_1 \tag{27}$$

$$u^g_{i,k-1} - u^g_{i,k} + u^g_{i,t} \le 1 \quad \forall i \in \mathcal{G} \wedge k \in \mathcal{T}^* \wedge t \in \mathcal{T}_2 \tag{28}$$

$$u^g_{i,k} - v^g_{i,k} = \omega^g_{i,k} - \omega^g_{i,k-1} \quad \forall i \in \mathcal{G} \wedge k \in \mathcal{T} \tag{29}$$

$$u^g_{i,k} + v^g_{i,k} \le 1 \quad \forall i \in \mathcal{G} \wedge k \in \mathcal{T} \tag{30}$$

where,

$$\begin{aligned} \mathcal{T}_1 &= \{t+1, \ldots, min\{t + MU^g_i - 1, lenght(\tau)\}\} \\ \mathcal{T}_2 &= \{t+1, \ldots, min\{t + MD^g_i - 1, lenght(\tau)\}\} \end{aligned} \tag{31}$$

Equation (24) ensures that the output power of dispatchable units remains within the acceptable operation limit. Equations (25) and (26) guarantee that the dispatchable units do no exceed their ramp-up and ramp-down limits. Please note that these ramping constraints are relaxed for units that participate in frequency control, since these units have a much faster response aligned with the frequency control requirements [12]. Minimum-up and minimum-down time limits are modelled in (27) and (28). Finally, coordination constraints are modelled in (29) and (30).

Furthermore, for ILS control, another constraint is included to ensure that each unit is operating at the same percentage of its full-load rating, as follows [12]:

$$P^g_{j,k} \sum_{i\in\mathcal{F}} \omega^g_{i,k} \bar{P}^g_i - \bar{P}^g_j \omega^g_{j,k} \sum_{i\in\mathcal{F}} P^g_{i,k} = 0 \quad \forall j \in \mathcal{F} \tag{32}$$

Since this constraint is nonlinear, it should be decomposed in its linear equivalent constraints; hence, a new auxiliary variable $\alpha^g_{i,j,k} = \omega^g_{i,k} P^g_{j,k}$ is defined, resulting in the following set of constraints:

$$\sum_{i\in\mathcal{F}} \alpha^g_{i,j,k} \bar{P}^g_i - \bar{P}^g_j \sum_{i\in\mathcal{F}} \alpha^g_{j,i,k} = 0 \quad \forall j \in \mathcal{F} \tag{33}$$

$$0 \le \alpha^g_{i,j,k} \le \omega^g_{i,k} \bar{P}^g_{j,k} \quad \forall i,j \in \mathcal{F} \tag{34}$$

$$P^g_{j,k} - \left(1 - \omega^g_{i,k}\right) \bar{P}^g_j \le \alpha^g_{i,j,k} \le P^g_{j,k} + \left(1 - \omega^g_{i,k}\right) \bar{P}^g_j \quad \forall i,j \in \mathcal{F} \tag{35}$$

3) ESS: The following set of constraints are included to properly model the ESS behaviour:

$$\underline{SOC}_i \le SOC_{i,k} \le \overline{SOC}_i \quad \forall i \in \mathcal{S} \wedge k \in \mathcal{T} \tag{36}$$

$$SOC_{i,k+1} - SOC_{i,k} = \left(P^{s,chg}_{i,k} \eta_i - \frac{P^{s,dch}_{i,k}}{\eta_i}\right) \Delta\tau \quad \forall i \in \mathcal{S} \wedge k \in \mathcal{T} \tag{37}$$

$$0 \le P^{s,chg}_{i,k} \le \bar{P}^s_i \left(1 - d^s_{i,k}\right) \quad \forall i \in \mathcal{P} \wedge k \in \mathcal{T} \tag{38}$$

$$0 \le P^{s,dch}_{i,k} \le \bar{P}^s_i d^s_{i,k} \quad \forall i \in \mathcal{P} \wedge k \in \mathcal{T} \tag{39}$$

The SOC maximum and minimum limit constraints and the SOC evolution model over time are modelled in (36) and (37). Also, (38) and (39) make sure that the ESS charge and discharge powers are within a certain range and would not take place simultaneously.

C. Reserve Constraints

Adequate reserves play a key role in proper frequency control, and hence it should be carefully modelled in the UC problem; however, most of the previous works that propose a linear UC either neglect reserve constraints or apply it only to the largest generator in the system. This approach is not adequate for either droop control or ILS control, because multiple generators participate in regulating the system frequency and the reserve constraint should be applied to all participants as an aggregate. Therefore, the reserve constraint is represented here as follows:

$$\sum_{i\in\mathcal{F}} \left(\omega^g_{i,k} \bar{P}^g_i - P^g_{i,k}\right) \ge RES_k \quad \forall k \in \mathcal{T} \tag{40}$$

In this paper, RES_k is considered to be 10% of $P_{L,k}$. Furthermore, at least one of the units that participate in frequency control should be committed at every dispatch time step, which can be enforced as follows:

$$\sum_{i\in\mathcal{F}} \omega^g_{i,k} \ge 1 \quad \forall k \in \tau \tag{41}$$

