Multi-objective Optimal Power Flows to Evaluate Voltage Security Costs in Power Networks

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***Abstract*— In this paper, new optimal power flow (OPF) techniques are proposed based on multi-objective methodologies to optimize active and reactive power dispatch while maximizing voltage security in power systems. The use of interior point methods together with goal programming and linearly combined objective functions as the basic optimization techniques is explained in detail. The effects of minimizing operating costs, minimizing reactive power generation and/or maximizing loading margins are then compared in both a 57-bus system and a 118-bus system, which are based on IEEE test systems and modeled using standard power flow models. The results obtained using the proposed mixed OPFs are compared and analyzed to suggest possible ways of costing voltage security in power systems.**

***Index Terms*— Voltage collapse, optimal power flow, bifurcations, multi-objective optimization, goal programming, interior point methods.**

I. Introduction

Several voltage collapse events throughout the world show that power systems are being operated closer and closer to their stability limits [1]. This problem can only be exacerbated by the application of open market principles to the operation of power systems, as stability margins are being reduced even further to respond to market pressures, which demand greater attention to reduced operating costs. As the overall stability limits can be closely associated with the voltage stability of the network, the incorporation of voltage collapse criteria and tools in the operation of power systems is becoming an essential part of new energy management systems (EMS) [2].

Typically, voltage collapse events can be related to a lack of a post contingency equilibrium point in the system, which in turn can be associated theoretically with either a saddle-node bifurcation (SNB) or a limit-induced bifurcation (LIB) [2]. Hence, analysis tools that are currently under use throughout the world are mostly based on bifurcation theory principles. However, in the last few years, the use of optimization techniques to study voltage stability problems has been gaining interest [3]; new voltage stability analysis tools that use optimization methods to determine optimal control parameters that maximize load margins and thus avoid voltage collapse problems are being introduced in EMS. Various uses of optimization methodologies applied to the voltage collapse problem can be found in the technical literature. For example, in [4], a voltage collapse computation problem is first formulated as an optimization problem, proposing the use of optimization techniques and tools to study power system collapse. More recently, in [5], reactive power margins to voltage collapse are determined based on interior point methods, and in [6], the maximum loadability of a power system is examined using interior point methods. The authors in [7] determine the closest bifurcation to the current operating point on the hyperspace of saddle-node bifurcation points. In [8], various techniques to determine optimal shunt and series compensation parameter settings to maximize the distance to a saddle-node bifurcation are presented. In [9], an interior point optimization technique is used to determine the optimal PV generator settings to maximize the distance to voltage collapse; the algorithms presented include constraints on the present operating conditions. Applications of optimization techniques to voltage collapse studies are discussed theoretically and numerically in [10], proposing a new technique to incorporate voltage stability into traditional optimal power flow (OPF) formulations; a Lagrangian based proof to show that optimization techniques allow to compute the maximum loading point for both SNBs and LIBs is also presented here. In [11], several Voltage Stability Constrained Optimal Power Flow (VSC-OPF) formulations are proposed considering both the current loading point and the maximum loading point into the formulation, so that voltage stability margins can be considered in the OPF problem. A similar Voltage Stability Constrained Optimal Power Flow formulation is presented in [12]. In [13], a method to incorporate both transient and voltage stability into an optimization problem is presented. Finally, in [14] a optimization based load shedding scheme to prevent voltage instability is presented.

The current paper presents a detailed analysis of the use of optimization techniques in the study of voltage stability problems, leading to the incorporation of voltage stability criteria in traditional OPF formulations. Numerical analysis using the IEEE 57 and 118 test systems, which are based on IEEE test systems that model portions of the American Midwest power system in the early sixties [15], are used to highlight the characteristics of these problems, and to analyze how voltage stability criteria influence operating costs.

