A Procurement Market Model for Reactive Power Services Considering System Security

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Abstract— This paper proposes a two-level framework for the operation of a competitive market for reactive power ancillary services. It is argued that the first-level, i.e. reactive power procurement, be on a seasonal basis while the second-level, i.e. reactive power dispatch, be close to real-time operation. To this effect, a reactive power procurement market model is proposed here taking into consideration system security aspects. This procurement procedure is based on a two-step optimization model. First, the marginal benefits of reactive power supply from each provider with respect to system security are obtained by solving an optimal power flow (OPF) that maximizes system loadability subject to transmission security constraints imposed by voltage limits, thermal limits and stability limits. Second, the selected set of generators is then determined by solving an OPF-based auction to maximize a societal advantage function comprising generators' offers and their corresponding marginal benefits with respect to system security, considering all transmission system constraints. The proposed procedure yields the selected set of generators and zonal price components, which would form the basis for seasonal contracts between the system operator and the selected reactive power service providers.

Index Terms- **System operation, ancillary services, electricity markets, pricing, reactive power management, system security.**

I. Introduction

RECENTLY, there has been a significant interest in reactive power as one of several ancillary services required to ensure system reliability and security. System operators and researchers have been looking for appropriate mechanisms for reactive power provision in the context of deregulation [1]-[4]. However, there are several issues concerning the existing provision policies and payment mechanisms for reactive power services that impede the full development of a competitive market. Hence, this paper proposes a possible market structure and related techniques to address some of the main issues associated with reactive power provision in competitive electricity markets.

In general, there are two classes of problems when analyzing reactive power provisions in the context of deregulated electricity markets, namely, *reactive power procurement* and *reactive power dispatch*. Reactive power procurement is essentially a long-term issue, i.e. a problem in which the independent system operator or ISO seeks optimal reactive power "allocations" from possible suppliers that would be best suited to its needs and constraints in a given season. This optimal set should ideally be determined based on demand forecasts and system conditions expected over the season. The criterion to be used for such procurement could be varied, but should essentially take into consideration the cost/price offers of reactive power provision. Some researchers have examined the reactive power procurement problem using optimal power flow (OPF) frameworks [5]-[10]. Reactive power dispatch, on the other hand, corresponds to the short-term, "real-time" allocation of reactive power generation required from suppliers based on current operating conditions. In this case, the ISO is interested in determining the optimal reactive power schedule for all providers based on a given objective that depends on system operating criteria, such as minimization of total system losses [11], minimization of reactive power cost [12]-[14], minimization of deviations from contracted transactions [15], or maximization of system loadability to minimize the risk of voltage collapse [16], [17].

Most of the reported works on reactive power management focus either on developing suitable pricing methods that can effectively reflect the cost of reactive power production [6], [7], [14], [18], or proposing appropriate models for optimal reactive power procurement and/or dispatch as discussed above. These models usually aim to achieve the extremum of a certain objective function (e.g. reactive power production cost minimization or social welfare maximization) using OPF models. An important requirement that has not been addressed in most of the existent or proposed models is the inclusion of system security in the reactive power procurement/dispatch process. The ISO typically seeks a reactive power solution that does not violate transmission security constraints, which are usually represented by voltage, thermal and stability limits [19].

In this paper, a unified framework for reactive power management in deregulated electricity markets using a two-settlement model approach is first presented. The proposed model works at two hierarchical levels and in different time horizons; the first level is the procurement market model which works in a seasonal time horizon, while the second level

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is the dispatch model which works in a 30 minutes to 1 hour window. The paper then presents the detailed formulation of a reactive power procurement market model, which corresponds to the first level of the proposed two-settlement model. The procurement market model aims at achieving optimal and secure reactive power provision that ensures a reliable and efficient network operation, while taking into account various market related issues. The detailed formulation of the second level (dispatch) of the proposed scheme will be reported in a future paper.

It is important to highlight the fact that, in this paper and adhering to existing FERC regulations, only reactive power support from generators is considered, as one of the six ancillary services eligible for financial compensation required to ensure system reliability and security. However, the proposed formulation is fairly generic and hence should be readily extendable to include other reactive power resources in addition to generators, such as capacitor banks and FACTS controllers, as recommended in [3]; this will be the focus of a future paper.

The rest of the paper is organized as follows: In Section II, a two-settlement reactive power management framework is discussed. A proposed model for reactive power service procurement is presented in Section III. The proposed reactive power procurement market model is then applied and tested on the CIGRE 32-bus system in Section IV, where two case studies are presented and discussed. Finally, in Section V, the main contributions of the paper are summarized.

II. Two-Settlement Reactive Power Management Framework

Currently, most power system operators use power flow studies to arrive at reactive power dispatches, primarily relying on operational experience. However, there are several complex issues involved in reactive power management in deregulated electricity markets which call for further systematic procedures to arrive at better solutions. These issues include market power being exercised by some reactive power service providers, considering the localized nature of reactive power support; the effect of reactive power on active power generation and on system security; and the possibility of reactive power price volatility when it is in the same time-frame as the spot energy market.

In general, many of the aforementioned issues can be resolved if reactive power services are optimally procured through long-term agreements between the ISO and the service providers. These long-term contracts would likely reduce the possibility of exercising market power by generators, and at the same time could solve the problem of price volatility that arises when reactive power services are priced on a real-time basis. This argument is supported by economic theories and empirical evidence [20]. Thus, in this work, a seasonal auction mechanism for reactive power procurement is proposed to determine the optimal set of forward contracts of the ISO with reactive power providers. These contracts then remain in place for the season, with pre-determined prices of reactive power.

A two-settlement reactive power management scheme is proposed here comprising two major activities at different hierarchical levels as depicted in Fig. 1. The first level consists of a long-term procurement market on a seasonal basis. In the second level, the ISO carries out the actual reactive power dispatch in a time frame of 30 minutes to 1 hour ahead of real-time, by solving an OPF with an appropriate objective function. Figure 1 illustrates how the active and reactive power markets can be decoupled from each other, placing them in different operating time frames, so that the ISO does not handle a reactive power auction in the same time frame as that of a real power auction. This minimizes the risk that might arise from price volatility, and thus help reduce market inefficiencies.

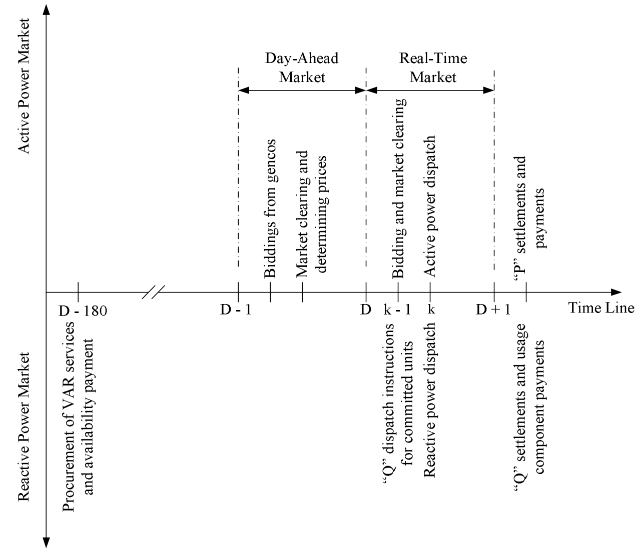

Fig. 1. Active and reactive market clearing and dispatch at day D and hour k.

