An Affine Arithmetic-Based Energy Management System for Isolated Microgrids

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***Abstract*—This paper presents a mathematical formulation of an energy management system (EMS) for isolated microgrids, which addresses uncertainty using the affine arithmetic (AA) method. The proposed EMS algorithm is based on an AA unit commitment (AAUC) problem for day-ahead dispatch, using uncertainty intervals of both load and renewable energy (RE) to provide robust commitment and dispatch solutions in AA form, which are feasible for all the possible realizations within the predetermined uncertainty bounds. A real-time dispatch solution is then found by the proposed algorithm, which computes the noise symbols values of the affine forms obtained by the AAUC, based on the current and actual load and RE power levels and available reserves. If the actual forecast error is outside the uncertainly bounds considered in the AAUC solution process, leading to possible load and/or RE curtailment, the AAUC is recalculated with updated forecast information. The proposed AA-based EMS is tested on a modified CIGRE microgrid benchmark and is compared against day-ahead deterministic, model predictive control (MPC), stochastic optimization, and stochastic-MPC approaches. The simulation results show that the proposed EMS provides robust and adequate cost-effective solutions, without the need of frequent re-calculations as with MPC-based approaches, or assumptions regarding statistical characteristics of the uncertainties as in the case of stochastic optimization.**

***Index Terms*—Affine arithmetic, economic dispatch, energy management system, microgrid, uncertainties, unit commitment.**

NOMENCLATURE

Indices

0	Center value
h	Noise symbols
i	Thermal units
j, k	Bus
l	Equality constraint
m	Inequality constraint
n	ESS unit
r	Load uncertainties
s	Scenarios
t	UC interval
t'	Dispatch interval
u	Wind uncertainties
v	Solar uncertainties

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Sets

$\mathcal{B}$	Buses
$\mathcal{I}$	Thermal units
$\mathcal{I}_b$	Subset of thermal units at bus j
$\mathcal{L}$	Equality constraints
$\mathcal{M}$	Inequality constraints
$\mathcal{N}$	ESS units
$\mathcal{N}_b$	Subset of ESS units at bus j
$\mathcal{S}$	Scenarios
$\mathcal{T}$	Time set for UC
$\mathcal{T}'$	Time set for dispatch

Parameters

C^{LC}	Load curtailment price
C^{sup},C^{sdn}	Start-up/shut-down cost
Δt	Absolute time between time periods
D^f	Demand forecast
$F(\cdot)$	Piecewise thermal unit cost
D^P	Active power demand
D^r	Demand realization (most recent available information)
η^{ch},η^{dch}	Charging/discharging efficiency of ESS units
$g_i^{on/off}$	Initial status of unit i
g^{up},g^{dn}	Minimum up-time/down-time
Kd	Reactive power demand factor of constant power factor load
$L_i^{up,min}$,$L_i^{dn,min}$	Number of periods that unit i is required to stay ON/OFF at the beginning of the optimization horizon
π	Probability of scenarios
p	Number of noise symbols
$P^{ch,max}$,$P^{dch,max}$	Maximum charging/discharging power limit of ESS units
$P^{ch,min}$,$P^{dch,min}$	Minimum charging/discharging power limit of ESS units
P^{min},P^{max}	Minimum/maximum generation capacity
P^{RC}	RE curtailment
PV	Photovoltaic unit output
R^L,R^W,R^{PV}	Power reserve requirements for load, wind power, and photovoltaic power variations
R^{up},R^{dn}	Ramp-up/ramp-down rate limits
SOC^{min},SOC^{max}	Minimum/maximum SOC of ESS units
θ	Y-bus matrix element angle
W	Wind turbine output
w	Objective function weight $\in [0, 1]$
Y	Y-bus matrix element

Variables

δ	Voltage angle
P,Q	Active/reactive power generated by thermal units
P^{dch},P^{ch}	Discharging/charging power of ESS units
P^{LC}	Load active power curtailment
Q^{ess}	ESS reactive power output
SOC	State of charge of ESS units
SU,SD	Start-up/shut-down decisions
ε	Noise symbols
V	Voltage magnitude
x	Scheduled state of thermal units
y,z	Binary variables to model the status of ESS units
$\hat{\cdot}$	Affine representation of an uncertain variable

I. Introduction

Microgrids have gathered significant attention over the last decade, due to their potential to integrate renewable energies (RE) into power systems in a reliable and efficient way, and their ability to provide sustainable energy supply solutions for remote areas without a main grid connection. Microgrids used for the latter application are known as isolated microgrids, since they are permanently operating in stand-alone mode. Isolated microgrids have specific technical features such as low inertia and a critical demand-supply balance constraint, which complicate their operation, especially in cases with high RE penetration [1].

In order to guarantee a reliable and economic operation of isolated microgrids, it is important to design adequate strategies and methods for their different control levels (e.g., primary and secondary controls) [1]. In these microgrids, secondary controls or energy management systems (EMS) have the main function of optimizing their operation through the solution of optimization problems such as unit commitment (UC) and/or economic dispatch (ED), for which different approaches have been reported in the literature. Some of the first works on microgrids EMS are based on deterministic formulations, where demand and RE are considered as deterministic variables, assuming a perfect forecast, with forecast errors being addressed through reserve constraints. For instance, in [2] the EMS problem of an isolated microgrid is formulated as a mixed-integer linear programming (MILP) problem, where generation costs and battery charge/discharge times are optimized based on RE and demand forecasts. An EMS algorithm is proposed in [3] to reduce the total cost of energy of an experimental isolated microgrid with energy storage systems (ESS) and demand response (DR). Such deterministic approaches do not directly take into account uncertainties, and thus their solutions may be either too conservative or unfeasible for some scenarios depending on the forecast accuracy, affecting the operation of microgrids.

Model predictive control (MPC) and coordinated control frameworks have been proposed in the literature to reduced the effect of forecast errors on microgrid operation. Thus, in [4], an EMS based in a double-layer coordinated control approach for a microgrid in grid-connected and stand-alone modes is presented; forecasting errors are taken into account by reserving adequate active power in the UC layer, and allocating that reserve in the dispatch layer. In [5], the energy management problem of a connected microgrid is formulated as an MILP problem, while a MPC framework is used to cope with forecast errors and disturbances. The authors in [6] proposed an EMS for isolated microgrids based on an MPC framework, with the energy management problem being decomposed in two stages in order to reduce the computational burden; the first stage corresponds to the UC formulated as an MILP problem, and the second corresponds to a three-phase optimal power flow (OPF) formulated as a nonlinear programming (NLP) problem. In [7], an integrated energy management system for isolated microgrids is proposed, where a coupled UC+OPF problem is solved, while an MPC technique is adopted to deal with uncertainties. Although the forecast errors are indirectly considered in the aforementioned papers, the uncertainty is not explicitly considered in the problem formulation, which requires defining an arbitrary reserve constraint that is typically overestimated to reduce the need for load shedding and/or RE curtailment.

Stochastic formulations of EMS for microgrids are also available in the literature. For instance, in [8] and [9], uncertainties are modeled by a scenario-based stochastic programming approach, where scenarios are generated by a roulette wheel mechanism. The resulting problem in [8] is solved by a multi-objective teaching-learning-based optimization technique, while in [9] it is solved with an optimization strategy based on an Adaptive Modified Firefly algorithm; both works look at the minimization of cost and emissions for a grid-connected microgrid. A two-stage stochastic energy management model for a connected microgrid is proposed in [10], with Monte Carlo simulations being used to generate wind and solar scenarios. In [11], a stochastic-predictive EMS for isolated microgrids is proposed, with the uncertainty being considered using a two-stage decision process combined with a receding horizon approach, where the first stage is the UC formulated as a stochastic MILP problem, and the second stage is an OPF problem formulated as an NLP problem with receding horizons. The main drawbacks of these approaches are the need to accurately identify probability distribution functions (pdfs), and the fact that feasibility is not guaranteed for all the uncertainties, especially when scenario reduction techniques are employed to reduce the computational burden.

