Optimal Power Flow Incorporating Voltage Collapse Constraints

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Abstract—The paper presents applications of optimization techniques to voltage collapse studies. First a "Maximum Distance to Voltage Collapse" algorithm that incorporates constraints on the current operating conditions is presented. Second, an Optimal Power Flow formulation that incorporates voltage-stability criteria is proposed. The algorithms are tested on a 30-bus system using a standard power flow model, where the effect of limits on the maximum loading point is demonstrated.

Keywords: Voltage Collapse, Optimal Power Flow, Bifurcations.

I. Introduction

As open-access market principles are applied to power systems, significant changes in their operation and control are occurring. In the new marketplace, power systems may be operated under higher loading conditions as market influences tend to demand greater attention to operating cost versus stability margin, increasing the emphasis on the use of a variety of new optimal power flow tools.

There have been several voltage collapse events throughout the world in recent years, as system are being operated with less stability margins, e.g., [1, 2]. Thus, the incorporation of voltage stability criteria in the operation of power systems has become essential [3]. In recent years, the application of optimization techniques to voltage stability problems has been gaining interest.

It is possible to restate many voltage collapse problems as optimization problems. Although, bifurcation methods are numerically well developed, the use of optimization based techniques has many advantages, including their ability to incorporate limits [4, 5]. This issue becomes even more important when considering limit-induced voltage collapse [6], which can not be easily defined using some of the traditional bifurcation-based computational techniques.

New voltage stability analysis techniques are being introduced using optimization methods that determine optimal control parameters to maximize load margins to a voltage collapse. In [7], optimal shunt and series compensation parameter settings are calculated to maximize the distance to a saddle-node bifurcation, which can be associated in some case with voltage collapse. In [8], a voltage-collapse point computation problem is formulated as an optimization problem, allowing the use of optimization techniques and tools. In [9], the reactive power margin from the point of view of voltage collapse is determined using interior point methods; the authors used a barrier function to incorporate limits. In [10], the authors determine the closest bifurcation to the current operating point on the hyperspace of bifurcation points. In [11], the maximum loadability of a power system is examined using interior point methods. In [4], an interior point optimization technique is used to determine the optimal PV generator settings to maximize the distance to voltage collapse. Furthermore, the algorithm presented in [4] includes constraints on the present operating conditions. Possible applications of optimization techniques to voltage collapse analysis are discussed in [5].

This paper presents an Optimal Power Flow (OPF) algorithm that incorporates voltage stability margins. Two main issues are considered: first, how limits affect maximum loading point computations; and second, how to include voltage stability criteria in the original OPF objective function. The role of limits, and power flow dependent and independent variables are demonstrated using a Lagrangian analysis. An OPF algorithm is then reformulated so as to increase the emphasis on voltage stability requirements as an operating point moves closer to voltage collapse.

The paper is structured as follows. In Section II, the basic background is reviewed for the Optimal Power Flow problem and issues related to voltage collapse and bifurcation methods are discussed. In Section III, the general proposed formulation to combine OPF and voltage stability is given, including an analysis of the effect of limits on the computations. Furthermore, a modification to the maximum distance to bifurcation problem is given which includes constraints and feasibility of the current operating point is given in this section. In Section IV, an OPF formulation with voltage stability constraints is given; here, voltage stability criteria is directly incorporated into an OPF objective function. The results of testing the algorithms on a 30-bus, 6 generator system are given in Section V. Finally, in Section VI, conclusions are given.

II. Background Review

A. Optimal Power Flow and Optimization Techniques

The optimal power flow problem was introduced in the early 1960's by Carpentier and has grown into a powerful tool for power system operation and planning. In general, the optimal power flow problem is a non-linear programming (NLP) problem that is used to determine the "optimal" control parameter settings to minimize a desired objective function, subject to certain system constraints [12, 13, 14]. OPF problems are generally formulated as nonlinear programming problems (NLP) as follows:

$$\begin{aligned} \min \quad & G(x) \\ \text{s.t.}: \quad & F(x) = 0 \\ & \underline{H} \le H(x) \le \overline{H} \\ & \underline{x} \le x \le \overline{x} \end{aligned} \tag{1}$$

where the mapping $G(x) : \Re^n \to \Re$ is the function that is being minimized and can include, for example, total losses in the system or generator costs; $F(x) : \Re^n \to \Re^m$ generally represents the load flow equations; and $H(x) : \Re^n \to \Re^p$ is usually transmission line limits, with lower and upper limits represented by $\underline{H}$ and $\overline{H}$, respectively. The vector of system variables, denoted by $x \in \Re^n$, typically includes voltage magnitudes and angles, generator power levels and transformer tap settings; their lower and upper limits are given by $\underline{x}$ and $\overline{x}$, respectively.