The decoupling of active and reactive power, also suggested in [11] and [12], implies that the OPF problem can be separated into two parts: an active power sub-problem that provides the active power dispatch and prices in real-time based on a cost minimization (or social welfare maximization) market settlement model; and a reactive power sub-problem, operating on different timeframes, that provides reactive power contracts, prices, and dispatch levels based on given optimization criteria. It is important to mention that the solution obtained from a coupled OPF model simultaneously dispatching active and reactive power is theoretically closer to the optimal. However, in addition to the market power and price volatility problems associated with handling them simultaneously, computational burden becomes an issue for practical sized power systems, since it would require solving a rather complex and large-scale non-linear programming (NLP) model. Decoupling the OPF problem provides the required flexibility for spot market applications, and avoids having to deal with the coupled model complexity, while retaining an acceptable level of accuracy.

III. Procurement Market for Reactive Power Services

As mentioned in Section II, the first level in the proposed hierarchical reactive power management scheme is the design of a procurement market model, which is the subject of this paper. The objective of the ISO in this case is essentially to

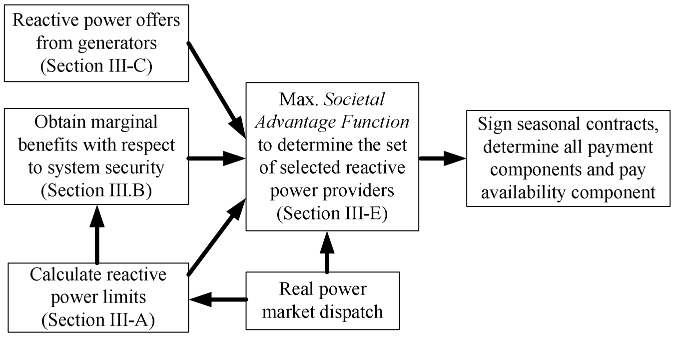

Fig. 2. Long-term procurement market for reactive power.

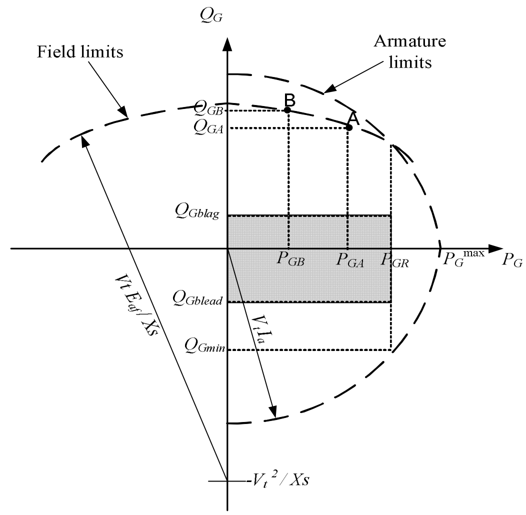

Fig. 3. Synchronous generator capability curve.

define and procure adequate long-term reactive power supplies for the system. The proposed procurement market would work as follows (see Fig. 2):

- The ISO calls for reactive power offers from the reactive power providers. The structure of these offers should ideally reflect their cost of providing reactive power; this issue is discussed in more detail below.
- Based on the received offers, the ISO carries out an auction settlement, i.e. solves an optimization model to maximize a societal advantage function (SAF) subject to system constraints that include system security.
- This procurement market settlement, i.e. the solution of the optimization model, yields a set of contracted generators, as well as the price components of reactive power. The contracted providers will have a seasonal obligation for reactive power provision, and receive an availability payment.

A. Determine Reactive Power Ancillary Service Limits

When real power and terminal voltage are fixed, the armature and field winding heating limits determine the reactive power capability of a generator [21]. These limits are illustrated in Fig. 3, where V_t is the voltage at the generator terminal bus, I_a is the armature current, E_f is the excitation voltage, X_s is the synchronous reactance, and P_G and Q_G are the real and reactive power outputs of the generator, respectively. The generator's MVA rating is the point of intersection of the two curves, and therefore its real power rating is given by P_{GR}. At an operating point A, with real power output P_{GA} such that $P_{GA}<P_{GR}$, the limit on Q_G is imposed by the generator's field winding heating limit; whereas, when $P_{GA}>P_{GR}$, the limit on Q_G is imposed by the generator's armature winding heating limit.

Three regions for reactive power generation can be identified in Fig. 3. In Region I ($Q_{Gmin} \le Q_G = Q_{G1} \le 0$), the generators are required to mandatorily provide a base leading reactive power support (Q_{Gblead} to 0). Any reactive power provided beyond Q_{Gblead} is eligible for an under-excitation payment component as an ancillary service. Such *mandatory* and *ancillary* classification of reactive power capability is in line with what most ISOs currently have in place for reactive power management (e.g. IESO Ontario requires 0.9 lead to 0.95 lag power factor). In the same way, in Region II ($0 \le Q_G = Q_{G2} \le Q_{GA}$), the mandatory lagging reactive power requirement is from 0 to Q_{Gblag}, and any reactive power provision beyond Q_{Gblag} is recognized as an ancillary service, and thus eligible for a payment for the increased losses in the windings; this payment component is referred to as *cost of loss payment*, and is currently recognized by most ISOs. The shaded area in Fig. 3 represents the mandatory base reactive power provision range set by the system operator. Because of the requirement that the reactive power generated should be within or on the limiting curve, in Region III ($Q_{GA} \le Q_G = Q_{G3} \le Q_{GB}$), any reactive output increase requested by the ISO beyond Q_{GA} will require a decrease in active power generation, and hence an *opportunity cost payment* to the reactive power service providers is expected.

B. Marginal Benefits of Reactive Power Supply with Respect to System Security

The idea of looking at the marginal benefits of reactive power has been proposed in [9], where the Lagrange multipliers associated with a loss-minimization model were used to represent these marginal benefits. However, it has been widely recognized by ISOs that system security, and particularly the impact of inadequate reactive power support on security, are important issues to be considered in system operation. In this context, a reactive power procurement model based on the marginal contributions of reactive power to system security is proposed in this paper.

The ISO needs to check the technical feasibility of potential transactions after energy market settlement; only those transactions that are within the transfer capabilities of the network are allowed. This is particularly important when dealing with reactive power, since it has a direct bearing on system security, and hence the power transfer capabilities of the transmission system [22].

Typically, the transfer capabilities of the system in main transmission corridors are defined using the concept of Available Transfer Capability *(ATC)*, which is in turn defined as the remaining transfer capability of the transmission system for further commercial activity over and above already committed uses [23], and is commonly expressed as:

$$ATC = TTC - TRM - ETC - CBM \tag{1}$$

where TTC is the Total Transfer Capability; TRM is the Transmission Reliability Margin, which is typically assumed to be a fixed value (e.g. 5% of TTC under normal operating conditions in the Western Electricity Coordinating Council *WECC* [24]); ETC is the Existing Transmission Commitments; and CBM is the Capacity Benefit Margin and is usually included in the ETC.

The TTC is typically defined as follows:

$$TTC = Min\left\{P_{Max_{I\lim}}, P_{Max_{V\lim}}, P_{Max_{S\lim}}\right\} \tag{2}$$

where $P_{Max_{I\lim}}$, $P_{Max_{V\lim}}$ and $P_{Max_{S\lim}}$ are the maximum powers the system can securely transmit considering thermal limits, voltage limits, and stability limits, respectively, based on at least an N-1 contingency criterion.

