New Techniques to Speed Up Voltage Collapse Computations Using Tangent Vectors

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Abstract—**This paper discusses various methods based on network partitioning and voltage stability indices to accelerate the computation of voltage collapse points using continuation techniques. Partitioning methods derived from right eigenvector and tangent vector information are thoroughly studied, identifying limitations and probable application areas; a mixed partition-reduction technique is then proposed to reduce computational burden. Also, tangent vectors are used to define a clustering method for the identification at any operating condition of the critical area at the collapse point, and a new voltage stability index is defined based on the identification of this critical area. Finally, a predictor-corrector methodology based on this index and the continuation method is proposed for fast computations of voltage collapse points. All the different methods are compared based on the results obtained for the IEEE 300-bus test system, and a methodology is recommended based on its prospective computational savings.**

Keywords: **Network partitioning, voltage stability index, voltage collapse, continuation power flows, on-line security analysis.**

I. Introduction

In power systems, which are a certain class of nonlinear systems, saddle-node bifurcations have been shown as one of the primary causes for voltage collapse problems, and hence have been thoroughly studied in many articles and reports written on the subject, e.g., [1, 2, 3, 4, 5]. Most of these references show that these types of voltage collapse problems are typically encountered in power systems due to stressed operating conditions. Thus, these stability problems are usually studied in terms of the system loading level.

A. *Voltage Collapse*

Several papers have shown the direct relation between saddle-node bifurcations and voltage collapse problems, e.g., [6, 7]. Saddle-node bifurcations, also known as turning points, are generic codimension one local bifurcations of nonlinear dynamical systems of the form

$$\dot{x} = f(x, \lambda) \qquad (1)$$

where $x \in \Re^n$ are the state variables, $\lambda \in \Re$ is a particular scalar parameter that drives the system to bifurcation in a quasi-static manner, and $f : \Re^n \times \Re \mapsto \Re^n$ is a nonlinear function [8]. System (1) exhibits a saddle-node bifurcation at the equilibrium point (x_o, λ_o), i.e., $f(x_o, \lambda_o) = 0$, if the corresponding system Jacobian $D_x f|_o = D_x f(x_o, \lambda_o)$ has a unique zero eigenvalue, and some particular transversality conditions hold at that equilibrium point [9, 10].

In power systems, the sate variables x and nonlinear function $f(\cdot)$ are typically defined in terms of the quasi-steady-state phasor models used in transient stability studies. Thus, generator angular speed deviations and phasor bus voltages, magnitudes and angles, are usually an important part of vector x. The parameter λ is typically used to represent changes in the system loading, regardless of the load model used [7].

Not all events of voltage collapse in power systems can be associated to saddle-node bifurcations, or other kinds of bifurcations for that matter [3]. Some voltage collapse problems may be caused by fast dynamic events that have nothing to do with bifurcation phenomena, such as large disturbances that push the system outside its stability region producing voltage problems. Furthermore, other bifurcations have also been shown to trigger collapse problems, such as Hopf bifurcations [10] or chaotic blue sky bifurcations [11]. This paper deals only with locations of saddle-node bifurcations; thus, the scope of application in power systems of the proposed methodologies is limited to this particular area of voltage stability analysis.

In [12, 13], the authors show that, under certain assumptions, the power flow equations may represent some of the dynamic behavior of a particular power system model about the system equilibrium points defined by the manifold $f(x, \lambda) = 0$ of (1). This power flow model is particularly useful to study slow changes in system variables that drive the system from one stable equilibrium point to another, until a singularity of the associated Jacobians is encountered, which can be directly related to a saddle-node bifurcation of the original dynamical system [13]. Thus, the typical approach is to solve the power flow equations as the load, or other parameters, change, and at the same time monitor "small" eigenvalues of the power flow Jacobian at the various solution points. However, better numerical techniques are available to detect these singular bifurcation points as discussed below.

B. *Detection Methods*

Direct and continuation methods are techniques typically used to identify the location of saddle-node bifurcations in nonlinear system models [8]. One of the main properties of these methods is that the corresponding equations are not singular at the saddle-node bifurcation point, making them very useful to calculate the location of such equilibrium points in power systems [13].

Direct methods find the bifurcation point directly from a known operating condition by modified Newton-Raphson iterations [8]. These methods find the actual bifurcation point; however, they require a good initial guess and may fail if all

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system limits are considered [9, 14]. Although recent applications of interior point methods to the solution process of the direct method equations have mitigated some of the numerical problems of handling limits [15, 16], the use of these methods for identification of voltage collapse points in realistic power systems is still somewhat problematic [17].

