Re-Defining the Reactive Power Dispatch Problem in the Context of Competitive Electricity Markets

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Abstract: This paper proposes a novel reactive power dispatch model that takes into account both the technical and economical aspects associated with reactive power dispatch in the context of the new operating paradigms in competitive electricity markets. The main objective of the proposed model is to minimize the total amount of dollars paid by the system operator to the generators for providing the required reactive power support. The real power generation is decoupled and assumed fixed during the reactive power dispatch procedures; however, due to the effect of reactive power on real power, a re-schedule in the real power generation is allowed within given limits. The 32-bus CIGRE benchmark system is used to illustrate the proposed reactive power dispatch technique. The developed model is generic in nature and designed to be adopted by system operators in any electricity market structure, as demonstrated by its application to Ontario's grid considering its market rules for reactive power payments.

1 Nomenclature

P_{GR}: Real power rating of a synchronous generator.

Q_{GR}: Reactive power rating of a synchronous generator.

Q_G^{min}: Minimum reactive power limit of a generator.

Q_{Gb}^{lead}: Base leading reactive power of a generator.

Q_{Gb}^{lag}: Base lagging reactive power of a generator.

Q_{GA}: Maximum reactive power limit of a generator without reduction in real power generation.

Q_{GB}: Maximum allowable reactive power limit of a generator with reduction in real power generation.

ρ_0: Availability price for generator g in \$.

ρ_1: Price of Losses in the under-excitation region for generator g in \$/Mvar.

ρ_2: Price of losses in the over-excitation region for generator g in \$/Mvar.

ρ_3: Loss of opportunity price for generator g in \$/Mvar^2.

Q_{G1}: Under-excitation reactive power of a generator in p.u.

Q_{G2}: Over-excitation reactive power of a generator in p.u.

Q_{G3}: Reactive power of a generator operating in the opportunity region in p.u.

m_1, m_2: Binary variables associated with Regions I and II of reactive power operation, respectively.

m_{3r}, m_{3f}: Binary variables associated with armature and field limits on reactive power generation, respectively.

X: Set of procured generators for reactive power.

Y: Set of available generators from market clearing.

ξ: Available set of generators for reactive power dispatch.

J: Total payment associated with the reactive power dispatch in \$/h.

J_1: Reactive power payment component in \$/h.

J_2: Balance services payment component in \$/h.

J_3: Loss payment/credit component in \$/h.

ρ_{B1}: Price of upward balance services P_B in \$/MW

ρ_{B2}: Price of downward balance services P_B in \$/MW

P_{B1i}: Upward balance service at bus i in p.u

P_{B2i}: Downward balance service at bus i in p.u.

P_L: Total system losses with proposed Q-dispatch, p.u.

P_{Lo}: Pre-determined total system losses after market clearing, p.u.

ΔP_{Gi} Reduction in real power at bus i due to increase in reactive power beyond heating limits, in p.u.

P_{Goi}: Market clearing pre-determined real power dispatch at bus i in p.u.

Q_{Gi}: Reactive power generation at bus i in p.u.

P_{Di}: Real power demand at bus i in p.u.

Q_{Di}: Reactive power demand at bus i in p.u.

V_i: Voltage magnitude in p.u. at bus i.

δ_i: Voltage angle in radians at bus i.

P_{ij}: Power flow from bus i to bus j in p.u.

Y_{ij}: Element ij of admittance matrix in p.u.;

$Y_{ij} = G_{ij} + j\,B_{ij} = |Y_{ij}|\,\angle\theta_{ij}$

c_i: Maximum allowed level of real power reduction at bus i.

P_{Gxg}: New real power dispatch for generator g.

Z: The set of generators in zone z.

K_z: The amount of reactive power reserves in zone z.

2 Introduction

Reactive power dispatch has been of great interest to researchers as well as system operators, especially after the restructuring of the power industry. This interest is mainly due to the significant effect that reactive power has on system security given its close relationship with the bus voltages throughout the power network. Insufficient reactive power supply can result in voltage collapse, which has been one of the reasons for some recent major blackouts; for example, the US-Canada Power System Outage Task Force states in its report that insufficient reactive power was an issue in the August 2003 blackout, and has recommended strengthening the reactive power and voltage control practices in all North American Electric Reliability Council (NERC) regions [1].

Traditionally, reactive power dispatch has always been viewed by researchers as a loss minimization problem, subject to various system constraints such as nodal real and reactive power balance, bus voltage limits, and power generation limits [2]-[5]. Another approach has been to dispatch reactive power with the objective of maximizing the system loadability in order to minimize the risk of voltage collapse [6], [7]. Multi-objective optimization models have also been proposed for the reactive power dispatch problem; in these models, reactive power is dispatched to achieve other objectives, in addition to the traditional loss minimization, such as maximizing voltage stability margin [8], or minimizing voltage and transformers taps deviations [9].

In deregulated electricity markets, the Independent System Operator (ISO) is responsible for the provision of ancillary services that are necessary to support the transmission of electrical energy while maintaining secure and reliable operation of the power system. According to the Federal Energy Regulatory Commission's (FERC) Order No.888, reactive power supply and voltage control from generators is one of six ancillary services that transmission providers must include in an open access transmission tariff [10]. FERC Order 2003 further states that a reactive power provider should not be financially compensated when operating within a power factor range of 0.95 lagging and 0.95 leading, but an ISO may change this range at its discretion [11].

Reactive power ancillary services in deregulated electricity markets can be provided based on a two-stage approach, namely, *reactive power procurement* and *reactive power dispatch*, as proposed by the authors in [12]. Reactive power procurement is essentially a long-term issue, where the ISO signs seasonal contracts with possible service providers that would best suit its needs and constraints in the given season [13], [14]. Reactive power dispatch, on the other hand, corresponds to the short-term allocation of reactive power generation required from already contracted suppliers based on "real-time" operating conditions [15], [16]. The issues associated with the first level of the proposed framework, i.e. reactive power procurement, were discussed by the authors in [17], where appropriate mechanisms were proposed for management and pricing of reactive power in the long term. The current paper, on the other hand, concentrates on the issues associated with the second level of the proposed framework, i.e. reactive power dispatch in the short term.

The "traditional" dispatch approaches do not consider the cost incurred by the system operator to provide reactive power. One of the reasons for this is that, in a vertically integrated system, all generators were under the direct ownership and control of the central operator, and hence reactive power payments were bundled in the energy price. The problem of reactive power dispatch in the context of competitive electricity markets, therefore, needs to be re-defined, since reactive power has been recognized as an ancillary service to be purchased separately by the ISO [10], [11], and hence has an economic effect on the market, playing an important role in the way it is operated.

In the context of competitive electricity markets, reactive power *dispatch* essentially refers to short-term allocation of reactive power required from suppliers (e.g. generators), based on current system operating conditions. The ISO's problem is to determine the optimal reactive power schedule for all providers based on a given objective that depends on system operating criteria. Different objective functions can be used by the ISO, besides the traditional transmission loss minimization, such as minimization of reactive power cost [15], [16], [18], or minimization of deviations from contracted transactions [19]. Any of the aforementioned objectives can be adopted, but since some of them are of a conflicting nature, the ISO needs to choose a criterion that best suits the market structure.

In this paper, a novel framework that re-defines the reactive power dispatch problem to suit the ISO requirements in the context of competitive electricity markets is proposed, based on the preliminary reactive power dispatch model proposed and discussed in [20]. The model seeks to minimize the ISO’s total payments which include payments for reactive power dispatched from service providers, payments for balance services needed to compensate for the deficit in real power supply due to possible changes in generation dispatch levels, and payments/credits associated with the increase/decrease in total system losses. The effect of scheduling reactive power on real power is also considered, by allowing limited re-scheduling of real power, as well as accounting for the availability of balance services (referred to operating reserves in some markets such as Ontario), with the objective of minimizing the overall payments for the ISO.