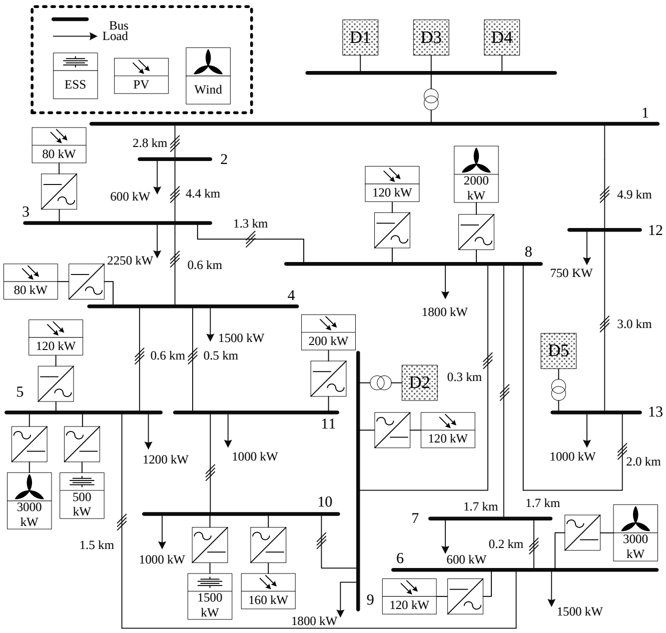

Fig. 3. Cigre benchmark system for medium voltage netwrok.

Finally, it should be ensured that the output of dispatchable units that participate in frequency control will remain within acceptable generation limits during the dispatch time interval; this can be enforced as follows:

$$\underline{P}_i^g \leq PE_{i,k}^g \leq \bar{P}_i^g \quad \forall i \in \mathcal{F} \wedge k \in \mathcal{T} \tag{42}$$

where $\boldsymbol{PE_{i,k}^g}$ is given from (15) as: $\boldsymbol{PE_{i,k}^g = P_i^g(\tau_{k+1})}$. Note that (42) is different from the conventional UC constraint that requires the dispatch values to be within an acceptable range, only at the beginning of the dispatch interval.

IV. Results and Discussions

To test and validate the efficiency of the proposed UC for isolated microgrids, a modified version of the CIGRE benchmark system for medium voltage networks is used [20], as shown in Fig. 3. The test system has a total installed capacity of 27 MW, with 5 diesel engines, ESS, and wind and PV based renewable energy resources. The peak load in the system is around 15 MW. Nominal ratings of the diesel engines are given in Table I; the nominal rating of the wind turbine is 8000 kW and of the PV unit is 1000 kW. Units D1, D3, and D4 participate in frequency control. Typical values are assumed for parameters and heat-rates corresponding to the diesel engines [19]. For the ESS, ESS1 has a maximum power rating of 1500 kW, a maximum energy rating of 5000 kWh, and a minimum allowable SOC of 300 kWh; and ESS2 has a maximum power rating of 500 kW, a maximum energy rating of 1000 kWh, and a minimum allowable SOC of 150 kWh. In all test cases, the wind, PV, and load profiles are based on high resolution (1 s) realistic measurements from an actual isolated/remote microgrid.

The performance of the proposed UC is tested over 24 h of operation, and a dispatch time interval of 5 min. The MIQP model is coded in GAMS [21], and is solved using the CPLEX solver [22]. The benefits of the proposed UC are demonstrated through several test case studies described next.

TABLE I
Diesel Generators Parameters

	D1	D2	D3	D4	D5
a_i (\$/kWh^2)	0.00015	0.00025	0.00015	0.00010	0.0005
b_i (\$/kWh)	0.2881	0.2876	0.2571	0.224	0.3476
c_i (\$/h)	7.5	0	25.5	45.5	0
Cst_i^g (\$)	15	7.35	45	95	10
Csh_i^g (\$)	5.3	1.44	8.3	15.3	0
ID_i (kW/Hz)	4000	-	2000	5000	-
$\bar{P}_i^g$ (kW)	5000	1500	4000	6000	1000
$\underline{P}_i^g$ (kW)	180	100	150	200	100

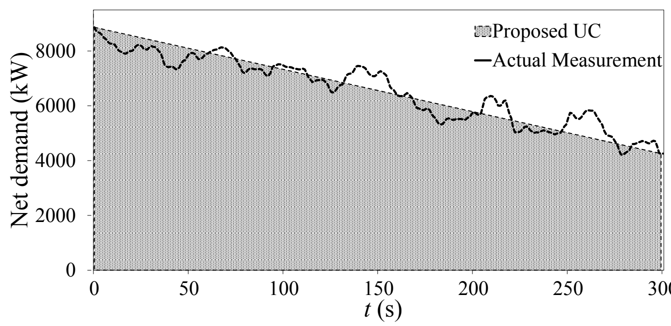

Fig. 4. Case A actual used measurements of and assumed net demand.