B. Voltage Collapse and Bifurcation Theory

Nonlinear phenomena and related bifurcations have been shown to be responsible for some stability problems in power systems [15].

For stability analysis, differential-algebraic mathematical models of power systems are developed as

$$\begin{aligned} \dot{z} &= f_d(z, y, \lambda) \\ 0 &= f_a(z, y, \lambda) \end{aligned} \tag{2}$$

where $z \in \Re^n$ is a vector of the state variables, $y \in \Re^m$ is a vector of algebraic variables, and $\lambda \in \Re$ is any parameter in the system that changes slowly, moving the system from one equilibrium point to another. When the Jacobian $D_y f_a(\cdot)$ of the algebraic constraints is non-singular along system trajectories, the system model can be reformulated, based on the Implicit Function Theorem [16], as

$$\begin{aligned} & y \equiv s(z, \lambda) \\ & \dot{z} \equiv f_d(z, s(z, \lambda), \lambda) \\ & \dot{z} = h(z, \lambda) \end{aligned}$$

If the Jacobian of the algebraic constraints becomes singular, then the model used to describe the system becomes invalid. In this case, the original model can be modified to consider dynamics ignored in the original model, resulting in the transformation of some algebraic constraints into differential equations [16, 17].

Equilibrium points are the values z_0, y_0, λ_0 where the rate of change of each state variable is zero, i.e.,

$$\left.\begin{aligned} 0 &= f_d(z_0, y_0, \lambda_0) \\ 0 &= f_a(z_0, y_0, \lambda_0) \end{aligned}\right\} \Rightarrow 0 = h(z_0, \lambda_0) \tag{3}$$

Of interest is where the system goes from being stable to unstable; from being unstable to stable; or where the number of equilibrium points changes with respect to a bifurcation parameter. These bifurcations are mathematically characterized by one of the eigenvalues of the Jacobian of $h(z, \lambda)$ with respect to z becoming zero (saddle-node, transcritical and pitchfork bifurcation), or by a pair of complex conjugate eigenvalues crossing the imaginary axis (Hopf bifurcation).

System limits, especially generator reactive power limits, have been shown to result in limit-induced bifurcations [18]. In particular, when the reactive power of the generator is reached, no local equilibria may exist for increased loading, typically resulting in voltage collapse. Voltage collapse has been shown to be strongly connected to saddle-node and limit-induced bifurcations [18, 19]. Therefore, in this paper, voltage collapse is defined by either a saddle-node bifurcation or a limit-induced bifurcations.

1. Saddle-Node Bifurcations

Saddle-node bifurcations are characterized by two equilibrium points of (2), typically one stable (s.e.p.) and one unstable (u.e.p.), merging at an equilibrium point for parameter value $\lambda = \lambda_*$. This equilibrium point has a simple and unique zero eigenvalue of $D_z h|_0$ [16, 20, 21]. If the two merging equilibria co-exist for $\lambda < \lambda_*$, the two equilibrium points locally disappear for $\lambda > \lambda_*$, or vice versa. Saddle node bifurcations are local bifurcations, occurring at the point where the equilibrium locally vanishes for further values of the bifurcation parameter.

2. Limit-Induced Bifurcations

Although saddle-node bifurcations can be shown to be generic in power systems, limits, especially generator reactive power limits, may restrict the space of feasible solutions. In this case, voltage collapse is not determined by a saddle-node bifurcation [6, 19]; this has a major effect on "measuring" the distance to voltage collapse.

Limit-induced bifurcations analyzed in [6], occur when generator models are changed from constant voltage and active power models, to constant active and reactive power models on encountering reactive power limits. The change in models represents a different set of equations, in some cases the new equations are unstable at the current operating point. Both the original model and the limit-induced model have the same equilibrium point when the limit is encountered but have different bifurcation diagrams.