Non-probabilistic methods such as fuzzy logic are also available in the literature [12], but it is difficult to specify an appropriate membership function in uncertain environments such as those of isolated microgrids [13]. Recently, Self-Validated Computing techniques (e.g., Interval optimization (IO), Affine Arithmetic (AA)), have been proposed for addressing uncertainty in power systems [14], [15]. These techniques have shown interesting features such as their capability of modeling the uncertainty in optimization problems without requiring information about the type of uncertainty in the parameters [16]; thus, pdfs are not required to obtain accurate and robust solutions. In [17], an AA-based approach to solve the OPF problem for microgrids is presented, and an EMS for connected microgrids based on AA to model uncertainties is proposed on [18]. In none of the aforementioned works UC constraints and energy storage systems (ESS) are considered, due to the challenges of integrating integer variables and intertemporal constraints in the AA formulation, which are addressed in the current paper.

In this paper, a mathematical formulation of an AA-based EMS for isolated microgrids is presented, where both real-time economic dispatch and UC problems are solved. This formulation is partially based on [19], where an AA-based UC (AAUC) method is introduced for the first time for transmission grids; furthermore, in the present work, ESS constraints are properly included in the model, and an iterative dispatch algorithm is proposed to reduce errors associated with the estimation of the uncertainty intervals. Hence, the main objectives and contributions of the current paper can be summarized as follows:

- A novel microgrid EMS formulation approach based on AA, considering ESS and intertemporal constraints, is proposed. The AA-based EMS solves an AAUC problem and then finds real-time economic dispatch solutions by applying a novel dispatch procedure.
- The presented approach is compared using a benchmark microgrid to deterministic, MPC, stochastic, and stochastic-MPC techniques, demonstrating that the AA-based EMS approach provides robust and cost-effective solutions, with adequate reserves for secure microgrid operation.

The rest of this paper is organized as follows: Section II briefly reviews relevant background material, introducing deterministic, stochastic, and MPC formulations, and various relevant AA concepts. Section III discusses the proposed AA-based EMS, and in Section IV, a modified CIGRE microgrid benchmark is used to assess and compare the performance of the proposed method. Finally, Section V summarizes the main conclusions and contributions of this paper.

II. Background Review

A. Deterministic EMS model

The main function of a practical EMS is to guarantee the optimal operation of the microgrid through the solution of a UC problem [20]. This problem is formulated as an MILP problem, whose objective function is given as:

$$\min_{P_{i,t},x_{i,t},SU_{i,t},SD_{i,t}} \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}}[F_i(P_{i,t})x_{i,t}+P_t^{LC}C^{LC}]\Delta t + C_i^{sdn}SD_{i,t}+C_i^{sup}SU_{i,t} \quad (1)$$

where all variables and parameters in this and other equations are defined in the Nomenclature section. The function $F_i(.)$ accounts for the cost of each thermal unit using a piecewise linear upper approximation of the convex cost curve. The constraints of this optimization problem are described next.

1) Generalized UC Constraints:

$$\sum_{i\in\mathcal{I}}P_{i,t}+W_t+PV_t-P_t^{RC}+\sum_{n\in\mathcal{N}}(P_{n,t}^{dch}-P_{n,t}^{ch}) = D_t^P-P_t^{LC} \quad \forall t\in\mathcal{T} \quad (2)$$

$$P_i^{min}x_{i,t}\le P_{i,t}\le P_i^{max}x_{i,t} \quad \forall\ t\in\mathcal{T},i\in\mathcal{I} \quad (3)$$

$$P_{i,t+1}-P_{i,t}\le R_i^{up}\Delta t+SU_{i,t+1}P_i^{min}\forall\ t,t+1\in\mathcal{T},i\in\mathcal{I} \quad (4)$$

$$P_{i,t}-P_{i,t+1}\le R_i^{dn}\Delta t+SD_{i,t+1}P_i^{min} \quad \forall\ t,t+1\in\mathcal{T},i\in\mathcal{I} \quad (5)$$

$$SU_{i,t}-SD_{i,t}=x_{i,t}-x_{i,t-1} \quad \forall\ t\in\mathcal{T},i\in\mathcal{I} \quad (6)$$

$$SU_{i,t}+SD_{i,t}\le 1 \quad \forall\ t\in\mathcal{T},i\in\mathcal{I} \quad (7)$$

$$x_{i,t}=g_{i,t}^{on/off} \quad \forall\ t\le(L_i^{up,min}+L_i^{dn,min}),i\in\mathcal{I} \quad (8)$$

$$\sum_{tt=t-g_i^{up}+1}^{t}SU_{i,tt}\le x_{i,tt} \quad \forall t\ \ge L_i^{up,min} \quad (9)$$

$$\sum_{tt=t-g_i^{dn}+1}^{t}SD_{i,tt}\le 1-x_{i,tt} \quad \forall\ t\ge L_i^{dn,min} \quad (10)$$

Here, constraints (2) are the power balance equations. Constraints (3) corresponds to minimum and maximum generation capacity limits of controllable units. Constraints (4) and (5) impose the ramp-up and ramp-down rates limits of the dispatchable generators, respectively. Constraints (6) and (7) associate the unit commitment decisions with the status variable, as well as ensuring that each unit is not turned-on and -off simultaneously. Finally, constraints (8)-(10) enforce the minimum up-time and the minimum down-time.

2) ESS Constraints:

$$SOC_{n,t+1}-SOC_{n,t}=\left(P_{n,t}^{ch}\eta_n^{ch}-\frac{P_{n,t}^{dch}}{\eta_n^{dch}}\right)\Delta t \quad \forall t,t+1\in\mathcal{T},n\in\mathcal{N} \quad (11)$$

$$SOC_n^{min}\le SOC_{n,t}\le SOC_n^{max} \quad \forall t\in\mathcal{T},n\in\mathcal{N} \quad (12)$$

$$P_n^{ch,min}y_{n,t}\le P_{n,t}^{ch}\le P_n^{ch,max}y_{n,t} \quad \forall t\in\mathcal{T},n\in\mathcal{N} \quad (13)$$

$$P_n^{dch,min}z_{n,t}\le P_{n,t}^{dch}\le P_n^{dch,max}z_{n,t} \quad \forall t\in\mathcal{T},n\in\mathcal{N} \quad (14)$$

$$y_{n,t}+z_{n,t}\le 1 \quad \forall t\in\mathcal{T},n\in\mathcal{N} \quad (15)$$

Constraints (11) are the energy balance constraints of the ESS. Constraints (12) enforce the SOC minimum and maximum limits of ESS units. Constraints (13) and (14) enforce the minimum and maximum charging/discharging power limit of ESS units, respectively. Finally, constraints (15) avoid simultaneous charging and discharging of ESS.

3) Reserve Constraint: As previously mentioned, a reserve constraint has to be include in a deterministic formulation of the energy management problem, in order to reduce the risk of stability problems or expensive load curtailments. This constraint can be defined as follows:

$$\sum_{i\in\mathcal{I}}(x_{i,t}P_i^{max}-P_{i,t})+\sum_{n\in\mathcal{N}}(P_n^{dch,max}-P_{n,t}^{dch}+P_{n,t}^{ch}) \geq R^L(D_t^P-P_t^{LC})+R^W W_t+R^{PV}PV_t \quad \forall t\in\mathcal{T} \quad (16)$$

The second term on the left is included here in order to allow ESS to provide both primary reserve and peak-shaving services [21].