TABLE II
Case A Proposed UC vs. Conventional UC

UC	P_1^g (kW)	P_3^g (kW)	P_4^g (kW)	Objective Function (\$)	Actual Cost (\$)	CPU Time (s)
Conv.	2777	1857	4231	471	299	1.45
Prop.	2591	2276	3998	303	296	1.45

A. Case A: Proof of Concept

This case is specifically conducted to demonstrate the basics of the proposed algorithm, and how it results in a more efficient dispatch solution. In the test system, only units D1, D3, and D4 are included as dispatchable, since the focus here is on frequency control impact, and these are the units that participate in frequency control; wind is considered as the renewable source with an average penetration of 46%. It is assumed that the diesel generators are operating in droop control mode. The performance of the proposed UC is compared with the conventional UC only for one dispatch time interval, i.e. $\mathcal{T} = \{1, 2\}$, of 5 min duration.

The solid line in Fig. 4 illustrates the actual measurements of the net demand D during the dispatch time interval; the net demand at $t = 0$ and $t = 300$ is 8865 kW and 4256 kW, respectively. Assuming perfect forecast, the dotted area in Fig. 4 illustrates the energy to be supplied by the proposed UC, which is 547 kWh; in a conventional UC, the required energy to be supplied would be 737 kWh.

The dispatch values, costs, and computation time of the proposed UC and the conventional UC are shown in Table II.

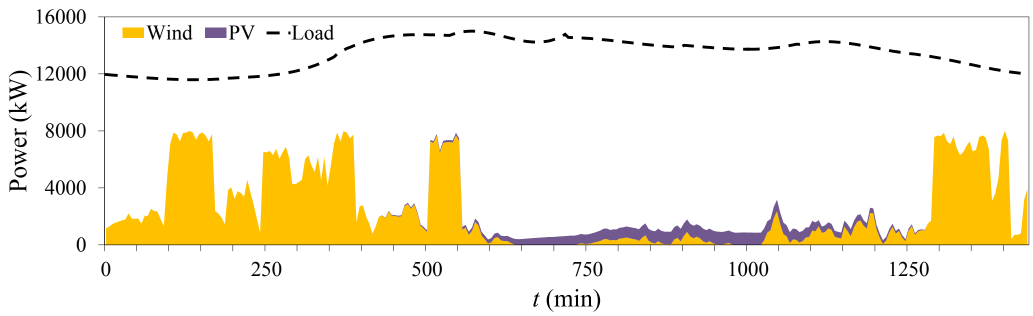

Fig. 5. Wind, PV, and load forecasted values used in Cases B, C, and D.

Observe that the objective function value of the conventional UC is considerably higher than the objective function value of the proposed UC, indicating that conventional UC overestimates the required energy during the dispatch time interval, dispatching 737 kWh compared to 547 kWh in the proposed UC. To calculate the actual operation cost, the output of the diesel generators for the 300 s interval, resulting from the measurements shown in Fig. 4, was used. As noted from Table II, compared to the conventional UC, the proposed UC yields around 1% cost savings. Note that the actual cost is very close to the solution obtained by the proposed UC, since the proposed UC obtained the solution by optimizing an amount of energy close to the actual required energy during the 300 s interval. Observe also that the computation time for both UC approaches are similar in an Intel(R) Xeon(R) CPU L7555 1.87GHz (4 processors) server.

B. Case B: ILS Control, Deterministic Forecast

This case considers the full test system, with all diesel and renewable generation units. To validate the advantage of the proposed UC compared to the conventional UC, a perfect forecast is assumed for the load and renewable outputs; the wind, PV, and demand forecasted values are shown in Fig. 5. Also, an extra constraint is added to ensure that the SOC of the batteries are equal at the beginning and end of the day. The test is conducted for 24 h of operation, with dispatch interval of 5 min.

Figure 6 shows the dispatch results obtained with the proposed and the conventional UC. Observe the differences in the UC and charging/discharging patterns of the ESS, which are magnified for two different dispatch windows; in addition, the dispatch difference of the diesel engines are shown in Fig. 7. Note that these differences are significant when the net demand fluctuation is high, and low when the net demand fluctuation is also low.

The computation time and operation costs are reported in Table III; the actual cost is calculated using the same method described in Case A. In this case, the reserve constraints described in Section III-C are used for both conventional and proposed UC. Observe in Table III that the actual cost of operating the system for 24 h in 5 min dispatch intervals for the proposed UC is \$168,148, whereas it is \$172,410 for the conventional UC; hence, under perfect forecast assumption and given that the system is operating in ILS control mode,

TABLE III
CASE B PROPOSED UC VS. CONVENTIONAL UC

UC	Obj. Function (\$)	Actual Cost (\$)	Time (s)
Conv.	172,445	172,410	21
Prop.	168,148	168,148	65

TABLE IV
CASE C PROPOSED UC VS. CONVENTIONAL UC

UC	Obj. Function (\$)	Actual Cost (\$)	Time (s)
Conv.	169,852	169,843	5951
Prop.	168,855	168,855	2781

dispatching the units using the proposed UC will reduce the operation cost by 2.47%, yielding a saving of \$4261 for the day. Also, note that the computation time for the proposed UC is similar to the conventional one.