It is important to highlight the fact that, in this paper, and adhering to existing FERC regulations, only reactive power support from generators is considered, as one of the six ancillary services eligible for financial compensation [10]. However, the proposed dispatch scheme is fairly generic and hence could be in principle extended to include other reactive power resources in addition to generators, such as capacitor banks and FACTS controllers, as recommended in [21]. This will be the focus of future research, considering that these reactive power sources are essentially different from generators. It is also relevant to mention that it is assumed here, as in previous Q-dispatch papers (e.g. [2]-[5]), that the generators have the means to maintain reactive power at the required dispatch levels, and that the system conditions do not change significantly between dispatch intervals. This is certainly not necessarily the case in practice, as “special” generator Q-controls would be required to accomplish this. Thus, the current practice in most ISOs, that typically do not have generator Q-control capabilities, is to use the results of the Q-dispatch processes, which in most cases are simply based on a power flow run for the desired P-dispatch levels as discussed in more detail below, to actually define the set points of the voltage regulators that resulted from the Q-dispatch procedure. These voltage levels are then maintained, with the generator Q-output changing according to the system conditions, until new Q-dispatch levels, i.e. new voltage regulator set points, are provided by the ISO. The actual generator Q-output is then paid ex-post according to the contractual agreement, which in this paper corresponds to the contracted prices resulting from the procurement process [12], [17].

The rest of paper is organized as follows: Section 3 provides a background review for reactive power production and associated costs from a synchronous generator. The proposed reactive power dispatch model is presented in Section 4, including detailed discussions on how the dispatch problem is re-defined in the context of competitive electricity markets. In Section 5, the proposed model is tested on the CIGRE 32-bus system, and several case studies are presented and discussed. Section 6 demonstrates the application of the proposed dispatch methodology to the Ontario grid, in view of the Ontario’s market rules for reactive power payments. Finally, in Section 7, the main conclusions and contributions of the presented work are summarized and highlighted.

3 Reactive Power from Synchronous Generator

The reactive power capacity of a synchronous generator is determined from its capability curve, which demonstrates the relationship between real and reactive power generation from this generator. When real power and terminal voltage are fixed, the armature and field winding heating limits determine the reactive power capability of the generator, as shown in Figure 1. The generator's MVA rating is the point of intersection of the two curves, and therefore its MW rating is given by P_{GR}. At an operating point A, with real power output P_{GA} such that $P_{GA}<P_{GR}$, the limit on Q_G is imposed by the generator's field heating limit; whereas, when $P_{GA}>P_{GR}$, the limit on Q_G is imposed by the generator's armature heating limit.

There is a mandatory amount of reactive power that each generator has to provide, which is shown by the shaded area in Figure 1. If the generator is called upon by the ISO for additional reactive power provision beyond this area, it is then eligible for payment to compensate for the increased costs associated with losses in the windings. Such mandatory and ancillary classification of reactive power capability is in line with what most system operators have in place for reactive power management nowadays.

According to the capability curves in Figure 1, the generator can provide reactive power until it reaches its heating limits (point A in Figure 1); any further increase in reactive power provision from the generator will be at the expense of a reduction in its real power generation. Hence, the generator is expected to receive an *opportunity cost payment* for providing reactive power beyond Q_{GA} in Figure 1, which accounts for the lost opportunity to sell its real power in the energy market and the associated revenue loss. Thus, the following three regions for reactive power generation can be identified in Figure 1 [12], [13]:

- Region I ($Q_G^{min} \leq Q_G = Q_{G1} \leq 0$) refers to the under-excitation region, in which the generator is required to absorb reactive power.
- Region II ($0 \leq Q_G = Q_{G2} \leq Q_{GA}$) refers to the over-excitation region, in which the generator is required to supply reactive power within its reactive power capability limits.
- Region III ($Q_{GA} \leq Q_G = Q_{G3} \leq Q_{GB}$) refers to the loss of opportunity region, in which the generator is asked to reduce its real power production in order to meet the system reactive power requirements. It is assumed here that P_{GB} would be the minimum amount of real power that the generator is able/willing to produce.

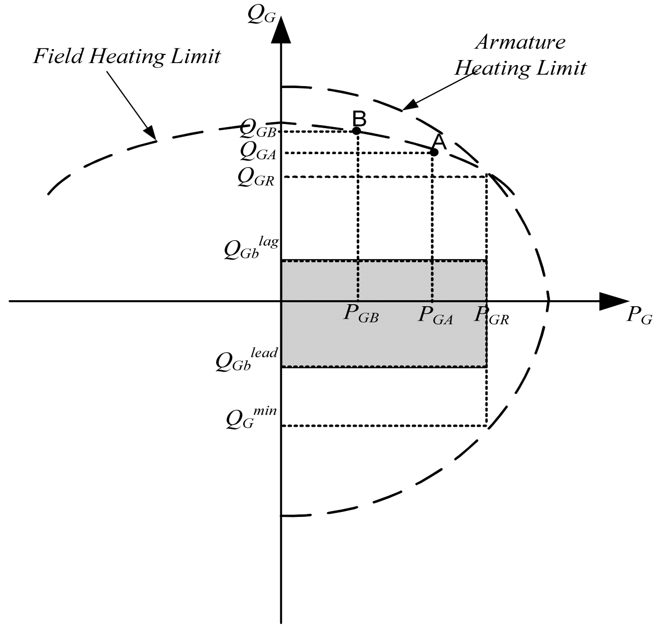

Figure 1 Synchronous generator capability curve [12], [13].

Based on the above three Regions of reactive power operation and the associated costs, a reactive power payment function (*QPF*) is formulated, as shown in Figure 2, including the following payment components: an availability payment component (with a price ρ_0), which is a fixed component to account for that portion of a supplier's capital cost that can be attributed to reactive power production; two losses payment components (with prices ρ_1 and ρ_2), which are assumed as linearly varying components to account for the increased winding losses as reactive power output increases, in the under- and over-excitation regions, respectively; an opportunity payment component (with a price ρ_3) to account for the lost opportunity cost associated with the operation in Region III. This opportunity component is assumed to be quadratic based on the assumption that real power costs are parabolic functions of output power, which may be considered to approximately change linearly with reactive power in Region III (from point A to B in Figure 1). Accordingly, *QPF* for each generator *g* in the system can be mathematically represented by the following equation [19]:

$$
\begin{aligned}
QPF_g &= \rho_{0g} + m_{2g}\rho_{2g}\left(Q_{G2g} - Q_{Gbg}^{lag}\right) - m_{1g}\rho_{1g}\left(Q_{G1g} - Q_{Gbg}^{lead}\right) \\
&\quad + \rho_{2g}\left(m_{3fg} + m_{3rg}\right)\left(Q_{G3g} - Q_{Gbg}^{lag}\right) \\
&\quad + 0.5\rho_{3g}\left(m_{3fg} + m_{3rg}\right)\left(Q_{G3g} - Q_{GA}\right)^2 \\
m_{1g} &= \begin{cases} 1 & \text{if } Q_{Gg}^{\min} \le Q_{Gg} = Q_{G1g} \le 0 \\ 0 & \text{otherwise} \end{cases} \\
m_{2g} &= \begin{cases} 1 & \text{if } 0 \le Q_{Gg} = Q_{G2g} \le Q_{GAg} \\ 0 & \text{otherwise} \end{cases} \\
m_{3fg} &= \begin{cases} 1 & \text{if } Q_{GAg} \le Q_{Gg} = Q_{G3g} \le Q_{GBg} \,\&\, P_{Ggo} < P_{GRg} \\ 0 & \text{otherwise} \end{cases} \\
m_{3rg} &= \begin{cases} 1 & \text{if } Q_{GAg} \le Q_{Gg} = Q_{G3g} \le Q_{GBg} \,\&\, P_{Ggo} > P_{GRg} \\ 0 & \text{otherwise} \end{cases}
\end{aligned}
\tag{1}
$$

In this equation, the binary variables are needed to reflect the fact that the generator operates in only one of the three regions defined in Figure 1.