Continuation methods, on the other hand, find the bifurcation point by tracing the bifurcation manifold $f(x, \lambda) = 0$ as λ changes, i.e., it traces the equilibrium points of the nonlinear system as the system parameters change, yielding an adequate approximation of the location of the bifurcation point. These methods, however, are time consuming, specially for large power systems [14]. Since continuation methods are typically used in power flow equations, they have become known as continuation power flows [18].

The authors in [19] suggest different techniques to reduce computational burden in the continuation process to expedite the calculation of the collapse point. The objective is to be able to quickly determine the location of the collapse point for a particular loading pattern using a continuation method, so that this technique can be utilized in an operating environment for On-line Security Analysis. In [19], a network partitioning technique base on reduce determinants and an equation reduction technique are proposed and studied using the IEEE 300 bus system described below. The current paper studies the limitations of new partitioning schemes based on eigenvector and tangent vector information, and a mixed partition-reduction method is proposed to speed up continuation power flows under certain conditions.

Voltage Stability indices are alternative techniques for detection of collapse points [2, 4]. These indices are scalar variables that are continuously monitored to determine how close a system is to a voltage collapse problem. Simple examples of voltage stability indices are the minimum singular value and real eigenvalue of the power flow Jacobian matrix; the closer these values are to zero, the closer the system is to collapse. Some indices, such as the singular values or eigenvalues, have the problem of being highly nonlinear, i.e., they change rapidly as the system approaches bifurcation, as illustrated for the 300 bus system below; therefore, they are not adequate for detection of proximity to collapse. The indices defined in [20] have a more predictable and smooth shape, the problem is that they require knowing the critical bus of the system at the collapse point. This critical bus corresponds to the bus with the largest entry in the right eigenvector associated to the zero eigenvalue at the point of collapse; thus, this bus cannot be identified in most cases until the system is rather close to collapse. A clustering technique based on tangent vector information and a new voltage stability index are proposed in this paper, so that critical buses may be identified early in the loading process to use the associated index to accelerate continuation power flow computations.

C. *Content*

The paper is organized as follows: Section II introduces general aspects of network partitioning based on several techniques used to identify weak areas of a power system for voltage collapse analysis [21, 22] . This section also discusses new power systems partitioning techniques for bifurcation analysis based on right eigenvector and tangent vector information, as well as the definition of a new voltage stability index, labeled here as the Tangent Vector Index (TVI). A new clustering technique is proposed in this section to detect, at any operating condition, the system's critical areas at the collapse point.

In section III, a mixed partition-reduction technique is proposed based on tangent vector information and the equation reduction methodology described in [19]. The use of the TVI for improving the computational performance of the continuation method is discussed in section IV. Section V shows the computational results in the IEEE 300-bus test system, applying and comparing the different methodologies. Finally, in section VI a summary of the limitations and areas of applications of the proposed techniques is presented to identify the most appropriate one.

II. Network Partitioning

From standard partitioning theory, let n be the number of nodes in a network, and C_{ij} the parameter that characterizes the connection between two nodes i and j. The definition of the parameter C_{ij} depends on the partitioning technique utilized, as discussed below. If C_{ij} is large, one can say that node i is strongly connected to node j; otherwise, nodes i and j are said to be weakly connected. A set of nodes strongly connected to each other is called a *block*. A network partitioning can be obtained based on the assumption that if nodes i and j are weakly connected, node i does not play an important role in the analysis of node j and can be, therefore, neglected. Thus, the determination of C_{ij} is crucial in network partitioning.

In [23], the author proposes an eigenvector-based decomposition technique. This method consists on partitioning the system by sorting the right eigenvector associated to the smallest eigenvalue of a connectivity matrix. Reference [24] employs this technique to obtain the initial partition and tries to improve it by using an interchange method. The technique consists on exchanging nodes among the different blocks, with the exchange being carried out up to a point where no more improvement is obtained for further interchanges.

For a power system partitioning, the trivial connection between two nodes is represented by the parameters of the transmission line that connects them. However, this measure does not directly identify whether two unconnected buses are strongly or weakly tied to each other. Hence, calculating C_{ij} in a power system may require to analyze a different "network" based on the original one. The authors in [21] define C_{ij} as the voltage variation in all load buses j with respect to a load variation in the bus of interest i. Reference [22] employs a reduced Jacobian determinant to define C_{ij}, showing good results in a small test system. In these papers, the partitioning is obtained in relation to the initial critical bus of the system, which is identified by the smallest reduced Jacobian determinant. In [25], however, the author shows that this technique may not work for realistic power systems.