C. Case C: ILS Control with MPC

In Case C, the same model as in Case B is used, but considering errors in the forecasted values of renewable energy and demand; these errors are computed assuming a cumulative density function (CDF) with a standard deviation obtained from a linear approximation of the difference between the current time step and the forecasted time step, as per [8]. To mitigate the impact of forecast inaccuracy, the optimal dispatch solutions obtained in each time step is only applied to the next time interval, and the problem is re-solved for the next time step with the updated forecast, repeating the process until the end of the 24 h, with a shrinking time horizon. This solution technique is referred to as the receding horizon approach [19].

Figure 8 shows the dispatch results obtained with the proposed and the conventional UC in this case, and the dispatch differences for the diesel engines are shown in Fig. 9. The solution and actual costs, and the computation times are reported in Table IV. Observe that using the proposed UC, the actual costs for operating the system would decrease by 0.6%, saving around \$1000 for the day. Note also that the computation time for the proposed method is reduced by around 50% compared to the conventional method.

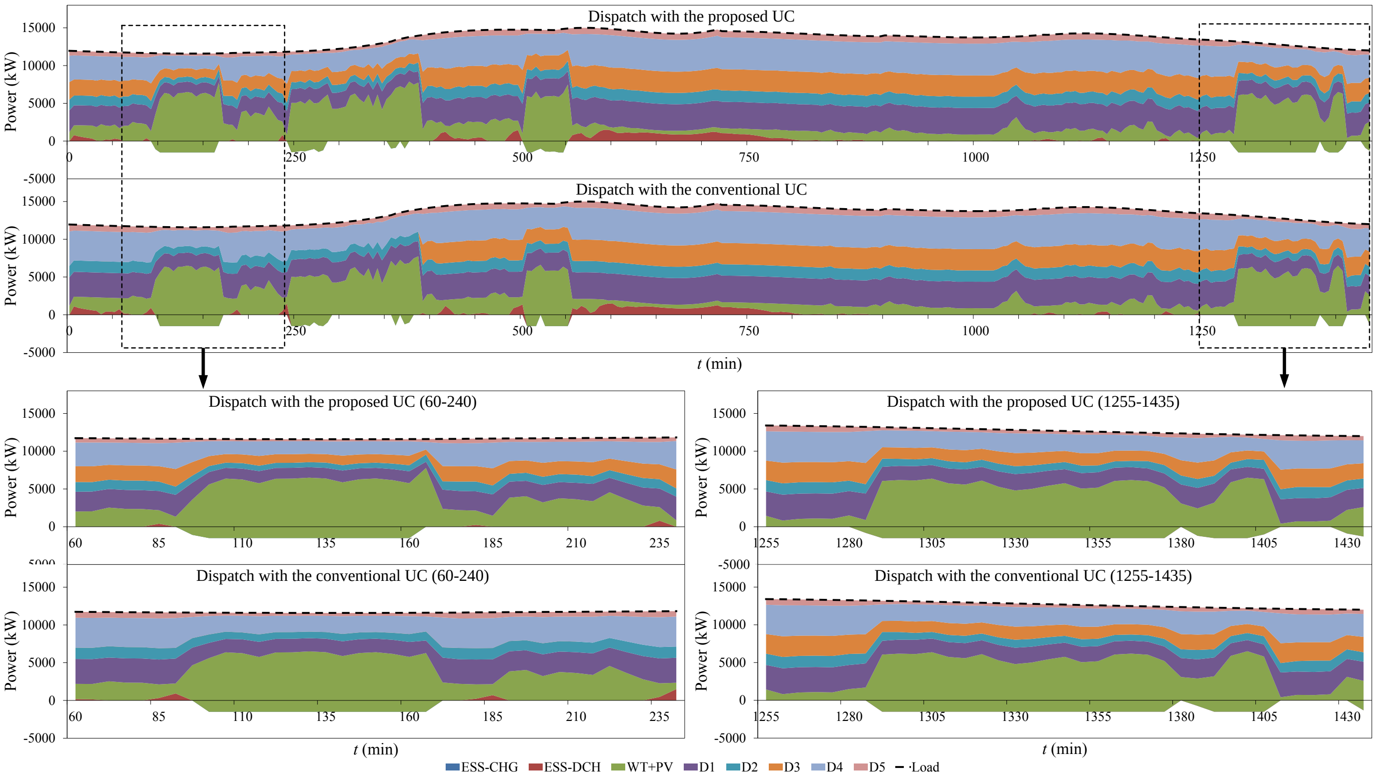

Fig. 6. Case B dispatch results with the proposed and conventional UC.

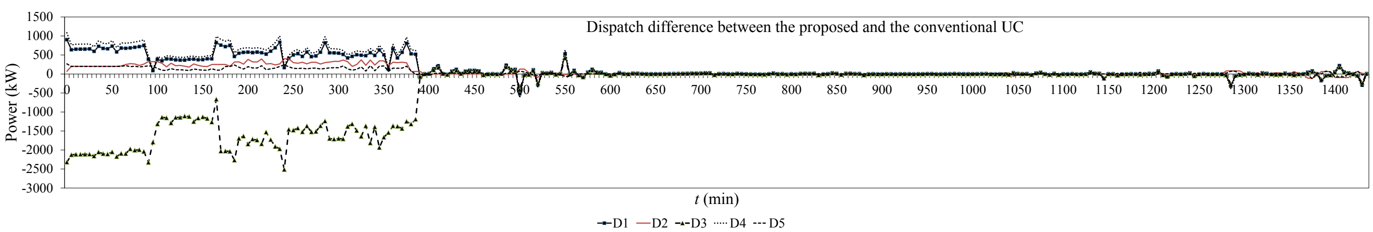

Fig. 7. Case B dispatch differences with the proposed and conventional UC.