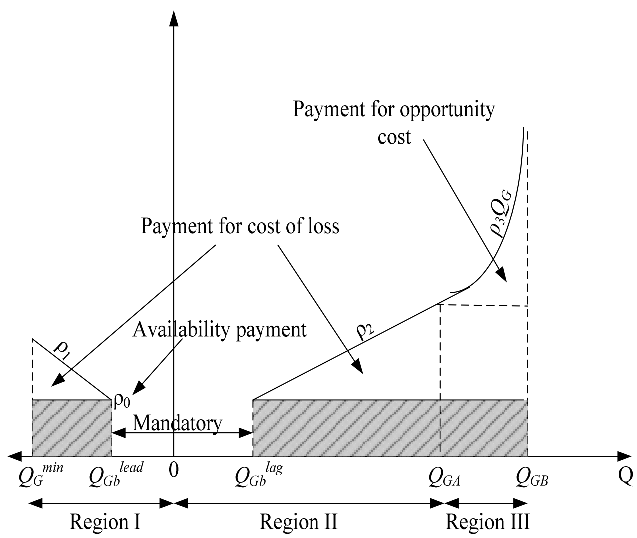

Figure 2 Reactive power payment function [19].

The above four reactive power price components are usually determined from a procurement stage, where the ISO signs long-term contracts with reactive power service providers, in which both parties agree on the prices and the payment mechanism. For example, in Ontario, the Independent Electricity System Operator (IESO) signs 36-month contracts with generators that are willing to provide reactive power services. Prices are based on costs of providing reactive power, which include additional costs from energy losses incurred by operating at non-unity power factor, and cost of running the generating units as synchronous condensers if requested by the IESO [22]. Generators that are asked to reduce their real power output in order to meet the reactive power requirements are further paid a lost opportunity component at the market clearing price. The ISO-New England, on the other hand, pays a capacity component for qualified generator reactive resources for the capability to provide reactive power services, in addition to the energy and lost opportunity components [23]; a “base VAR rate” of 2.32 $/Kvar-yr has been newly incorporated for qualified generators available for reactive power provision below 0.95 leading or 0.95 lagging power factors.

In [12], the authors propose a two-settlement reactive power market structure, which segregates reactive power management in two distinct time-frames: a reactive power procurement stage carried out on a seasonal basis, and a reactive power dispatch stage that determines the reactive power levels in “real-time”. The authors propose an optimal procurement model in [17] that considers both the technical and economical issues associated with reactive power provision in the long-term. The solution of this OPF-based model yields a set of contracted generators and the four reactive power price components, namely, availability, under- and over-excitation, and opportunity components, which reflect current reactive power pricing practices. Hence, regardless of the procurement procedure, it is assumed here that these four price components are known to the ISO during the dispatch process.

4 Proposed Reactive Power Dispatch Scheme

In practice, real power and reactive power have generally been handled separately by most power system operators. Typically, real power dispatch is carried out using a linear programming model associated with an Economic Load Dispatch (ELD) calculation that maximizes social welfare, while guaranteeing that system security constraints are met [24]. Reactive power, on the other hand, is dispatched based on power flow studies and operational experience. However, there are several complex issues involved in reactive power service provision in deregulated electricity markets that call for further systematic procedures to arrive at better solutions [12]. Ideally, reactive power should be dispatched from generators in an economical manner that minimizes the ISO’s payment burden, while considering system security constraints.

Figure 3 illustrates the proposed schematic procedure for short-term dispatch of reactive power. The scheme is based on the assumption that a pre-determined set of contracted (or procured) generators with corresponding reactive power price components, which should be obtained within a long-term framework to avoid the adverse impacts of energy price volatility on these prices [12], [17], is available as previously discussed. In line with current reactive power dispatch approaches, the ISO would carry out the dispatch procedure several minutes (e.g. one hour to half-hour) ahead of real-time. Based on the set of procured/contracted generators and the generating units available from short-term energy market clearing, the ISO would determine the available units for reactive power dispatch, and then properly dispatch the units using an OPF-based model that minimizes total payments associated with reactive power dispatch, subject to appropriate system security constraints. Finally, the payments would be calculated post real-time operation, based on the actual usage and dispatch requested by the ISO. The following sub-sections provide detailed discussion on each of the blocks in Figure 3.

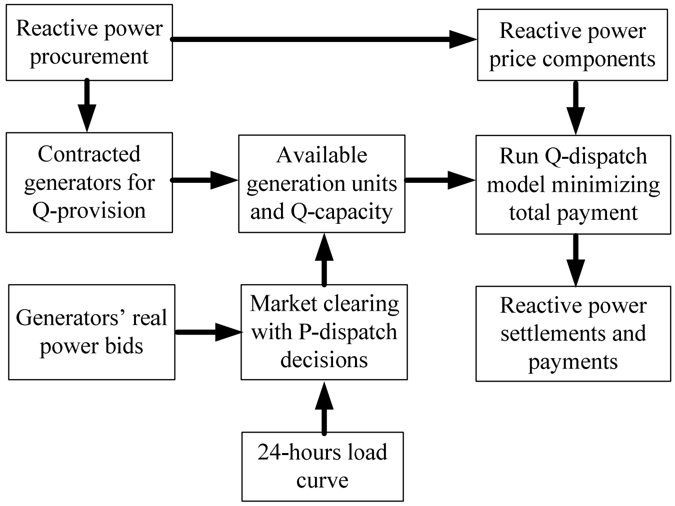

Figure 3 Flow chart for the proposed reactive power dispatch scheme.

4.1 Reactive Power Procurement

As explained earlier in Section 3, an ISO typically makes long-term plans for reactive power procurement, in which it determines the expected reactive power capacity required to ensure a secure and reliable operation of the power system. Currently, most system operators depend on power flow studies and operational experience in determining a set of contracted generators for reactive power service provision. The ISO then signs seasonal contracts with these generators, in which both parties agree on reactive power prices as well as the payment mechanism. It is assumed here that the procurement process determines the prices needed in the proposed real-time dispatch procedures. This is discussed in more detail for the applications presented in Sections 5 and 6.

It is important to highlight the main differences between the long-term Q-procurement methodology proposed in [17], and the short-term Q-dispatch procedure proposed herein. Thus:

- The time horizons for the two problems are different, with [17] discussing the first level of Q-management (months in advance), whereas the current paper discusses the second level (real-time), as defined in [12].
- The mathematical optimization models are significantly different, with the model in [17] concentrating on the maximization of a Societal Advantage Function that comprises the marginal benefits of reactive power from generators with respect to system security, and Q-offers from generators, subject to appropriate security constraints. The model here, on the other hand, is based on a minimization of the ISO's total payments associated with real-time reactive power dispatch, consisting of reactive power payments to generators, balance service payments and credit/payments for system losses, and subject to several additional constraints not considered in [17], as discussed in detail below.
- In [17], the reactive power price components are outcomes of the optimization process, whereas these prices are considered as inputs to the Q-dispatch model discussed herein.
- Zonal reserves and the effect of reactive power on real power and system losses are considered here in the dispatch model, whereas these issues are not considered at all in [17].