Currently, electricity markets are usually operated under stressed loading conditions due to the increased demand and power transfers, thereby increasing the probability of the system experiencing stability problems. Under such conditions, system stability limits can be approximated through voltage stability limits [25]. In this context, the TTC can be evaluated using the system Loading Factor (LF), which is defined as the amount of additional loading of a given transmission corridor for a given dispatch pattern that does not violate thermal limits, bus voltages limits or voltage stability limits [26], and can be expressed as:

$$LF = LF_c - LF_0 \tag{3}$$

where LF_0 is the existing loading (the ETC), while LF_c is the system loading at the maximum loading point. Hence, using a $TRM = 0.05*TTC$, the ATC can be approximately expressed in terms of LF as follows:

$$ATC \approx 0.95LF_c - LF_0 \tag{4}$$

It is worth mentioning here that the concept of a "system-wide" ATC, as proposed in [25], has been adopted in this paper. This approach, however, does not preclude using (4) to determine the ATC values of particular transfer corridors, since this can be readily accomplished by properly defining power "sources" and "sinks" in the computation of LF_c.

Typically, an N-1 contingency criterion is used in the TTC calculation, which consists of studying single contingency cases one by one. For each contingency, LF_c is calculated, and the minimum LF_c defines the "worst" contingency.

The LF_c can be readily obtained from the system PV curves, as this represents the change in load between the operating point and the nose of the curve, which corresponds to how much the system may be loaded before reaching its thermal, voltage magnitude or voltage stability limits. A typical PV curve is illustrated in Fig. 4 showing the relation between the aforementioned ATC definitions and its relation with the system LF. This PV curve can be obtained using Continuation Power Flow (CPF) methods [26], which allow calculating complete voltage profiles, and hence determining the value of LF_c for a given "direction" of generation and loading increase considering different contingencies. On the other hand, the LF_c can also be computed by reformulating the conventional OPF with the objective function of maximizing LF [27], [28]. OPF-based models not only yield the value of LF_c, but also provide Lagrange multipliers that can be used as sensitivities equivalent to those computed by solving a CPF [29]. System security can, therefore, be introduced in the reactive power procurement market model by solving the following OPF model:

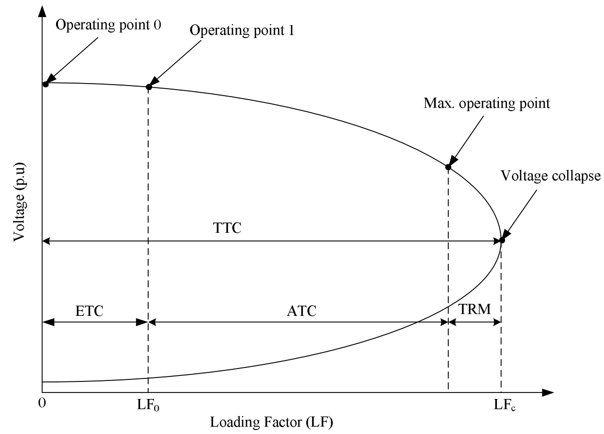

Fig. 4 A Typical PV curve.

$$\max. \quad LF \tag{5}$$

$$\text{s.t.} \quad P_{Gi}(1 + LF + K_G) - P_{Di}(1 + LF) = \sum_j V_i V_j Y_{ij} \cos\left(\theta_{ij} + \delta_j - \delta_i\right) \qquad \forall i \tag{6}$$

$$Q_{Gi} - Q_{Di}(1 + LF) = -\sum_j V_i V_j Y_{ij} \sin\left(\theta_{ij} + \delta_j - \delta_i\right) \; : \lambda_{gi} \qquad \forall i \tag{7}$$

$$Q_{Gg} \le Q_{Gg}^{\max} \quad : \gamma_g \qquad \forall g$$

$$Q_{Gg}^{\max} = \begin{cases} \sqrt{\left(\dfrac{V_{tg} E_{fg}}{X_{sg}}\right)^2 - \left(P_{Gg}\right)^2} - \dfrac{V_{tg}^2}{X_{sg}} & for\, P_{Gg} < P_{GR} \\ \sqrt{\left(V_{tg} I_{ag}\right)^2 - \left(P_{Gg}\right)^2} & for\, P_{Gg} > P_{GR} \end{cases} \tag{8}$$

$$Q_{Gg} \ge Q_{Gg}^{\min} \quad : \mu_g \qquad \forall g \tag{9}$$

$$V_i^{\min} \le V_i \le V_i^{\max} \qquad \forall i \tag{10}$$

$$\left|P_{ij}\left(V,\delta\right)\right| \le P_{ij}^{\max} \qquad \forall ij \tag{11}$$

$$P_{Gi}(1+LF+K_G) \le P_{Gi}^{\max} \qquad \forall i \tag{12}$$

$$(Q_{Gg}-Q_{Gg}^{\min})\cdot v_{ga} \le 0 \quad \forall g \tag{13}$$

$$(Q_{Gg}^{\max}-Q_{Gg})\cdot v_{gb} \le 0 \quad \forall g \tag{14}$$

$$V_g = V_{g0} + v_{ga} - v_{gb} \qquad \forall g \tag{15}$$

$$v_{ga}, v_{gb} \ge 0 \qquad \forall g \tag{16}$$

where g denotes a generator bus. The following are the model variables to be determined by the solution of the optimization problem:

LF: Loading factor.
K_G: Variable to model a distributed slack bus.
Q_{Gi}: Reactive power generation at bus i in p.u.
V_i: Bus i voltage magnitude in p.u.
δ_i: Bus i voltage angle in radians.
P_{ij}: Power flowing from bus i to bus j, in p.u.
v_{ga}, v_{gb} : Two auxiliary variables representing the changes in generator g bus voltage due to reactive power limits.

The following are the model parameters which are input into the optimization model:

P_{Gg}: Active power generation at bus g in p.u.
P_{Di}: Active power demand at bus i in p.u.
Y_{ij}: Element of admittance matrix in p.u.
θ_{ij}: Angle associated with Y_{ij} in radians.
Q_{Di}: Reactive power demand at bus i in p.u.
V_{tg}: Terminal voltage of generator g at which its capability curves are calculated, in p.u. (assumed here to be 1.05 p.u.)
I_{ag}: Rated armature current of generator g at which its capability curves are calculated, in p.u.
E_{fg}: Excitation voltage of generator g, in p.u.
X_{sg}: Synchronous reactance of generator g, in p.u.
Q_{Gg}^{min}: Minimum reactive power of generator g, in p.u.
V_i^{max}: Maximum allowable voltage at bus i, in p.u.
V_i^{min}: Minimum allowable voltage at bus i, in p.u.
P_{ij}^{max}: Maximum power flow from bus i to bus j, in p.u.
V_{g0}: Generator g terminal voltage corresponding to operating point 1 in Fig. 4, in p.u.

In the above OPF model, (6) and (7) are the nodal active and reactive power flow equations, where the variable K_G is used to model a distributed slack bus to be able to better represent the distribution of system losses. The field and armature winding heating limits are imposed by (8). Equation (10) constrains all bus voltage to be within appropriate limits, while (11) imposes transmission line thermal limits, with P_{ij} representing the power flowing from bus i to bus j. Finally, (12) guarantees that generator active power dispatch levels will not be exceeded.

In order to account for the effect of reactive power limits on generator voltage settings and properly model the generators' voltage regulators, constraints (13)-(16) are added to the model [30]. These constraints ensure that all the generators will be operating at their terminal voltage settings, defined by operating point 1 in Fig. 4, as long as the reactive power is within its limits; in this case, the two variables v_{ga} and v_{gb} will be equal to zero to satisfy (13) and (14). If the reactive power output of any of the generators hits its maximum limit, set by (8), constraints (14) and (16) will force v_{gb} to have a positive value, therefore reducing the voltage at this generator bus according to (15). Similarly, if the lower limit of reactive power output for any generator is reached, v_{ga} will have a positive value, hence increasing the voltage at this generator bus. Note that v_{ga} and v_{gb} may still have a zero value even if reactive power limits are reached; these variables only simulate the loss of voltage control due to limits.

