Smart Operation of Centralized Temperature Control System in Multi-Unit Residential Buildings

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SUMMARY

This paper presents the mathematical modelling of a Centralized Temperature Control System (CTCS) of a Multi-Unit Residential Building (MURB) and its optimal operation considering electricity prices and weather variations. The model considers comfort levels, preference settings and activity level of residents in different units of the building to determine the optimal operation schedules of the CTCS, minimizing its total energy consumption cost considering multi-objective operation when residents in different units have conflicting interests. The proposed model is a Mixed Integer Non Linear Programing (MINLP) model, where some of the constraints are linearized to reduce computational complexity for real-time applications. The model is studied for various customers preferences using a realistic MURB model. Simulation results show that significant cost savings can be achieved using the proposed model.

KEYWORDS

Smart grid, energy management system, multi-unit residential building, central temperature control system

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1. INTRODUCTION
Demand Side Management (DSM) and Demand Response (DR) programs have been envisioned as tools for unlocking the full potential of smart grids, and achieving energy and cost savings through the proper use of Energy Management Systems (EMS) [1]. Advanced intelligent communication technologies and superior control systems have enabled the grid to link customers and utility control centers in real-time. Thus energy usage optimization in different load sectors is becoming an important part of smart grids [2]-[3]. In the residential sector, Multi-unit Residential Buildings (MURB) can achieve significant cost savings through smart energy management of the Heating, Ventilation and Air-conditioning (HVAC) system, since it accounts for a major share of the monthly electricity cost. A MURB can be equipped with distributed split-type temperature control systems for different units or a Centralized Temperature Control System (CTCS) for all its units depending on its geographical location and availability of other necessary resources.

Various methods have been proposed in the literature to model the MURB’s CTCS for controlling the temperature of each unit simultaneously. The mathematical modeling of temperature control systems and other energy elements such as lighting, various electrical appliances of a single residential house is described in [4]. A hygrothermal model for HVAC system and multi-zone building model is proposed in [5], which simulates combined heat, vapor and liquid transfer in porous elements and the HVAC system; however, the computational burden makes it unsuitable for integration in real-time supervisory control systems. Optimization of energy consumption based on electricity and natural gas price is proposed in [6], which includes thermal and electrical storage options. A smart sensor web network model is proposed as an EMS in [7], which minimizes energy consumption of the centralized AC of a residential building based on ambient room temperature setting. Simulation results show that application of the EMS can significantly reduce the energy consumption of AC, which consequently reduces peak load of the grid and its carbon footprint. A simplified model of a district heating/cooling system of a MURB complex is developed in [8] and its accuracy in quantifying the heating/cooling demand is validated through a series of simulations. Energy supply and distribution in the system is optimized considering the heating/cooling demand estimated using their historical data. This approach can lead to excessive heating or cooling in some units, since it provides heating or cooling to all units simultaneously and uniformly.

In the present paper, a novel mathematical model of a CTCS of a MURB is proposed to control the inside temperature of different units simultaneously, while maintaining the various customers' preferred comfort levels. The model seeks to minimize the energy consumption and costs of a MURB through the smart operation of CTCS, taking into account weather forecast, real-time energy price information, and customers' activity levels. Two-way communication between the utility and MURB facilitates information updates for customers regarding energy price and weather data.

The rest of the paper is organized as follows: Section II describes the smart operational framework; description of the CTCS of a MURB and the conceptual framework of the supervisory control system. Section III presents the mathematical model of the CTCS incorporating the MURB's physical properties. In Section IV, a MURB example and inputs to the model are presented, and the results of case studies are discussed, and Section V highlights the main contributions of this work.