TABLE V
CASE D PROPOSED UC VS. CONVENTIONAL UC

UC	Obj. Function ($)	Actual Cost ($)	Time (s)
Conv.	169,149	169,134	656
Prop.	167,193	167,193	1001

D. Case D: Droop Control with MPC

In Case D, the same model as in Case B and Case C is used, except that the diesel units are assumed to operate in droop control mode. Note that the errors for the forecasted values of renewable energy and demand are computed using the same CDF as in Case C, although yielding different error values compared to the previous case. Figure 10 shows the dispatch results obtained with the proposed and the conventional UC in this case, and the dispatch differences for the diesel engines are shown in Fig. 11. The solution and actual costs and the computation times are reported in Table V. Observe that using the proposed UC, the actual costs of operating the system would decrease by 1.15%, saving $1941 for the day. However, in this case the computation time is higher for the proposed UC.

V. DISCUSSIONS

A. Performance of Droop Control and ILS Control

From the results presented in Case C and Case D, in Section IV, it may seem that the UC formulation with droop control outperforms the UC formulation with ILS control, since the UC in the former case yields a smaller objective function and takes less computational time. However, this cannot be generalized, as the performance of the UC in droop control mode and ILS control mode depends on several factors such as the droop coefficients, the generators nominal rating, the forecasted values of renewable energy and demand, and other parameters.

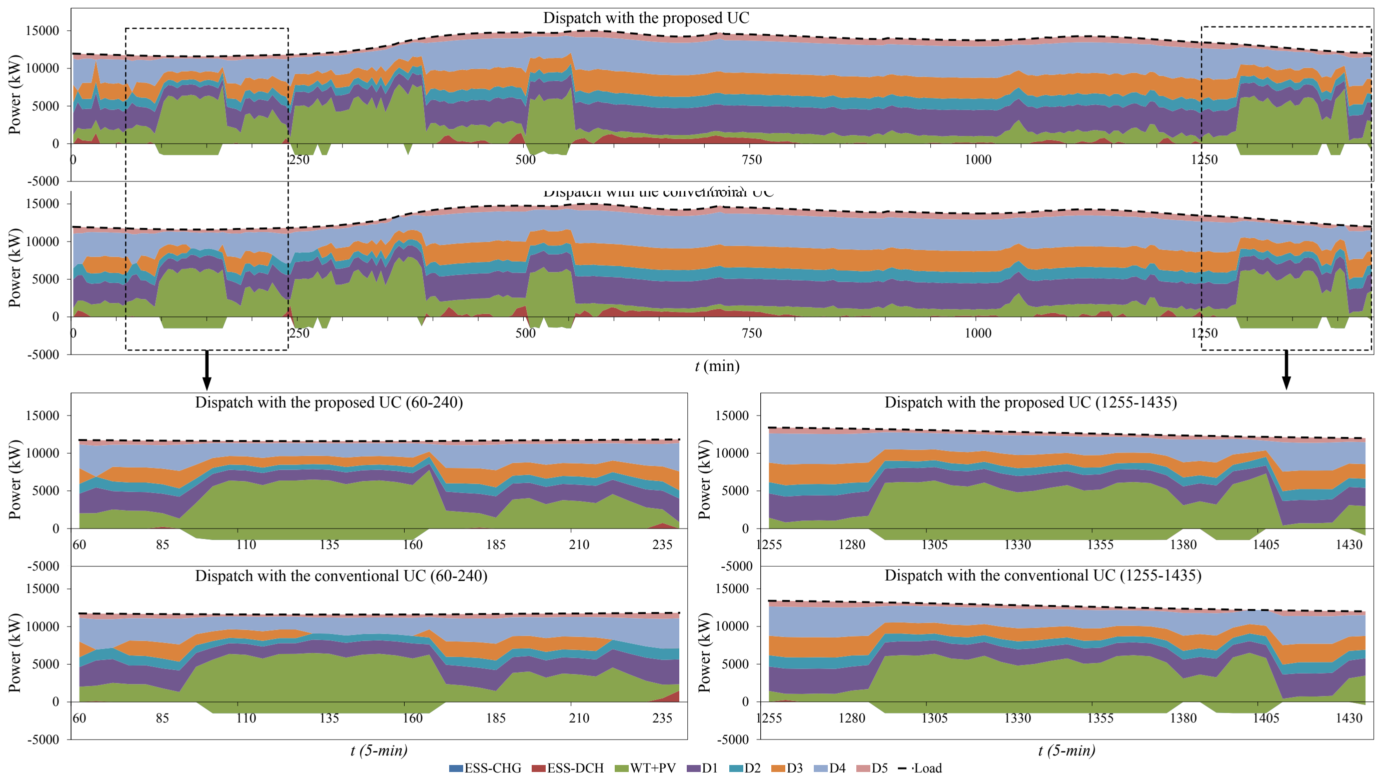

Fig. 8. Case C dispatch results with the proposed and conventional UC.