B. Model Predictive Control

In an MPC approach the main idea is to solve the EMS problem given by (1)-(16) over a finite-horizon T, based on the available day-ahead forecast of demand and RE, and the current state of the system. The process defines the state of the variables at time $t=t+\Delta t$ only, discarding the solutions obtained for the rest of the horizon time $t+2\Delta t, ..., T$, and the whole process is repeated every Δt.

C. Stochastic Optimization Formulation

In a stochastic programming approach, a set of scenarios is used to model the uncertainty of the random variables (e.g., RE generation, load). In this case, the following objective function is used, while enforcing constraints (1)-(16) for each scenario:

$$\min_{P_{i,t},x_{i,t},SU_{i,t},SD_{i,t}}\sum_{s\in\mathcal{S}}\pi_s\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}}\big[F_i(P_{i,t,s})x_{i,t} + P_{t,s}^{LC}C^{LC}\big]\Delta t+\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}}C_i^{sdn}SD_{i,t}+C_i^{sup}SU_{i,t} \quad (17)$$

This formulation can be modified in order to include risk measures such as conditional value at risk, expected load not served, or worst-case regret, allowing to modify the solutions conservativeness. Since this topic is not the focus of the current paper, the readers are referred to [22], where risk measures are reviewed.

D. Optimal Power Flow

In some cases, such as heavily-loaded microgrids, both active and reactive power flows have to be considered in the energy management problem to guarantee a feasible dispatch and thus adequate operation of the microgrid. In this case, power flow constraints have to be included in the EMS, resulting in a Mixed-Integer Non-linear Programming problem (MINLP), which is usually decomposed into a UC problem, described previously, and an microgrid OPF problem, since the full problem is generally hard to solve. The microgrid OPF problem consists in minimizing (1), while satisfying the constrains (3)-(16), plus reactive power, voltages magnitude, and feeder current limits, and the following active and reactive power balance constraints for each bus:

$$\sum_{i\in\mathcal{I}_b}P_{i,t'}+W_{j,t'}+PV_{j,t'}-P_{j,t'}^{RC}+\sum_{n\in\mathcal{N}_b}(P_{n,t'}^{dch}-P_{n,t'}^{ch}) -(D_{j,t'}^P-P_{j,t'}^{LC})=V_{j,t'}\sum_k V_{k,t'}Y_{j,k}\cos(\delta_{j,t}-\delta_{k,t}-\theta_{j,k}) \quad \forall t'\in\mathcal{T}', j,k\in\mathcal{B} \quad (18)$$

$$\sum_{i\in\mathcal{I}_b}Q_{i,t'}+\sum_{n\in\mathcal{N}_b}Q_{n,t'}^{ess}-Kd_{j,t'}(D_{j,t'}^P-P_{j,t'}^{LC}) =V_{j,t'}\sum_k V_{k,t'}Y_{j,k}\sin(\delta_{j,t}-\delta_{k,t}-\theta_{j,k}) \quad \forall t'\in\mathcal{T}', j,k\in\mathcal{B} \quad (19)$$

This problem is typically solved for each instant of time t' in a UC interval t, using the discrete variables values obtained from the UC solution as input parameters. Note that unbalancing conditions have not been explicitly considered here; however, this problem can be readily addressed in the presented formulation by replacing the aforementioned balanced equations with their equivalent three-phase representations, as discussed in [6].

E. Elements of AA

AA is a range analysis technique introduced in [23], which handles both external (e.g., imprecise or missing input data, uncertainty in the mathematical modeling) and internal (e.g., round off and truncation errors) uncertainty sources. AA is similar to standard Interval Mathematics (IM) [24], but this paradigm provides narrower bounds in the computing process by keeping track of correlations between the input and the computed quantities [25]. In AA, each uncertain variable χ has an affine representation $\hat{\chi}$, as follows:

$$\hat{\chi}=\chi_0+\chi_1\varepsilon_1+\chi_2\varepsilon_2+...+\chi_p\varepsilon_p=\chi_0+\sum_{h=1}^{p}\chi_h\varepsilon_h \quad (20)$$

where χ_0 is the center value (more likely value) of the variable χ; ε_h are p symbolic real variables assumed to be unknown but bounded in the interval [-1,1], which represent an independent component of the total uncertainty of the variable χ; and χ_h are the coefficients defining the magnitude of the corresponding uncertainty components. One noise symbol can contribute to the uncertainty of multiple quantities (e.g., inputs, outputs); this sharing of noise symbols represents some partial dependency between two uncertain variables in affine form $\hat{\chi}$ and $\hat{\psi}$, determined by the coefficients χ_h and ψ_h [25]. As proposed in [15], noise symbols are used here to represent the uncertainties associated with load and RE source forecast errors, which affect all variables of the dispatch in a measure defined by the corresponding affine form coefficients, as explained in detail in the next section.

In order to perform mathematical operations in the AA domain, it is necessary to replace the elementary real-number operators by equivalent mappings between affine forms, as explained in [16]. For linear functions, the corresponding affine extension is obtained by expanding and rearranging only the noise symbols characterising the affine forms $\hat{\chi}$ and $\hat{\psi}$. However, if the mapping is non-linear, the corresponding affine extension cannot be described by an affine combination of the "primitive" noise symbols ε_h; in this case, it is necessary to identify an affine function, which approximates the function reasonably well over its domain. The reader is referred to [25] and [26] for further information of the definition of affine and non-affine operations.

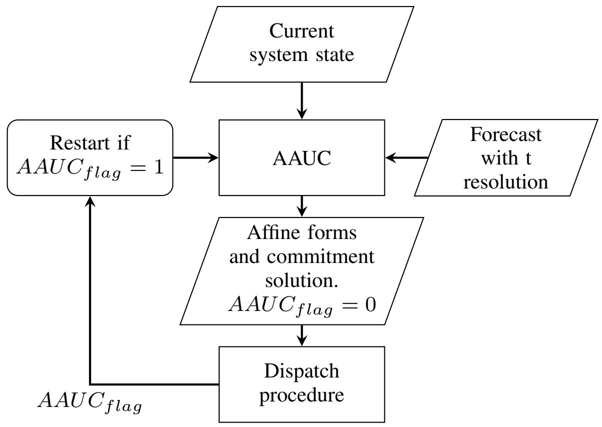

Fig. 1. Proposed EMS structure.

A new theoretical framework to solve uncertain optimization problems based on AA is introduced in [27]. More specifically, this framework aims to solve the following non-linear constrained optimization problem under data uncertainties represented as affine forms:

$$\begin{array}{llll} \min\limits_{\hat{z}} & \hat{f}(\hat{z}) & & \\ \text{s.t.} & \hat{g}_l(\hat{z}) & = & 0 \quad \forall l \in \mathcal{L} \\ & \hat{h}_m(\hat{z}) & < & 0 \quad \forall m \in \mathcal{M} \end{array} \tag{21}$$

where $\hat{z} = (\hat{z}_1, ..., \hat{z}_{N_z})$ represents the unknown affine form of the optimization variables, which include both dependent and control variables; $\hat{f}$ is the affine, continuous, and differentiable function describing the problem objectives; and $\hat{g}_j$ and $\hat{h}_k$ are continuous and differentiable affine functions representing the equality and inequality constraints, respectively. This theoretical framework proposes an extension into the affine domain of the minimization operator and the main comparison operators ($<, >, \leq, \geq$, and $=$), as well as a two stage decomposition algorithm to solve the problem given by (21). The details of these operators and the two stage decomposition algorithm can be found in [19] and [28].