The paper is structured as follows: In Section II, the basic background of the OPF problem is reviewed, discussing also the basic concepts associated with voltage collapse and bifurca-

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tion analysis as well as the methodologies used to evaluate voltage security based on optimization techniques. In Section III, a general formulation for a VSC-OPF is proposed, presenting a modification to the "maximum distance to bifurcation" problem by including constraints at the current operating point; several OPF formulation considering voltage stability criteria are also presented and discussed in this section. An analysis of the results obtained from applying the proposed formulations to two test systems is presented in Section IV, concentrating in particular on discussing possible ways to consider voltage security costs in system operation. Finally, Section V summarizes the main contributions of the current paper and discusses future research directions.

II. Background Review

The basic background behind the proposed OPF problem with voltage stability constraints is presented in this section. Some fundamental concepts behind voltage stability analysis are briefly discussed first to understand better how optimization techniques can be used to study the voltage collapse problem.

A. Optimal Power Flow

The OPF problem was introduced in the early 1960's by Carpentier and has grown into a powerful tool for power system operation and planning. In general, the optimal power flow problem is a non-linear programming (NLP) problem that is used to determine the "optimal" control parameter settings to minimize a desired objective function, subject to certain system constraints [16], [17], [18]. OPF problems are generally formulated as nonlinear programming problems (NLP) as follows:

$$\begin{aligned} \min \quad & G(x) \\ \text{s.t.}: \quad & F(x) = 0 \\ & \underline{H} \le H(x) \le \overline{H} \\ & \underline{x} \le x \le \overline{x} \end{aligned} \qquad (1)$$

where $G(x) : \Re^n \rightarrow \Re$ is the objective function that typically includes total generator costs (active power dispatch) or total losses in the system (reactive power dispatch); $F(x) : \Re^n \rightarrow \Re^m$ generally represents the load flow equations; and $H(x) : \Re^n \rightarrow \Re^p$ usually represents transmission line limits, with lower and upper limits represented by $\underline{H}$ and $\overline{H}$, respectively. The vector of system variables, denoted by $x \in \Re^n$, typically includes voltage magnitudes and angles, generator power levels and transformer tap settings; their lower and upper limits are given by $\underline{x}$ and $\overline{x}$, respectively.

Throughout the years, the OPF problem (1) has been solved using a variety of nonlinear optimization techniques, as discussed in [17]. Nowadays, Interior Point Methods (IPs) have become popular for solving this problem, given their computational advantages when dealing with large systems that include a variety of operational and control limits [18]. Hence, in this paper, IPs are used to solve all the proposed optimization problems.

B. Voltage Stability and Bifurcation Theory

Nonlinear phenomena, especially bifurcations, have been shown to be responsible for a variety of stability problems in power systems (e.g. [19]). In particular, the lack of post contingency equilibrium points, typically associated with SNBs and LIBs, have been shown to be the main reason behind several voltage collapse problems throughout the world [2].

In general, bifurcation points can be basically defined as equilibrium points where changes in the "quantity" and/or "quality" of the equilibria associated with a nonlinear set of dynamic equations occur with respect to slow varying parameters in the system [20]. Since power systems are modeled by sets of nonlinear differential equations, various types of bifurcations are generically encountered as certain system parameters vary.

A typical power system model used for stability studies can be represented by the following set of differential-algebraic equations (DAEs):

$$\begin{aligned} \dot{z} &= f(z, y, p, \lambda) \\ 0 &= g(z, y, p, \lambda) \end{aligned} \qquad (2)$$

where $z \in \Re^n$ is a vector of state variables (e.g. generator angles) associated with the set of differential equations defined by the nonlinear function $f : \Re^n \rightarrow \Re^n$; $y \in \Re^m$ is a vector of algebraic variables (e.g. load bus voltages) associated with the set of algebraic equations defined by the nonlinear function $g : \Re^m \rightarrow \Re^m$; $p \in \Re^k$ is a vector of "controlling" parameters (e.g. AVR set points); and $\lambda \in \Re^\ell$ is a vector of "uncontrolled" slowly varying parameters in the system (e.g. active and reactive load powers), which make the system move from one equilibrium point to another. When the Jacobian $D_y g(\cdot)$ of the algebraic constraints is non-singular along system trajectories, the system model can be reformulated based on the Implicit Function Theorem as [21]

$$\begin{aligned} & y \equiv s(z, p, \lambda) \\ \Rightarrow \quad & \dot{z} \equiv f(z, s(z, \lambda), \lambda) \\ & \phantom{\dot{z}} = h(z, p, \lambda) \end{aligned}$$