A. *Partitioning by Right Eigenvector*

The idea of a network partitioning by an eigenvector-based technique has been exploited in [23, 24, 26], where clustering techniques are proposed, depending on the desired number of blocks. In this paper, the number of blocks is not previously defined, but calculated as a function of a C_{ij} defined below, and a specified threshold value k. Thus, if $C_{ij} \geq k$, for a given value of k, the two buses i and j are assumed to be strongly connected. Based on the knowledge that the "zero" right eigenvector (the eigenvector associated to the zero eigenvalue at the saddle-node bifurcation point) provides the rank of the critical buses in voltage collapse analysis, the following partitioning algorithm is proposed:

1. Find the right eigenvector associated to the smallest eigenvalue of the system Jacobian matrix for a known operating point, and sort the entries of this eigenvector in order of absolute magnitude.

2. Based on the most critical bus (largest absolute entry on the right eigenvector), say bus i, find an initial critical area formed by the first neighbors to this bus (level 1).

3. Add new levels of neighboring buses to this basic critical area, one at a time. For each level, the ratio between the eigenvector entries associated to each neighbor bus j and the critical bus i is defined as the partitioning parameter C_{ij}. Thus, the final critical area associated to the initial critical bus is obtained when the largest C_{ij} is smaller than an arbitrarily specified k, or when this value is larger than the corresponding value at the previous level.

4. Repeat the process for the buses of the remaining system to identify new clusters.

The buses that do not belong to the critical area but connect this area to the rest of the system (border buses), are added to the critical area by treating them as constant voltage buses, i.e., PV buses. This is done based on the assumption that their steady-state voltage magnitudes are not significantly affected by the parameter changes in the system.

For the 300-bus test system used in this paper, a value of the partitioning parameter k of 0.75 yielded the best results. This value of k has produced similar results in other test systems.

B. *Partitioning by Tangent Vector*

The technique described in the foregoing section depends on the determination of a right eigenvector, which could be expensive, particularly if the smallest real eigenvalue of the Jacobian matrix is not "close" to zero. Furthermore, as the system is loaded, this eigenvector may change significantly, which has been the authors experience [25], as is clearly demonstrated for the 300-bus test system below. Therefore, a cheaper and more reliable technique is desirable.

The tangent vector to the bifurcation manifold $f(x, \lambda) = 0$, i.e., $\mathrm{d}x/\mathrm{d}\lambda$, is proposed here to replace the eigenvector as the partitioning parameter in the algorithm described above. This vector is defined at a particular equilibrium point (x_*, λ_*) as

$$\left.\frac{\mathrm{d}x}{\mathrm{d}\lambda}\right|_* = -[D_x f|_*]^{-1} \left.\frac{\partial f}{\partial \lambda}\right|_* \tag{2}$$

From this equation, it is clear that the tangent vector is computationally inexpensive, as it can be computed at the maximum cost of one additional Newton-Raphson iteration of the power flow equations. Furthermore, based on (2), it can be readily demonstrated that the tangent vector converges to the zero right eigenvector at the bifurcation point.

The tangent vector $\mathrm{d}x/\mathrm{d}\lambda$ contains important information regarding how the system variables x are affected by changes on the parameter λ. Hence, one would expect better results when using this vector for system partitioning, as demonstrated by the results for the 300-bus test system shown below.

Using a similar partitioning algorithm as the one described for the right eigenvector, the tangent vector based methodology generates several clusters or blocks at the initial operating condition. In this case, a value of $k = 0.5$ has produced adequate results in several test systems, particularly for the 300 test system used in this paper. The first two or three clusters identified by the this partitioning method are then used to define a voltage stability index with respect to the corresponding critical bus of the block, in a similar fashion as for the test functions described in [20]. The Tangent Vector Index (TVI) is defined then as

$$TVI_i = \left|\frac{\mathrm{d}V_i}{\mathrm{d}\lambda}\right|^{-1} \tag{3}$$

where $\mathrm{d}V_i/\mathrm{d}\lambda$ is the entry in the tangent vector $\mathrm{d}x/\mathrm{d}\lambda$ corresponding to the bus voltage magnitude V_i for a block's critical bus i. Observe that as the collapse point is approached, $\mathrm{d}V_i/\mathrm{d}\lambda \to \infty$ and, hence, $TVI_i \to 0$. After carrying out several tests on different systems, a "quadratic" profile was detected for TVI_i, when i corresponds to the system's critical bus at the collapse point, similar to the behavior reported in [20] for several other indices. Hence, if the bifurcation critical area is in the set identified by the clustering technique, then the corresponding TVIs show a predictable shape that can be used to quickly determine the collapse point using a continuation methodology, as explained in section IV.