III. Proposed Affine Arithmetic-Based EMS

The proposed EMS is based on the algorithm illustrated in Fig. 1, where an AA-based UC (AAUC) is solved first, providing commitment and dispatch solutions in AA form. Then, a final dispatch solution is found by a dispatch procedure that computes the noise symbols values of the obtained affine forms, based on the most recent available information of demand, RE, and power reserves. If the actual forecast error is outside the uncertainly bounds considered in the AAUC solution process, resulting in load and/or RE curtailment, a re-calculation signal $AAUC_{flag}$ is then sent to the AAUC problem, and the whole process is repeated with the new forecast information. The AAUC formulation and the dispatch procedure are explained in detail next.

A. AA-based UC

Active power demand and RE generation uncertainties can be represented by affine forms as follows:

$$\hat{D^P}_t = D^P_{0,t} + \sum_{r=1}^{p_d} D^P_{r,t}\varepsilon_{r,t} \quad \forall t \in \mathcal{T} \tag{22}$$

$$\hat{W}_t = W_{0,t} + \sum_{u=1}^{p_w} W_{u,t}\varepsilon_{u,t} \quad \forall t \in \mathcal{T} \tag{23}$$

$$\hat{PV}_t = PV_{0,t} + \sum_{v=1}^{p_s} PV_{v,t}\varepsilon_{v,t} \quad \forall t \in \mathcal{T} \tag{24}$$

Note that the affine forms $\hat{D^P}_t$, $\hat{W}_t$, and $\hat{PV}_t$ do not share any noise symbol, since the uncertainty sources for the load and the RE generation are assumed to be independent. The number of noise symbols and the values of the partial deviations of the affine forms in (22)-(24) can be obtained by a characterization of the statistical properties of random variables associated with the uncertainties [27].

Based on (22)-(24), the continuous variables of the UC problem, i.e., generator active power, SOC of ESS, and charging/discharging power of ESS, can be represented by the following affine forms:

$$\hat{P}_{i,t} = P_{0,i,t} + \sum_{r=1}^{p_d} P_{r,i,t}\varepsilon_{r,t} + \sum_{u=1}^{p_w} P_{u,i,t}\varepsilon_{u,t} + \sum_{v=1}^{p_s} P_{v,i,t}\varepsilon_{v,t} \quad \forall t \in \mathcal{T}, i \in \mathcal{I} \tag{25}$$

$$\hat{SOC}_{n,t} = SOC_{0,n,t} + \sum_{r=1}^{p_d} SOC_{r,n,t}\varepsilon_{r,t} + \sum_{u=1}^{p_w} SOC_{u,n,t}\varepsilon_{u,t} + \sum_{v=1}^{p_s} SOC_{v,n,t}\varepsilon_{v,t} \quad \forall t \in \mathcal{T}, n \in \mathcal{N} \tag{26}$$

$$\hat{P}^{ch/dch}_{n,t} = P^{ch/dch}_{0,n,t} + \sum_{r=1}^{p_d} P^{ch/dch}_{r,n,t}\varepsilon_{r,t} + \sum_{u=1}^{p_w} P^{ch/dch}_{u,n,t}\varepsilon_{u,t} + \sum_{v=1}^{p_s} P^{ch/dch}_{v,n,t}\varepsilon_{v,t} \quad \forall t \in \mathcal{T}, n \in \mathcal{N} \tag{27}$$

where the first term of each affine form is the central value of the corresponding variable, the second term is the deviation of the variable because of the load forecasting errors, and the third and forth terms are the deviation of the variable associated with the RE generation forecasting errors. The main idea of the AAUC approach is to find the commitment status of dispatchable resources and the parameters of the affine forms (25)-(27), which simultaneously minimize the base-case scenario cost and the corrective dispatch cost, while satisfying all the generators and system constraints. The resulting problem is more complicated than the problem given by (21), due to the integer variables (e.g., $x_{i,t}$, $SU_{i,t}$ and $SD_{i,t}$) and intertemporal constraints (e.g., ESS energy balance, ramp constraints) on the AA-based UC formulation. These integer variables and intertemporal constraints were not considered in the two-stage decomposition algorithm proposed in [27] to solve an AA OPF problem; this approach cannot be used to solve the AAUC problem, as the solution of the first stage does not guarantee the feasibility of the second stage due to the integer variables. Therefore, the AA-based UC is here formulated as an MILP multi-objective optimization problem, with the following objective function:

$$\min_{\hat{P}_{i,t},x_{i,t},SU_{i,t},SD_{i,t}} \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}} w\Big[\big(F_{0,i}(\hat{P}_{i,t})x_{i,t}+P_{0,t}^{LC}C^{LC}\big)\Delta t + C_i^{sdn}SD_{i,t}+C_i^{sup}SU_{i,t}\Big] + (1-w)\Big[\sum_{h=1}^{p}\big|F_{h,i}(\hat{P}_{i,t})\big|x_{i,t} + \big|P_{h,t}^{LC}\big|C^{LC}\Big]\Delta t \quad (28)$$

where $p=p_d+p_w+p_s$; the first objective term represents the operating cost of the base-case or affine central value of the function cost, which is the sum of the cost of the dispatch $F_{0,i}$, the commitment cost, and the load curtailment price, for the central value; and the second objective term corresponds to the affine radius of the function cost, which comprise the re-dispatch $F_{h,i}$ and the variations on load curtailment $P_{h,t}^{LC}$ costs. The value of w can be chosen according to the decision maker's degree of conservativeness with respect to the uncertainties; thus, a value of w close to 1 leads to cost-effective solutions that are sensitive to uncertainties, while a value of w close to 0 leads to solutions that are more expensive but less sensitive to uncertainties.

The problem constraints are (6)-(10) and (15), as well as constraints (2)-(5) and (11)-(14), but formulated in affine form. Based on the operators of the theoretical framework proposed on [27], constraint (2), which is a non intertemporal equality constraint, has the following formulation in AA form:

$$\sum_i P_{h,i,t}+W_{h,t}+PV_{h,t}-P_{h,t}^{RC}+\sum_n(P_{h,n,t}^{dch}-P_{h,n,t}^{ch}) = D_{h,t}^{P}-P_{h,t}^{LC} \quad \forall h\in(0,...,p), t\in\mathcal{T}, i\in\mathcal{I}, n\in\mathcal{N} \quad (29)$$

Also, the inequality constraints (3) can be formulated in AA form as follows:

$$P_i^{min}x_{i,t}\le P_{0,i,t}-\sum_{h=1}^{p}\big|P_{h,i,t}\big| \quad \forall t\in\mathcal{T}, i\in\mathcal{I} \quad (30)$$

$$P_{0,i,t}+\sum_{h=1}^{p}\big|P_{h,i,t}\big|\le P_i^{max}x_{i,t} \quad \forall t\in\mathcal{T}, i\in\mathcal{I} \quad (31)$$

and similarly for (12)-(14).