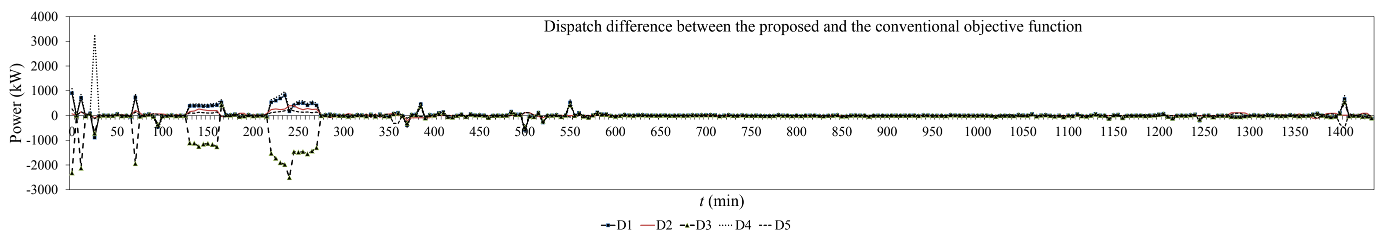

Fig. 9. Case C dispatch differences with the proposed and conventional UC.

B. Control Hierarchies

It is important to properly classify the proposed UC in the hierarchy of active power control. The active power control in isolated microgrids usually consists of two control levels [2]. The first level, or primary control, consists of local turbine governors and inverter controls that respond immediately to changes in the system, bringing the rate of change of frequency to zero in a few seconds. Thus, primary controls are in charge of maintaining the system frequency stability.

In bulk power systems, Automatic General Control (AGC), or secondary control, takes care of frequency deviation, considering the power exchange among control areas. In isolated/islanded microgrids, AGC is not necessary in ILS mode, since the primary control does not cause frequency deviations in steady state. For droop control, the dispatch of units is used to recover the steady-state frequency; hence, the UC plays the role of AGC or secondary control in isolated/islanded microgrids, as discussed in [2].

C. Primary Control Performance of Droop vs. ILS

As mentioned in Section III-A, primary control regulates the frequency, ensuring frequency stability in real-time. The proposed UC mathematically models and considers the impact of primary frequency control, so that it more accurately optimizes each DER output. From the primary control perspective, ILS control, in principle, exhibits a superior performance compared to droop control, since there is no steady state frequency deviation; however, in this case, DERs need to communicate their power output, which imposes additional communication requirements not needed in droop control. Note that in droop control operation, if the droop coefficients are chosen in reverse proportion to the generation units nominal rating, the load sharing would be exactly the same as in the ILS control mode; hence, there would be very little difference in the immediate response of the two control techniques.