4.2 Real Power Market Clearing

Most ISOs use dc-OPF models for real power market clearing and dispatch, with iterative mechanisms to guarantee system security [24]. In this paper, because the emphasis is on reactive power dispatch, it can safely be assumed that the energy market clearing (and hence the market price), and the resulting P-dispatch are available to the ISO.

It should be noted that an ac-OPF can also be used in lieu of the dc-OPF to arrive at the real power market clearing and dispatch. However, the computational burden of such OPF is large for practical sized power systems, since it requires solving a rather complex and large-scale non-linear programming (NLP) model every few minutes (e.g. every 5 minutes in Ontario). Thus, decoupling the OPF problem and handling reactive power dispatch separately [15], [25], [26], helps alleviate market power and price volatility which may come into play if reactive power dispatch is handled in the same time-frame as that of real power market clearing [12].

4.3 Available Generators and Q-Capacity

As mentioned earlier, an important input to the dispatch model is the set of available generators for real-time reactive power dispatch. Letting X be the set of contracted/procured generators for reactive power obtained at the procurement stage, and Y be the set of available generators from market clearing, then the available set of generators for reactive power dispatch will be given by $\xi = X \cap Y$.

The reactive power capacity of the available generating units is predetermined based on the capability curves of these generators (Figure 1). Accordingly, for each generator, the upper and lower limits of the three reactive power operating regions are assumed to be known.

4.4 Reactive Power Dispatch Model

Equipped with the information regarding the set of available generators, reactive power limits of the three operating regions, and the four reactive power price components, the ISO should now be able to execute an optimal dispatch program to arrive at the required amount of reactive power in real-time operation stage. A reactive power dispatch model is proposed as follows, taking into account both the economic and technical issues associated with service provisions in a competitive electricity market:

4.4.1 Objective Functions

The proposed model seeks to minimize the following objective function J, which represents the total payments associated with reactive power dispatch:

$$
\begin{aligned}
J &= J_1 + J_2 + J_3 \\
J_1 &= \sum_g \begin{pmatrix} \rho_{0g} + m_{2g}\rho_{2g}\left(Q_{G2g} - Q_{Gbg}^{lag}\right) \\ - m_{1g}\rho_{1g}\left(Q_{G1g} - Q_{Gbg}^{lead}\right) \\ + \rho_{2g}\left(m_{3fg} + m_{3rg}\right)\left(Q_{G3g} - Q_{Gbg}^{lag}\right) \\ + 0.5\rho_{3g}\left(m_{3fg} + m_{3rg}\right)\left(Q_{G3g} - Q_{GA}\right)^2 \end{pmatrix} \\
J_2 &= \sum_i \left(\rho_{B1} \cdot P_{B1i} + \rho_{B2} \cdot P_{B2i}\right) \\
J_3 &= \rho_{MC} \cdot \left(P_L - P_{Lo}\right)
\end{aligned}
\tag{2}
$$

where all components are defined in Section 1. These payments can be divided into the following three main categories:

- Payment associated with reactive power provided from generators (J_1), which is a function of the predetermined price components associated with each region of operation (ρ_0, ρ_1, ρ_2, and ρ_3), as explained in Section 3. Accordingly, payment components are determined for Q_{G1}, Q_{G2} and Q_{G3} corresponding to operation in Region I, II, and III, respectively, plus an availability payment.
- Payment associated with energy balance services (operating reserves) that are required to compensate for re-scheduling of real power (J_2), i.e. the effect of Q-dispatch on P-dispatch. This will apply only under conditions when some generators are required to supply reactive power in Region III, where they need to reduce their real power generation in order to meet the system reactive power requirement. Consequently, there is a need for rescheduling of their real power (ΔP_G), and a balance service (P_B) is required at certain buses to compensate for real power deviations from already dispatched values (P_{Go}). The energy balance services from available providers might be an upward or downward service, i.e. P_{B1} and P_{B2}, respectively; the corresponding prices ρ_{B1} and ρ_{B2} are assumed to be predetermined from an energy balance market [27].
- Payment/credit associated with the change in the total system losses due to reactive power dispatch and the re-scheduling of real power generation (J_3). This component can be positive (payment) or negative (credit) depending on the difference between the value of the total losses calculated within the Q-dispatch model (P_L), and the value associated with the real power market clearing dispatch (P_{Lo}). The payment/credit can be calculated by multiplying this difference by the market clearing price (ρ_{MC}).

It is important to highlight the fact that the proposed payment function J given in (2) is of a generic nature, and can be modified to fit other payment schemes adopted by system operators. For example, it was mentioned earlier that both the IESO in Ontario and the ISO-New England compensate the generators operating in the opportunity region by directly paying them a lost opportunity component at the market clearing price; hence, in this case, the objective function can be readily modified as follows to represent such a payment mechanism:

$$J^* = \sum_g \begin{pmatrix} \rho_{0g} + m_{2g}\rho_{2g}\left(Q_{G2g} - Q_{Gbg}^{lag}\right) \\ - m_{1g}\rho_{1g}\left(Q_{G1g} - Q_{Gbg}^{lead}\right) \\ + \rho_{2g}\left(m_{3fg} + m_{3rg}\right)\left(Q_{G3g} - Q_{Gbg}^{lag}\right) \end{pmatrix} + \sum_g \rho_{MC} \cdot \Delta P_{Gg} + \sum_i \left(\rho_{B1} \cdot P_{B1i} + \rho_{B2} \cdot P_{B2i}\right) \tag{3}$$

Observe here that the quadratic opportunity cost term in (2) has been replaced by a direct payment for re-scheduled power $\rho_{MC}\Delta P_G$. Furthermore, loss payments/credits are not included, since this is currently not a common practice in "standard" reactive power dispatch approaches.

4.4.2 Power Balance Equations

The following nodal real power balance equation is modified to include ΔP_G, P_{B1} and P_{B2}:

$$P_{Goi} - \Delta P_{Gi} + P_{B1i} - P_{B2i} - P_{Di} = \sum_j V_i V_j Y_{ij} \cos\left(\theta_{ij} + \delta_j - \delta_i\right) \quad \forall i \tag{4}$$

$$Q_{Gi} - Q_{Di} = -\sum_j V_i V_j Y_{ij} \sin\left(\theta_{ij} + \delta_j - \delta_i\right) \quad \forall i \tag{5}$$

4.4.3 *System Security Limits*

These limits include bus voltage limits and line power transfer capacity constraints:

$$V_i^{\min} \le V_i \le V_i^{\max} \qquad \forall i \tag{6}$$

$$\left|P_{ij}\left(V, \delta\right)\right| \le P_{ij}^{\max} \qquad \forall i, j \tag{7}$$

Note that the line limits are being represented by real power limits, instead of apparent power or current limits, which is in line with current practices by most system operators, who typically represent system security limits by defining power transfer limits determined as the minimum of thermal limits and voltage and angle stability limits for the "worst" contingencies (N-1 contingency criterion) [28]. However, the apparent power or current limits could also be readily considered in the present model.

4.4.4 Reactive Power Limits

The three regions of reactive power production identified from the generator's capability curves (Figure 1) are introduced as follows:

$$\begin{aligned} & m_{1g} Q_{Gg}^{\min} \le Q_{G1g} \le 0 \\ & 0 \le Q_{G2g} \le m_{2g} Q_{GAg} \qquad\qquad \forall g \\ & \left(m_{3fg} + m_{3rg}\right) Q_{GAg} \le Q_{G3g} \le \left(m_{3fg} + m_{3rg}\right) Q_{GBg} \end{aligned} \tag{8}$$

$$m_{1g} + m_{2g} + m_{3fg} + m_{3rg} \le 1 \qquad \forall g \tag{9}$$

These constraints guarantee that only one of the three regions, out of Q_{G1}, Q_{G2} and Q_{G3}, is selected at a time for each generator.