For intertemporal constraints such as (4), (5), and (11), the theoretical framework cannot be directly applied, since these operators assume that the involved affine forms have the same noise symbols, which is not the case for these constraints, since ε_{t+1} is not necessary equal to ε_t. For these intertemporal inequalities, based on basic theory of interval analysis, if the upper limit of the affine form $\hat{\chi}_t=\chi_{0,t}+\sum_{h=1}^{p}\chi_{h,t}\varepsilon_{h,t}$, given by $\chi_{0,t}+\sum_{h=1}^{p}\big|\chi_{h,t}\big|$, is less than the lower limit of the affine form $\hat{\psi}_{t+1}=\psi_{0,t+1}+\sum_{h=1}^{p}\psi_{h,t+1}\varepsilon_{h,t+1}$, given by $\psi_{0,t+1}-\sum_{h=1}^{p}\big|\psi_{h,t+1}\big|$, then $\hat{\chi}_t$ is less than $\hat{\psi}_{t+1}$ in affine form, regardless of the value of their noise symbols. Thus, constraints (4), after representing $P_{i,t}$ in affine form using (25) and reorganizing, can be expressed as:

$$P_{0,i,t+1}+\sum_{h=1}^{p}P_{h,i,t+1}\varepsilon_{h,t+1}\le P_{0,i,t}+R_i^{up}\Delta t+ SU_{i,t+1}P_i^{min}+\sum_{h=1}^{p}P_{h,i,t}\varepsilon_{h,t} \quad \forall t,t+1\in\mathcal{T}, i\in\mathcal{I} \quad (32)$$

which can be reformulated as follows:

$$P_{0,i,t+1}+\sum_{h=1}^{p}\big|P_{h,i,t+1}\big|\le P_{0,i,t}+R_i^{up}\Delta t+ SU_{i,t+1}P_i^{min}-\sum_{h=1}^{p}\big|P_{h,i,t}\big| \quad \forall t,t+1\in\mathcal{T}, i\in\mathcal{I} \quad (33)$$

and similarly for (5).

For intertemporal equality constraints, one can only say that $\hat{\chi}$ is equal to $\hat{\psi}$ in affine form, if and only if $\varepsilon_t=\varepsilon_{t+1}$; thus, these kinds of constraints have to be treated differently. Therefore, constraints (11) are approximated here in the AA domain by equalizing the central values and the radius of the affine forms as follows:

$$SOC_{0,n,t+1}=SOC_{0,n,t}+\left(P_{0,n,t}^{ch}\eta_n^{ch}-\frac{P_{0,n,t}^{dch}}{\eta_n^{dch}}\right)\Delta t \quad \forall t,t+1\in\mathcal{T}, n\in\mathcal{N} \quad (34)$$

$$\sum_{h=1}^{p}\big|SOC_{h,n,t+1}\big|=\sum_{h=1}^{p}\big|SOC_{h,n,t}\big|+\left(\sum_{h=1}^{p}\big|P_{h,n,t}^{ch}\big|\eta_n^{ch} - \frac{\sum_{h=1}^{p}\big|P_{h,n,t}^{dch}\big|}{\eta_n^{dch}}\right)\Delta t \quad \forall t,t+1\in\mathcal{T}, n\in\mathcal{N} \quad (35)$$

which will not guarantee equality between the affine forms for all possible values of the noise symbols, but will assure that the operating limits (e.g minimum and maximum SOC) will be respected during all the optimization horizon, even for the extreme scenarios associated with the noise symbol values -1 and 1.

The resulting mathematical model has absolute values in the objective function and constraints, which can be linearized with additional variables and linear constraints [29], so that the optimization problem becomes an MILP problem that can be readily solved by using commercial solvers (e.g., CPLEX).

B. Dispatch Procedure

Except for the commitment decisions, all the outputs of the first step (e.g., SOC, active powers) are represented by affine forms. These affine forms can be transformed either into intervals or into particular solutions for given realizations of the random variables. Thus, a specific value in the interval $[-1,1]$ can be assigned to the affine forms noise symbols, based on the actual uncertain variable values at a given time t', as follows, for power demands:

$$\varepsilon_{h,t'}=\frac{D_{t'}^{r}-D_t^{f}}{D_{h,t}^{P}} \quad (36)$$

The same can be done for wind and solar power realizations. Note that t' and t are in principle different, since t' represent the dispatch interval (e.g., every 5 min, 15 min), while t represents the AAUC solution interval (e.g., every 30min, 1 h).

Once the values of the noise symbols are computed, a feasible economic dispatch solution for active powers can be found by replacing these values on the affine forms resulting from the solution of the AAUC problem. However, if the actual value of a noise symbol is out of the interval $[-1, 1]$ due to an underestimation of the forecast error, or the dispatch solution violates operating limits, as determined from power flow computations, load or RE power curtailment might be necessary. Therefore, the algorithm in Fig. 2 is proposed here to deal with these cases. Thus, for each time t', the algorithm checks if the forecast error is between the considered uncertainty bounds of the AAUC problem and checks for power-flow feasibility. If these conditions are met, the algorithm adjusts the dispatch solution using the "AA reserve", which is the power inside the operating interval provided by the AAUC solution for each dispatchable resource, as depicted with an illustrative example in Fig. 3, guaranteeing that the AAUC solution remains within the given AA intervals. If the AA reserve is not enough and/or power flow limits are violated, the actual reserve is accommodated, and if necessary, load or RE curtailment is made, guaranteeing that power-flow feasibility is maintained. In this case, the AAUC problem has to be solved once more, as illustrated in Fig. 1, since the AAUC solution is not "AA optimal" anymore for the rest of the optimization horizon.

The UC solution obtained by solving the AAUC problem can be used to produce dispatch set points using OPFs, as with the other EMS approaches presented here, using the balanced formulation discussed in Section II-D or an unbalanced one as in [6]. The dispatch procedure proposed here, based on balanced (or unbalanced if necessary) power flows, takes advantage of the features of the AAUC solution to reduce computational costs and facilitate the implementation of the proposed EMS, which are relevant issues in practical microgrid EMS as discussed in [6].

IV. NUMERICAL RESULTS

To validate and compare the proposed AA-based EMS approach, the modified CIGRE distribution network benchmark shown in Fig. 4 is used; the parameters of the distributed energy resources, as well as the load and RE profiles are the same as in [30]. A load curtailment price of $\$5/kWh$ is assumed here [11]. A simplified RE and load prediction model is used here, which assume a linear behavior of forecasting errors, as shown in Fig. 5, representing 2 standard deviations of a normal distribution used to obtain load, wind, and solar power forecasts. The forecasting performance indices considered here are from a remote microgrid in Huatacondo, Chile [31]. The presented formulations were all implemented in GAMS 23.3.3 [32], solving the UC and OPF problems with the CPLEX 12.1.0 and MINUS solvers [33], respectively. Simulations were performed on an Intel® Xeon® CPU L7555 1.87GHz 4-processor server.