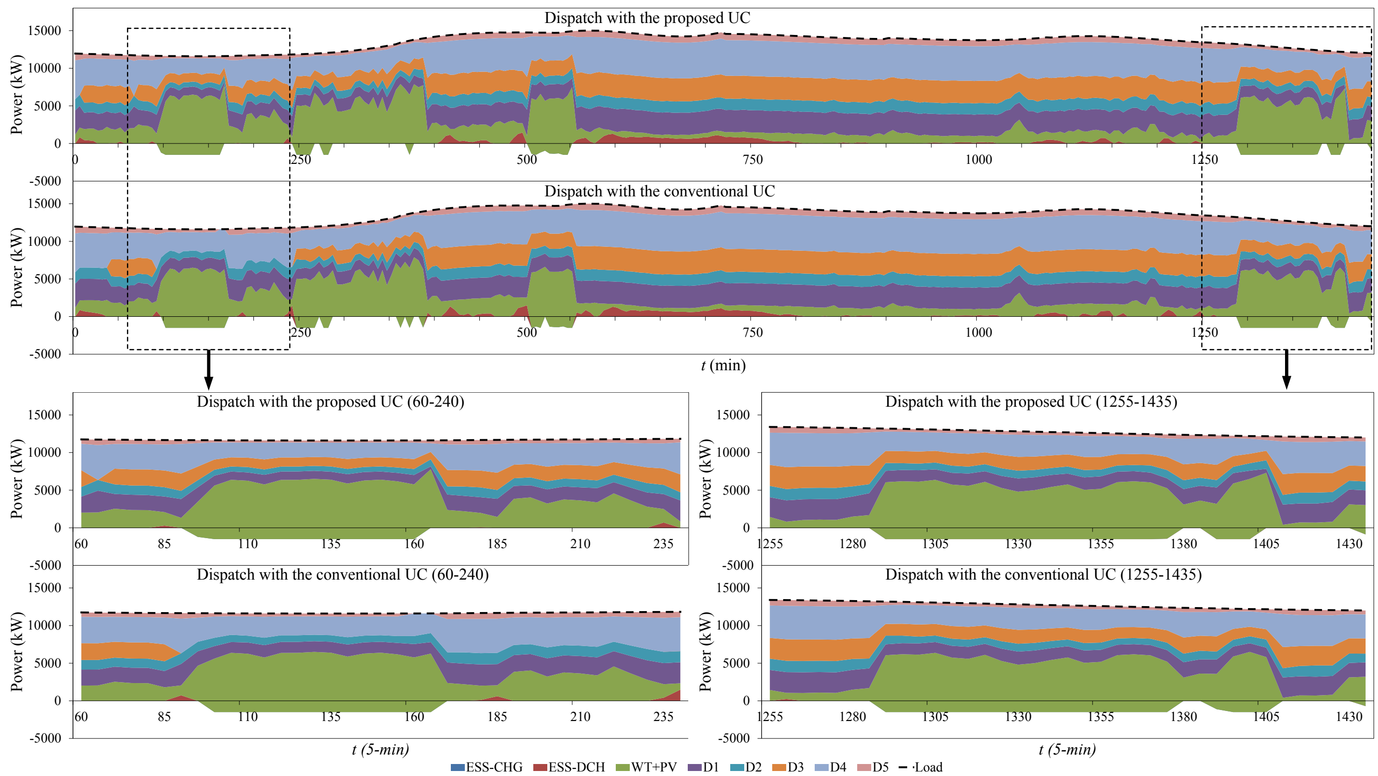

Fig. 10. Case D dispatch results with the proposed and conventional UC.

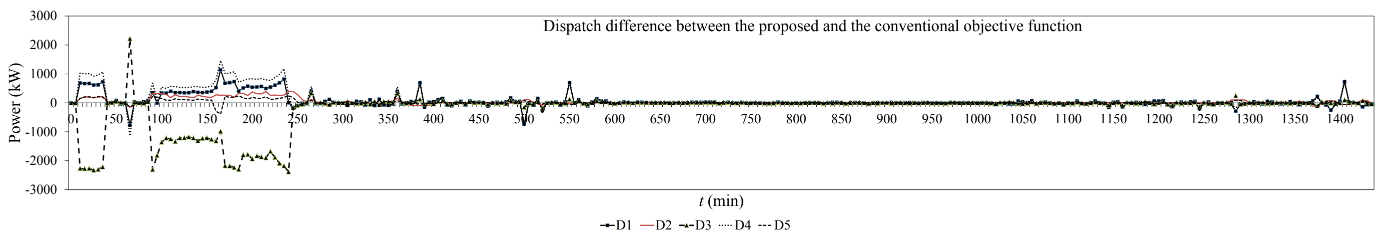

Fig. 11. Case D dispatch differences with the proposed and conventional UC.

VI. Conclusions

In this paper, a mathematical model that represented changes in the dispatchable unit outputs was proposed for isolated microgrids, for various frequency control techniques. The developed mathematical model was integrated into a UC model considering various operational constraints for a system with DERs and ESS. In addition, typical reserve constraints were modified to ensure the feasibility of frequency regulation in the system. The mathematical model was kept linear to allow its integration into current utility EMS. It was shown through several case studies in a complex isolated test microgrid that the conventional UC either over-estimated or under-estimated the energy required during each time interval, yielding dispatch levels that were not necessarily optimized for the actual required energy. By properly modelling the changes in the dispatchable unit outputs considering their frequency regulation characteristics, it was demonstrated that the proposed method results in dispatch settings that were better optimized, yielding savings in operational costs. Finally, it was shown that adopting the proposed UC had no significant impact on computation times with respect to current utility practices.

References

[1] A. G. Expósito, A. J. Conejo, and C. A. Cañizares, *Electric Energy Systems: Analysis and Operation*. New York, US: CRC Press, 2008.

[2] D. E. Olivares *et al.*, "Trends in microgrids control," *IEEE Trans. Smart Grid*, vol. 5, no. 4, pp. 1905–1919, May 2014.

[3] UCTE ad-hoc Group, "Frequency Quality Investigation," ENTSO-e, Brussels, Belgium, Tech. Rep., Aug. 2008, [Online]. Available: https://www.entsoe.eu/fileadmin/user_upload/_library/publications/ce/otherreports/090330_UCTE_FrequencyInvestigationReport_Abstract.pdf.

[4] C. A. H. Aramburo, T. C. Green, and N. Mugniot, "Fuel consumption minimization of a microgrid," *IEEE Trans. Ind. Appl.*, vol. 41, no. 3, pp. 673–681, June 2005.

[5] Z. Ding, W. Lee, and J. Wang, "Stochastic resource planning strategy to improve the efficiency of microgrid operation," *IEEE Trans. Ind. Appl.*, vol. 51, no. 3, pp. 1978–1986, Dec. 2014.

[6] H. Kanchev, F. Colas, V. Lazarov, and B. Francois, "Emission reduction and economical optimization of an urban microgrid operation including dispatched pv-based active generators," *IEEE Trans. Sustain. Energy*, vol. 5, no. 4, pp. 1397–1405, July 2014.

[7] R. Palma-Behnke, C. Benavides, F. Lanas, B. Severino, L. Reyes, J. Llanos, and D. Saez, "A microgrid energy management system based on the rolling horizon strategy," *IEEE Trans. Smart Grid*, vol. 4, no. 2, pp. 994–1006, Jan. 2013.