2. SMART OPERATIONAL FRAMEWORK OF MURBS
2.1 Centralized Temperature Control System (CTCS)
The CTCS is divided into two sub-systems, as illustrated in Fig. 1. The first is the primary system which comprises one air-cooled water chiller, one natural gas fired heater, a primary water circulation loop, and two water circulation pumps. The chiller and the heater come into operation according to the seasonal demand and cannot be in operation simultaneously. In the summer, the chiller comes in operation and maintains the main loop water temperature between 5°C and 7°C, and the water pumps keep the water circulating through the primary water loop.
Each zone has its own secondary system which is connected to the primary system through a cooling/heating coil. The secondary system comprises three components: mixing box, cooling/heating coil, and supply fan. In case of intra-zoning, one unit could have several zones according to need, comfort level, and the building's geographical location. The heat exchange coils are responsible for cooling or heating the incoming air from the mixing box, which mixes the indoor air partially with incoming fresh air, maintaining ASHRAE standards [9], and then circulates it in the respective zones; frequently, ON/OFF based control systems are used for all zones. The programmable thermostat S determines the zone temperature at every interval and acts on the valve opening, if necessary. The effect of partial recirculation of inside air is modeled in this work in a simplified manner so that the energy saving strategy can be implemented for real-time control.

2.2 Proposed Supervisory Control Strategy
Real time electricity price and accurate weather forecast information are available day ahead and updated every few hours. As the MURB is connected to the utility through two-way communication system, their updated information are available to the proposed Supervisory Control System (SCS). These forecasted data are used in the supervisory module to improve the operation and scheduling of the CTCS. The hierarchical control strategy proposed (Fig. 2) has two main parts: the SCS where the developed mathematical model is integrated and external data are given as input signals, second is the existing control system in which decision signals are processed and ON/OFF devices according to these signals are implemented. Feedback from the existing controller goes to the new supervisory controller to ensure smooth operation of the temperature control system. If there are large differences between measured and calculated parameters, the feedback is used to run the optimization model again and determine new operational set points for each device based on a Model Predictive Control (MPC) approach [10].

The scheduling horizon (24 hours) is divided into 96 intervals; each interval (τ) is 15 minutes in duration. At any interval t, the SCS gets the real-time energy price and weather data as well as forecasted data from t+1 to t+N-1 or T. Then, the SCS generates the optimal operational schedules of the CTCS according to the objective function of the model; it can be either minimization of energy cost/consumption or maximizing comfort level of customers. The operation of the CTCS at time t maintains the optimal schedule of that time. At time t+1, the SCS updates information of forecasted data for the next N intervals (from t+1 to t+N) and runs the optimization model again, to generate the new optimal set points for the CTCS. After that, the scheduling horizon moves forward by one time slot for the new optimization.
3. SYSTEM MODELING

3.1 Objective Functions

Minimization of Temperature Deviations: It seeks to track the user defined temperature of zone \( z \) at time \( t \), \( \theta(t) \), by minimizing the sum of the absolute value of temperature deviation from a given set point \( \theta^{\text{set}} \), at each interval, as follows:

\[
J_1 = \sum_{z} \sum_{t} |\theta_z(t) - \theta^{\text{set}}_z|
\]  

(1)

This objective is used to represent the existing operation strategy of CTCS for comparison purposes and \( z \) represents the number of zones and \( T \) total scheduling horizon (24 hours).

Minimization of Total Energy Cost of MURB: This takes place over the scheduling horizon, as follows, assuming that residents in different units have the same objective, but may have different temperature settings;

\[
J_2 = \sum_{z} \sum_{t} \sum_{i \in \text{A}} \tau C_{\text{ed}}(t) P_i(z) S_i(z,t)
\]  

(2)

where \( \tau \) is the 15 minutes time interval; \( C_{\text{ed}}(t) \) is the electricity price ($/kWh); \( P_i \) is the rated power of a device (kW); \( S_i(z,t) \) is the ON/OFF decision of different devices at \( t \)-th time interval; and \( \text{A} \) is the set of devices: central AC (ac), central heater (ht), cooling valves (cv), heating valves (hv) and air mixing units (m).

