Optimal Operation of Distribution Feeders in Smart Grids

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***Abstract*—This paper presents a generic and comprehensive Distribution Optimal Power Flow (DOPF) model that can be used by Local Distribution Companies (LDCs) to integrate their distribution system feeders into a Smart Grid. The proposed three-phase DOPF framework incorporates detailed modeling of distribution system components and considers various operating objectives. Phase specific and voltage dependent modeling of customer loads in the three-phase DOPF model allows LDC operators to determine realistic operating strategies that can improve the overall feeder efficiency. The proposed distribution system operation objective is based on the minimization of the energy drawn from the substation while seeking to minimize the number of switching operations of load tap changers and capacitors. A novel method for solving the three-phase DOPF model by transforming the mixed-integer non-linear programming problem to a non-linear programming problem is proposed which reduces the computational burden and facilitates its practical implementation and application. Two practical case studies, including a real distribution feeder test case, are presented to demonstrate the features of the proposed methodology. The results illustrate the benefits of the proposed DOPF in terms of reducing energy losses while limiting the number of switching operations.**

***Index Terms*—Unbalanced distribution systems, distribution power flow, optimal feeder operation, Smart Grid**

I. Nomenclature

Parameters

α, β, γ	Scalar weights of the objective function components
ΔQ	Size of each capacitor block in capacitor banks, p.u.
ΔS	Percentage voltage change for each LTC tap
θ	Load power factor angle, rad
A, B, C, D	Three-phase ABCD parameter matrices, p.u.
Io	Load phase current at specified power and nominal voltage, p.u.
K	A constant multiplier
N	Total number of nodes
NC	Total numbers of controllable capacitor banks
Ni	Total numbers of integer variables
Nl	Total number of series elements
NL	Total number of loads
$Nmax$	Number of capacitor blocks available in capacitor banks
Nt	Total number of controllable tap changers
P	Active power of load, p.u.
Q	Reactive power of load/capacitor banks, p.u.
Vo	Nominal phase voltage, p.u.
W	The 3x3 identity matrix
X	Reactance of capacitor, p.u.
Z	Load impedance at specified power and nominal voltage, p.u.

Indices

a, b, c	Phases
C	Controllable capacitor banks, $C = 1, 2, ...NC$
C_n	Controllable capacitor banks at node n
d	Row of candidate solution matrix
h	Hours, $h = 1, 2...24$
l	Series elements, $l = 1, 2, ...Nl$
L	Loads, $L = 1, 2, ...NL$
L_n	Loads at node n
n	Nodes, $n = 1, 2, ...N$
ni	Integer variables, $ni = 1, 2, ...Ni$
p	Phases, $p = a, b, c$
r	Receiving-end
r_n	Receiving-ends connected at node n
s	Sending-end
s_n	Sending-ends connected at node n
t	Controllable tap changers, $t = 1, 2, ...Nt$

Variables

cap	Number of capacitor blocks switched in capacitor banks
E_{sub}	Energy drawn from substation, Mwh
I_p	Line current phasor, p.u.
$I_{p,p}$	Line-to-line current phasor, p.u.
$\overline{I}$	Vector of three-phase line current phasors, p.u.
J, J'	Objective functions
$Psub$	Power drawn from substation, p.u.
tap	Tap position
V_p	Line voltage phasor, p.u.
$V_{p,p}$	Line-to-line voltage phasor, p.u.
$\overline{V}$	Vector of three-phase line voltage phasors, p.u.
w	Continuous variables
x_o	Initial solution set
$\overline{x}$	Set of upper bound integers close to x_o
$\underline{x}$	Set of lower bound integers close to x_o

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X_B Candidate solution matrix
X_F Feasible solution matrix

II. Introduction

WITH the evolving concept of Smart Grids in electric power systems, Local Distribution Companies (LDCs) are gradually integrating, into their distribution systems, the use of advanced metering and two-way communication and automation technologies, among others . The U.S. Department of energy states: “Think of the Smart Grid as the internet brought to our electric system. Devices such as wind turbines, plug-in hybrid electric vehicles and solar arrays are not part of the Smart Grid. Rather, the Smart Grid encompasses the technology that enables us to integrate, interface with and intelligently control these innovations and others” [1]. These Smart Grids, because of environmental concerns and incentives from regulators, are expected to integrate high levels of Distributed Energy Resources (DERs). Furthermore, dynamic pricing schemes will force LDCs to integrate into their systems the customers participating in Demand Side Management, Demand Response and Demand Control programs [2]–[4], thereby rendering the distribution systems to be smart networks, where intelligent operation of resources will result in enhanced benefits to customers, DERs and LDCs [5], [6]. Some Smart Grid technologies may also adversely impact distribution systems. For example, high penetration of DERs may impact distribution losses, voltage profiles and equipment loadings, or single-phase DGs and PHEVs may increase phase unbalancing in distribution networks [7]. Also, with the introduction of dynamic pricing, the customer demand would be highly uncertain and system dependent [8], [9].