The proposed clustering technique cannot be "theoretically" guaranteed to correctly identify the critical bus at the collapse point, especially when limits are encountered, as these have a significant effect on the tangent direction. However, the experience of the authors with applying the proposed tangent vector clustering technique to several "theoretical" and "realistic" systems has been very consistent with regards to the clear identification of the system critical areas at any operating condition, obtaining good results even when limits have a large effect on the tangent vector, as clearly demonstrated in section V for the 300-bus test system.

One important issue to consider in the eigenvector and tangent vector partitioning techniques is the singularity of the Jacobian $D_x f|_*$ in (2). In [20], the authors show that the power system must be rather close to the bifurcation point in order for this matrix to be "numerically" singular, and this is clearly depicted below for the 300-bus test system. It is also the experience of the authors, and an interesting and somewhat expected observation, that the larger the system, the more numerically stable the Jacobian matrix is. Nevertheless, to avoid singularity problems, local parameterization techniques similar to the ones proposed in [8] could be used, e.g., interchange one entry in x with the parameter λ.

C. *Load Partitioning*

The main unresolved problem with the proposed partitioning techniques is how to handle the active power injections and reactive power limits of the border buses that have been transformed into PV buses, since the partition subsystem has to behave in the same way as the original system, not only at the operating point where the partitioning is carried out, but for all values of λ up to the bifurcation point. For this reason, the partitioning techniques proposed here are only used to identify clusters where the load is allowed to changed, while the load buses on the rest of the system remain unchanged, with the exception of large system loads; the basic structure of the original system stays the same during the continuation process. In other words, only the load variation that drives the system to collapse is somewhat altered based on the blocks identified by the partitioning techniques. Observe that this technique does not completely change the direction of load increase in all buses, i.e., the amount by which particular loads are changed is the same as in the original system; the proposed methodology is designed to eliminate load buses that have no major effect on the system variables as the collapse point is approached. This technique is different than other techniques that have been proposed for the identification of the closest bifurcation point, where the left eigenvector is used to iteratively and fully define the direction of load increase [27]. The tangent vector partitioning approach plus the reduction technique described below, i.e., a mixed partition-reduction methodology, can be used to reduce the number of equations and variables during the continuation process to yield computational savings, as shown for the test system in section V.

III. System Reduction Techniques

The reduction technique proposed in [19] is used here to reduce the number of variables x and related equations as the system approaches the collapse point. This technique consists on eliminating the system variables that suffer little change during the calculation of the bifurcation manifold, i.e., a variable $x_i \in x$ is kept fixed at its last equilibrium value if

$$|x_i^l - x_i^{l-1}| \leq u \qquad (4)$$

where l is a step of the continuation process and u is a specified tolerance. The value of $u = 0.001$ has been determined to be adequate from several systems tests. Observe that this technique is basically based on tangent vector information, as variables with small entries in $\mathrm{d}x/\mathrm{d}\lambda$ are basically eliminated from the set of system equations.

A drawback of this method is the possible elimination of variables associated to system controls with limits, particularly bus voltages with large reactive power support. These voltages change slowly, or do not change at all, until voltage control is lost when minimum or maximum limits are reached. Hence, variables that are eliminated in the early stages of the continuation process, might become later a source of computational error. One may minimize this problem by reducing the system only every U number of steps of the continuation process, and by also considering the generator reactive power equations as part of the system set. From several systems tests, a good value has been determined to be $U = 10$.

Notice that in this technique, the whole system is used at the beginning of the process, as opposed to the partitioning techniques that reduce the loading areas of the system before the continuation power flow is applied. Thus, if the partitioning and reduction techniques are applied concurrently, further savings can be expected, as demonstrated by the results obtained for the test systems below. Hence, the mixed partition-reduction algorithm can be described as follows:

1. Find the tangent vector at a known operating point, and sort the entries of this vector in order of absolute magnitude.

2. Proceed to identify load clusters using steps 2, 3 and 4 of the partitioning algorithm described in section II.

3. Allow to change only "large" loads and loads in the first one or two clusters identified in step 2. Loads in all other system buses remain fixed.

4. Apply a continuation method to compute the collapse point, reducing the system equations every U steps based on (4). Additional load sub-partitions may be identified as the system moves closer to the collapse point by using the clustering procedure described above, based on the tangent vector information available during the continuation process.