Minimization of Total Energy Consumption: In this case, the total energy consumption of the system over scheduling horizon is minimized using the following objective function:

\[
J_3 = \sum_{z} \sum_{t} \sum_{i \in \text{A}} \tau P_i(z) S_i(z,t)
\]  

(3)

3.2 Model Constraints

A. Primary System Modelling

The primary system model includes the central heating and cooling unit, main water loop, and the effect of each valve opening on the temperature of the main water loop. The heat balance equation between the primary and secondary systems can be represented as follows:

\[
\theta_p(t) = \theta_p(t-1) + \frac{\tau}{mS} \left[ \sum_{z} \gamma_z S_{cv}(z,t-1) - \sum_{z} \kappa_z S_{cv}(z,t-1) - \gamma_{ac} S_{ac}(z,t-1) + \kappa_{ht} S_{ht}(z,t-1) \right]
\]  

(4)
where \( \theta(t) \) is the temperature of main water loop at \( t \)-th time interval; \( S_c(t) \) and \( S_h(t) \) the ON/OFF decisions of cooling and heating valves; \( \gamma \) and \( \kappa \) are cooling and heating capacity (KJ/hr/C) respectively; \( m \) is the maximum water flow rate (kg/hr) in the main water loop; and \( S \) is the specific heat of water (KJ/(kgC)). The first term on the right hand side of (4) represents the inter-temporal dependency of inside temperature. The second and third terms model the effect of zonal valve openings on the main water loop temperature. The last two terms represent the effect of central AC and central heater operation on the main water loop temperature. When the temperature of water in the main water loop reaches the pre-defined upper temperature limit (UB), the central AC is turned ON to bring down the temperature of the water below this limit. A similar control mechanism takes place in the case of the central heater for a pre-defined lower temperature limit LB. The following constraints ensure successful implementation of these conditions:

\[
S_{ac}(t) = \frac{\theta_p(t) - UB}{C_1} ; \quad S_{ht}(t) = \frac{LB - \theta_p(t)}{C_1} \tag{5}
\]

where \( C_1 \) is an auxiliary constant to scale down the temperature.

### B. Secondary System Modelling

In addition to the heat loss through walls, the secondary system model also includes the heat loss through ventilation; activity level in different units; cooling and heating rate of zonal valve; cooling effect of fan operation; ambient room temperature; and the maximum temperature deviations that a resident is willing to tolerate. Since operational characteristics of cooling and heating systems are the same, these devices are modeled together, but only one of them operates at a time. Thus the room temperature is modelled as follows:

\[
\theta_z(t) = \theta_z(t-1) + \frac{\tau}{C_z} \left[ \gamma_z S_{hv}(z,t) + \beta_{act} A L_z(t) - \gamma_z S_{cv}(z,t) + q_{f,z}(t) + \alpha_z(t) (\theta_{out}(t) - \theta_z(t)) \right] \tag{6}
\]

where \( \beta_{act} \) is the heat generation co-efficient (k/J/hr/C) due to activity level; \( C_z \) is the total heat capacity (kJ/C) of a zone; \( \alpha_z(t) \) is the thermal leakage (kJ/hr/C) of a zone; \( \theta_{out}(t) \) is the forecasted outside temperature (˚C) at the \( t \)-th time interval. The first term of (6) represents the inter-temporal dependency of the room temperature. The second and fourth term represents the effect of zonal valve operation on inside temperature. The third term models the generated heat due to the activity level \( AL_z(t) \) of the residents, as proposed in [4]. Finally, the fifth term \( q_{f,z}(t) \) represents the effect of forced air-circulation in the zone, and the last term represents the effect of heat loss through the outside wall of the unit. The zonal supply fan equation is modelled as follows:

\[
q_{f,z}(t) = \beta_z S_f \left[ S_m(z,t) \theta_{out}(t) - \theta_z(t) \right] \tag{7}
\]

which includes the effect of air mixing and zonal fan's air circulation capability, where \( \beta_z \) is the cooling effect (kJ/hr-C) of zonal supply fan; \( S(z,t) \) and \( S_a(z,t) \) are ON/OFF decisions of zonal supply fan and zonal air mixing unit respectively. It is assumed that the amount of fresh air entering the zone at any instant is equal to the amount of air exhausting the respective zone; \( S_m(t) \) and \( S_h(t) \) are binary variables, i.e. 1 (ON) or 0 (OFF); while \( S_c(t) \) and \( S_a(t) \) are continuous variables ranging between 0 and 1. The parameters \( \alpha_z, \beta_z, \gamma_z, \kappa_z, \gamma_{ac} \) and \( \kappa_{ac} \) for the primary and secondary system models can be calculated by measurements or through simple performance tests.