Conventional distribution system operation has been treated as a volt/var control problem over the years. To realize this control in distribution feeders, transformer load tap changers (LTCs) and Switched Capacitors (SCs) were the main controllable devices, with their controls being based on local measurements, while wide-area controls and measurements are an evolving concept. In this context, distribution loss minimization has been the operational objective in most cases. Some other possible distribution system operational objectives are discussed in [10], which include maintaining a unity power factor along the feeder, minimizing the power drawn from substation, maximizing the revenue for systems with DERs, and others. A “conservation biased” voltage control concept is proposed to reduce the power demand by reducing distribution system voltage within acceptable ranges [11]. The solution to these operational problems typically comprise: a Distribution Load Flow (DLF) solution, and an iterative optimization procedure. The DLF solution is usually obtained using a Newton-Raphson method [12], [13], or a fast-decoupled method adapted for distribution systems [14]. Special ladder network methods have also been proposed [15], [16], which employ forward and backward sweeps providing faster convergence in radial network configurations. Various solution methods for the optimization procedure have been developed, including fuzzy dynamic programming [17], artificial neural network-fuzzy dynamic programming [18], combinatorial optimization [19], and others. More recently, integrated optimization models and solution approaches have been proposed [20], [21], in which the DLF model is treated as a part of the optimization model and solution, and not independently. The present paper deals with the proposed optimization problem in the same integrated manner.

In optimization problems [17]–[21], the distribution system has been typically assumed to be a balanced three-phase system, and hence single-phase equivalents are used to reduce the computational burden. However, such an assumption for distribution feeders is not very realistic because of un-transposed three-phase feeders, existence of single-phase laterals and unbalanced loads. Thus, there is a need to consider three-phase models of distribution systems for more precise operational decisions in the context of an integrated optimal approach, as is the case in the present paper.

The implementation of real-time information systems, Advanced Metering Infrastructure (AMI), improved communication capabilities [22], more sensors, and improved infrastructure for control systems is envisaged to transform the conventional distribution system into a “Smart Grid” [23], which will bring in flexibility in distribution system operations via centralized control of distribution components such as LTCs, SCs and switches [24]. In this Smart Grid environment, the distribution system operator will have to consider various operational objectives, the need for which may arise from the flexible operational possibilities available to different players. This research proposes a general optimization framework where various objectives and constraints can be incorporated into distribution system operation; Fig.1 presents a schematic of the proposed framework. Observe that, real-time information systems will allow customers access to information such as energy price, emissions, incentives signals and weather; these data are essential components of the customer’s Energy Management System (EMS). The AMI, and communication technologies will provide information to LDCs regarding load profiles and the distribution system operating status. The improved control infrastructure would allow LDCs to control their equipment in distribution grids in real-time, based on a variety of operational objectives, as proposed here.

In view of the above discussions, the main objectives and contributions of the present work are:

1) To develop a decision-making framework for distribution systems operating with multi-faceted players with flexible operating possibilities. The proposed framework considers comprehensive models of distribution system components considering unbalanced conditions and voltage dependent loads. Such a three-phase Distribution Optimal Power Flow (DOPF) model is developed, for the first time, with the possibility to consider various operating objectives and constraints of the system and the various players.
2) Three-phase DOPF model so developed is a mixed

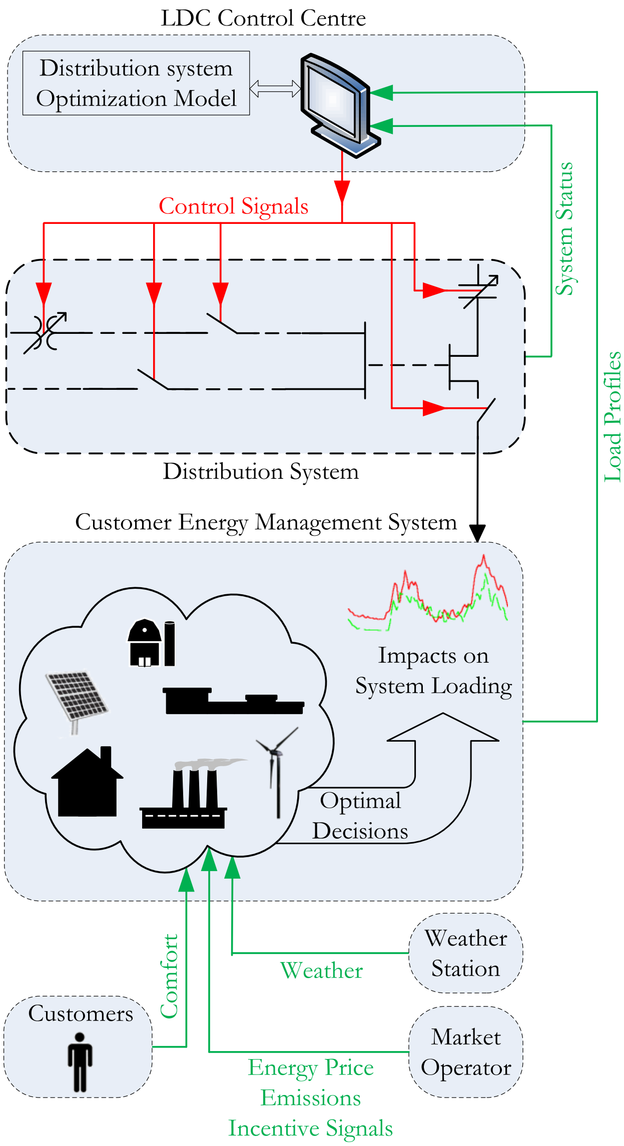

Fig. 1. Schematic diagram of the proposed research.

integer non-linear programming (MINLP) problem, because of the presence of integer variables associated with the discrete nature of LTCs and switched capacitors. In order to simplify the solution of such an MINLP problem, a solution method based on [20], [21] is adopted to relax the integer variables and convert the problem into a non-linear programming (NLP) one.