III. Voltage Collapse and Optimal Power Flow

For the remainder of the paper, a static model for the power system of the form

$$0 = F(x, \rho, \lambda) \tag{4}$$

is used, where the vector $x \in \Re^N$ represents the system's dependent variables, normally non-generator bus voltage magnitudes and angles, reactive power levels of generators when using PV generator models, and real and reactive power levels of the slack bus generator. The vector $\rho \in \Re^m$ represents the independent variables in the system; in a simple model, this would include generator active power settings and terminal voltage levels. The parameter $\lambda \in \Re$ represents the loading factor in the system, generally referred to as the bifurcation parameter [15]. Typically, the loading factor is used to linearly model the direction of load increase in the system.

For this system model, an OPF problem that incorporates voltage collapse criteria can be generically written as

$$\begin{array}{ll} \min & G(x_p, \rho, \lambda_p, \lambda_*) \\ \text{s.t. :} & F(x_p, \rho, \lambda_p) = 0 \\ & F(x_*, \rho, \lambda_*) = 0 \\ & \underline{H_p} \leq H(x_p) \leq \overline{H_p} \\ & \underline{H_*} \leq H(x_*) \leq \overline{H_*} \\ & \underline{\rho} \leq \rho \leq \overline{\rho} \end{array} \tag{5}$$

where the subscripts p and $*$ indicate the current and critical operating points respectively. The dependent and independent variables in the model are given as x and ρ and λ is the loading factor. $G(x_p, \rho, \lambda_p, \lambda_*)$ is the function to be minimized, its OPF portion may be dependent on (x_p, ρ, λ_p), and the voltage stability portion is a function of λ_* and possibly of λ_p. It is assumed that the inequality constraints defined by the limits on $H(x_p)$ and $H(x_x)$, can be separated into separate constraints on the current and critical loading points and the control variables. The vectors of lower and upper limits on the power system independent variables ρ are given by $\underline{\rho}$ and $\overline{\rho}$ respectively.

Using a Logarithmic Barrier approach [14], the first order KKT optimality conditions to problem (5) is given to demonstrate when the maximum loading point is defined by a limit-induced bifurcation or a saddle-node bifurcation. Using slack variables problem (5) can be rewritten as

$$\begin{array}{ll} \min & G(x_p, \rho, \lambda_p, \lambda_*) \\ \text{s.t. :} & F(x_p, \rho, \lambda_p) = 0 \\ & F(x_*, \rho, \lambda_*) = 0 \\ & H(x_p) - \underline{H_p} - s_1 = 0 \\ & \overline{H_p} - H(x_p) - s_2 = 0 \\ & H(x_*) - \underline{H_*} - s_3 = 0 \\ & \overline{H_*} - H(x_*) - s_4 = 0 \\ & \rho - \underline{\rho} - s_5 = 0 \\ & \overline{\rho} - \rho - s_6 = 0 \\ & s_1, s_2, s_3, s_4, s_5, s_6 \geq 0 \end{array} \tag{6}$$

where $s_1, s_2, s_3, s_4 \in \Re^p$ and $s_5, s_6 \in \Re^m$ are the primal non-negative slack variables used to transform the inequality constraints to equalities. The non-negativity constraints are now incorporated into the objective function using a logarithmic barrier as follows:

$$\begin{array}{l} \min G(x_p, \rho, \lambda_p, \lambda_*) - \mu \sum_{i=1}^{m} (log\ s_5[i] + log\ s_6[i]) \\ \quad - \mu \sum_{i=1}^{p} (log\ s_1[i] + log\ s_2[i] + log\ s_3[i] + log\ s_4[i]) \end{array}$$

$$\begin{array}{lrcl} \text{s.t. :} & F(x_p, \rho, \lambda_p) & = & 0 \\ & F(x_*, \rho, \lambda_*) & = & 0 \\ & H(x_p) - \underline{H_p} - s_1 & = & 0 \\ & \overline{H_p} - H(x_p) - s_2 & = & 0 \\ & H(x_*) - \underline{H_*} - s_3 & = & 0 \\ & \overline{H_*} - H(x_*) - s_4 & = & 0 \\ & \rho - \underline{\rho} - s_5 & = & 0 \\ & \overline{\rho} - \rho - s_6 & = & 0 \end{array} \tag{7}$$

where μ is the barrier parameter and $s[i]$ represents the i^{th} element of the vector s. The Lagrangian function of the modified barrier problem (7) is then defined as