If the Jacobian of the algebraic constraints becomes singular along any system trajectories, the model basically "breaks down". In this case, the original model can be modified to consider dynamics ignored in the original model, resulting in the transformation of some algebraic constraints into differential equations [21], [22].

Equilibrium points (z_0, y_0) where $\dot{z} = 0$, for given parameter values p_0 and λ_0, are defined by the following set of nonlinear equations:

$$\left. \begin{aligned} 0 &= f(z_0, y_0, p_0, \lambda_0) \\ 0 &= g(z_0, y_0, p_0, \lambda_0) \end{aligned} \right\} \Rightarrow 0 = h(z_0, p_0, \lambda_0)$$

Given the nonlinear nature of the system and its associated equations, the system typically has multiple equilibrium points. Of interest are the equilibrium points and parameter values where the system goes from being stable to unstable, from being unstable to stable, or where the number of equilibrium points changes with respect to the bifurcation parameters λ;

these points are defined as bifurcation points. These bifurcations are mathematically characterized by the eigenvalues of the Jacobian $D_z h|_0 = D_z h(z_0, p_0, \lambda_0)$ changing. Thus, when two equilibrium points "merge" and one of the eigenvalues becomes zero, one has a saddle-node (SNB), a transcritical or a pitchfork bifurcation; when a conjugate pair crosses the imaginary axis, one has a Hopf bifurcation (HB). Generally, one can expect to encounter SNBs or HBs as the bifurcation parameters change, as transcritical and pitchfork bifurcations can only occur if the system contains some particular "symmetries" [19].

In power systems, control limits, in particular generator reactive power limits, have been shown to yield special bifurcations known as limit-induced bifurcations (LIBs) [23], [24]. LIBs are also generic bifurcations, i.e. are typically encountered in power systems, and are characterized by two merging equilibrium points and an instantaneous "jump" of the eigenvalues from the left-half plane to the right-half plane; there are no singularities of $D_z h|_0$ associated with this bifurcation. As in the case of SNBs and LIBs associated to certain control limits, the system equilibria locally disappear for an additional increase or decrease, depending on the direction of change, of the bifurcation parameters λ. For example, when reactive power limits of certain generators are reached, no local equilibria may exist for increased loading conditions [2].

Voltage collapses have been shown to be strongly connected to SNBs and LIBs [2], [24]. As the system approaches the SNB/LIB point, also referred to as the voltage collapse point, the stability region of the system decreases until it becomes "zero" at the SNB/LIB point, resulting in a system collapse due to lack of equilibria. Thus, a "voltage stability" margin is defined as the "distance", with respect to the bifurcation parameters λ, from the "current" operating point to the voltage collapse or SNB/LIB point; the system is assumed to be voltage secure if this margin is "reasonably" greater than zero. In practical systems, operators would be interested in maintaining the system with a "given" voltage stability margin, so that contingencies do not make the system unstable [2].

C. *Voltage Stability and Optimization Techniques*

Given the definitions of voltage stability used in this paper, which are basically based on a steady state model of the power network, a static model of the power system is used here. Thus, typically, one assumes that the solution to the following set of nonlinear *power flow* equations define the system equilibria:

$$0 = F(x, \rho, \lambda) \tag{3}$$

where the vector $x \in \Re^N$ represents the system's dependent variables, which are normally bus voltage angles δ and non-generator bus voltage magnitudes V_L, reactive power levels Q_G of generators modeled as PV buses, and real P_S and reactive power Q_S levels of the slack bus generator (some of these variable are directly associated with the variables z and y in (2)). Thus,

$$x = [\delta \; V_L \; Q_G \; P_S \; Q_S]^T$$

The vector $\rho \in \Re^M$ represents the independent or controlled variables in the system (associated with p in (2)); for the models considered in this paper, these are generator active power settings P_G and terminal voltage levels V_G, and transformer tap settings a, i.e.