IV. Tangent Vector Index Method

The TVI defined in (3) is used here to change the predictor stage of the continuation method, so that the number of steps needed to obtain the collapse point can be reduced, yielding significant savings in CPU time as demonstrated for the test system below. The main idea is to determine the "size" of the predictor step based on the quadratic profile of the TVI.

The standard predictor step of a continuation method can be described as follows [14, 19]:

1. Compute the tangent vector $\mathrm{d}x/\mathrm{d}\lambda|_*$ at a known operating point $*$ using (2).

2. Define the size of the step $(\Delta x, \Delta\lambda)$ based on

$$\begin{aligned} \Delta\lambda &= \frac{K}{||\mathrm{d}x/\mathrm{d}\lambda|_*||_1} \\ \Delta x &= \Delta\lambda \left.\frac{\mathrm{d}x}{\mathrm{d}\lambda}\right|_* \end{aligned} \qquad (5)$$

   Typically, $K = 1$ generates adequate results; however, by increasing its value, the continuation process can be accelerated.

3. The predicted values (x_p, λ_p) are then defined as $\lambda_p = \lambda_* + \Delta\lambda$ and $x_p = x_* + \Delta x$

Based on (x_p, λ_p) the corrector step proceeds to compute the actual of values of (x, λ) such that $f(x, \lambda) = 0$. Although several techniques are available to solve the corrector problem [8], the simplest and probably most practical for large power systems is to make $\lambda = \lambda_p$ and solve $f(x, \lambda_p) = 0$ for x, i.e., solve a power flow for a given load value. If no convergence is attained, the value of $\Delta\lambda$ can be reduced, say by halves, until convergence is attained. Observe that when "close" to the collapse point, the proposed corrector step may have convergence difficulties due to a singularity of the Jacobian matrix $D_x f(x, \lambda)$. In practice, however, this is not the case, due to the highly nonlinear behavior of the Jacobian smallest eigenvalues, i.e., the system must be practically at the collapse point for a Jacobian eigenvalue to be close to zero; this is clearly depicted in the next section for a test system. Nevertheless, if singularity problems are a concern, a simple parameterization, consisting of interchanging λ with an $x_i \in x$ that has a large entry in $\mathrm{d}x/\mathrm{d}\lambda$, solves the problem [14, 19].

When two steps of the continuation method are available, a quadratic equation can be computed for λ as a function of TVI_i, where i corresponds to the critical buses identified by the tangent vector clustering method, i.e.,

$$\lambda = a TVI_i^2 + c \qquad (6)$$

A prediction $\lambda_{o_p} = c$ of the value of λ at the collapse point can be easily determined from (6), since $TVI_i = 0$ at λ_{o_p}. The value λ_{o_p} is then used to compute $\Delta\lambda$ and Δx in the predictor

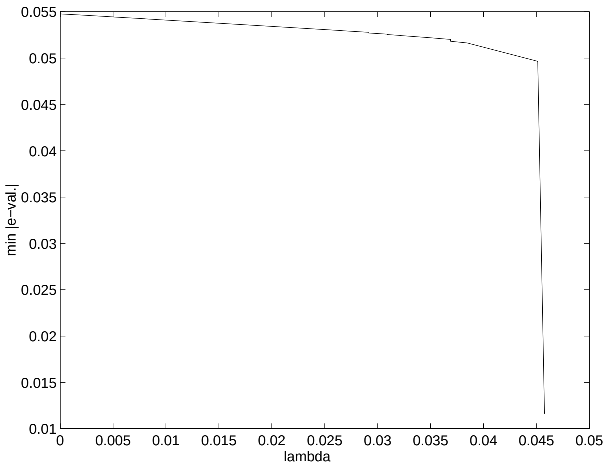

Fig. 1. Smallest absolute eigenvalue for global load increase.

step. Thus, equation (5) is replaced by

$$\Delta\lambda = c - \lambda_* \tag{7}$$
$$\Delta x = \Delta\lambda \left.\frac{\mathrm{d}x}{\mathrm{d}\lambda}\right|_*$$

The TVI algorithm may then be summarized as follows:

1. Using a continuation method based on a tangent predictor, and a corrector with fixed λ and step cutting techniques, compute two operating points on the bifurcation manifold.

2. Based on the tangent vector computed at each predictor step, identify the critical clusters in the system at every continuation step and determine the corresponding TVIs.

3. After two continuation steps switch to equations (6) and (7) in the predictor step, and continue until the collapse point is identified when $TVI_i \leq \epsilon$.

V. Test Results

The IEEE 300-bus test system, with 69 generators, 3 areas, 51 regulating transformers and 411 transmission elements, was used to illustrate the application of the proposed partitioning and reduction techniques, and determine the effectiveness of the different methodologies[1].