3) In order to guarantee a feasible solution for the NLP problem, which is discontinuous in nature, a novel local search method is proposed, where the search space is restricted to the nearest integer bounds of the NLP solution and a combinatorial search algorithm is implemented.
4) A new distribution system operating strategy to minimize the energy purchased from the external grid and reduce the number of switching operations of LTCs and capacitors is proposed, tested, and validated using the IEEE 13-node test feeder [25] and a Hydro One distribution feeder. Therefore, studies are carried out on the test feeders to examine the impact of the proposed technique on voltage profiles, energy savings and number of LTC and capacitor switching operations; computational burden analyses are also presented.

It should be noted that this research considers variations on the load profiles at each node in response to external inputs in a Smart Grid context, since these load profiles will be affected by signals such as weather information, energy prices, system-wise emission levels, incentive signals from system operators, etc., as depicted in Fig.1. Note that, DERs are not explicitly considered in the optimization framework discussed in this paper, and are the subject of research work currently underway.

The rest of the paper is organized as follows: Section III describes the proposed distribution system components modeling, the quadratic penalty function method adopted to relax the integer variables, and the optimization objectives proposed for distribution system operation. Section IV presents and discusses the results of the various case studies carried out for two realistic test systems to analyze the applicability of the proposed framework. A summary of the presented work and the main contributions of this paper is discussed in Section V.

III. Optimization Model

A. Three-phase Distribution System Components Modeling

The mathematical models of the distribution system components considered in this work, and basically described in [26] are discussed next. Thus, conductors/cables, transformers, LTCs and switches are series components modeled using ABCD parameters. Conductors and cables are modeled as π-equivalent circuits. Single-phase, two-phase, three-wire three-phase, and four-wire three-phase conductors and cables are represented. Switches are modeled as zero impedance series components.

Three-phase transformer models depend on the connection type (wye or delta), with the most common types of distribution system transformers being considered, i.e. single-phase and three-phase wye grounded-wye grounded, delta-wye grounded, and open wye-open delta connections. Voltage regulating transformers in distribution systems are equipped with LTCs. In this paper, single-phase LTCs and wye-connected three-phase LTCs are modeled, and three-phase LTCs are modeled with individual phase control and group control options.

Shunt components (loads and capacitors) are modeled for individual phases separately to represent unbalanced three-phase loads, since single-phase loads and single phase capacitors are common in distribution feeders. A polynomial load model is adopted, where each load is modeled as a mix of constant impedance, constant current, and constant power components. Capacitors are modeled as constant impedance loads. Capacitor banks are modeled as multiple capacitor units with switching options. Wye-connected and delta-connected loads and capacitors are represented.

B. Three-phase Distribution Optimal Power Flow

A three-phase DOPF model is developed based on the component models described in Section III-A. The developed model is a generic optimization model where any objective function can be selected for distribution system operations. In general, the optimization objective depends on the voltages/currents in series and shunt components, the tap position of LTCs and the number of capacitors switched as discussed in detail in the next Section. The constraints of this proposed three-phase DOPF are discussed in detail next.

For each series element, a set of equations based on the ABCD parameters are used, which relate the three-phase voltages and currents of the sending-end and receiving-end as follows:

$$\begin{bmatrix} \overline{V}_{l,s} \\ \overline{I}_{l,s} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \overline{V}_{l,r} \\ \overline{I}_{l,r} \end{bmatrix} \qquad \forall l \tag{1}$$

All parameters, indices and variables in the equations presented here are properly defined in Section I.

The ABCD parameters of all series elements are constant except for LTCs, which depend on the setting of tap positions during operation. The following additional set of equations is needed to represent the A and D matrices in (1) for each LTC:

$$A_t = W \begin{bmatrix} 1 + \Delta S_t \, tap_{a,t} \\ 1 + \Delta S_t \, tap_{b,t} \\ 1 + \Delta S_t \, tap_{c,t} \end{bmatrix} \qquad \forall t \tag{2}$$

$$D_t = A_t^{-1} \qquad \forall t \tag{3}$$

The variables $tap_{a,t}$, $tap_{b,t}$, and $tap_{c,t}$ take only integer values (e.g. -16 to +16 for a 32-step LTC). Equations (2) and (3) are for a tap changer with per-phase tap controls. For a three-phase tap changer, the following additional equation is used to make sure that all tap operations are the same:

$$tap_{a,t} = tap_{b,t} = tap_{c,t} \tag{4}$$

The next set of equations is used to represent each type of wye-connected load on a per-phase basis. Thus, for constant power loads:

$$V_{p,L} \, I^*_{p,L} = P_{p,L} + j \, Q_{p,L} \qquad \forall p, \forall L \tag{5}$$