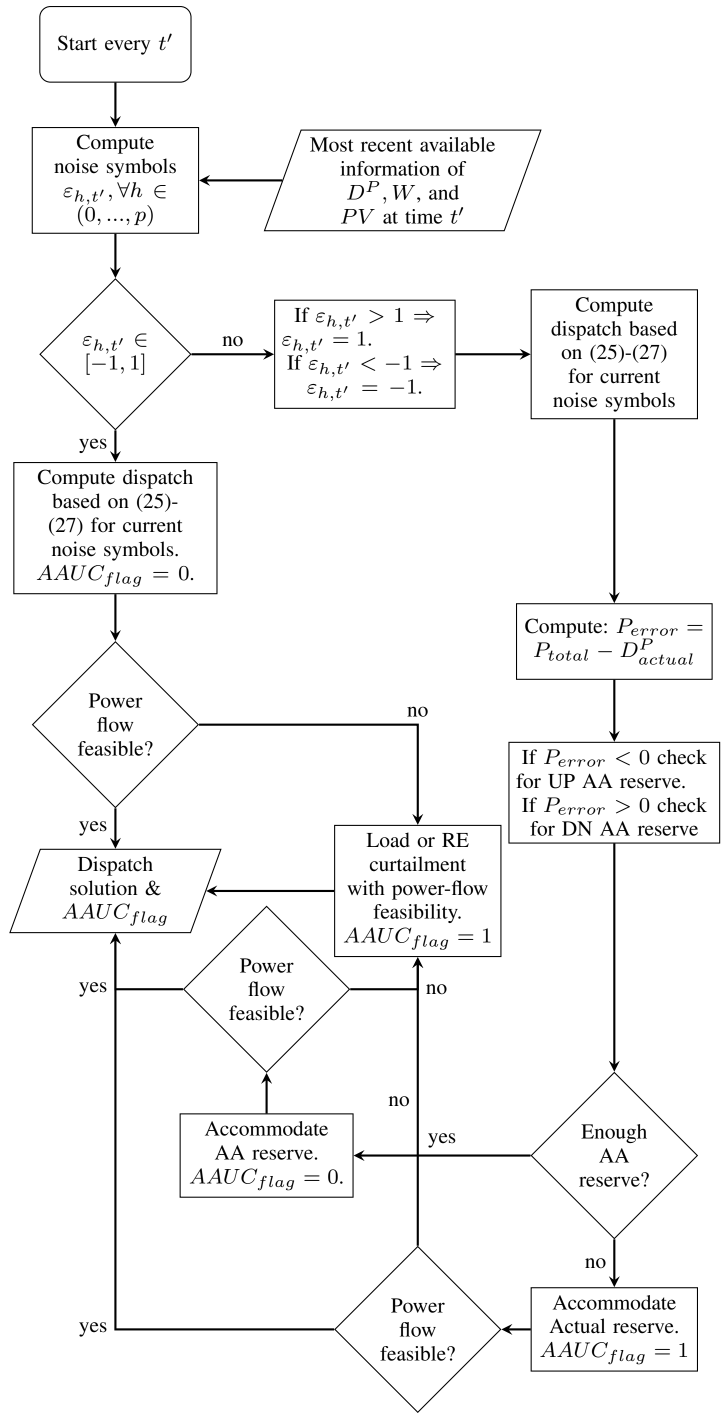

Fig. 2. Dispatch procedure in Fig. 1.

A. Study Cases

The study cases explained next allows assessing the performance of the proposed AAUC.

1) Deterministic (Det): For this case, a day-ahead UC problem with the formulation described in Section II-A and using forecasts with 1 h steps and a "middle" forecast error, as per Fig. 5, is computed first. Then, an OPF is computed each 5 min using the formulation described in Section II-D, with the SOC levels at the beginning and end of each hour and the binary variables obtained from the deterministic day-

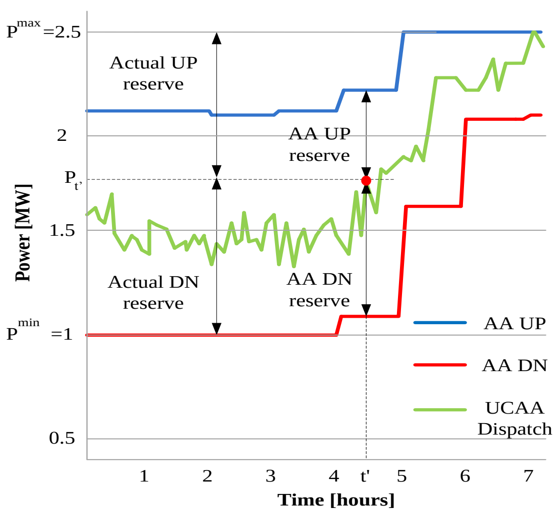

Fig. 3. Example of AA reserves.

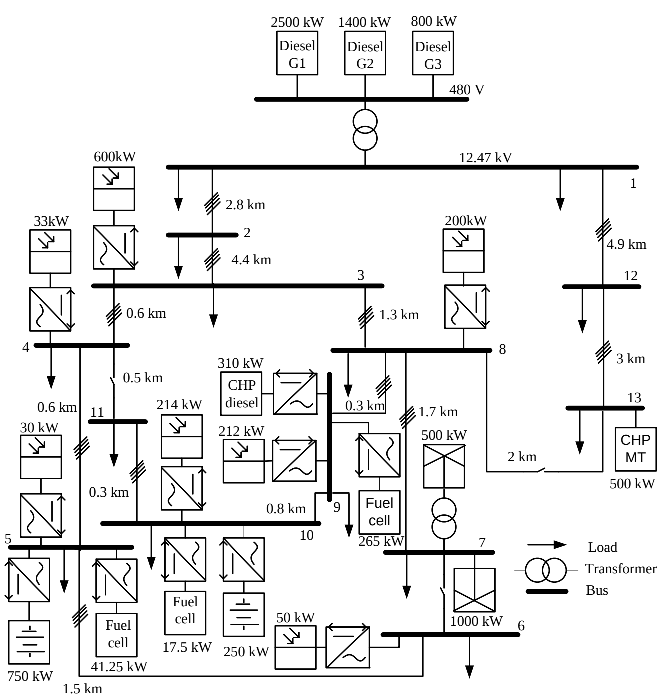

Fig. 4. Modified CIGRE microgrid benchmark [6].

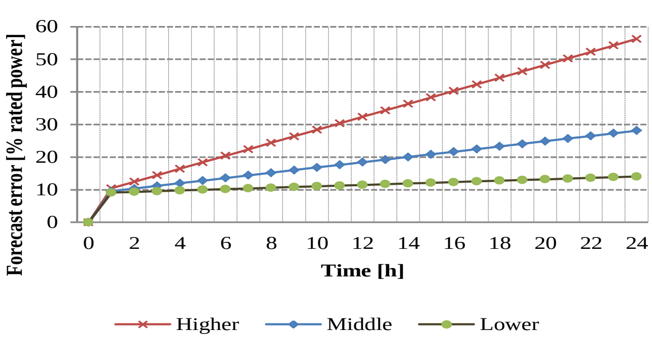

Fig. 5. Wind forecasting errors; and similarly for solar and demand powers

ahead UC solution. For the reserve constraint (16), R^L, R^W, and R^{PV} are assumed to be equal to 10%, 50%, and 25% of the output powers, respectively, which are typical reserve values used in isolated microgrids. The especial case of the deterministic formulation with a perfect 5 min forecast, i.e., the actual realization, over the 24 h horizon (Det-perf), is used as the "ideal" solution for comparison purposes.

2) MPC: The approach described in Section II-B is used here, with a re-calculation time of 5 min, and using a day-ahead forecast with 5 min steps with a "middle" forecast error. An OPF is computed at each 5 min step to guarantee a feasible dispatch solution.

3) Stochastic (Stoch): The formulation described in Section II-C is used in this case to obtain a UC solution, considering 100 day-ahead 1 h step scenarios of both RE and load, which are obtained from a scenario reduction process executed in the SCENRED2 tool of GAMS [34], using the backward reduction method, where a first set of 1000 scenarios, generated based on the aforementioned prediction model with a "middle" forecast error, is reduced to the final set used in the optimization process. An OPF solution is then obtained each 5 min using the formulation described in Section II-D, with the SOC levels at the beginning and end of each hour being equal to the mean of the SOC levels of all considered scenarios, and with the schedule obtained from the stochastic UC solution.