All the results shown here were mainly obtained using the continuation power flow program PFLOW [14], which allows to carry out voltage collapse studies in ac/dc/FACTS systems, yielding a variety of voltage stability indices, tangent vectors and eigenvectors at different loading levels[2]. MATLAB was used “off-line” to produce graphics and to carry out certain calculations related to the system partitioning methods. Thus, the CPU times reported in some of the tables below correspond only to the computational time needed to obtain the nose curves with PFLOW in a Sun SPARCstation LX. The partition techniques did not take more that a few seconds, based on the eigenvector and tangent vectors generated by PFLOW, and hence should not increase considerably the reported CPU time once these techniques are integrated in the continuation power flow. The proposed reduction technique was coded within PFLOW, so that it can be applied as the continuation process takes place.

[1] This test system was originally developed by the IEEE/PES Test Systems Task Force under the direction of M. Adibi and can be obtained through anonymous ftp from *wahoo.ee.washington.edu*.

[2] A copy of this program, including ac/dc/FACTS sample systems and a tutorial, for DOS, Windows and UNIX can be obtained from *http://iliniza.uwaterloo.ca*, or through anonymous ftp from *iliniza.uwaterloo.ca*.

TABLE I
Critical bus identification for IEEE 300 bus system

λ p.u.	*Critical bus E-vector*	*Bus 526 Ranking*	*Critical bus Tang. Vector*
0	9042	59	526
0.0050	9042	58	526
0.0175	9042	56	526
0.0309	9042	34	526
0.0369	9042	28	526
0.0451	9042	19	526
0.0458	526	1	526

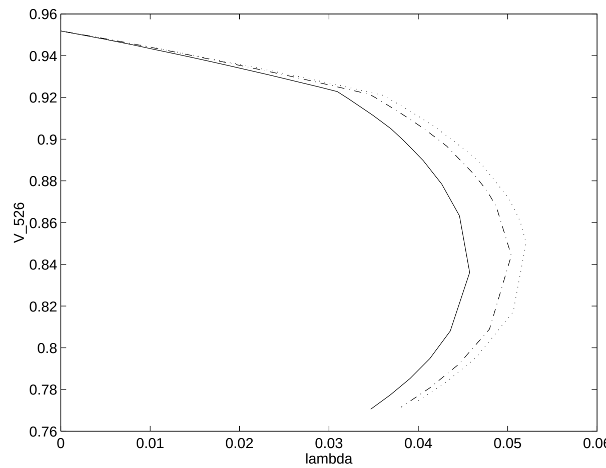

Fig. 2. Voltages profiles at the critical bus for global load increase. The solid line corresponds to the full system, the dashed line depicts the results of applying the tangent vector load-partitioning technique, and the dotted line illustrates the results obtained with the mixed method.

The critical bus at the bifurcation point, bus 526, was identified for this particular system using the zero right eigenvector at the collapse point. The tangent vector clustering technique was able to detect this critical bus in the first cluster for all loading conditions. This is illustrated in Table I, where the critical bus voltages are identified using eigenvectors and tangent vectors for different loading levels up to the collapse point. In this table, the first column depicts the loading level represented by λ, with a maximum loading value of $\lambda_o = 0.0458$ p.u. The second column corresponds to the bus number identified as critical using the right eigenvector associated to the smallest Jacobian eigenvalue, whereas the third column shows the eigenvector ranking of the actual critical bus at the collapse point. The last column indicates the number of the bus identified as critical using tangent vectors.

The inadequacy of the right eigenvector for the detection of the critical bus may be explained based on the behavior of the associated eigenvalue as the load changes, as depicted in Fig. 1. Notice the insensitivity of the smallest eigenvalue to load increase; moreover, the system Jacobian does not become singular until rather close to the collapse point. The latter is exploited here to speed-up the continuation method.

The following tests were carried out for global and local

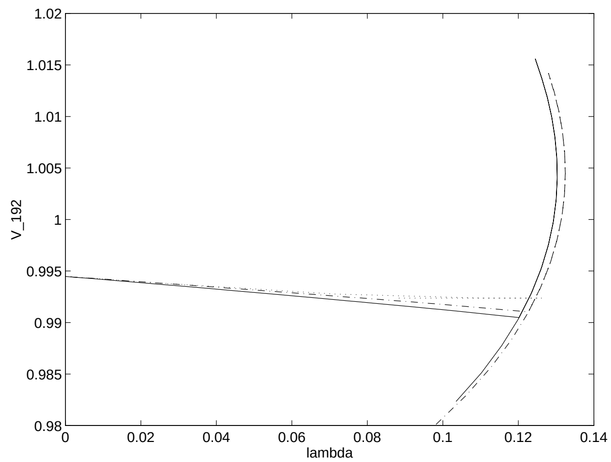

Fig. 3. Voltages profiles at the critical bus for local area load increase. The solid line corresponds to the full system, the dashed line depicts the results of applying the tangent vector load-partitioning technique, and the dotted line illustrates the results obtained with the mixed method.

area load variations:

1. Right eigenvector partitioning technique.
2. Load clustering based on both partitioning techniques.
3. Mixed partition-reduction technique.
4. Continuation method using TVI predictor.