For constant impedance loads:

$$V_{p,L} = Z_{p,L} \, I_{p,L} \qquad \forall p, \forall L \tag{6}$$

For constant current loads:

$$\left| I_{p,L} \right| (\angle V_{p,L} - \angle I_{p,L}) = \left| Io_{p,L} \right| \angle \theta_{p,L} \qquad \forall p, \forall L \tag{7}$$

The following mathematical models represent each wye-connected capacitor bank with multiple capacitor blocks:

$$V_{p,C} = X_{p,C} \, I_{p,C} \qquad \forall p, \forall C \tag{8}$$

$$X_{p,C} = \frac{-j \, Vo^2_{p,C}}{cap_{p,C} \, \Delta Q_{p,C}} \qquad \forall p, \forall C \tag{9}$$

$$Q_{p,C} = Nmax_{p,C} \, \Delta Q_{p,C} \qquad \forall p, \forall C \tag{10}$$

The variables $cap_{p,C}$ take only positive integer values in the range 0 to $Nmax_{p,C}$.

For delta-connected loads and capacitors banks, line-to-line voltages and currents need to be used. In that case, equations (5)-(10) can be used by replacing the line variables with line-to-line variables. The following additional equations are also needed to properly relate line-to-line variables to line variables:

$$\begin{bmatrix} V_{a,b} \\ V_{b,c} \\ V_{c,a} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_{a,c} \\ I_{b,a} \\ I_{c,b} \end{bmatrix} \tag{12}$$

The following equation corresponds to the line current balance at each node and phase:

$$\sum_l I_{p,l,r_n} = \sum_l I_{p,l,s_n} + \sum_L I_{p,L_n} + \sum_C I_{p,C_n} \qquad \forall p, \forall n \tag{13}$$

Also, at each node and phase, the voltages of the elements connected to that node are equal to the corresponding nodal voltage:

$$V_{p,l,s_n} = V_{p,l,r_n} = V_{p,L_n} = V_{p,C_n} = V_{p,n} \qquad \forall p, \forall n \tag{14}$$

Equations (1)-(12) correspond to the component models and (13)-(14) represent the circuit laws. Equations (1)-(14) define the equality constraint of the three-phase DOPF, which is a novel detailed representation of the distribution grid. In addition, distribution system operating constraints are also required. For example, distribution voltages at the point of load connection are required to be maintained between 0.95-1.05 p.u. Observe that, feeder current limits, transformer capacity limits, limits on the number of switchings of capacitors and LTCs and other appropriate limits, could be readily represented as inequality constraints in this three-phase DOPF model.

C. Proposed Optimization Objective

Electrical loads in a distribution system are voltage dependent. By operating the distribution system towards the lower limit of the acceptable voltage range, i.e. 0.95 p.u., the total energy consumption and system peak demand can be reduced. A Pacific Northwest study [11], [27] reveals that operating a distribution systems in such a way can bring about 3% reduction in energy consumption and 4% reduction in demand. Thus, available LTCs and capacitor banks can be employed to maintain the distribution system voltage close to the acceptable lower limit of 0.95 p.u.

As demand changes over a day, the LTCs and capacitors are required to switch frequently to maintain the voltage around the lower limit. However, since switching of these devices is associated with maintenance costs, frequent switching operations are not very desirable.

As per the previous discussions, a novel objective function is defined here to minimize the energy drawn from the substation as well as minimize the number of switching operation of LTCs and capacitors. This function can be defined as following:

$$J = \alpha \sum_{h} Psub_h + \sum_{p} \sum_{t} \left(\beta_t \sum_{h=2}^{24} \left| tap_{p,t,h} - tap_{p,t,h-1} \right| \right) + \sum_{p} \sum_{C} \left(\gamma_C \sum_{h=2}^{24} \left| cap_{p,C,h} - cap_{p,C,h-1} \right| \right) \quad (15)$$

The parameters α, β_t and γ_C are the weights given in the optimization process to energy drawn from the substation, LTC switchings and capacitor switchings, respectively. Selection of the weights depends on the level of priority attached to energy cost and control effort by the distribution system operators.

D. Solution Method

In the three-phase DOPF model, LTC and capacitor switching actions are discrete operations, which essentially render the proposed model be an MINLP problem. The number of continuous and integer variables increase with the size of the distribution system. The number of variables also increases significantly when distribution system operation decisions are optimized over a 24-hour timeframe as in (15). It should be noted that commercially available solvers for MINLP problems, in particular BARON and DICOPT, did not perform well to solve the proposed problem, in terms of solution time and convergence characteristics.

A method proposed in [20], [21] is adopted in this work that alleviates the use of integer variables and thus transform the three-phase DOPF into an NLP problem. A quadratic penalty term is augmented to the objective function (15), resulting in the following modified objective function:

$$J' = J + \sum_{ni} K_{ni} \left(w_{ni} - round(w_{ni}) \right)^2 \quad (16)$$

where w_{ni} represents the *tap* and *cap* variables. The quadratic term adds a high penalty value to the objective function at non-integer solutions, and thus drives w_{ni} close to its corresponding integer value $round(w_{ni})$. By employing the above method, the MINLP problem is converted into an NLP problem. The parameter K_{ni} needs to be carefully selected, as discussed in [20], [21], to obtain an optimal integer solution to the NLP problem (16), plus (1)-(14) equality constraints, and inequality constraints (operational limits).