4) Stochastic-MPC (Stoch-MPC): For this case, the stochastic UC problem described above is solved each 1h, using the current forecast information for the scenario generation process. In each iteration, only the commitment solution obtained for the next hour is applied and an OPF solution is obtained each 5 min using this solution.

5) AAUC: This case is based on the method proposed on Section III for values of w equal to 0.9, 0.5, and 0.1, in order to show the effect of this parameter on the results [19]. The values of $D^P_{r,t}$, $W_{u,t}$, and $PV_{v,t}$ in (22)-(24) are equal to the load, wind, and photovoltaic power forecast errors at time $t = 24$ for a "middle" forecast error, respectively, in order to avoid frequent re-calculations of the AAUC problem. The number of noise symbols for the study case in the paper is $p = 3$, since it is assumed that the wind speed, solar radiation, and loads have similar statistical behavior, given the proximity of the RE generators and the assumed similar characteristics of loads typical of microgrids; therefore, a noise symbol is used to represent wind uncertainty, another for solar uncertainty, and a third one for load uncertainty. However, if after a statistical analysis, important differences between the behavior of different groups of RE generators or loads are identified, more noise symbols could be readily added to the system model, affecting to some degree the computational performance.

B. Results

Table I shows the operating costs for all study cases. The dispatches obtained with the AAUC method for $w = 0.9$ (shown in Fig. 6), $w = 0.5$, and $w = 0.1$ have a lower total cost than the deterministic and stochastic approaches at reasonable computational costs, since the AAUC method

TABLE I
DISPATCH COSTS FOR DIFFERENT EMS FORMULATIONS

	Operating cost [$]	Curtailment price [$]	Total cost [$]	Simulation time over 24h [s]
Det-perf	18,711.22	0	18,711.22	191.92
Det	18,580.48	2,825.03	21,405.51	95.74
MPC	18,564.78	1,673.27	20,238.06	1,135.88
Stoch	18,614.39	2,281.40	20,895.79	2,571.29
Stoch-MPC	18,682.80	1,076.62	19,759.42	12,599.53
AAUC $w = 0.9$	18,742.59	1,084.86	19,827.45	56.18
AAUC $w = 0.5$	19,329.37	979.90	20,309.27	81.10
AAUC $w = 0.1$	19,577.07	813.29	20,390.36	64.94

TABLE II
DISPATCH COSTS FOR DIFFERENT EMS FORMULATIONS AND FORECASTING ERRORS

	Total cost [$]		
	Forecast errors		
	Lower	Middle	Higher
Det	21,383.92	21,405.51	23,225.01
MPC	19,217.65	20,238.06	21,186.02
Stoch	20,188.62	20,895.79	22,120.21
Stoch-MPC	19,841.70	19,759.42	19,863.47
AAUC $w = 0.9$	19,756.77	19,827.45	19,763.68

TABLE III
DISPATCH COSTS FOR DIFFERENT AAUC INTERVALS

Interval factor	Operating cost [$]	Curtailment price [$]	Total cost [$]	Number of AAUC recalculations [$]
0.5	18,679.63	2,880.93	21,560.56	20
1	18,742.59	1,084.86	19,827.45	15
2	18,980.36	64.86	19,045.22	6

provides more robust solutions, resulting in a lower power curtailment prices than these methods. However, the solutions provided by the AAUC approach, for $w = 0.5$ and $w = 0.1$, are slightly more expensive than that of the MPC, since the latter has a shorter re-calculation time, using better forecasting information but resulting in higher computational costs. The stochastic-MPC method shows the most cost-efficient solution, but at large computational costs. Observe also that for small values of w such as 0.1, more robust solutions can be obtained, as less load curtailment is needed; however, these solutions have a higher operating cost, resulting, for this study case, in higher total costs than the solutions for larger values of w such as 0.9. This shows the importance of properly selecting the value of w, in order to obtain adequate solutions.

Table II shows the different approaches total costs for the forecasting error margins shown in Fig. 5. Note that the total costs of the deterministic and MPC methods are considerably affected when the quality of the forecast is reduced, since these methods do not directly take into account uncertainties in the formulation. For the other methods, the total costs do not show considerable changes for the different forecasting error levels considered here, since these accommodate enough reserves to respond to variations in the forecasting errors.

Table III shows the effect in the results of the proposed EMS as the AA partial deviations $D^P_{r,t}$, $W_{u,t}$, and $PV_{v,t}$ are multiplied by the factor in that table. Note that if the considered uncertainly intervals are too narrow, the method loses robustness, resulting in more expensive demand curtailments. In practice, the width of these bounds can be readily chosen based on historical data.

For the methods compared here and based on their inherent characteristics and the results obtained for the benchmark microgrid, which is more complex than existing microgrids in terms of number of components and feeder features, one can conclude the following: For cases where pdfs are well defined, UC stochastic optimization methods could to be a good choice, since these provide an accurate representation of the uncertainty resulting in adequate dispatch solutions, especially if an MPC approach is used, but at relatively high computational costs that may make them impractical for online applications. For microgrids with accurate forecast systems, the deterministic UC MPC approach could offer adequate dispatch solutions at very low computational costs. The proposed AAUC method provides cost-effective solutions at low computational costs, only requiring the identification of forecasting error intervals, in order to define the partial deviations to be used that can be obtained from simple statistical analyses (e.g., confidence intervals), thus offering adequate dispatch solutions for cases where pdfs cannot be readily identified; however, the method does require properly choosing, offline, the required weight w value to obtain reasonable results.

V. CONCLUSIONS

This paper proposed a novel microgrid EMS formulation based on an AA, considering ESS and intertemporal constraints. The AAUC was compared against deterministic, MPC, stochastic and stochastic-MPC approaches, using a microgrid benchmark system, with the results showing that the AAUC is able to provide robust and adequate cost-effective solutions at reasonable computational costs, without the need of assumptions regarding the statistical characteristics of the uncertainties. Finally, it was also shown that the AAUC solution conservativeness depends on the presumed uncertainty intervals and the value of the parameter w.

REFERENCES

[1] D. E. Olivares *et al.*, "Trends in Microgrid Control," *IEEE Trans. Smart Grid*, vol. 5, no. 4, pp. 1905–1919, Jul 2014.
[2] H. Morais *et al.*, "Optimal scheduling of a renewable micro-grid in an isolated load area using mixed-integer linear programming," *Renew. Energy*, vol. 35, no. 1, pp. 151–156, Jan 2010.
[3] M. Marzband *et al.*, "Experimental validation of a real time energy management system for microgrids in islanded mode using a local day-ahead electricity market and MINLP," *Energy Convers. Manag.*, vol. 76, pp. 314–322, Dec 2013.
[4] Q. Jiang, M. Xue, and G. Geng, "Energy Management of Microgrid in Grid-Connected and Stand-Alone Modes," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 3380–3389, Aug 2013.
[5] A. Parisio, E. Rikos, and L. Glielmo, "A Model Predictive Control Approach to Microgrid Operation Optimization," *IEEE Trans. Control Syst. Technol.*, vol. 22, no. 5, pp. 1813–1827, Sep 2014.
[6] D. E. Olivares, C. A. Canizares, and M. Kazerani, "A Centralized Energy Management System for Isolated Microgrids," *IEEE Trans. Smart Grid*, vol. 5, no. 4, pp. 1864–1875, 2014.