$$\rho = [P_G \; V_G \; a]^T$$

The variable $\lambda \in \Re$ is a scalar bifurcation parameter that is typically known as the "loading factor", as it represents the system loading level in the system for a linearly increasing, constant power factor load model, i.e.

$$\begin{aligned} P_L &= \lambda \, P_{Lo} \\ Q_L &= \lambda \, Q_{Lo} \end{aligned}$$

where P_{Lo} and Q_{Lo} are "base" load power values. Observe that in this case, λ stands for only one parameter instead of several, i.e. the load is assumed to change in only one known direction, which is a reasonable assumption based on an adequate load forecast at an "initial" operating point x_0 associated with λ_0.

Typically, SNBs and LIBs can be found using direct and/or continuation methods [2], [20]. However, for the given power flow model (3), the collapse point corresponding to a SNB or LIB point, may also be determined using the following optimization procedure [3], [8], [9]:

$$\begin{aligned} \min \quad & -|\lambda - \lambda_0| \\ \text{s.t.}: \quad & F(x, \rho_0, \lambda) = 0 \\ & \underline{H} \le H(x) \le \overline{H} \\ & \underline{x} \le x \le \overline{x} \\ & \underline{\rho} \le \rho \le \overline{\rho} \end{aligned} \tag{4}$$

where λ_0 and ρ_0 are initial parameter values associated with an initial operation point x_0, i.e. $F(x_0, \rho_0, \lambda_0) = 0$; $\underline{x}$ and $\overline{x}$, and $\underline{\rho}$ and $\overline{\rho}$ represent lower and upper limits, respectively, on the independet variables x and the the control parameters ρ. The function $H(\cdot)$ is used to represent lower $\underline{H}$ and upper $\overline{H}$ limits in line currents and/or power flows, which are basically functions of x. The loading parameter λ is a variable in the optimization problem, i.e. it is free to change.

Solutions of (4), as well as the VSC-OPF problems described below, basically correspond to a maximum loading point, which corresponds to a collapse point, i.e. a SNB or LIB. (The proof outlined in [9] and detailed in [10] is based on the Lagrangian associated with a logarithmic-barrier Interior Point solution process of the optimization formulations discussed here.) However, if operating limits such as bus voltage, line current and power flow limits are modeled, the critical point does not necessarily correspond to a collapse point, as reaching these limits do not directly lead to stability problems in the network; for this reason, the maximum loading point will be referred to hereafter as the "critical" point, as opposed to simply the collapse point.

III. OPF WITH VOLTAGE STABILITY CRITERIA

In general and for the given power flow model assumed in this paper, an OPF problem that incorporates voltage stability criteria can be generically written as

$$\begin{aligned} \min \quad & G(x_p, \rho, \lambda_p, \lambda_*) \\ \text{s.t.}: \quad & F(x_p, \rho, \lambda_p) = 0 \\ & F(x_*, \rho_*, \lambda_*) = 0 \\ & \underline{H_p} \le H(x_p) \le \overline{H_p} \\ & \underline{H_*} \le H(x_*) \le \overline{H_*} \\ & \underline{x_p} \le x_p \le \overline{x_p} \\ & \underline{x_*} \le x_* \le \overline{x_*} \\ & \underline{\rho} \le \rho \le \overline{\rho} \\ & \underline{\rho_*} \le \rho_* \le \overline{\rho_*} \end{aligned} \tag{5}$$