### 3.3 Operational Constraints

The central AC and the heater cannot be ON at the same time, and also the ON/OFF times are limited to a maximum number \( (C_2) \) over the scheduling horizon \( T \) for increasing the lifetime of central AC and heater as follows:

\[
S_{ac}(t) + S_{ht}(t) \leq 1 ; \quad \sum_{t \in T} S_{ac}(t) + \sum_{t \in T} S_{ht}(t) \leq C_2 \tag{8}
\]

The cooling and heating valve operating states, \( S_c(t) \) and \( S_h(t) \), are dependent on the operational states of \( S_m(t) \) and \( S_a(t) \) respectively. The zonal supply fan is ON when the respective zonal valve (heating or cooling) is open. If the zonal supply fan is not ON, but the zonal valve is open, the cooling or heating coil can be damaged. Moreover, if any zonal supply fan is ON, the air mixer of that zone should also be ON, and vice-versa.
In each zone, the zonal supply fan, the mixing box and the cooling/heating coil valves operate simultaneously but independently from those of other zones. However, if the outside temperature is less than \( \theta_{out}^{min} \), the zonal supply fan will stop circulating the outside air; this is enforced as follows:

\[
S_f(z,t)\left(\theta_{out}(t) - \theta_{out}^{min}\right) \geq 0
\]  

Finally, the inside temperature of each zone is bounded by the upper and lower limits, \( \theta_z^u(t) \) and \( \theta_z^l(t) \) respectively, set by MURB residents to define their own comfort levels, and are represented as follows:

\[
\theta_z^l(t) \leq \theta_z(t) \leq \theta_z^u(t)
\]

The developed MURB model is a Mixed-Integer Non-Linear Programming (MINLP) problem. Hence, to reduce the computational burden of the model, some constraints are linearized by replacing the bi-linear terms; this linearization makes the model suitable for real-time application [11].

4. CASE STUDIES

An actual MURB from [5] is modified to set up realistic case studies. The MURB considered has three floors, each floor has only one unit that is treated as a single zone, and each unit is \( 13m \times 10m \times 3m \) in size. Since no intra-zoning is considered within a unit, no intra-zonal heat flow takes place. The floor heat flow is considered using the parameter \( \alpha_z \), denoting heat loss through walls. All the walls, floors and the roof are assumed to have the same physical properties. All case studies have been carried out for summer season, and thus, only central AC comes into operation. The central cooling unit size is 124 kW, which provides necessary cooling for all the units. Each zonal supply fan is rated at 0.7457 kW. The air mixing unit consumes very little energy when closing and opening the valves; hence, its energy consumption is neglected. The heat exchange between primary and the secondary system is considered to be ideal.

Two different electricity price schemes are used: Time of Use (TOU) and Real Time Pricing (RTP) for Ontario. There are three price periods in TOU scheme, peak, mid-peak and off-peak which vary with season and day of the week. The RTP is the Hourly Ontario Energy Price (HOEP), which is the pricing scheme that applies to large customers who participate in the wholesale electricity market [12]. A fixed pricing scheme is considered for natural gas at 0.130806 $/m^3 [13]. Fifteen minute intervals are considered as time steps for the model simulations to determine the optimal set-points of the decision variables. The effect of customers’ activity and occupancy of the unit are modelled through the Activity Level parameter proposed in [4]. Temperature limits are set by the units’ residents as per their own preferences for comfort and energy savings; in this work, the limits are chosen based on [14]. Outside temperature data for the Toronto region is used.