Figures 2 and 3 depict in a solid line the voltage profiles or "nose" curves for the original system obtained with the standard continuation method, for global and local area loading schemes. Only the bifurcation diagrams of the corresponding critical buses are illustrated, i.e., bus 526 for global loading and bus 192 for area loading. Observe that for the system with local area load increases, the system presents a Q-limit instability, i.e., the voltage collapses due to loss of voltage control before the system gets to the saddle-node bifurcation point. For a detailed explanation of this phenomenon the reader is referred to [28, 29].

Local area loading increase is studied here due to the interest of utilities in investigating this problem, so that the effect of particular load areas in the overall system can be studied. As it is demonstrated below, the proposed partitioning and mixed techniques yield adequate results when applied to this type of problem.

A. Network Partitioning

The results of applying the right eigenvector partitioning technique to obtain independent subsystems were rather poor, as expected, obtaining errors in the computation of λ_o in the order of +73% for the case of global load variations, where the error is defined as

$$Error = \frac{\lambda_o^{partition} - \lambda_o}{\lambda_o}$$

The tangent vector partitioning technique yielded slightly better results. The large errors in the maximum loadability margin λ_o are basically due to the previously discussed problems of not being able to accurately define the power injections and limits in the neighbor PV buses as the system loads change.

TABLE II
Critical area load change

Case		λ_o	*Error*	*PQ*	*CPU*
Load	*Partition*	*[p.u.]*	*[%]*	*buses*	*time [s]*
Global	No	0.04576	0	193	65.3
	E-vec.	0.05467	+19.5	153	78.9
	Tg.vec.	0.05041	+10.2	74	79.4
Area	No	0.12021†	0	49	84.4
	E-vec.	0.12021†	0	49	84.4
	Tg.vec.	0.12269†	+2.1	25	82.2

† Q-limit instability.

The partitioning techniques to obtain subsystems were not applied to other system conditions, in spite of the significant savings in CPU time, as the large errors do not justify the use of these methods for voltage collapse studies. Hence, this techniques are only used here to identify critical areas where the load is allowed to change, reducing the number of PQ buses involved in the continuation process. The results of applying this methodology for both partitioning techniques are shown in Table II.

Several observations can be made based on the results shown in Table II. First, the partitioning techniques increase the computational burden of tracing the nose curves for the system with global load changes, with relative large errors in λ_o. The errors are clearly due to the different loading schemes used, whereas the time difference is mainly due to more refactorizations of the Jacobian matrices within PFLOW. However, it is interesting to see in this case that the tangent vector load-partitioning yields significantly better results than the eigenvector technique, as the tangent vector is identifying the critical area practically from the initial operating point. Better results can be obtained for both methods for an initially more loaded system; nevertheless, the CPU times and the errors in λ_o do not justify the use of these methods when loads are significantly changed all throughout the system. When the load is only changed in a particular area of the system, the results for the tangent vector technique are significantly better for λ_o, with a slight reduction in CPU time.

B. Partition-Reduction

Table III illustrates the results of applying the equations reduction method to the test system, for both loading conditions. The results obtained with the mixed technique are only shown for tangent vector partitioning, as this is a better method than the eigenvector. For global load changes, once again the CPU time and errors in λ_o do not justify the use of this method. Notice that, even though the number of equations is reduced, the computational burden increases due to having to refactorize the matrix when the equations change. Better results are expected with improve factorization routines in PFLOW.

In the case of area load increase, the results are certainly promising, as the number of equations are reduced to one third and the CPU time is cut in half, with small errors in λ_o. The mixed methodology yields additional savings, which could become a significant factor in larger systems.