The Lagrange multipliers that represent the marginal benefit/contribution of each reactive power source with respect to system security for the above model are λ_g, γ_g and μ_g. The Lagrange multiplier λ_g is the dual of the nodal reactive power balance constraint (7), denoting the sensitivity of the system security (LF) to a change in reactive power demand at a bus i; γ_g is the dual of reactive power constraint (8) of generator g, indicating by how much LF will change for a unit change in reactive power capability of this generator; and μ_g is the dual of the under-excitation constraint (9). Accordingly, all of the three Lagrange multipliers will have a zero value for any generator as long as its Q_G lies within the limits given by (8) and (9); whereas, either γ_g or μ_g will have a non-zero value for any generator if its Q_G touches the upper or lower limits, respectively, and the corresponding λ_g in this case will be equal in magnitude but might be with opposite sign. This is due to the fact that if Q_G for any generator is within its limits, it will be capable of compensating for any increase in reactive power demand at this generator bus without affecting LF, and hence λ_g will be zero in this case. Notice that the sign of these multipliers depends on the nature of the optimization problem, and the way the associated constraints are treated in the solution process.

Observe that the optimization model (5)-(16) is solved for each significant contingency to determine the Lagrange multipliers associated with the worst contingency, as per the N-1 contingency criterion. It is important to highlight the fact that there might exist other contingencies other than the worst one, where some generators' reactive power outputs may have more significant effect on system loadability, i.e. higher values of λ_g, γ_g and μ_g; however, according to NERC's security criterion, operators are not required to act unless the worst contingency conditions are violated. Hence, a contingency that does not violate security limits, regardless of the positive effect that Q_G of a particular generator may have on improving system security for that given operating condition, is not relevant for the purpose of improving system security from the N-1 contingency criterion point of view. It should also be highlighted, as explained further below, that this proposed procurement model is solved for multiple operating conditions that are representative of the given season of interest, and hence a variety of worst contingencies and associated sensitivities are taken into account in the proposed procurement process.

C. Reactive Power Offers from Generators

The different reactive power cost components discussed in Section III-A form the basis for the procurement procedure proposed here. Hence, the reactive power price offers to be submitted by generators should comprise the following three parts (Fig. 5) [10]:

- Availability price offer (m_o, \$): A fixed component to account for that portion of a supplier's capital cost that can be attributed to reactive power production.
- Cost of loss offer (m_1, m_2, \$/Mvar): An assumed linearly varying component to account for the increased winding losses as reactive power output increases, in the under- and over- excitation ranges respectively [9].
- Opportunity offer (m_3, \$/Mvar/Mvar): A quadratic component to account for the lost opportunity cost when a supplier is constrained from producing its scheduled real power in order to increase its reactive power production. This is based on the assumption that active power costs are parabolic functions of output power, which may be considered to approximately change linearly with reactive power in Region III (from point A to B in Fig. 3).

Based on this offer structure, the long-term nature of the proposed procurement market model, and the local characteristics of reactive power, reactive power prices for each of the components of the reactive power offers may be determined.

D. Reactive Power Prices

In case of traditional offer-based commodity markets, the two pricing approaches usually adopted are: *pay as bid* (first price auction), where selected participants are paid as per their respective bid; or *uniform price market* (second price auction), where all selected participants are paid a uniform price, which is the price of the highest accepted offer. It has been argued by economists that the uniform price markets provide an incentive to participants to bid their true costs and hence such auctions promote competition.

Applying the uniform price to reactive power markets would be a natural extension to the real power auction mechanisms already existing. However, given the localized nature of reactive power control [31], and the issues of market power associated with the limited number of providers in reactive power markets, it would be more pertinent to disaggregate the uniform price of reactive power into zonal components. This has been argued earlier in [32], where the power system is split into different voltage control areas or zones based on electrical distances or sensitivity analyses. Such a *zonal uniform price* mechanism for reactive power markets would reduce the impact of market power exercised by certain gaming generators, and should hence restrict them only to their given zones. Zonal pricing is also discussed in detail in the context of a general market structure for reactive power services in [33].

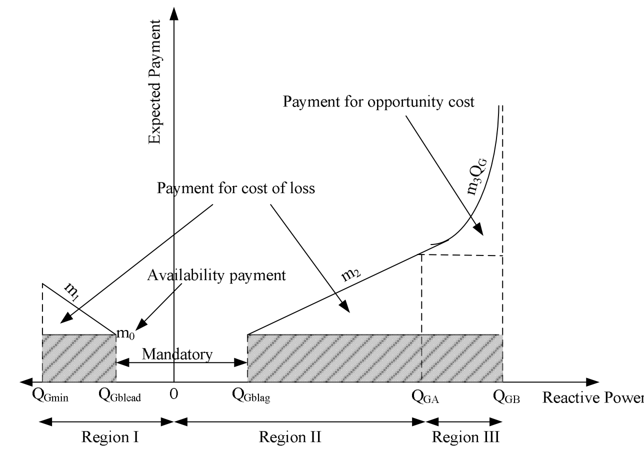

Fig. 5. Reactive power offers from generators and the three operating regions.

E. Societal Advantage Function Maximization

Once the reactive power ancillary service limits and the marginal benefits (λ, γ and μ) of each provider with respect to system security are determined, as explained in Section III-B, and reactive power offers are received, the ISO is in a position to carry out a procurement market settlement where its sole objective is to maximize a Societal Advantage Function or *SAF*. The classical concept of social welfare from economic theory is extended here to formulate a reactive power *SAF* which is based on the determination of aggregate system benefits accrued from reactive power services minus the expected payment by the ISO. The proposed SAF is formulated on a zonal basis and can be expressed as follows:

$$\begin{aligned} SAF_k = & -\sum_{g\in K}\rho_{ok} - \sum_{g\in K}\left(C_L\left|\mu_g\right| - \rho_{1k}\right)\left(Q_{G1g} - Q_{Gblead_g}\right) \\ & + \sum_{g\in K}\left(C_L\left|\lambda_g\right| - \rho_{2k}\right)\left(Q_{G2g} - Q_{Gblag_g}\right) \\ & + \sum_{g\in K}\begin{pmatrix}\left(C_L\left|\gamma_g\right| - \rho_{2k}\right)\left(Q_{G3g} - Q_{Gblag_g}\right) \\ -0.5\rho_{3k}\left(Q_{G3g} - Q_{GAg}\right)^2\end{pmatrix} \end{aligned} \tag{17}$$

In (17), the subscript g denotes a generator in the system, while K refers to the set of generators in zone k, considering that the system is divided into voltage control zones. The three reactive power generation components Q_{G1}, Q_{G2}, and Q_{G3} for a generator g correspond to the three region of operations defined earlier in Section III-A and shown in Figs. 3 and 5. Observe in (17) that only reactive power generation beyond the mandatory region, i.e. between Q_{Gblead} and Q_{Gblag}, is considered for financial compensation of the generators, as per the payment structure shown in Fig. 5. It can also be noticed that, for (17) to hold correctly, the quadratic term ($Q_{G3} - Q_{GA}$) for each generator should exist only if Q_{G3} of this generator has a value, i.e. the generator is operating in Region III. Hence, a binary variable should be assigned to Q_{G3} in order to represent such a condition. The variables ρ_{1k} (in \$/Mvar) and ρ_{2k} (in \$/Mvar) are the under- and over-excitation prices for reactive power in zone k, respectively; similarly ρ_{3k} (in \$/Mvar/Mvar) is the zonal uniform opportunity price component; and ρ_{ok} (in \$) is the zonal availability price component. The constant C_L is a "loadability" cost parameter (in \$/MWh) denoting the economic worth of increasing the system loadability, which represents, in this model, the expected worth of active power for the season of interest, and hence can be defined by the ISO using historical data and appropriate forecasting methods; in this paper, C_L is assumed to be equal to 100 \$/MWh, which is a typical "high" price figure in the Ontario electricity market.