4.1 Results and Analysis

The effectiveness, performance and capabilities of the proposed model are examined considering the following three cases:

- Case 0: Seeks to maximize the comfort level of residents by minimizing the temperature deviation from their respective set points. This is a realistic base case, as it reflects the operation of the existing temperature control systems, and establishes a reference for comparison purposes.
- Case 1: Minimize energy consumption cost of CTCS, assuming that residents in different units have the same rational objective of minimizing their energy cost.
- Case 2: Minimize energy consumption of the CTCS only, which is independent of the pricing scheme and reflects a typical Load Distribution Companies’ (LDC) point of view.

Figure 3 shows the ON/OFF decisions of the cooling valve and the variation of indoor temperature of Zone 1 for Case 0, 1, and 2, considering TOU pricing. Figure 4 presents the corresponding operational schedules of the central AC for Case 0, 1 and 2, respectively. Since the customers’ comfort level is maximized in Case 0, the operation schedule of the cooling valve is not optimized, and it has a large number of operations (see Fig. 3 Case 0), thus increasing the main water loop temperature rapidly, which leads to a higher number of operations of the central AC (see Fig. 4 Case 0). On the other hand, as shown in Figs. 3 (Case 1 and Case 2), when energy cost and energy consumption are minimized, respectively, the optimal operation of the cooling valve results in fewer number of operations of the central AC than in Case 0 (Figs. 4 Case 1 and Case 2). Note that the optimization
model turns ON the central AC during the low price periods. Optimal operation schedules of the cooling valve, supply fan and air mixing unit for the respective zones are also shown in these figures.

Table 1 presents a comparison of the total electrical energy consumption and cost for the cases for two pricing schemes. Observe that, for HOEP and TOU, Case 2 results in the lowest cost as compared to other cases, while the maximum total cost is obtained in Case 0. The objective function of Case 1 is cost minimization, and of Case 2 is energy consumption minimization, but the lowest cost is obtained for Case 2, because of the presence of multiple local optimal solutions to the optimization problem. Table 2 presents a comparison of savings in energy consumption and cost with respect to Case 0; note that over 40% savings in electricity cost and energy consumption can be observed in both Case 1 and Case 2.

### Table 1: Comparison of Electrical Energy Consumption and Cost

<table>
<thead>
<tr>
<th>Cases</th>
<th>HOEP</th>
<th>TOU</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cost ($)</td>
<td>Energy (kWh)</td>
</tr>
<tr>
<td>Case 0</td>
<td>19.308</td>
<td>572.541</td>
</tr>
<tr>
<td>Case 1</td>
<td>10.871</td>
<td>287.762</td>
</tr>
<tr>
<td>Case 2</td>
<td>6.696</td>
<td>196.999</td>
</tr>
</tbody>
</table>

### Table 2: Comparison of Savings in Total Energy Consumption and Cost

<table>
<thead>
<tr>
<th>Cases</th>
<th>HOEP Savings</th>
<th>TOU Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cost</td>
<td>Energy</td>
</tr>
<tr>
<td>Case 0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Case 1</td>
<td>43.696</td>
<td>49.739</td>
</tr>
<tr>
<td>Case 2</td>
<td>65.32</td>
<td>65.57</td>
</tr>
</tbody>
</table>

5. **CONCLUSIONS**

A comprehensive mathematical model has been proposed in this paper for the smart operation of the CTCS of a MURB. The proposed model seeks to determine, in real-time, the optimal operation schedules of the CTCS that optimized various customers' desired objectives. The developed model considered weather forecast, electricity price information and the end-users' dynamic preference settings, and was implemented as a supervisory control system. The simulation results showed that significant cost savings can be achieved using the proposed model. The impacts of conflicting user objectives in a MURB were also studied based on multi-objective optimization approach, demonstrating that in this case, there is no Pareto-optimal solution.

**BIBLIOGRAPHY**


Fig. 3: Indoor temperature variation of Zone 1 with TOU.

Fig. 4: Optimal schedule of central AC, with TOU.