The effect of applying all these partitioning and reduction techniques on the voltage profiles are illustrated in Figs. 2 and 3 for the critical buses. Observe that for the area load

TABLE III
SYSTEM REDUCTION

Case		λ_o	*Error*	*#.*	*CPU*
Load	*Reduction*	*[p.u.]*	*[%]*	*Eqs.*	*time [s]*
Global	No	0.04576	0	600	65.3
	Red.	0.04907	+7.2	532	76.2
	Part.-Red.	0.05205	+13.7	504	70.9
Area	None	0.12021†	0	600	84.4
	Red.	0.12110†	+0.7	211	43.4
	Part.-Red.	0.12426†	+3.7	204	42.2

† Q-limit instability.

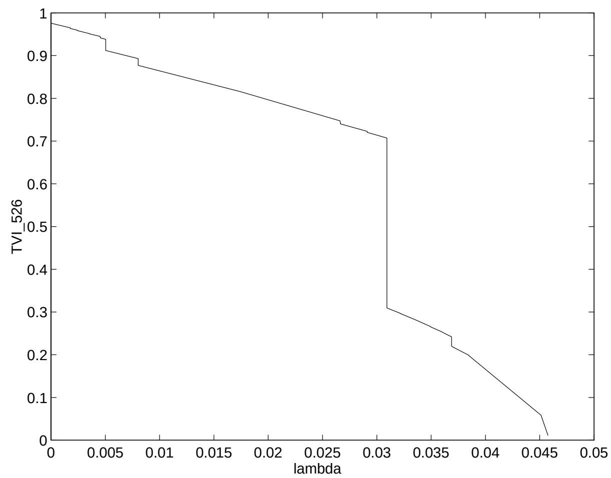

Fig. 4. TVI for the critical bus. Observe the large changes due to system limits and the "quadratic" behavior when close to the collapse point.

changes, the tangent vector partitioning and reduction techniques produce the best results at less computation costs.

C. TVI Continuation Method

As the critical bus can be detected using the tangent vector clustering technique described above, the corresponding TVI can be readily monitored as the system loading changes with λ. The TVI profile for critical bus 526 is depicted in Fig. 4, for global load increases in the 300-bus test system; observe the large changes on the TVI value due to system limits, and the quadratic shape of the index when approaching the bifurcation. Similar behavior has been observed by the authors for a variety of systems and loading conditions. However, the quadratic shape is more evident in systems with large reactive support, where limits do not have a significant effect on the collapse point. For system buses not belonging to the critical area, the corresponding TVI profiles are highly nonlinear, presenting profiles similar to the one depicted in Fig. 1 for the eigenvalue.

The quadratic shape of the critical bus TVI is used here to change the predictor stage of the continuation method, so that larger steps can be taken in order to rapidly locate the collapse point. The results of applying this modified continuation methodology are depicted in Table IV. First, observe that, even though the TVI does not present a global quadratic profile, there is a significant reduction on the number of steps required to compute the maximum loading margin λ_o; this has a direct effect on the computational burden of locating the collapse point, yielding large savings in CPU time. Second, the

TABLE IV
TVI CONTINUATION METHOD

Predictor Type	λ_o *[p.u.]*	TVI_{526}	*Cont. Steps*	*CPU time [s]*
Standard	0.04576	0.01079	39	65.3
TVI	0.04578	0.00233	6	18.5

TVI based predictor allows the continuation method to obtain a better approximation of the collapse point, as reflected by the corresponding value of TVI_{526} which is closer to zero; this practically eliminates the need for using direct methods to determine the collapse point. Finally, based on the TVI profile, system operators can make quick on-line predictions of the distance to collapse.

VI. Conclusions

A review of network partitioning by right eigenvector is presented, and the results of its application to voltage collapse analysis are discussed, demonstrating the inadequacy of this typical partitioning technique for these types of studies. A new partitioning technique based on the tangent vector to the bifurcation manifold is then proposed, yielding better results than the standard eigenvector method. Partitioning techniques used for generating independent subsystems are shown to be inadequate for voltage collapse studies, as the system partitions are obtained at a particular loading level and are not able to capture the system behavior for broad load variations; this is certainly true for any partitioning technique. Nevertheless, the tangent vector clustering technique is shown to be useful for the identification of the critical area with respect to the collapse point at any loading conditions, which is not possible to do with eigenvectors.

A reduction technique, together with a load clustering technique are also presented and studied. Although the results for a test system are poor when the load is increased through the whole system, the computational savings and small errors obtained for the more realistic case of local load variations may justify the use of these techniques for voltage collapse studies of these particular cases.

Finally, the tangent vector clustering technique is applied to the computation of the new voltage stability index TVI, which is then used to significantly speed up the computation of the collapse point. Of all the techniques presented, the TVI based continuation method is by far the most simple and useful, as good approximations of the collapse point can be rapidly computed.

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