[8] B. V. Solanki, A. Raghurajan, K. Bhattacharya, and C. A. Cañizares, "Including smart loads for optimal demand response in integrated energy management systems for isolated microgrids," *IEEE Trans. Smart Grid*, in print.

[9] Y. C. Wu, M. J. Chen, J. Y. Lin, W. S. Chen, and W. L. Huang, "Corrective economic dispatch in a microgrid," *Int. J. Numer. Model. Electron. Netw. Devices Fields*, vol. 26, no. 2, pp. 140–150, June 2012.

[10] J. W. O'Sullivan and M. J. O'Malley, "A new methodology for the provision of reserve in an isolated power system," *IEEE Trans. Power Syst.*, vol. 14, no. 2, pp. 519–524, May 1999.

[11] G. W. Chang, C. Ching-Sheng, L. Tai-Ken, and W. Ching-Chung, "Frequency-regulating reserve constrained unit commitment for an isolated power system," *IEEE Trans. Power Syst.*, vol. 28, no. 2, pp. 578–586, Aug. 2012.

[12] A. H. Hajimiragha, M. R. D. Zadeh, and S. Moazeni, "Microgrids frequency control considerations within the framework of the optimal generation scheduling problem," *IEEE Trans. Smart Grid*, vol. 6, no. 2, pp. 534–547, March 2013.

[13] G. X. Guan, F. Gao, and A. J. Svoboda, "Energy delivery capacity and generation scheduling in the deregulated electric power market," *IEEE Trans. Power Syst.*, vol. 15, no. 4, pp. 1275–1280, Nov. 2000.

[14] Y. Yang, J. Wang, X. Guan, and Q. Zhai, "Subhourly unit commitment with feasible energy delivery constraints," *Appl. Energy*, vol. 96, pp. 245–252, Aug. 2012.

[15] G. Morales-Espana, A. Ramos, and J. Garcia-Gonzalez, "An MIP formulation for joint market-clearing of energy and reserves based on ramp scheduling," *IEEE Trans. Power Syst.*, vol. 29, no. 1, pp. 476–488, May 2013.

[16] C. Yuen, A. Oudalov, and A. Timbus, "The provision of frequency control reserves from multiple microgrids," *IEEE Trans. Ind. Electron.*, vol. 58, no. 1, pp. 173–183, Jan. 2011.

[17] P. Kundur, *Power System Stability and Control*. New York, US: McGraw-hill Professional, 1994.

[18] C. A. Floudas, *Nonlinear and Mixed-Integer Optimization: Fundamentals and Applications*. Oxford, UK: Oxford University Press, 1999.

[19] D. E. Olivares, C. A. Cañizares, and M. Kazerani, "A centralized energy management system for isolated microgrids," *IEEE Trans. Smart Grid*, vol. 5, no. 4, pp. 1864–1875, April 2014.

[20] K. Strunz, "Developing benchmark models for studying the integration of distributed energy resources," in *Proc. IEEE Power Eng. Soc. Gen. Meeting*, Montreal, QC, July 2006.

[21] R. E. Rosenthal, *GAMS – A User's Guide*. Washington, DC, USA: GAMS Development Corporation, 2016.

[22] "IBM ILOG CPLEX V12.1 User's Manual for CPLEX," International Business Machines Corporation, Tech. Rep., 2009, [Online]. Available: ftp://public.dhe.ibm.com/software/websphere/ilog/docs/optimization/cplex/ps_usrmancplex.pdf.

Mostafa Farrokhabadi (S'12) received the B.Sc. degree in electrical engineering from Amirkabir University of Technology, Tehran, Iran, in 2010, and the M.Sc. degree in Electric Power Engineering from KTH Royal Institute of Technology, Stockholm, Sweden, in 2012. He is currently a Ph.D. candidate in the Electrical and Computer Engineering Department at the University of Waterloo, ON, Canada. His research interests includes modeling, control, and optimization in microgrids.

Claudio A. Cañizares (S'85, M'91, SM'00, F'07) received in April 1984 the Electrical Engineer Diploma from the Escuela Politécnica Nacional (EPN), Quito-Ecuador, where he held different teaching and administrative positions from 1983 to 1993. His MS (1988) and PhD (1991) degrees in Electrical Engineering are from the University of Wisconsin-Madison. He has held various academic and administrative positions at the E&CE Department of the University of Waterloo since 1993, where he is currently a full Professor and the Hydro One Endowed Chair. His research activities concentrate on the study of modeling, simulation, control, stability, computational and dispatch issues in sustainable power and energy systems in the context of competitive markets and smart grids.

Kankar Bhattacharya Kankar Bhattacharya (M'95, SM'01) received the Ph.D. degree in Electrical Engineering from the Indian Institute of Technology, New Delhi, India in 1993. He was in the faculty of Indira Gandhi Institute of Development Research, Mumbai, India, during 1993-1998, and then the Department of Electric Power Engineering, Chalmers University of Technology, Gothenburg, Sweden, during 1998-2002. He joined the E&CE Department of the University of Waterloo, Canada, in 2003 where he is currently a full Professor. His research interests are in power system economics and operational aspects.