$$\begin{aligned} L \;=\; & G(x_p, \rho, \lambda_p, \lambda_*) - \mu \sum_{i=1}^{m} (log\ s_1[i] + log\ s_2[i]) \\ & -\mu \sum_{i=1}^{p} (log\ s_3[i] + log\ s_4[i]) + log\ s_5[i] + log\ s_6[i]) \\ & -\gamma_1^T (F(x_p, \rho, \lambda_p)) - \gamma_2^T (F(x_*, \rho, \lambda_*)) \\ & -\nu_1^T (H(x_p) - \underline{H_p} - s_1) - \nu_2^T (\overline{H_p} - H(x_p) - s_2) \\ & -\nu_3^T (H(x_*) - \underline{H_*} - s_3) - \nu_4^T (\overline{H_*} - H(x_*) - s_4) \\ & -\zeta_1^T (\rho - \underline{\rho} - s_5) - \zeta_2^T (\overline{\rho} - \rho - s_6) \end{aligned} \tag{8}$$

where $\gamma_1, \gamma_2 \in \Re^n$, $\nu_1, \nu_2, \nu_3, \nu_4 \in \Re^p$ and $\zeta_1, \zeta_2 \in \Re^m$ are the Lagrange multipliers. The vector $y = (x_*, x_p, \lambda_*, \rho, s_1, s_2, s_3, s_4, s_5, s_6, \gamma_1, \gamma_2, \nu_1, \nu_2, \nu_3, \nu_4, \zeta_1, \zeta_2)$ is introduced to simplify the expression. The Karush-Kuhn-Tucker (KKT) first-order necessary conditions are used to define the local minimum of equation (7),

$$\nabla_y L = \begin{bmatrix} \nabla_{x_*} L \\ \nabla_{x_p} L \\ \nabla_{\lambda_*} L \\ \nabla_{\rho} L \\ -\mu I e + S_1 \nu_1 \\ -\mu I e + S_2 \nu_2 \\ -\mu I e + S_3 \nu_3 \\ -\mu I e + S_4 \nu_4 \\ -\mu I e + S_5 \zeta_1 \\ -\mu I e + s_6 \zeta_2 \\ F(x_p, \rho, \lambda_p) \\ F(x_*, \rho, \lambda_*) \\ H(x_p) - \underline{H_p} - s_1 \\ \overline{H_p} - H(x_p) - s_2 \\ H(x_*) - \underline{H_*} - s_3 \\ \overline{H_*} - H(x_*) - s_4 \\ \rho - \underline{\rho} - s_5 \\ \overline{\rho} - \rho - s_6 \end{bmatrix} = 0 \tag{9}$$

where S_1 through S_6 are diagonal matrices with elements of the corresponding vector s_1 through s_6 on the diagonal, $I \in \Re^{p\times p}$ is an identity matrix, $e \in \Re^p$ is a vectors of ones, and

$$\begin{aligned}
\nabla_{x_*} L &= \gamma_2^T \nabla_{x_*} F(x_*, \rho, \lambda_*) - (\nu_3^T + \nu_4^T) \nabla_{x_*} H(x_*) \\
\nabla_{x_p} L &= \nabla_{x_p} G(x_p, \rho, \lambda_p, \lambda_*) + \gamma_1^T \nabla_{x_p} F(x_p, \rho, \lambda_p) \\
&\quad -(\nu_1^T + \nu_2^T) \nabla_{x_p} H(x_p) \\
\nabla_{\lambda_*} L &= \nabla_{\lambda_*} G(x_p, \rho, \lambda_p, \lambda_*) - \gamma_2^T \nabla_{\lambda_*} F(x_*, \rho, \lambda_*) \\
\nabla_\rho L &= G(x_p, \rho, \lambda_p, \lambda_*) - \gamma_1^T \nabla_\rho F(x_p, \rho, \lambda_p) \\
&\quad -\gamma_2^T \nabla_\rho F(x_*, \rho, \lambda_*) - \zeta_1 + \zeta_2
\end{aligned}$$

The issue of collapse due to limit-induced bifurcation versus saddle-node bifurcation can now be explained as follows. The first condition in (9), $\nabla_{x_*} L$, includes the Jacobian of the system model at the maximum loading point multiplied by γ_2, which can be considered to be equivalent to an eigenvector of the Jacobian. Therefore, the first condition corresponds to a singular Jacobian if $(\nu_3^T + \nu_4^T)\nabla_{x_*} H(x_*) = 0$. This condition would imply the dependent variables are not at their limits, since ν_3 and ν_4 are zero when their corresponding limits are not active. If dependent variables of the critical point are at their limits, then ν_4 and ν_3 become non-negative, i.e., the load flow Jacobian is non-singular. In this case, the system has reached a limit-induced bifurcation point.