4.4.5 Effect of Q-dispatch on P-Dispatch

The required reduction in real power dispatch (ΔP_G) is determined as follows:

$$P_{Gxg} = m_{3fg}\sqrt{\left(\frac{V_t E_{af}}{X_s}\right)^2 - \left(Q_G + \frac{V_t^2}{X_s}\right)^2} + m_{3rg}\sqrt{(V_t I_a)^2 - Q_{Gg}^2} + \left(m_{1g} + m_{2g}\right)P_{Gog} \tag{10}$$

$$P_{Gog} - P_{Gxg} = \Delta P_{Gg} \tag{11}$$

$$\Delta P_{Gi} \le c_i P_{Goi} \tag{12}$$

Observe that ΔP_G will have a non-zero value only if the generator is operating in Region III, i.e. if the generator hits its field limit ($P_{GA}<P_{GR}$) or armature limit ($P_{GA}>P_{GR}$); otherwise, P_{Gx} in (9) will be equal to P_{Go} and hence, according to (10), ΔP_G will be zero. In order to minimize the effect on real power dispatch, a "cap" on reduction in real power is imposed (e.g. between 5 to 15%) as per constraint (12).

4.4.6 Energy Balance Limits

The ISO will require some balancing mechanism to compensate for the reschedule in real power and changes in system losses as a result of the reactive power dispatch. In this work, it is assumed that such a market mechanism, i.e. upward and downward balance services (operating reserves), is already in place as an ancillary service, which is typically the case in most markets, and thus can be used by the ISO within the proposed reactive power dispatch framework. A limit on the maximum upward and downward balance service available at each system bus is assumed as follows:

$$P_{B1,2i} \le P_{B1,2i}^{\max} \tag{13}$$

4.4.7 Zonal Reactive Power Constraints

Based on typical voltage control approaches, the following constraints are included to ensure sufficient reactive power reserves within a voltage control zone:

$$\sum_g Q_{Gg} \le K_z \sum_g Q_{GAg} \qquad \forall g \in Z, \forall z \tag{14}$$

where Z denotes the set of generators in zone z. The amount of reactive power reserves in zone z is denoted by K_z; for example, K_z = 0.9 implies a 10% reactive power reserve in the zone. The sum of Q_{GAg} in any zone defines the maximum total amount of reactive power available in this zone. Observe that the value of Q_{GA} instead of Q_{GB} was used to define the zonal reactive power reserve to be more conservative, i.e. extra reactive power coming from the operation in the opportunity region is not considered in the zonal reserve constraints, as this is unknown at the start of the dispatch process.

4.4.8 Losses

The total system losses P_L in (2) are calculated as follows:

$$P_L = 0.5\sum_{i,j}\left(G_{i,j}\left(V_i^2 + V_j^2 - 2V_iV_j\cos\left(\delta_j - \delta_i\right)\right)\right) \tag{15}$$

4.5 Computational Issues

The proposed dispatch model (2)-(15) captures both the technical and economical aspects of reactive power dispatch. However, from the optimization point of view, this model represents a difficult optimization problem, since it is essentially a mixed-integer NLP (MINLP) problem, due to the presence of binary variables required to properly select only one out of the three regions of reactive power operation in the model.

One approach to solve the proposed model (2)-(15) is to solve the problem as an explicit non-convex MINLP problem; however, the solution of these types of problems is very challenging due to the presence of both the integer variables and the non-convexities of the model itself [28], [30], [31]. The capability of available solvers for MINLP problems is still rather limited and most of them require a substantial amount of computational time even for small case studies, and might not yield an optimal solution within many CPU hours for a large case study [30]. Moreover, most of the available MINLP solvers (e.g. DICOPT) are not able to handle line flow limits, which are of great importance to represent system security, in various test cases tried by the authors. Hence, solving (2)-(15) using non-convex MINLP techniques is not the most appropriate choice, especially when realistic sized power systems are considered.

To address these issues, a Generator Reactive Power Classification (GRPC) algorithm proposed by the authors in [17] was used here. This GRPC algorithm solves the proposed optimization problem by re-formulating the MINLP problem (2)-(14) into a series of NLP sub-problems, and hence alleviating the need for binary variables. The algorithm starts with reactive power values obtained from an initial feasible power flow solution, to identify an initial region of reactive power operation for each generator. The solution process then goes into a series of sequential updates until no further improvement in the objective function can be achieved. The main steps of the GRPC algorithm are given in the Appendix for the readers’ convenience; more details can be found in [17].

The solution of the proposed dispatch model yields the required reactive power support from each generator; the amount of real power to be rescheduled in order to meet the system reactive power requirements; the amount of energy balance services needed to compensate for the change in real power resulting from the reactive power dispatch; and the total payment of the ISO to the service providers.

5 Implementation and Test Results

The proposed reactive power dispatch model (2)-(15) is tested on the CIGRE 32-bus system (Figure 4), and the associated results are presented and discussed in this section. The optimization models, which are transformed into NLP problems as previously discussed, are modeled in GAMS and solved using the MINOS solver [32]. Without any loss of generality, the power flow limits are considered to be dependent on the transmission voltage levels; thus, limits of 2000 MW for the 400 kV lines, 350 MW for 220 kV lines, and 250 MW for 130 kV lines are assumed. The test system has 20 generators and a total demand of 10,940 MW. The system is split into three zones or voltage control areas as reported in [33]. To simplify the analysis and without loss of generality, all generators are assumed to be eligible for payments in all the three regions of operation, which implies that Q_{Gb}^{lead} and Q_{Gb}^{lag} in Figure 1 are equal to zero for all generators. Finally, a 30% reactive power reserve is guaranteed for each zone, i.e. $K_z = 0.7$ for all three zones.

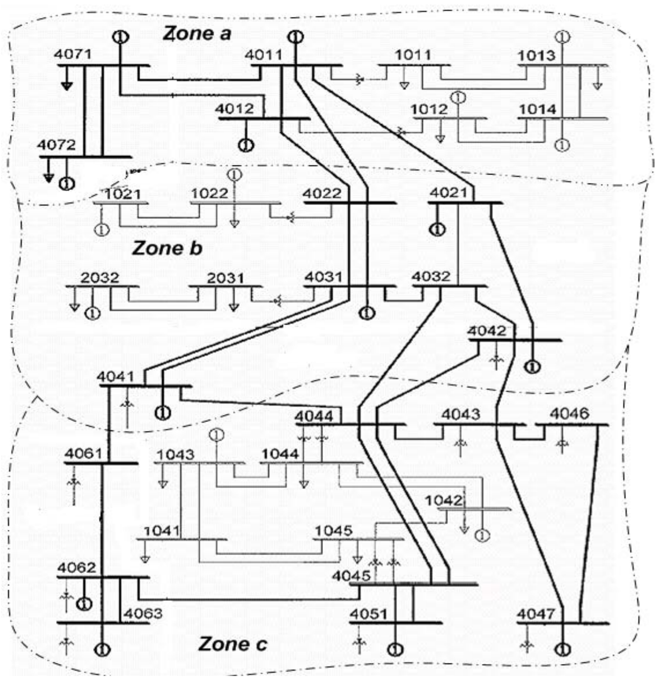

Figure 4 CIGRE 32-bus system.