Even after the MINLP to NLP conversion, commercially available NLP solvers (e.g. MINOS) do not guarantee reaching a feasible solution because of the presence of the discontinuous quadratic penalty term in the objective function. The feasibility is not guaranteed because it is possible that the integer variables $round(w_{ni})$ may lie outside the feasible region even though the variables w_{ni} lie in the feasible region of the optimization problem. This could happen, in particular, when the optimal solution is close to the boundary of the feasible region. To address this problem, a local search technique is proposed that searches the two integer numbers closest to w_{ni} that yield a feasible solution. Note that the solution thus obtained is locally sub-optimal, which is reasonable in practice. The solution obtained without a local search would be more accurate, if and only if, a feasible solution could be reached, which was not possible only using commercially available solvers as previously indicated, this did not allow making comparisons with and without local search.

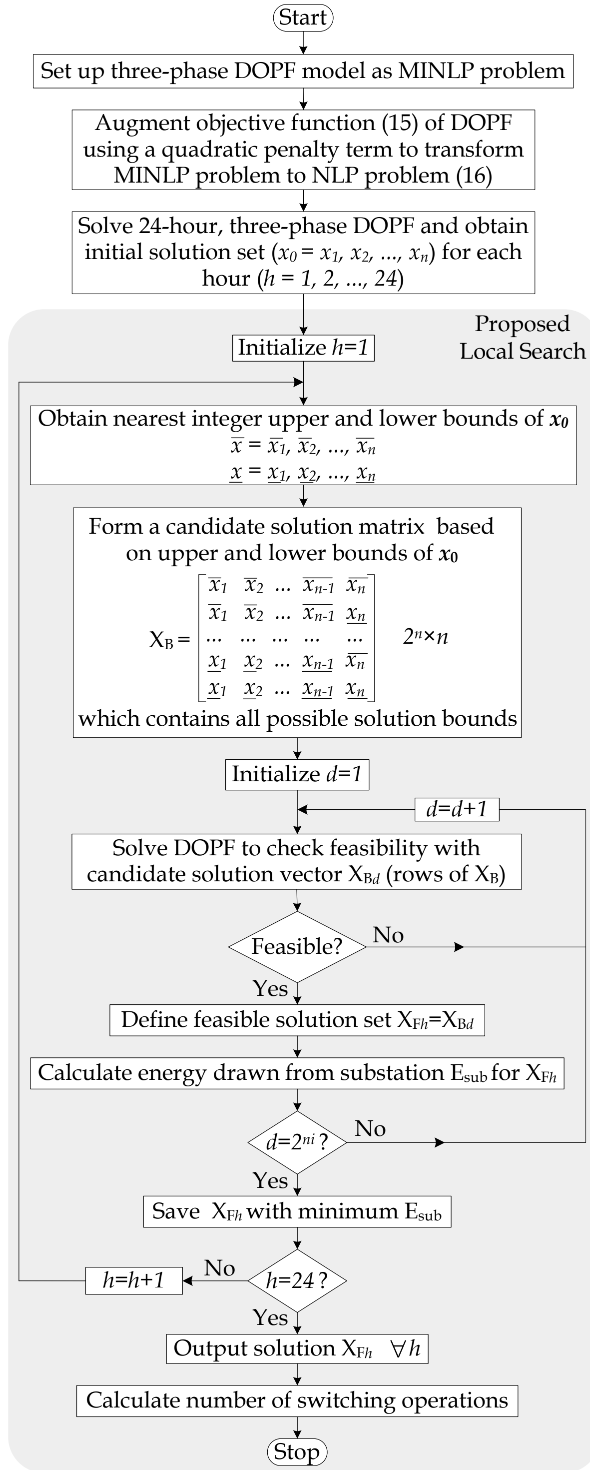

Fig. 2. Flowchart of the proposed DOPF and solution procedure.

The proposed three-phase DOPF, with the NLP approximation and local search procedure, is still computationally intensive because of the size of the search space resulting from the 24-hour timeframe analysis, since for *ni* integer variables, the search space is $2^{(24\,ni)}$. To reduce the search space, an hourly local search approach is implemented as explained in Fig.2; in this case, the search space is substantially reduced to $2^{ni} \times 24$. In this process, mathematical precision is somewhat compromised for the sake of reducing computational burden. However, in practical applications this is a reasonable sub-optimal approach that allows for the implementation of the proposed technique in the real-time.

IV. Case Studies

The result of applying the three-phase DOPF and the solution procedure proposed in Section III to the IEEE 13-node test feeder and a Hydro One distribution feeder are shown and discussed in this section. The proposed model is developed in GAMS [28], a high-level optimization problem modeling tool and solved using the MINOS solver [29]. For the simulations reported here, the DOPF is formulated using the objective function (16), plus equality constraints (1)-(14) and inequality constraints to represent voltage limits, since feeder current limit information was not available.