The above derivation demonstrates when the inequality constraints can be separated based on the dependent and independent variables of the load flow model, the maximum loading point may be a limit-induced point only when constraints based on the dependent variables become active. The independent variables, ρ, being at their limits, do not directly affect the type of bifurcation.

A particular example of this optimization problem is the "Maximum Distance to Collapse" with constraints included on the current and critical loading point. This problem can be written as

$$\begin{aligned}
\min \quad & -\tfrac{1}{2}(\lambda_p - \lambda_*)^2 \\
\text{s.t.}: \quad & F(x_p, \rho, \lambda_p) = 0 \\
& F(x_*, \rho, \lambda_*) = 0 \\
& \underline{H_p} \le H(x_p) \le \overline{H_p} \\
& \underline{H_*} \le H(x_*) \le \overline{H_*} \\
& \underline{\rho} \le \rho \le \overline{\rho}
\end{aligned} \tag{10}$$

This problem maximizes the distance to a saddle-node or limit-induced bifurcation. Including the current loading point into the constraints ensures that, when independent variables are calculated to maximize the distance to voltage collapse, feasibility and inequality constraints at the current loading point are met. For example, increasing generator voltage magnitude settings generally increase the distance to collapse but, under lighter loading conditions, the increased levels may lead to over-voltages. Incorporating the current operating point into the optimization problem can eliminate this problem; however, it also reduces the space of feasible solutions. This formulation differs from existing formulations, for example [7, 11], since constraints are placed in the critical loading point and the current loading point. Furthermore, the constraint in [7] forcing the maximum loading point be a saddle-node bifurcation is removed in this case.

IV. Optimal Power Flow with Voltage Stability Constraints

With the current loading point included into the optimization problem, it is now possible to incorporate voltage stability constraints into an OPF formulation at the current loading point. As the operating point moves closer to a point of voltage collapse, more emphasis must be placed on stability criterion versus operating cost minimization.

The algorithm indirectly scales the traditional optimal power flow problem with the inverse of the difference between the current value of the loading parameter and its value at the voltage collapse point. Therefore, as the system moves closer to the bifurcation point, more weight is given to the voltage stability versus generation cost. A technique utilizing voltage stability indices is examined in [4]; but, since voltage stability indices have very nonlinear characteristics due to limits, this technique is not generally adequate. However, since the maximum loading point of the system is a variable in the optimization problem, it is possible to accurately use a measure of the distance to collapse as a means of shifting the weighting between cost minimization and voltage stability security. The following formulation is proposed to remove the use of voltage collapse indices:

$$\min \; G(x_p, \rho, \lambda_p)(\Phi) \tag{11}$$

$$\begin{aligned}
\text{s.t.}: \quad & F(x_p, \rho, \lambda_p) = 0 \\
& F(x_*, \rho, \lambda_*) = 0 \\
& \Phi\,(\lambda_* - \lambda_p)^2 = 1 \\
& \underline{H_p} \le H(x_p) \le \overline{H_p} \\
& \underline{H_*} \le H(x_*) \le \overline{H_*} \\
& \underline{\rho} \le \rho \le \overline{\rho}
\end{aligned}$$

where $G(x_p, \rho, \lambda_p)$ represents an optimal power flow objective function, whereas the function Φ is defined so that it tends to infinity as λ_p approaches λ_*. If the current loading point λ_p is at the bifurcation point λ_*, then the algorithm fails since the inverse of $\lambda_p - \lambda_*$ is infinity. Although, it is unlikely that $\lambda_p = \lambda_*$, some numerical problems may occur if they are "close". If the system is effectively at λ_*, then a strict maximum distance to collapse algorithm should be utilized.