Observe that in (17), *SAF* is not solely a function of reactive power price components, as in the case of traditional social welfare functions used in energy market auctions; rather, *SAF* is based on the notion of a "marginal security benefit", i.e. the economic worth of reactive power support with respect to system security. For example, λ_{gk} denotes the change in *LF* per Mvar change in reactive power demand at a bus *g* in zone *k*; since *LF* is dimensionless, λ_{gk} is scaled by the total system MW demand, resulting in MW/Mvar units. Hence, the term $C_L|\lambda_{gk}|$ represents the hourly marginal benefit to the ISO in \$/Mvar, with respect to system security, from a change in reactive power demand at bus *g* in zone *k*. Similarly, the marginal benefit with respect to system security from a generator operating at its armature or field current limit, or at the limit of its under-excitation mode are given by $C_L|\gamma_{gk}|$ and $C_L|\mu_{gk}|$ respectively. On the other hand, the four price components ρ_o, ρ_1, ρ_2, and ρ_3 represent the cost burden (total expected payment or TEP) of the ISO to provide reactive power support. Therefore, the proposed procurement algorithm is based on the following OPF model:

$$\text{max.} \quad SAF = \sum_k SAF_k \tag{18}$$

$$\text{s.t.} \quad P_{Gi} - P_{Di} = \sum_j V_i V_j Y_{ij} \cos\left(\theta_{ij} + \delta_j - \delta_i\right) \quad \forall i \tag{19}$$

$$Q_{Gi} - Q_{Di} = -\sum_j V_i V_j Y_{ij} \sin\left(\theta_{ij} + \delta_j - \delta_i\right) \quad \forall i \tag{20}$$

$$V_i^{\min} \le V_i \le V_i^{\max} \qquad \forall i \tag{21}$$

$$\left|P_{ij}\left(V, \delta\right)\right| \le P_{ij}^{\max} \qquad \forall ij \tag{22}$$

$$\left.\begin{aligned} Q_{Gg}^{\min} &\le Q_{G1g} \le 0 \\ 0 &\le Q_{G2g} \le Q_{GAg} \\ Q_{GAg} &\le Q_{G3g} \le Q_{GBg} \end{aligned}\right\} \quad \forall g \tag{23}$$

$$\left.\begin{aligned} Q_{G1g} Q_{G2g} &= 0 \\ Q_{G2g} Q_{G3g} &= 0 \\ Q_{G1g} Q_{G3g} &= 0 \end{aligned}\right\} \quad \forall g \tag{24}$$

$$Q_{Gg} = Q_{G1g} + Q_{G2g} + Q_{G3g} \qquad \forall g \tag{25}$$

$$\begin{aligned} & m_{1g} \le \rho_{1k} \\ & \quad if\ Q_{Gg}^{\min} \le Q_{Gg} \le 0 \qquad \forall\, g \in K;\ \forall k \end{aligned} \tag{26}$$

$$\begin{aligned} & m_{2g} \le \rho_{2k} \\ & \quad if\ 0 \le Q_{Gg} \le Q_{GBg} \qquad \forall\, g \in K;\ \forall k \end{aligned} \tag{27}$$

$$\begin{aligned} & m_{3g} \le \rho_{3k} \\ & \quad if\ Q_{GAg} \le Q_{Gg} \le Q_{GBg} \qquad \forall\, g \in K;\ \forall k \end{aligned} \tag{28}$$

$$\begin{aligned} & m_{og} \le \rho_{ok} \\ & \quad if\ Q_{Gg} \ne 0 \qquad \forall\, g \in K;\ \forall k \end{aligned} \tag{29}$$

In this model, the three regions of reactive power production identified from the generator's capability characteristic, shown in Fig. 3, are introduced through (23)-(25). It is to be noted that the two constraints (24) and (25) guarantee that only one of the three regions (out of Q_{G1}, Q_{G2} and Q_{G3}) will be selected at a time, for each generator. Therefore, this is a non-convex NLP problem with complementarity constraints, which requires special solvers and/or solution techniques. The approach used in this paper for solving this optimization problem is discussed in detail in the following sub-section.

The constraints (26)-(29) ensure that the highest offer price for each of the four components of reactive power determines the four reactive power price components in each zone. According to (26)-(28), only the offers from contracted generators for each region of reactive power operation are considered when determining the corresponding price component. On the other hand, (29) ensures that the zonal availability price component (ρ_{ok}) will have a non-zero value if there is any contracted generator in that zone. Therefore, constraints (26)-(29) require the association of binary variables in order to satisfy such conditions on the regions of reactive power operation.

It is important to highlight here that the *SAF* in (16) is somewhat similar to the one proposed in [9], [10] and [32] in the sense that both extend the classical concept of social welfare from economic theory, based on the determination of aggregate system benefits accrued from reactive power services minus the expected payment. However, there is a clear and important difference between the two objective functions, which is the incorporation of system security in the proposed reactive power procurement model in this paper; this is achieved by developing a novel approach to obtain the marginal benefits of reactive power with respect to system security, and the inclusion of the line flow limits given by constraint (22), which were not considered in the previous formulations.

The solution of the above procurement model (18)-(29) yields the set of contracted generators as well as the four zonal uniform price components. This procurement auction needs to be solved for different cases (e.g. light load, heavy load, contingencies, etc.), thereby properly representing the various expected system operating conditions for the season of interest.

F. Generator Reactive Power Classification Algorithm

The proposed procurement model (18)-(29) captures both the technical, i.e. transmission security constraints, and economical, i.e. marginal benefits and payments, aspects of reactive power procurement. However, from the optimization point of view, this model represents a difficult optimization problem, since it is not just an NLP with complementarity constraints (24), but it is essentially a mixed-integer NLP (MINLP) problem, where binary variables are required to select only one out of the three reactive power operating regions, in order to satisfy the conditions associated with the opportunity region in (17), and the three regions of reactive power operation in constraints (26)-(29). Therefore, one approach to solve the proposed model (18)-(29) is to formulate the problem as an explicit non-convex MINLP problem. However, the solution of this type of problems is very challenging due to the presence of both the integer variables and the non-convexities of the model itself [34], [35]. Solution techniques for this type of problem may get trapped at suboptimal solutions or even fail to yield a feasible point [36]. The number of available solvers for MINLP problems is still rather small, and according to [35], most of these solvers require a substantial amount of computational time for a small case study and might not yield an optimal solution within many CPU hours for a large case study. Moreover, most of the available MINLP solvers (such as DICOPT) have not been able to handle transfer capability constraints, which are of great importance to represent system security, in various test cases tried by the authors. Hence, solving (18)-(29) using non-convex MINLP techniques is not the most appropriate choice, especially when realistic sized power systems are considered.

A *generator reactive power classification (GRPC)* algorithm is proposed in this paper to solve the proposed procurement model (18)-(29). The idea of the proposed GRPC is re-formulate the problem so that it becomes a series of NLP sub-problems, which is some what similar to other existing MINLP solution approaches, making use of the fact that an integer solution is not required in this case, as the main objective is to choose only one region of reactive power operation to satisfy the conditions associated with the quadratic term in (17) and the constraints (26)-(29). This solution approach also has the advantage of using the structure of the problem itself to eliminate the complementarity constraints from the sub-problem solved in each iteration, which allows the use of standard well-tested NLP solvers, and the incorporation of all essential security constraints. The proposed GRPC algorithm is depicted in Fig. 6 and Fig. 7.