A. IEEE 13-node Test Feeder

The proposed three-phase DOPF model is first tested considering the IEEE 13-node test feeder shown in Fig.3 (the number of phases in the feeders are depicted with cross bars) [25]. To demonstrate and test the proposed three-phase DOPF, an optimal loss minimization solution is obtained for a given hour by allowing the LTC positions to vary, while keeping the capacitor banks fixed. As expected, the simulation result shows that losses can be reduced to 107.79 kW from 111.06 kW. The LTC positions for each tap obtained in this case are 16, -1, and 16, respectively. The corresponding node voltages obtained from loss minimization using the proposed three-phase DOPF model versus the base DLF solutions reported in [25] are compared in Fig.4. Observe that in the base DLF solution, the phase *b* voltage at node 675 is above the upper limit of 1.05 p.u. This can be attributed to the conventional voltage control procedures used in distribution systems, where LTCs are switched to control the voltage at a fictitious load center rather than maintaining voltages at all load nodes. In contrast, the load voltages obtained from the three-phase DOPF model are all within the range of 0.95-1.05 p.u.

To demonstrate the application of the proposed three-phase DOPF model in a 24-hour timeframe in the context of Smart Grids, in which load behavior responds to external inputs and thus cannot be considered to be the similar at each node as is typically done in distribution system studies, twenty-four hourly randomly generated load profiles at each node are considered. Each bus load representation comprises constant impedance, constant current, and constant power load model; the load data provided in [25] are assumed to be peak loads and the load profile reported in [30] is used. The random-load scenarios are generated using a typical load profile and a normal distribution function; thus, at each hour, a random number is generated to scale the load profiles below or above the nominal value using a normal distribution function (μ=1, σ=0.1). To shift load profiles in time, discrete random numbers that take integer values from -2 to +2 using a uniform distribution function are used; the load is then shifted ahead or behind in the time axis, where the integer number represents the number of hours to be shifted. This random load changes in magnitude and time yield the load profiles depicted in Fig.5.

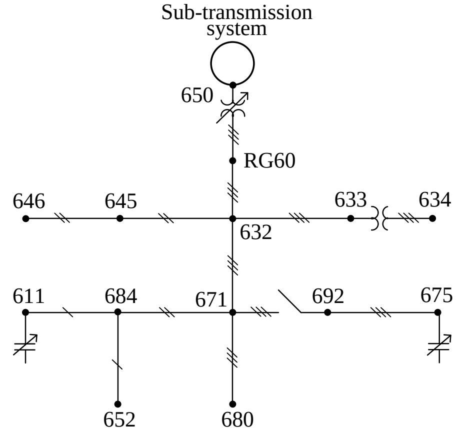

Fig. 3. IEEE 13-node test feeder [25].

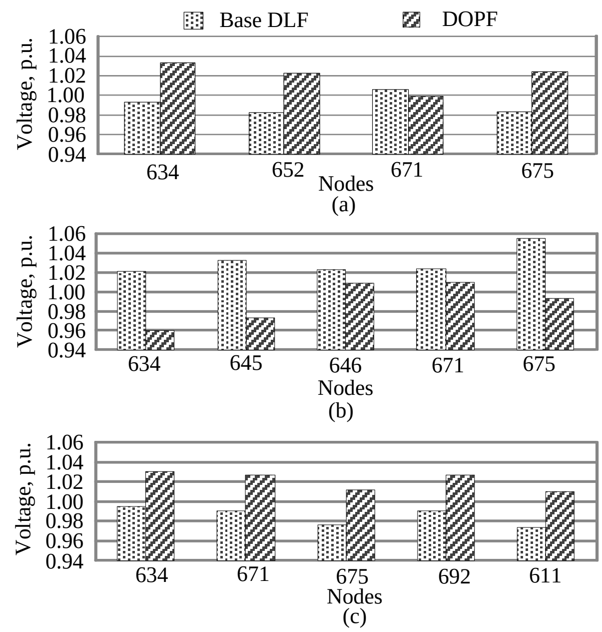

Fig. 4. Three-phase load voltage profiles for IEEE 13-node test feeder: (a) phase *a*, (b) phase *b*, and (c) phase *c*.

The capacitors available in IEEE-13 node test feeders are single units with fixed values. To demonstrate the applicability of the proposed method considering SCs, the given capacitor data are modified assuming that five blocks of 100 kVar capacitors are connected at node 675 in each phase, and five blocks of 50 kVar capacitors were connected at node 611 in phase *c*. The LTC and the two assumed capacitor banks are considered to be controllable. In the proposed objective function (15), equal weights are given to the switching operations of LTC and capacitors and are

TABLE I
SIMULATION RESULTS IN IEEE 13-NODE TEST FEEDER

Case	α	Number of switching operations				Energy from substation (Mwh)	Energy loss (Mwh)	% Reduction in energy from substation	% Reduction in number of switching operations
		tap_1	cap_1	cap_2	Total				
1	—	10	8	14	32	67.684	2.129	—	—
2	1.0	8	16	6	30	62.886	1.986	7.09	6.25
3	0.2	16	2	2	20	67.081	2.125	0.89	37.5
4	0.0	12	0	0	12	67.730	2.138	-0.07	62.5

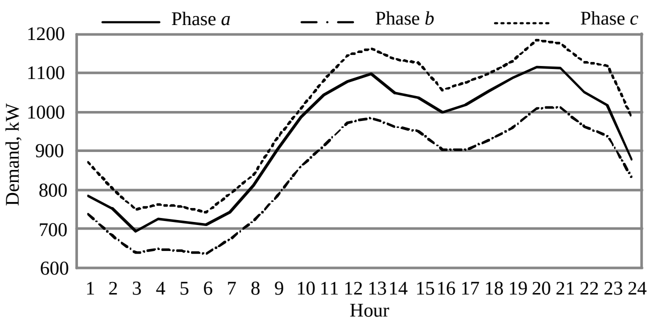

Fig. 5. Load profiles for IEEE 13-node test feeder.