where the subscripts p and $*$ indicate the current and critical points, respectively. $G(x_p, \rho, \lambda_p, \lambda_*)$ is the objective function to be minimized, which has an OPF component, i.e. production costs or losses, that may be dependent on (x_p, ρ, λ_p), and a voltage stability component that is a function of λ_* and possibly of λ_p, as discussed below. It is assumed that the inequality constraints defined by the limits on $H(x_p)$, $H(x_*)$, x_p, and x_* can be separated into separate constraints at the current and critical points. Finally, ρ_* is used to map the control variables at the current operating point, defined by ρ, into the critical point to account for certain system changes. Thus, generators at the critical point are assumed to have the same terminal voltage set points as at the base loading point, and their power levels are represented based on the following distributed slack-bus model:

$$P_{G*} = P_{Gp}(1 + K_{G*})$$

where K_{G*} is a scalar variable that distributes the generated powers at the critical point proportionally to the value of the independet control variable P_{Gp}; $K_{G*} \in x_*$, i.e.

$$x_* = [\delta_* \; V_{L*} \; Q_{G*} \; K_{G*} \; Q_{S*}]^T$$

Hence,

$$\rho_* = [P_{G*}(P_{Gp}, K_{G*}) \; V_{Gp} \; a_p]^T$$

Observe that the tap settings a_p are assumed to be the same for the current and maximum loading points.

The variable λ_* is a variable in the optimization problem, i.e. it is fully free to change during the solution process; on the other hand, λ_p is given a fixed value. Thus, the critical point is affected by changes in the control variables ρ, due to the relationship between ρ and ρ_*.

It is important to highlight the fact that in (5), λ stands for only one parameter instead of several, contrary to what is proposed in [7], i.e. the optimization is done in a particular direction of load change. This is not a problem, given that the optimization would be typically done several times a day during the operation of the system, as in the case of any other OPF procedure. This assumption simplifies the numerical solution process of the optimization problem, which is already a difficult numerical problem, given the highly nonlinear behavior of the system constraints and the effect of limits associated with the inclusion of the critical conditions $*$.

Depending on the definition of the objective function $G(\cdot)$ in (5), one can pursue different optimization strategies and hence obtain solutions to a variety of distinct problems, as discussed below.

A. *Maximum Loading Distance*

The Maximum Loading Distance problem with constraints incorporated on the current and critical loading point is a particular example of using (5) to enhance voltage security [9], [10]. The objective function in this case can be written as

$$G(\lambda_p, \lambda_*) = -(\lambda_* - \lambda_p) \tag{6}$$

where $\lambda_* > \lambda_p > 0$. The main idea here is to maximize the distance to a critical point, while guaranteeing the feasibility of the current operating point x_p associated with the load level defined by λ_p, as well as the feasibility of all control and operating limits. For example, increasing generator voltage magnitude settings generally increases the distance to the critical point, improving voltage security; but under lighter loading conditions, higher voltage settings may lead to over-voltages. Incorporating the current operating point into the optimization problem can eliminate this problem; however, it also reduces the space of feasible solutions.

B. *Voltage Stability Constrained OPF*

With the current loading point included into the optimization problem, it is possible to incorporate voltage stability criteria into an OPF formulation at the "current' operating point x_p. As the operating point moves closer to a critical point, i.e. as x_p approaches x_*, more emphasis must be placed on maximizing voltage stability as opposed to minimizing operating costs.

A first approach to this problem introduced in the objective function voltage stability indices as indicators of the proximity to voltage collapse, as explained in [9]; however, since voltage stability indices present rather nonlinear characteristics, especially when limits are considered, this technique did not produced adequate results. In [10], the difference between λ_p and λ_* is used as a measure of the distance to the critical point or maximum loading; this measure is then used to automatically shift the weighting between cost minimization and voltage stability security depending on the current system conditions p. This formulation tends to emphasize voltage stability when the system is closer to a critical point, but since there is no direct control on the relative weighting assigned to stability versus costs, there is no way to guarantee that this will actually occur.

In order to incorporate voltage stability constraints into a traditional OPF formulation, various multi-objective optimization formulations are proposed here, based on standard optimization concepts [25].