For brevity of presentation, only two different operating scenarios are presented and discussed here, illustrating the performance, validity and robustness of the proposed reactive power dispatch model. The first scenario is a base loading condition, whereas the second is a "stressed" operating condition in which the system loading level is increased by 10% with respect to base load, with one generating unit out of service. For each of these scenarios, the following test cases are studied:

- *Case I ("standard" reactive power dispatch):* In this case, common ISO-practices are applied, where the real power dispatch from energy market clearing is used to solve an ac-power flow to determine the required reactive power dispatch. In most cases, operators use their own experience to "tune" the ac-power flow until a feasible and secure solution that does not violate voltage and line flow limits is achieved. Hence, this approach is simulated here to obtain a "standard" reactive power dispatch, and the associated payments are then calculated based on the proposed payment function J_1 in (2).
- *Case II (proposed reactive power dispatch):* In this case, the real power dispatch from energy market clearing, together with the set of contracted generators and the four reactive power price components from a procurement stage, are used to solve the proposed reactive power dispatch model. The solution of this model simultaneously yields the required reactive power dispatch and the associated payment components. In order to demonstrate the generality of the proposed dispatch model, the results are compared for two objectives, one with the function J given in (2), denoted here by Case II.a, and the second with the more "realistic" objective J^* given in (3), denoted

here by Case II.b.

- *Case III ("ideal" reactive power dispatch):* This case simulates an ideal scenario in which a "typical" security-constrained ac-OPF, minimizing the total real power cost, is used to simultaneously dispatch P and Q [34]. This would be an ideal solution because it achieves the least-cost solution; however, such an approach is not used by ISOs in practice, because of the complexity associated with solving a coupled, large-scale, non-convex NLP model every few minutes. Furthermore, possible adverse effects on market prices associated with simultaneous dispatch of real and reactive power within a competitive market environment could be a significant problem [14]. Once the reactive power dispatch is obtained for this ideal scenario, the associated reactive power payment is calculated based on the proposed J_1 in (2).

In the above cases, the four reactive power price components are assumed to be available from a given reactive power procurement stage (e.g. [17]); these are given in Table 1, and are assumed to remain unchanged for all test cases. A market clearing price ρ_{MC} = 100 \$/MWh is assumed, which is a typical "high" price figure in the Ontario electricity market. The prices of energy balance services ρ_{B1} = 90 \$/MWh and ρ_{B2} = 110 \$/MWh are assumed to be pre-determined from a given energy balance auction, which can be assumed to be typical values, since these are usually around the value of ρ_{MC}.

Table 1 Contracted generators and the associated reactive power prices

Zone	Contracted Generators	Zonal Reactive Power Prices			
		ρ_o	ρ_1	ρ_2	ρ_3
a	4072	0.78	0.74	0.57	0.35
	4011				
	1013				
	1012				
b	4021	0.92	0.91	0.90	0.36
	4031				
	2032				
	1022				
	1021				
c	4063	0.85	0.53	0.81	0.26
	4051				
	1043				
	1042				

It should be noted that reactive power limits for the "standard" reactive power dispatch Case I, and the "ideal" reactive power dispatch Case III do not include the opportunity region of operation. This is due to the fact that in the "traditional" reactive power dispatch approaches, which both of these cases represent, generators are typically modeled using fixed reactive power limits (Q_{GR} in Figure 1). The re-defined reactive power dispatch proposed herein (Case II), on the other hand, is based on the concept of operating a generator in the opportunity region in return for adequate financial compensation. This allows for extended reactive power support from generators, which is important for power systems today, since grids are operating in more stressed conditions and hence closer to their limits.

5.1 Base Load Condition

The solution of the three case studies under base loading conditions is given in Table 2. A set of 13 generators out of 20 are assumed to be contracted for reactive power provision (Table 1). Generators with negative Q_G values are operating in the under-excitation region (Region I), while those in bold are operating in the opportunity region (Region III), which there are none in this case. For the purpose of the simulations presented here, the real power market clearing and dispatch, which is required to initiate the proposed reactive power dispatch procedure, is obtained using a "standard" dc-OPF model, which minimizes the cost of energy production, subject to system security constraints. Transmission losses are modeled as a function of generator shift factors and real power injections [35].

Table 2 Solution for the three cases under base loading condition

BUS	Case I AC-PF		Case II Proposed dispatch model				Case III AC-OPF	
			A		b			
	P_G (MW)	Q_G (Mvar)	P_G (MW)	Q_G (Mvar)	P_G (MW)	Q_G (Mvar)	P_G (MW)	Q_G (Mvar)
4072	1380.6	484	1380.6	179.3	1380.6	179.6	1590.6	394.5
4011	900 - 5.7	-100	900	-89.2	900	-77.5	539.8	-100
4021	270	-2.4	270	-30	270	-30	270	-30
4031	315	-23.1	315	-40	315	-40	315	-40
4063	1035.4	103.4	1035.4	92.4	1035.4	97.1	1080	106.2
4051	630	97.4	630	83.6	630	90.7	630	92.9
2032	760	168.9	760	10.9	760	15	765	23.3
1013	275.5	-50	275.5	0	275.5	-50	383.9	-50
1012	720	57.4	720	0	720	275.8	720	70
1022	225	-25	225	0	225	0	225	-25
1021	350.1	51.7	350.1-7.1	0	350.1	0	430.2	84.4
1043	180	69.2	180	55	180	63.9	180	65.1
1042	360	-37.4	360	-40	360	-38.9	360	-38.8

It can be seen from the results in Table 2 that in Case I, when reactive power is dispatched using an ac-power flow (AC-PF), a 5.7 MW reduction in the real power of the slack bus (4011) takes place to adjust for the lower losses. Note also that the same P_G dispatch (obtained from dc-OPF) applies to both Case II.a and Case II.b. None of the generators are operating in Region III for these sub-cases, and hence ΔP_G is zero for all the generators; however, due to the fact that real power generation levels are kept constant and there is no slack bus in Case II, a downward balance service of 7.1 MW is required at generator Bus 1021 for Case II.a to account for the reduction in the total system losses. The difference in Q_G between these two sub-cases is due to the difference in payment structure of the two objective functions. Finally, observe the significantly different P_G and Q_G values obtained from the security constrained ac-OPF approach in Case III, which minimizes the total real power generation costs.

Observe in Table 2 that there are significant differences in reactive power dispatch using the proposed approach (Case II) with respect to more "traditional" techniques (Cases I and III). This is a notable change arising from the proposed philosophy of reactive power dispatch, which is a basic paradigm shift.

The reactive power payment, the balance payment, and the total system losses for all cases are given in Table 3. Observe that the reactive power payment resulting from the proposed dispatch model (Case II) is the lowest, since the objective function is to minimize reactive power payments. It is to be noted that this base-load scenario does not induce any reactive power dispatch of generators in the opportunity region, and therefore the difference in opportunity payment components between Cases II.a and II.b is not clearly brought out. A stressed operating condition is considered next to clearly bring out this issue.

Table 3 Losses and payment components for the three cases under base loading condition

	Case I AC-PF	Case II: Proposed dispatch model		Case III AC-OPF
		a	b	
Losses (MW)	433.5	432.2	439.2	424.4
Q-Payment ($/h)	1,900	1,570	1,780	1,920
Balance-Payment ($/h)	515	640	0	0

The balance payment associated with Case II.a is due to the downward balance service required at generator Bus 1021 illustrated in Table 2, which arises from the need to account for the change in system losses associated with the corresponding reactive power dispatch. This balance-payment is lower in Case I, where a 5.7 MW reduction in the losses are accounted for by Generator 4011 (slack bus). Note that Case III yields the lowest value of the system losses, since both P_G and Q_G are simultaneously dispatched using an ac-OPF. On the other hand, the system losses in Case II.a are lower than that in Case II.b, since for the former the objective function J explicitly includes a loss-payment component.