The algorithm begins with initial allocations of reactive power to generators (Q_{Gg}) obtained from an initial feasibility analysis of the system (e.g. power flow solutions from given dispatch schemes). The initial region of reactive power operation of each generator can be identified according to the values of γ_g and the initial values of Q_{Gg} as depicted in Fig. 6. If Q_{Gg} is less than zero (see Fig. 1) the generator is operating in Region I; hence, Q_{G2} and Q_{G3} for this generator will be zero, and Q_{G1} will be a variable within its lower and upper limits.

On the other hand, if Q_{Gg} is greater than zero and the value of γ_g is equal to zero, this implies that the generator is operating in Region II; hence, Q_{G1} and Q_{G3} for this generator will be zero, and Q_{G2} will be a variable within its lower and upper limits. Finally, Region III is selected if a generator has a value of γ_g not equal to zero. Therefore, constraints (23)-(25) can be replaced by one of:

$$Q_{Gg} = \begin{cases} Q_{G1g}, & Q_{Gg}^{\min} \le Q_{Gg} \le 0 \\ Q_{G2g}, & 0 \le Q_{Gg} \le Q_{GAg} \\ Q_{G3g}, & Q_{GAg} \le Q_{Gg} \le Q_{GBg} \end{cases} \quad \forall g \tag{30}$$

This new constraint is of a simpler form, and is directly solvable using standard NLP solvers.

Once the new set of $Q_{Gg}s$ is obtained, an update of the solution is required for each generator if Q_{Gg} hits the limits in any region, as shown in Fig. 7. For example, if the reactive power of a generator g in Region II hits its lower limit, the problem is re-solved with this generator operating in Region I, and the new optimal value of *SAF* is calculated. If this value is higher than the old one, this generator is now in Region I; otherwise, the generator remains in Region II. Similarly, if a generator hits the upper limit of Region II, then the problem is re-solved with this generator operating in Region III. Note that the updating process is applied to only one generator at a time. The updating process is repeated for a certain number of iterations as shown in Fig. 7. Observe that the updating process stops if no change in *SAF* is achieved after a complete iteration.

The proposed *GRPC* algorithm avoids the need for binary variables, thus keeping the optimization problem as an NLP, which can be applied to realistic power systems while incorporating all transmission system security constraints. The only issue is its dependence on the choice of the initial set of Q_G values and the order of the generators, which is also an issue with other non-convex MINLP solution approaches which are not concerned with finding a global optimum, but rather obtaining a practical feasible solution that meets typical ISO requirements. With regard to the choice of initial conditions, in this paper, we have used the ISO's "best practice" approach as the initial solution point, wherein the initial values of Q_G are readily obtained from a power flow solution associated with the values of P_G obtained from the active power dispatch process. This initial solution point significantly improves the convergence and speed of the algorithm. With regard to the order of generators, it is expected that the updating process will be affected by this order, and hence random orders of generators were adopted in this paper. Almost the same set of contracted generators and zonal reactive power prices were obtained in each case, but after different number of iterations. However, this cannot be generalized, as the order in which the generators are updated might affect the final solution for other test systems; this is to be expected, as the optimization problem is non-convex, and thus only local optimal solutions can be guaranteed [37]. Note that the issue of obtaining local optima when solving non-convex NLP optimization problems applies to most practical optimization models in power systems (e.g. the ac-OPF problem); the same argument applies to the optimization model (5)-(16). Finally, it is important to highlight the fact that the proposed reactive power procurement model is to be

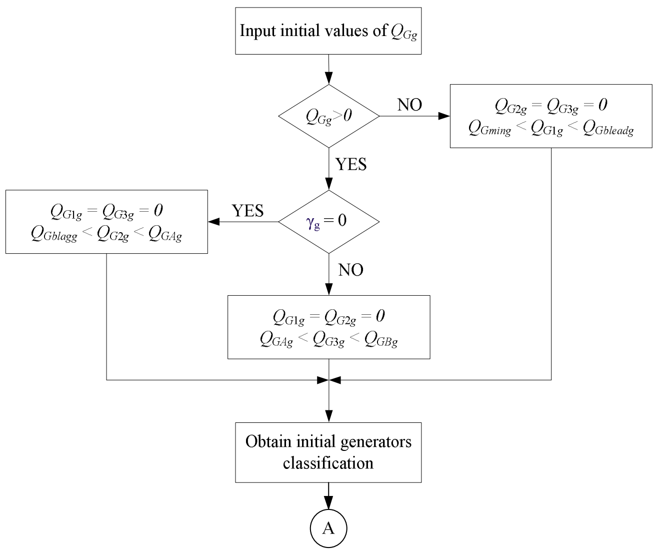

Fig. 6. Identifying the region of Q_{Gi}.

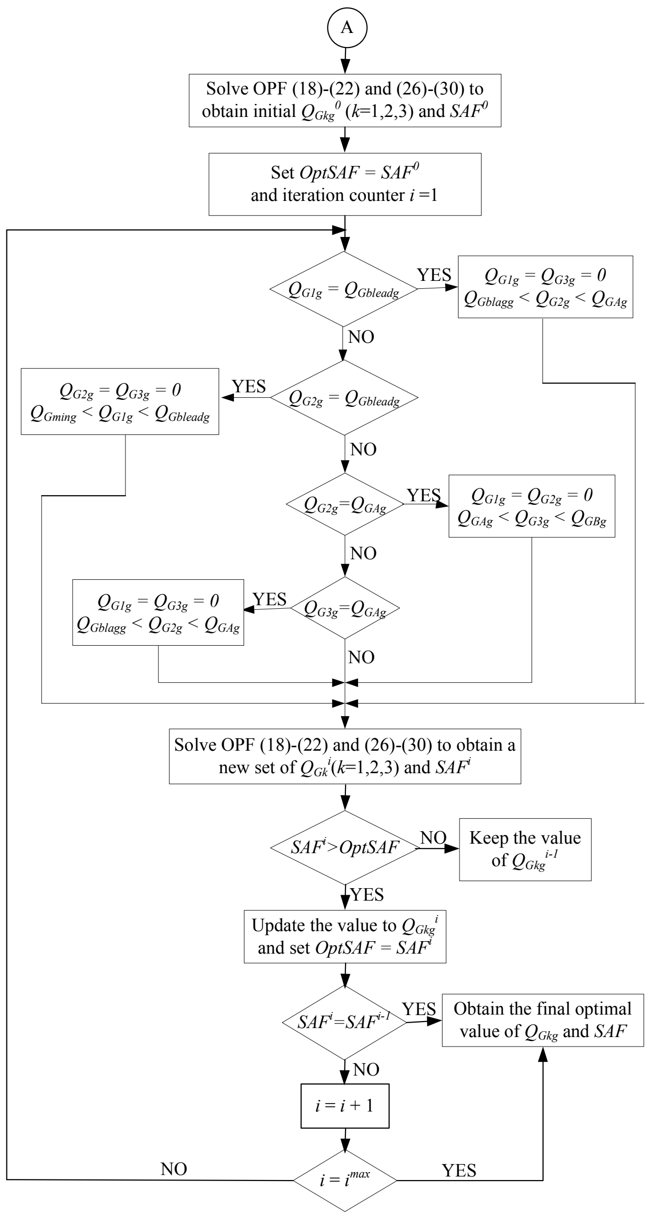

Fig. 7. Updating the solution to (18)–(22) and (26)–(30).

carried off-line, and hence computational burden is not a major issue in this case, regardless of the system size.