1) Linear Combination: In this formulation the maximum loading is directly incorporated into the following objective function:

$$G(x_p, \rho, \lambda_p, \lambda_*) = \omega_1 \, g(x_p, \rho, \lambda_p) - \omega_2 \, (\lambda_* - \lambda_p) \tag{7}$$

subject to the constraints in (5). Observe that this requires the introduction of two weighting factors ω_1 and ω_2 to balance the emphasis placed on maximizing stability, i.e. $(\lambda_* - \lambda_p)$, versus minimizing costs, which are represented by $g(x_p, \rho, \lambda_p)$ in (7). Generally, ω_2 must be significantly larger than ω_1, as the relative difference in the magnitudes of each term in the objective function is large; it is assumed that $\omega_1 + \omega_2 = 1$ to normalize their values. Values obtained from previous OPF and Maximum Loading Distance analysis can be used to determine reasonable values of ω_1 and ω_2 at different loading conditions.

2) Fixed Loading Margin: An alternative approach to assigning a cost to voltage stability is to include a voltage stability inequality constraint. In this formulation, the objective function is the traditional OPF cost minimization with the following equality constraint added to (5):

$$\lambda_* - \lambda_p \geq \Delta\lambda_{min}$$

where $\Delta\lambda_{min}$ represents the minimum acceptable margin of stability for the system and is defined by the system operator.

3) Modified Goal Programming: In the Linear Combination formulation, it is not possible to set a value for the loading margin. In the Fixed Loading Margin formulation, it may be possible to define a loading margin for which there is no solution to the optimization problem, as the margin of stability may be greater than what the system can provide. These limitations can be overcome using Goal Programming, where a desired "goal", $\Delta\lambda_g$, can be explicitly declared for the loading margin. In this case, the objective function of (5) is defined as

$$G(x_p, \rho, \lambda_p, \beta_1, \beta_2) = \omega_1\, g(x_p, \rho, \lambda_p) + \omega_2\, \beta_1 + \omega_3\, \beta_2 \quad (8)$$

with the following additional equality constraint

$$(\lambda_* - \lambda_p) - \Delta\lambda_g = \beta_1 - \beta_2$$

where the relative weights ω_1, ω_2 and ω_3 are used to vary the emphasis put on the desired loading margin, and the new variables $\beta_1, \beta_2 > 0$, which are minimized, define the actual loading margin. If β_1 and β_2 are equal to zero, then the loading margin equals the desired value $\Delta\lambda_g$. In this case and based on basic Goal Programming optimization concepts [25], the loading margin $(\lambda_* - \lambda_p)$ is forced towards its desired value to avoid the penalty term associated with the minimization of β_1 and β_2.

In the above formulation, the loading margin $(\lambda_* - \lambda_p)$ can be less or greater than the desired margin $\Delta\lambda_g$, depending on the proximity of the system to the critical point and the relative weights. This formulation allows to vary the "cost" assigned to the loading margin; a higher cost can be assigned if the margin is less than the desired value.

4) Reactive Power Costs: The final formulation consists in modifying (8) to add reactive power "costs" to the previous formulation based on a possible market environment (e.g. [26]). Here, it is assumed that some GENCOs operate at a constant power factor, as it is the case with certain generating units that can negotiate this operating condition with the ISO in the Italian electricity market [27]. Hence, goal programming is used to minimize the difference between the actual power factor of some selected generators and their desired power factor by modifying (5) as follows:

$$G(x_p, \rho, \lambda_p, \beta_1, \beta_2, \beta_3, \beta_4) = \quad (9)$$
$$\omega_1\, g(x_p, \rho, \lambda_p) + \omega_2\, \beta_1 + \omega_3\, \beta_2 + \sum_i (\omega_4 \beta_{3_i} + \omega_5 \beta_{4_i})$$

with the additional equality constraints

$$\begin{aligned} (\lambda_* - \lambda_p) - \Delta\lambda_g &= \beta_1 - \beta_2 \\ Q_{G_i} - \tan(cos^{-1}(pf))P_{G_i} &= \beta_{3_i} - \beta_{4_i} \end{aligned}$$

where the desired power factor is represented by pf; i stands for the index of the selected generators in the system; ω are weights used for varying the relative emphasis on operating costs, loading margin and power factor (reactive power "costs"); and β_3 and β_4 are vectors used for measuring the difference between the actual and the desired power factors for the selected generators. The formulation is such that generators will try to operate close to the desired power factor; otherwise, a penalty cost is automatically assigned [25].