5.2 Stressed Operating Condition

Table 4 depicts the dispatch results obtained for the stressed system conditions. Observe in this case that the reactive power requirements have significantly increased for all the studied cases. In Case I, a re-scheduling in real power generation of three generators is needed to achieve a feasible power flow solution; in this case, the 9 MW (1.25%) reduction in P_G from Generator 1022 is picked up by Generators 4072, and an additional 9.7 MW is supplied by the slack bus 4011 to account for the increase in system losses associated with the reactive power dispatch.

Table 4 Solution for the three cases under stressed condition

BUS	Case I AC-PF		Case II Proposed dispatch model				Case III AC-OPF	
	P_G (MW)	Q_G (Mvar)	a P_G (MW)	b Q_G (Mvar)	a P_G (MW)	b Q_G (Mvar)	P_G (MW)	Q_G (Mvar)
4072	2310.3+9	303.5	2310.3	319.3	2310.3	297.4	2279.3	422.3
4011	900+9.7	-100	900	-100	900	-32.5	804.1	-100
4021	270	-30	270	-30	270	-30	270	-30
4031	315	-40	315	0	315	0	315	128.8
4063	1080	152.7	1080	156.8	1080	159.1	1080	163.6
4051	630	247.9	630	98.5	630	95.6	630	122.8
2032	765	175.6	765	201.1	765	179.9	765	212.5
1013	350.1	236.7	350.1	76.9	350.1	45.7	481.9	-50
1012	720-9	349	720-11.7	**367**	720-11.7	**367**	720	348.7
1022	0	0	0	0	0	0	0	0
1021	391.4	88	391.4	156.5	391.4	157.8	540	213.6
1043	180	87.2	180	87.2	180	87.2	180	87.2
1042	360	16.3	360	0	360	0	360	-7.8
1041	*0*	*0*	*11.4*	*0*	*12.1*	*0*	*0*	*0*

In Cases II.a and II.b, the generator at Bus 1012 is dispatched in the opportunity region (shown in bold), and consequently a 11.7 MW (1.63%) reduction in its real power output is necessary to maintain reactive power generation within its field limits. This re-scheduling in real power is compensated by an upward balance service of 11.4 MW in Case II.a and 12.1 MW in Case II.b, both at load Bus 1041 (in italics in Table 4). The difference between the MW reduction and the balance service is accounted for by the change in the total system losses. In Case III, there is no re-scheduling of real power generation, since P_G and Q_G are simultaneously dispatched.

The reactive power and balance payments, as well as the total system losses for all cases are given in Table 5. Observe that the lowest reactive power payment is achieved in Case II.a, with the payment being significantly different from that in Case II.b because of the difference in the opportunity payment component in the respective objective functions. Note that the balance payment component is present in Cases I and II due to the required re-scheduling in real power generation. The difference in the balance-payment for Cases II.a and II.b can be attributed to the slight difference in the total system losses associated with the reactive power dispatch.

Table 5 Losses and payment components for the three cases under stressed condition

	Case I AC-PF	Case II: Proposed dispatch model		Case III AC-OPF
		a	b	
Losses (MW)	567.5	557.4	558.2	575
Q-Payment ($/h)	2,420	2,190	3,340	2,510
Balance-Payment ($/h)	2,060	1,250	1,330	0

It is interesting to note that for the stressed system conditions, the proposed dispatch of Case II.a reduces system losses with respect to the ac-OPF dispatch of Case III. This could be attributed to the fact that the proposed dispatch model has the possibility to extend the reactive power limits to a generator's opportunity region, while an ac-OPF based reactive power dispatch constraints these to fixed rated limits.

Comparing the reactive power dispatch results of Case II.a for the two system conditions, it is observed that for base load (Table 2), 4 generators are operating in the under-excitation region, and 4 are not dispatched for reactive power support, while none are required to operate in Region III. On the other hand, from Table 4, a significant increase is observed in reactive power output from generators under stressed operating condition, as expected, since only two generators are operating in the under-excitation region, while the generator at Bus 1012 is operating in Region III. The total amount of injected reactive power into the system has increased from 220 Mvar at base loading condition to 1333 Mvar under stressed operating condition.

6 Application to Ontario

The main objective of this section is to implement and test the proposed competitive framework for reactive power dispatch in the Ontario electricity market, considering the existent pricing regulations for reactive power services in the province. Thus, reactive power supply zones and associated prices are first determined using the procurement methodology proposed in [17]. These prices are then used to dispatch the IESO's generators at minimum costs using the approach described herein. The system studied is the dispatch model of the IESO-controlled gird, which comprises numerous generators located in different transmission zones in Ontario. It is shown here that the proposed structure and models should enable the IESO to properly determine zonal reactive power prices ahead of real-time, allowing it to develop a minimum-cost reactive power dispatch procedure in real-time that also guarantees system security.

The IESO-controlled grid includes all transmission lines at voltage levels equal to or greater than 50 kV located within the Ontario control area, including all distribution lines and loads in Ontario. The total length of transmission lines is nearly 31,000 km and the installed generation capacity present within the control area amounts to 31,000 MW, with a peak system demand of approximately 27,000 MW [36]. The Ontario system interconnects with Michigan, Minnesota, New York, Manitoba, and Quebec. A reduced version of the IESO-controlled grid utilized for dispatch purposes by the IESO was used for the studies presented and discussed here. It consists of 2,833 buses and 4,205 branches, including transmission lines and two- and three-winding transformers.

The system peak load of 27,000 MW was used for the presented studies. Normal system conditions and several main critical contingencies were analyzed. The list of the most critical contingencies for the loading conditions studied here was provided by the IESO, comprising thirteen N-1 and N-2 cases; these include single and double bus outages as well as transmission line and transformer outages and special protection schemes.

The procurement procedure described in [17] was used to identify the reactive power supply zones, independently from the IESO predefined zones, as well as the associated zonal prices and "contracted" generators. From the 13 studied contingencies, the critical (worst) contingency corresponding to the minimum value of the maximum loadability of the system was indentified to be a double-bus outage of buses located near the Bruce power station in Western Ontario. The generators were then ranked with respect to their effect on the system maximum loadability and the reactive power output levels, yielding the five V-zones for the IESO-controlled grid depicted in Figure 5, where a few relevant generators are highlighted. Observe that the most important generators are located in Western Ontario, Southwestern Ontario and the Greater Toronto Area (GTA), whereas the least important are located in Northeastern and Northwestern Ontario.