IV. Implementation and Test Results

In this section the complete reactive power procurement model described in Section III is implemented and the details of the solution procedure are also discussed. The simulations are carried out considering the CIGRE 32-bus test system (Fig. 8) [38], since this allows for direct comparisons with results available in the literature. Thermal limits were assumed for all transmission elements, and generators are assumed to be eligible for financial compensation in all of the three regions of operations defined in Section III-A, i.e. Q_{Gblead} and Q_{Gblag} in Fig. 3 are assumed to be equal to zero for all generators without any loss of generality. The optimization models, which are essentially NLP problems, are modeled in GAMS and solved using the MINOS solver [39].

Once the reactive power marginal benefits have been determined, the next step is to obtain the optimal set of reactive power providers using the *GRPC* algorithm proposed in Section III-F to maximize the *SAF*. The initial regions of reactive power operation are first identified using initial values of Q_{Gg} together with the value of γ_g, as depicted in the flow chart given in Fig. 6. This initial classification of the generators into three operating regions is then used to solve the OPF model (17)-(21) and (25)-(29) to obtain the first solution set, which is then updated following the algorithm depicted in Fig. 7. The updating process yields the final solution that includes the required set of generators to be contracted for reactive power service provision, and the zonal uniform reactive power price components.

The reactive power procurement market model is examined considering the following two cases:

- Case I: Unstressed condition, with seasonal “low” load and no contingencies. The Lagrange multipliers λ_g, γ_g and μ_g (shown in Table I) were computed without considering system contingencies.
- Case II: Stressed condition, with increased load with respect to Case I and considering contingencies. The Lagrange multipliers λ_g, γ_g and μ_g were calculated for the worst contingency, as explained in Section III-B.

For brevity of presentation, only two “extreme” loading cases are considered here to demonstrate the different procurement plans obtained from the proposed optimization model. However, in practice, *multiple* unstressed and stressed conditions should be studied by the ISO to arrive at its own set (or sets) of contracted generators. The ISO should then decide, based on its policies, directives and market structure, whether to contract the universal set of generators determined from all scenarios, or only those associated with the worst contingencies, or only those associated with peak load conditions, or others.

A. Case I

The ISO should consider a typical low demand that is expected in the system, which could be derived from a seasonal demand forecast. This should provide the ISO with a condition where generators in general are expected to be

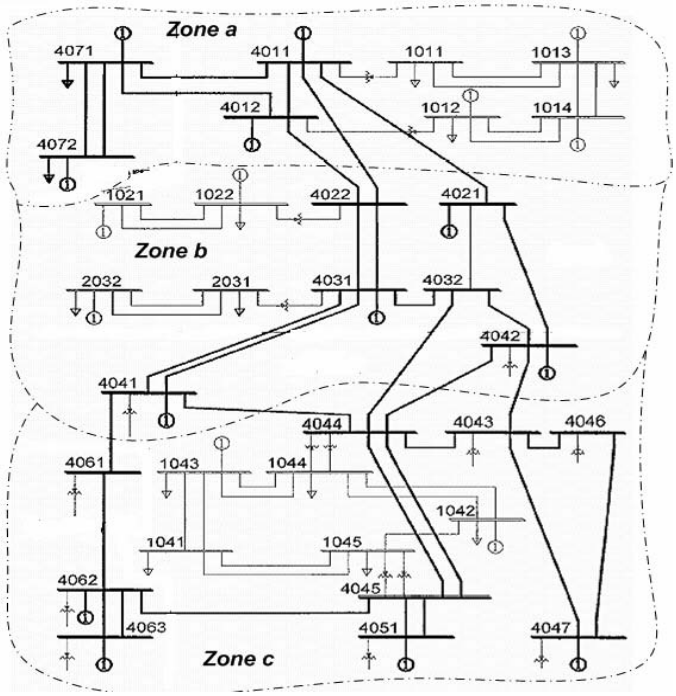

Fig. 8. CIGRE 32-Bus System.

under-excited, since actual demand in the system should remain higher than these low loading conditions for most of the time.

Applying the methodology explained in Section III, the initial value obtained for *SAF* was $1,766. The solution was then improved using the GRPC algorithm depicted in Fig. 7, where two updates were required in the first iteration, increasing the value of *SAF* to $1,969. This value remained the same for several iterations, indicating that this is the best solution that could be reached.

The final solution depicted in Table II provides the list of generators contracted by the ISO, and the zone-wise uniform reactive power price components for this case of low demand condition. As it can be observed, 15 generators are required for reactive power service provisions in this case. Generators with negative values of Q_G are operating in Region I (8 of the 15 in this case), which represents the under-excited mode of operation; no generator is operating in Region III.

Since no generator is contracted to operate in Region III, none of them will be expected to receive the opportunity payment component. For the other three price components, the highest reactive power offer from contracted generators within a zone is selected as the zonal uniform price. For example, in zone *a* only two generators, 4012 and 1012, are contracted to operate in Region I; and hence the under-excitation price (ρ_1) is 0.59 $/Mvar, which is the higher of the two generators' offer prices for this component, as shown in Table I.

B. Case II

There will be instances when the power system is heavily stressed due to contingencies and/or high demand conditions. In order to ensure a secure operation of the system in this case, reactive power provisions should be determined considering worst case scenarios, such as the case presented and discussed here.

TABLE I

VALUES OF λ_g, γ_g AND μ_g FROM *LF* MAXIMIZATION

Zone	Bus	Price offers from each generator				Lagrange multipliers from OPF (5)-(16)		
		a0	*m*1	*m*2	*m*3	λ_g	γ_g	μ_g
a	4072	0.58	0.57	0.57	0.21	0	0	0
	4071	0.70	0.84	0.84	0.33	0	0	0
	4011	0.78	0.74	0.74	0.29	0	0	0
	4012	0.61	0.57	0.57	0.21	0	0	0
	1013	0.40	0.40	0.40	0.18	0	0	0
	1012	0.59	0.59	0.59	0.35	0	0	0
	1014	0.86	0.88	0.88	0.50	0	0	0
b	4021	0.80	0.91	0.91	0.31	-0.853	0.853	0
	4031	0.92	0.90	0.90	0.36	-0.737	0.737	0
	4042	0.68	0.69	0.69	0.23	-1.076	1.076	0
	4041	0.51	0.56	0.56	0.24	-0.431	0.431	0
	2032	0.87	0.86	0.86	0.29	0	0	0
	1022	0.75	0.68	0.68	0.23	-0.729	0.729	0
	1021	0.54	0.55	0.55	0.20	-1.303	1.303	0
C	4062	0.85	0.93	0.93	0.33	0	0	0
	4063	0.73	0.66	0.66	0.39	0	0	0
	4051	0.85	0.81	0.81	0.26	-1.811	1.811	0
	4047	0.62	0.60	0.60	0.23	0	0	0
	1043	0.48	0.49	0.49	0.20	-2.503	2.503	0
	1042	0.58	0.53	0.53	0.24	0	0	0

TABLE II

FINAL SOLUTION FOR UNSTRESSED CONDITION (CASE I)

Zone	Gen.	Q_G	Zonal Uniform Prices			
			ρ_o	ρ_1	ρ_2	ρ_3
a	4072	2.383	0.78	0.59	0.74	NC
	4011	3.106				
	4012	-1.60				
	1013	0.893				
	1012	-0.80				
b	4021	-0.30	0.92	0.91	0.86	NC
	4031	-0.40				
	4041	-2.00				
	2032	1.689				
	1022	-0.25				
	1021	1.351				
c	4063	0.544	0.85	0.53	0.81	NC
	4051	0.240				
	1043	-0.20				
	1042	-0.40				
Total Marginal Benefit with respect to system security (TMB)			$4,914			
Total expected payment by the ISO (TEP)			$2,945			
Objective Function, SAF (*SAF* = TMB – TEP)			$1,969			

NC = No Contracted Generator

Table III shows the values of the three Lagrange multipliers λ_g, γ_g and μ_g obtained by solving the OPF (5)-(16) for the worst contingency, i.e. the one that yields the minimum LF_c, which in this case corresponds to the outage of the transmission line connecting buses 4031 and 4032. These values were then used to solve the OPF model (18)-(22) and (25)-(30), obtaining an initial value of *SAF* of \$23,486. The solution was then updated using the GRPC algorithm, with four updates in the first iteration increasing the value of *SAF* to \$39,984. In the second iteration, three more updates for the solution took place improving the value of *SAF* to \$55,403. This value remained the same for several iterations indicating that no further improvements were possible and the best solution was reached.