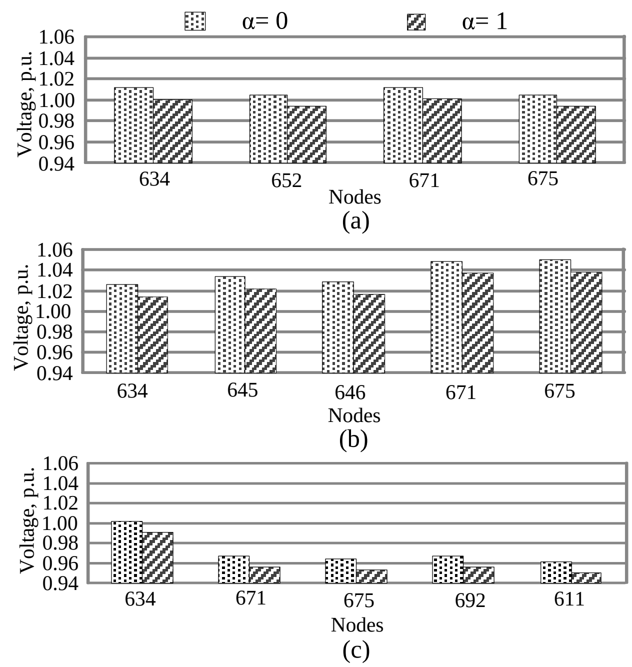

Fig. 6. Comparison of load voltages at peak load for cases $\alpha = 1$ and $\alpha = 0$: (a) phase *a*, (b) phase *b*, and (c) phase *c*.

considered complementary to the weight attached to the energy drawn from substation, i.e.

$$\beta_1 = \gamma_1 = \gamma_2 = 1 - \alpha \tag{17}$$

Table I presents the results from the simulation cases considering different value of α. In this table, Case 1 corresponds to a base DLF solution, so that appropriate comparisons can be made. Case 2 represents the minimization of energy drawn from the substation. In this case, the energy drawn from the substation is reduced by 7.09% compared to the base case, while the number of switching operations is reduced only by 6.25%. Case 4 represents the minimization of switching operations, thus resulting in the total number of switching operations of LTC and capacitors being only 12; in this case, the load voltages are not maintained near the lower limit of 0.95 p.u., thus the energy drawn from the substation is increased to 67.73 MWh. Compared to the base case, Case 4 requires 62.5% less switching operations while there is a 0.07% increase in the energy drawn from the substation, which is to be expected given the DOPF objective used. Case 3 represents a mixed minimization of the energy drawn from the substation and the total number of switching operations. This case is a compromise solution between Case 2 and Case 4 based on the weight α provided, and thus as expected, yielding a higher energy than Case 2 and more switching operations than Case 4. The choice of α depends on the distribution system operator's preference, based on the energy price and maintenance cost of LTC and capacitor switching mechanisms so that the overall operating cost can be optimized.

It is important to mention here that given the practical solution procedure used to deal with the integer variables, one cannot guarantee that the obtained solution is the local optimum. There might be better solutions in terms of both energy and number of operations as α changes. This is the trade-off between mathematical precision and computational effort. However, at least for $\alpha = 1$ and $\alpha = 0$, no better solutions than the minimum energy and minimum number of switching operations, respectively, are found.

Fig.6 shows the comparison of load voltages at peak load for two cases $\alpha = 1$ and $\alpha = 0$. Observe that, for $\alpha = 1$, when the distribution system operator is minimizing the energy drawn from the substation, the voltages are close to 0.95 p.u.; this is because of the frequent LTC and capacitor switchings carried out to maintain the voltage close to the lower acceptable boundary to reduce power demand. For $\alpha = 0$, the operator seeks to minimize the number of switching operations, and the voltage profile now no longer remains at the lower bounds, which thereby leads to increased system loading.