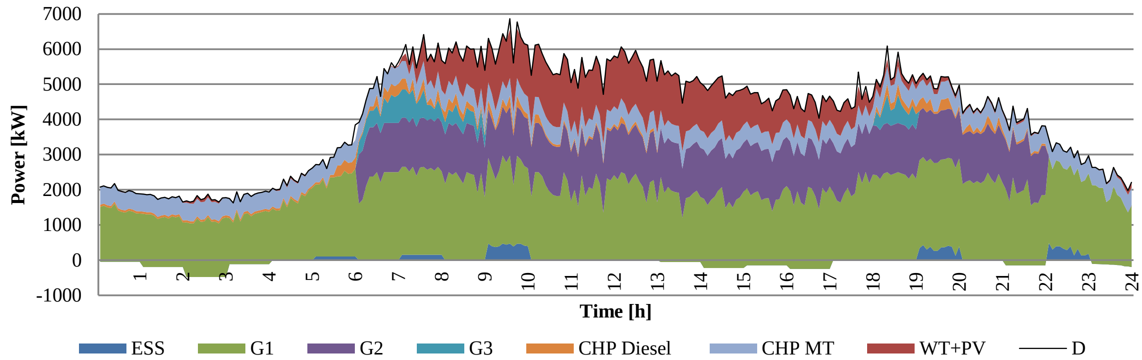

Fig. 6. AAUC dispatch results for $w = 0.9$.

[7] B. V. Solanki *et al.*, "Including Smart Loads for Optimal Demand Response in Integrated Energy Management Systems for Isolated Microgrids," *IEEE Trans. Smart Grid*, vol. PP, no. 99, pp. 1–10, 2015.

[8] T. Niknam, R. Azizipanah-Abarghooee, and M. R. Narimani, "An efficient scenario-based stochastic programming framework for multi-objective optimal micro-grid operation," *Appl. Energy*, vol. 99, pp. 455–470, Nov 2012.

[9] S. Mohammadi, S. Soleymani, and B. Mozafari, "Scenario-based stochastic operation management of MicroGrid including Wind, Photovoltaic, Micro-Turbine, Fuel Cell and Energy Storage Devices," *Int. J. Electr. Power Energy Syst.*, vol. 54, pp. 525–535, Jan 2014.

[10] W. Su, J. Wang, and J. Roh, "Stochastic Energy Scheduling in Microgrids With Intermittent Renewable Energy Resources," *IEEE Trans. Smart Grid*, vol. 5, no. 4, pp. 1876–1883, Jul 2014.

[11] D. E. Olivares *et al.*, "Stochastic-Predictive Energy Management System for Isolated Microgrids," *IEEE Trans. Smart Grid*, vol. 6, no. 6, pp. 2681–2693, Nov 2015.

[12] A. Chaouachi *et al.*, "Multiobjective Intelligent Energy Management for a Microgrid," *IEEE Trans. Ind. Electron.*, vol. 60, no. 4, pp. 1688–1699, Apr 2013.

[13] C. Jiang *et al.*, "A nonlinear interval number programming method for uncertain optimization problems," *Eur. J. Oper. Res.*, vol. 188, no. 1, pp. 1–13, Jul 2008.

[14] L. Wu, M. Shahidehpour, and Z. Li, "Comparison of Scenario-Based and Interval Optimization Approaches to Stochastic SCUC," *IEEE Trans. Power Syst.*, vol. 27, no. 2, pp. 913–921, May 2012.

[15] A. Vaccaro, C. A. Canizares, and K. Bhattacharya, "A Range Arithmetic-Based Optimization Model for Power Flow Analysis Under Interval Uncertainty," *IEEE Trans. Power Syst.*, vol. 28, no. 2, pp. 1179–1186, May 2013.

[16] J. Stolfi and L. De Figueiredo, "An Introduction to Affine Arithmetic," *TEMA - Tendências em Matemática Apl. e Comput.*, vol. 4, no. 3, pp. 297–312, Dec 2003.

[17] M. Pirnia *et al.*, "An affine arithmetic approach for microgrid dispatch with variable generation and load," in *2014 Power Syst. Comput. Conf.* IEEE, Aug 2014, pp. 1–8.

[18] V. Mohan *et al.*, "Microgrid Energy Management Combining Sensitivities, Interval and Probabilistic Uncertainties of Renewable Generation and Loads," *IEEE J. Emerg. Sel. Top. Circuits Syst.*, pp. 1–9, 2017.

[19] D. F. Romero and C. A. Canizares, "Affine Arithmetic Formulation of the Unit Commitment Problem Under Uncertainty," in *IREP'2017 Symposium*, Aug 2017.

[20] A. H. Hajimiragha and M. R. D. Zadeh, "Research and development of a microgrid control and monitoring system for the remote community of Bella Coola: Challenges, solutions, achievements and lessons learned," in *2013 IEEE Int. Conf. Smart Energy Grid Eng.*, Aug 2013, pp. 1–6.

[21] L. Sigrist, E. Lobato, and L. Rouco, "Energy storage systems providing primary reserve and peak shaving in small isolated power systems: An economic assessment," *Int. J. Electr. Power Energy Syst.*, vol. 53, pp. 675–683, Dec 2013.

[22] Q. P. Zheng, J. Wang, and A. L. Liu, "Stochastic optimization for unit commitment-a review," *IEEE Transactions on Power Systems*, vol. 30, no. 4, pp. 1913–1924, jul 2015.

[23] J. Stolfi and L. H. D. Figueiredo, "Self-validated numerical methods and applications," in *Proc. of the Monograph for 21st Brazilian Mathematics Colloquium, Citeseer*, 1997.

[24] R. E. Moore, *Methods and Applications of Interval Analysis*, ser. Studies in Applied and Numerical Mathematics. Society for Industrial and Applied Mathematics, 1979.

[25] L. H. de Figueiredo and J. Stolfi, "Affine arithmetic: Concepts and applications," *Numer. Algorithms*, vol. 37, no. 1-4, pp. 147–158, Dec 2004.

[26] A. Vaccaro, C. Pisani, and A. Zobaa, "Affine arithmetic-based methodology for energy hub operation-scheduling in the presence of data uncertainty," *IET Gener. Transm. Distrib.*, vol. 9, no. 13, pp. 1544–1552, 2015.

[27] A. Vaccaro and C. Canizares, "An Affine Arithmetic-based Framework for Uncertain Power Flow and Optimal Power Flow Studies," *IEEE Trans. Power Syst.*, vol. 8950, no. c, pp. 1–15, 2016.

[28] A. Vaccaro, "Affine arithmetic for power and optimal power flow analyses in the presence of uncertainties," Ph.D. dissertation, University of Waterloo, 2015.

[29] "Absolute Values". [Online]. Available: http://lpsolve.sourceforge.net/5.1/absolute.htm

[30] B. V. Solanki, K. Bhattacharya, and C. A. Canizares, "Integrated energy management system for isolated microgrids," in *2016 Power Syst. Comput. Conf.* IEEE, Jun 2016, pp. 1–7.

[31] R. Palma-Behnke *et al.*, "A Microgrid Energy Management System Based on the Rolling Horizon Strategy," *IEEE Trans. Smart Grid*, vol. 4, no. 2, pp. 996–1006, Jun 2013.

[32] R. E. Rosenthal, *GAMS–A user's guide*, September 2016. [Online]. Available: http://www.gams.com/help/topic/gams.doc/userguides/GAMSUsersGuide.pdf

[33] *GAMS –Solver manuals*. [Online]. Available: https://www.gams.com/latest/docs/S_MAIN.html

[34] "SCENRED2/GAMS documentation". [Online]. Available: https://www.gams.com/latest/docs/tools/scenred2/index.html

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