V. Numerical Simulations

The maximum distance to collapse and OPF with voltage stability constraints algorithms presented in Sections III and IV are tested on a 30-bus, 6 generator system that is based on the IEEE 30-bus test system [22]. Transmission line limits are not included in the inequality constraints. In normal operating conditions, the bus

TABLE I
OPF with Voltage Stability Constraints using Distance to Collapse Measure (No active ρ limits)

λ_p	λ_*
0.9	3.2513
1.2	3.2800
1.3	3.2883

TABLE II
OPF with Voltage Stability Constraints using Distance to Collapse Measure (Active Q_{gen} limits)

λ_p	λ_*
0.9	1.8342
1.1	1.9209
1.2	2.0290

voltages are to be within 1.0 ± 0.1 p.u. A number of simulations have been performed to analyze how the current loading point influences the algorithm and how limits effect the maximum loading point. A nonlinear primal-dual predictor-corrector interior point method [23] written in Matlab is used to perform the numerical analysis.

Initially, the reactive power limits of the generators are relaxed, such that their limits are not reached when the problems are tested at several loading levels, as defined by λ_p. The "Maximum Distance to Collapse" algorithm calculates an optimum λ_* of 3.3762 p.u. The results for the "OPF with Voltage Stability Constraints" algorithm for different current loading levels are given in Table I. In general, for the voltage stability constrained algorithm, as the current loading level increases, the algorithm places less emphasis on cost optimization and more on voltage stability security. For example, for $\lambda_p = 0.9$, the algorithm places more emphasis on minimizing generator cost versus stability margin, obtaining a ρ that yields a voltage collapse point at $\lambda_* = 3.2513$; when the current loading point is raised to $\lambda_p = 1.3$, more emphasis is placed on voltage stability versus cost minimization, resulting in ρ values for a voltage collapse point at $\lambda_* = 3.2883$. The change in λ_* demonstrates that, as the current loading point is increased, more emphasis is given to increasing the value of λ_* at which the system bifurcates. For each case, the maximum loading point corresponds to a saddle-node bifurcation. In general, the problem tends to push the generator voltage settings to their limits and generator active power settings vary depending on the current operating point.

If the reactive power limits on the generators are reduced such that they become active, the problem calculates maximum loading points that correspond to limit-induced bifurcations; the power flow Jacobian is nonsingular in this case. A summary of the results are given in Table II. Due to the generator reactive power limits, the maximum loading point is reduced, as the space of feasible solutions is "smaller". For active and non-active reactive power limits, a comparison of some generator variables, at λ_*, when $\lambda_p = 0.9$ is given in Table III. The reactive power of the generators Q_{gen} are dependent

TABLE III
Results of Reactive Power Limits on Various System Variables (At λ_* when $\lambda_p = 0.9$)

Parameter	Without Reactive Power Limits (p.u.)	With Reactive Power Limits (p.u.)
Q_{gen_1}	2.7950	1.0000*
Q_{gen_2}	2.8373	0.7000*
Q_{gen_3}	3.9500	0.5000*
V_{gen_1}	1.1000*	1.0962
V_{gen_2}	1.0700*	1.0700*
V_{gen_3}	1.1000*	1.0200

* indicates the parameter is at its limit

TABLE IV
OPF with Voltage Stability Constraints using Distance to Collapse Measure (Active ρ limits)

λ_p	λ_*
0.9	1.7000
1.1	1.7300
1.2	1.7500

variables, x, in the power flow model, and the generator voltage magnitudes V_{gen} are independent variables, ρ, in the power flow model. In both cases the problems tended to push the generator voltage magnitudes towards their upper limits, until reactive power limits are reached.

Finally, if "operational limits" are placed on all bus voltages, at both the critical and current loading conditions, the algorithm calculates the control parameters ρ that maximize the operating region. A summary of the results are given in Table IV. Due to both reactive power and bus voltage limits, the maximum loading point is defined by a limit-induced bifurcation. By including "operational limits" on all voltage magnitudes the problem becomes more "practical" from the system operation point of view. If reactive power limits do not become active, then the maximum loading point may still be a stable point.

VI. Conclusions

This paper demonstrates that voltage stability and optimal power flow studies can be performed concurrently. Furthermore, it is shown that incorporating constraints on the current operating point in the maximum distance to collapse problem reduces the space of feasible solutions, resulting in different optimal solutions. The conditions for saddle-node bifurcation versus limit-induced bifurcation are demonstrated. An optimal power flow algorithm that incorporates voltage stability criteria is proposed and implemented on a test system. The results indicate that the algorithm successfully shifts the importance of generation cost minimization and voltage stability security for different loading levels.

The future direction of this research is to reformulate the system model to incorporate a distributed slack bus. Furthermore, more numerical simulations will be performed to study the effect of including "operational limits" in a variety of test systems.

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