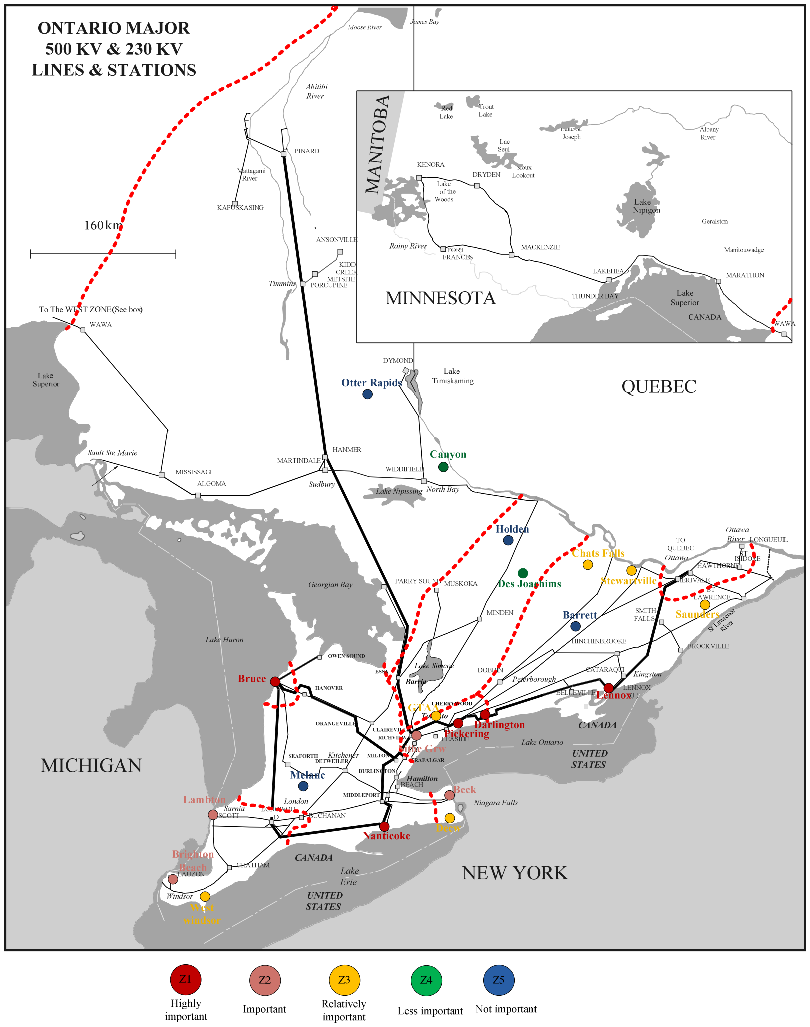

Figure 5 V-zones of the IESO-controlled grid

Two price components for reactive power services, i.e. for under-excitation operation and for over-excitation, were considered here, taking into consideration the IESO’s reactive power payment policies as per (3) with $\rho_0 = 0$ (no availability price is recognized in Ontario). The price offers used for each generator were obtained randomly from a uniformly distributed function in the range of 3 to 13 \$/Mvar. Generators operating in the opportunity region were assumed to also receive a payment of ρ_{MC} = \$120/MWh, which is a typical Hourly Ontario Energy Price or HOEP for peak-load periods. Table 6 shows the zonal reactive power prices obtained for the under-excitation and over-excitation regions. It is interesting to note that there are 47 generating units within Ontario that are forced to operate in the opportunity region in this case.

Table 6 Zonal reactive power prices

V-zone	Under-excitation Prices ρ_1 (\$/Mvar)	Over-excitation Prices ρ_2 (\$/Mvar)
1	10.21	10.32
2	10.67	10.24
3	9.71	10.52
4	11.07	9.98
5	10.98	11.24

Based on the V-zones and the corresponding reactive power prices obtained from the procurement studies, the dispatch optimization problem was then solved. Since no slack buses are considered in this model, balancing services account for the need of any real power rescheduling associated with system losses changes and generators operating in the opportunity region (Region III). The upward and downward balancing services were assumed to be priced at ρ_{B1} = \$90/MWh, and ρ_{B2} = \$110/MWh, respectively, which are typical operating reserves costs in Ontario (these costs are the result of the operating reserve prices plus the HOEP). A zonal reactive reserve limit of 70% was assumed to ensure sufficient var reserves for each V-zone, and a maximum 15% MW deviation of each generator from the dispatched value was assumed.

Table 7 presents a summary of the Q-dispatch solution obtained. In this case, 29 generators operate in Region III, and there is a total reduction of 143 MW in active power with respect to the original system dispatch; the upward balancing services provide 88 MW and the system losses are reduced by 55 MW.

Table 7 Dispatch problem main results

VARIABLE	VALUE
Active power reduction (MW)	143
Upward balance services (MW)	88
Downward balance services (MW)	0
Losses reduction (MW)	55
Reactive power payment (\$)	18,681
Opportunity costs (\$)	17,160
Balance service payment (\$)	9,685

The procurement and dispatch models were implemented in AMPL and solved using the KNITRO and IPOPT solvers. The iterative solution process described in the Appendix was applied to the MINLP procurement and dispatch models. This requires a reasonable initial guess; thus, an ac power flow solution was used to obtain the initial reactive power outputs of all generators. The solution of the MINLP dispatch model required more iterations per NLP problem and significantly more epochs, where an epoch refers to the stage where the algorithm checks if the solution improves when the reactive power operating region of a given generator changes; this is to be expected, since active power needs to be re-dispatched in this case. The first epoch required significant effort to solve, since many generators (up to 100) reached the reactive power limits of the initial reactive power operating region. The following epochs ran faster since fewer generators (as low as 25) reached their limits. The average total CPU time needed to obtain a solution of the dispatch model was approximately 60 min. Table 8 summarizes the computational burden for both models. Observe that this CPU times were obtained using "off-the-shelf" programs and solvers, which are certainly not designed and optimized to solve the particular problems at hand; furthermore, the studies were performed for the worst system loading conditions. Hence, it is reasonable to expect that the CPU times could be significantly reduced in an actual implementation of the proposed methodologies, so that their application should be feasible in practice.

Table 8 Typical computational burden for the solution of the MNLP optimization models using AMPL

Model	Range of Number of Iterations per Epoch	Range of CPU Times per Epoch [s]	Number of Epochs	Total Average CPU Time [min]
Procurement	74-94	13-16	1	0.25
Dispatch	91-205	660-1100	3-5	60

7 Conclusions

This paper has re-defined the reactive power dispatch problem from the perspective of an ISO operating in competitive electricity markets. A novel reactive power dispatch model that incorporates the ISO's composite payment burden associated with the provision of reactive power support, while considering all operating aspects pertinent to a competitive environment, was proposed and studied.

From the analysis of results obtained from various case studies, it was shown that the proposed reactive power dispatch model yields the lowest reactive power payments amongst all cases considered. It was also observed that the proposed reactive power dispatch approach yields better overall results than current dispatch practices. However, an integrated ac-OPF-based real and reactive power dispatch approach was better, as expected, even though it might not be practical at the moment. Finally, the application of the proposed procedure to the Ontario system and market showed its feasibility for practical applications.

The proposed technique seeks to minimize the total payment by the ISO to reactive power providers, while ensuring a secure and reliable operation of the power system, and presents a reasonable compromise between current and ideal reactive power dispatch practices. One important contribution of this model is that it considers the effect of reactive power on real power by internalizing the calculation of the reduction in real power output of a generator due to an increase in its reactive power supply. Furthermore, in order to ensure that re-scheduling of real power because of reactive power supply requirements from generators is kept at a minimum, a payment component for balance services is included in the objective function.

8 Appendix

The steps of the proposed GRPC algorithm are as follows [17]:

1. Start with a feasible initial reactive power dispatch, which can be obtained from a power flow solution associated with the real power dispatch obtained from an energy market clearing process.

2. Use this initial reactive power dispatch to obtain an initial classification for the regions of reactive power operation for each generator. For example, if the initial Q_{Gg} is less than zero, the generator is operating in Region I, and hence, Q_{G2} and Q_{G3} for this generator will be zero, and Q_{G1} will be a variable within its lower and upper limits; similarly for Regions II and III.
3. Solve the NLP formulation of the proposed dispatch model (2)-(15) using this initial classification, where the binary variables are no longer needed, since for each generator only one region of reactive power operation is identified.
4. Check if Q_G for any generator g has reached its limits for its region of operation. For example, if Q_G of a generator g in Region I is at its upper limit, the problem is re-solved (Step 3) with this generator operating in Region II, i.e. Q_{G1} and Q_{G3} for this generator are zero, and Q_{G2} is now a variable within its lower and upper limits. A similar update procedure is used for generators in Regions II and III, which are operating at their limits.
5. A new optimal value of the objective function (2) or (3) is calculated. If this new value is higher than the previous one, this generator is now considered to be in Region II; otherwise, the generator is kept in Region I. The solution is only updated if an improved value of the objective function is achieved.
6. The algorithm terminates after a certain number of iterations or if no further improvements in the objective function are achieved.

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