The final solution for this case is given in Table IV, where 12 generators are contracted for reactive power service provision; 4 of these (generators shown in bold) are expected to operate in Region III. However, as none of these generators are located in zone *a*, no generators in this zone are expected to receive an opportunity payment component. Observe also that 3 of these 4 generators are located in zone *b*, where the worst contingency took place.

Comparing the results of Case I and Case II, the following can be observed:

- As the system is stressed, the reactive power requirements from generators also increase resulting in more generators operating in Region III (4 generators in Case II versus none in Case I).
- The value of the objective function *SAF* in the stressed case is much higher than the unstressed case. To explain this, notice that *SAF* has two components, namely, the total marginal benefit (TMB) with respect to system security, and the total expected payment (TEP) by the ISO. It can be seen from Table II and IV that the TEP only increased by \$361 (from \$2,945 in Case I to \$3,306 in Case II), while the TMB increased significantly (from \$4,914 to \$58,709). The TMB "jump" is due to the fact that the benefit to the system from reactive power support is much more significant when the system is heavily stressed; the more the system is stressed, the higher is its need for reactive power support to maintain system security.
- For the two cases considered, no generator located in zone *a* is expected to receive an opportunity payment since none of the generators from this zone is contracted to operate in Region III. This was expected, since the values of the Lagrange multipliers λ_g, γ_g and μ_g obtained from the LF maximization analysis for both Cases I and II are all zeros for all generators in this zone (Table I and III); this indicates that the reactive power from generators in this zone does not have any effect on system security, and hence the procurement model does not seek much reactive power from these units.
- From the optimization point of view, both models have the same number of variables and equations, as indicated by the computational statistics depicted in Table V. Observe that Case II requires about two and a half the CPU time of Case I

TABLE III
VALUES OF λ_g, γ_g AND μ_g FOR THE WORST CONTINGENCY IN CASE II

Zone	Bus	Lagrange multipliers from OPF (5)-(16)		
		λ_g	γ_g	μ_g
a	4072	0	0	0
	4071	0	0	0
	4011	0	0	0
	4012	0	0	0
	1013	0	0	0
	1012	0	0	0
	1014	0	0	0
b	4021	-0.033	0.033	0
	4031	-0.903	0.903	0
	4042	-1.404	0	1.404
	4041	-1.310	1.310	0
	2032	0	0	0
	1022	-0.558	0.558	0
	1021	-0.741	0.741	0
C	4062	0	0	0
	4063	0	0	0
	4051	0	0	0
	4047	-1.774	0	1.774
	1043	-1.862	1.862	0
	1042	0	0	0

TABLE IV
FINAL SOLUTION FOR STRESSED CONDITION (CASE II)

Zone	Gen.	Q_G	Zonal Uniform Prices			
			ρ_o	ρ_1	ρ_2	ρ_3
a	4072	2.339	0.78	0.74	0.57	NC
	4011	-0.773				
b	4021	-0.30	0.92	0.91	0.90	0.36
	4031	**1.911**				
	4041	**1.638**				
	2032	1.045				
	1022	**1.365**				
	1021	1.515				
c	4063	2.112	0.85	0.53	0.81	0.20
	4051	1.824				
	1043	**1.092**				
	1042	-0.001				
Total Marginal Benefit with respect to system security (TMB)			\$58,709			
Total Expected Payment by the ISO (TEP)			\$3,306			
Objective Function (SAF) (*SAF* = TMB – TEP)			\$55,403			

NC = No Contracted Generator

to arrive at the solution, since the *SAF*-maximization model (18)-(22) and (26)-(30) is solved 16 times for the later compared to only 8 times for the former. This is to be expected, as more generators are pushed to operate in

Regions II and III in Case II due to the higher loading conditions, as illustrated on Tables II and IV, and hence solution updates are required in this case.

C. Comparison of Zonal Uniform Pricing with System Wide Uniform Pricing

In order to emphasize the advantages of the zonal uniform pricing scheme over a system-wide uniform price, the above two cases were solved with only one uniform price for each of the four reactive price components, i.e. the whole system is treated as a single zone only. The resulting prices for the two case studies are shown in Table VI.

Comparing the results in Table VI with those obtained earlier in Table II and IV, it can be observed that there is a reduction in the value of the *SAF* for both the cases when a system-wide uniform pricing scheme is adopted (e.g. the *SAF* with zonal uniform pricing is $1,969 in Case I, and $1,649 with system-wide uniform pricing). In Case II, this reduction is not very significant, as the value of Q_G in both pricing approaches remains the same, and hence the marginal benefit component in *SAF*, which is high in Case II, does not change. However, the expected payments, which are functions of price components, are greatly affected; thus, the TEP increases 11% for the unstressed case for the uniform price approach with respect to the zonal price approach, and 17% for the stressed case. This shows that using a zonal pricing mechanism not only reduces the risk of market power, but it also reduces the payment burden on the ISO.

After the zonal uniform price components are determined for a variety of system operating conditions, the ISO signs contracts with the selected generators in which generators receive an availability payment component. The generator will also receive a "usage" component, applicable in the real-time dispatch stage based on the actual reactive power supplied.

V. CONCLUSIONS

Based on the current practices for reactive power provision by various ISOs in competitive electricity markets, this paper has proposed a hierarchical reactive power market structure. The proposed market design comprises two stages, namely procurement of reactive power resources on a seasonal basis, and a real-time reactive power dispatch. The proposed procurement market model, which is the main focus of the paper, is based on a two-step optimization process; the first step consists of the determination of the marginal benefits of reactive power with respect to system security, which are then used in the second step to maximize a reactive power societal advantage function considering bids from service providers.

A sample system was used to demonstrate the feasibility of the proposed procurement market model and proposed solution technique. The solution of this model yields a set of generators and zonal prices that would form the basis of contractual agreements for seasonal reactive power provision.

The next stage is to develop reactive power dispatch procedures based on the proposed reactive power market framework, wherein the ISO will call for reactive power services from the already contracted generators according to real-time operating conditions. Furthermore, other reactive power providers, such as capacitor banks and FACTS controllers, will be also considered as possible reactive power service providers, with the objective of enhancing competition and thus help alleviate market power issues in these kinds of markets.

TABLE V

SAF-MAXIMIZATION MODEL (18)-(22) AND (26)-(30) STATISTICS

	Case I Unstressed condition	Case II Stressed condition
No. of variables	10,323	10,323
No. of equations	11,950	11,950
No. of *SAF-maximization* models solved	8	16
Total solution (CPU) time (sec)	14.35	35.56

TABLE VI

RESULTS FOR A SYSTEM-WIDE UNIFORM PRICING MECHANISM

Price component	Case I	Case II
Availability Price	0.92	0.92
Under-excitation Price	0.91	0.91
Over-excitation Price	0.86	0.90
Opportunity Price	0	0.36
SAF	$1,649	$54,849
TEP	$3,265	$3,860

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