B. Hydro One Distribution Feeder

Simulations are also carried out considering a practical distribution feeder. For this purpose, an unbalanced

TABLE II
SIMULATION RESULTS IN HYDRO ONE DISTRIBUTION FEEDER

Case	α	Number of switching operations				Energy from substation (Mwh)	Energy loss (Mwh)	% Reduction in energy from substation	% Reduction in number of switching operations
		tap_1	tap_2	tap_3	Total				
1	—	12	14	50	76	291.619	6.090	—	—
2	1	28	6	12	46	286.976	6.058	1.59	39.47
3	0.6	10	12	20	42	293.793	6.264	-0.75	44.73
4	0	4	14	14	32	293.987	6.265	-0.81	57.89

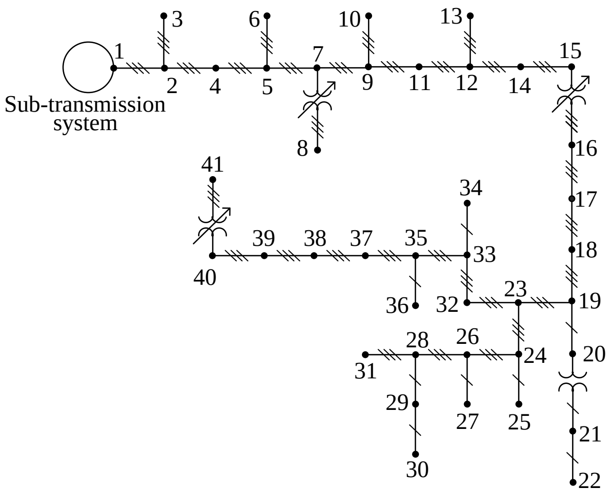

Fig. 7. Hydro One distribution feeder.

distribution feeder, which is a part of the distribution network of Hydro One Inc., is used [30]. The system configuration is shown in Fig.7.

The provided load data are considered to be peak loads, and 24-hour load profiles at each node are defined using the same random procedure used in the previous test system. In this case, a constant impedance load model is considered, directly calculated from the active and reactive powers at nominal voltage.

The system has three three-phase transformers and a single phase transformer. It is assumed that all three-phase transformers are equipped with LTCs, and these are the only controllable devices in the network.

Similar to Section IV-A, equal weights are attached to the switching operations of LTCs and are assumed to be complementary to the weight attached to the energy drawn from substation, as follows:

$$\beta_1 = \beta_2 = \beta_3 = 1 - \alpha \tag{18}$$

Table II presents the results from the simulation cases considering different values of α. In this table, Case 1 corresponds to a non-optimized base case. Case 2 represents the minimization of energy drawn from the substation resulting, as expected, in the maximum reduction of energy compared to the base case. Case 4 represents minimization of switching operations only, leading to the maximum reduction in the number of switching operations compared to the base case. Finally, Case 3 is a compromise solution between Case 2 and Case 4.

It should be noted from the analysis of the results for the two test systems that the energy losses in the distribution network are minimum when the operator's objective is to minimize the energy drawn from the substation (Case 2). On the other hand, Case 4 seeks to minimize the LTC and capacitor operations, and hence the losses increase.

C. Computational Burden Analysis

For the simulation cases discussed in the previous sections, the computational burden is discussed here. Thus, Table III shows a comparison of the number of variables in the MINLP and NLP problems; the size of 24-hour and 1-hour search space; and the computational time required to solve the case studies. Observe that the size of the search space is greatly reduced by using the proposed hourly search space. Also, note that the size of the Hydro One feeder three-phase DOPF problem is almost 3 times the size of the IEEE 13-node test feeder, however, the computational time, on an Intel machine with eight 2.83 GHz 32-bit virtual processors, and 3GB memory running Windows Server 2003, is only about twice as much in all cases.

The reported results of the proposed algorithm for a real Hydro One distribution feeder demonstrates the practical application of the method, and results in a large and realistic optimization problem. For the proposed optimization problem, multiple feeders need not to be considered simultaneously, since individual feeders can be optimized separately. It can be seen that in all cases, the computational time required to solve the problem is such that the real-time application of the proposed methodology is feasible considering that non-optimized, over-the-counter software tools are used. Also, despite only yielding sub-optimal solutions, the results are such that it would certainly improve feeder operation.

V. CONCLUSION

In this paper a generic distribution optimal power flow (DOPF) model is proposed and tested for unbalanced distribution systems. The novel three-phase DOPF model incorporates single-phase, two-phase and three-phase representations for feeder, transformers, switches and LTCs within an optimization framework. Customer loads are realistically modeled as voltage dependent loads, so that the energy consumption profile can be suitably modified by optimal control actions. The integer decision variables

TABLE III
COMPUTATIONAL BURDEN ANALYSIS

System		IEEE 13-node Test Feeder				Hydro One Distribution Feeder			
Case		1	2	3	4	1	2	3	4
Computational Time (min)		2.33	4.76	3.65	3.18	6.20	10.08	9.67	7.15
MINLP	Continuous Variables	9792				26784			
	Integer Variables	168				216			
NLP Continuous Variables		9960				27000			
Search Space	24-Hour	4.72×10^{21}				4.72×10^{21}			
	Hourly	192				192			

present in the optimization model are treated as continuous variables using an appropriate solution methodology that transforms the original MINLP problem into an NLP, which is solvable using commercially available solvers.

The application of the proposed procedure to two realistic problems demonstrate that the desired objectives of minimizing the energy drawn from the substation as well as limiting the number of switching operations of the control devices can be feasibly achieved. Furthermore, the computational burden analysis show that the real-time, centralized, and optimal control of practical distribution feeders can be readily achieved using the proposed technique. The proposed methodology will certainly help to make distribution networks Smart Grids.

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