5) Choosing the Weighting Factors: In the Linear Combination, Modified Goal Programming, and Reactive Power Costs formulations, the need to assign values to the weighting factors is required. The factors serve two related purposes; the first, equating the relative differences between operating costs and the loading margin, and second, changing the emphasis placed on stability versus operating costs.

Values obtained from previous OPFs and stability analysis can be used to determine reasonable initial values of the weights. Then depending on the objectives of the operator, additional emphasis can be placed on one of the components. A disadvantage of each of these methods is the effect of the weights is not exactly known until after the problem has been solved. For example, in the Linear Combination formulation, increasing ω_2 relative to ω_1 will increase the emphasis on stability versus operating costs; but by how much, compared to other values of the weights, is not know in advance.

The Fixed Loading Margin formulation overcomes the dependency on the use of weights. It allows the operator to explicitly define a loading margin which is incorporated as an inequality constraint. In the numerical results presented in the paper, different values of the weights have been used to demonstrate the characteristics of the formulations when the weights are changed.

IV. Numerical Analysis

The Maximum Loading Distance and VSC-OPF formulations presented in Section III are tested on two sample systems, one based on the IEEE 57-bus test system, and the second one based on the IEEE 118-bus test system [15]. Two different size systems are chosen to thoroughly test the proposed optimization formulations as well as the computational procedures used to obtain the results presented in this section. A number of simulations are performed to analyze how the current loading point and system limits influence the optimal solution. Based on the algorithm discussed in [18], a nonlinear primal-dual predictor-corrector interior point method written in MATLABis used to perform the numerical analysis. Simulations are performed considering various operating limits at both the current operating point p and the critical point $*$.

The results presented here were obtained through the following computational procedure: First system models were constructed symbolically in MAPLE, so that its differentiation tools could be used to calculate the vectors and matrices required for the optimization method. The set of equations describing the models were exported to text files using an export tool in MAPLE[28], and then modified using a MATLABscript file to form data files. MATLABroutines written to access the data files

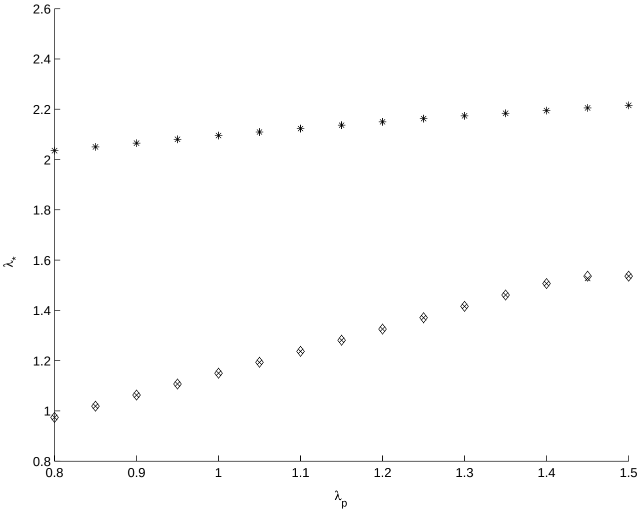

Fig. 1. Maximum loading versus current operating point using the Maximum Loading Distance problem for the 57-bus test system. The symbols $*$, $\diamond$, and $\times$ correspond to solutions for the system with no limits, generator P and Q limits, and both bus voltage limits and generator P and Q limits, respectively, at the maximum loading point; operating limits are always enforced at the current operating point.

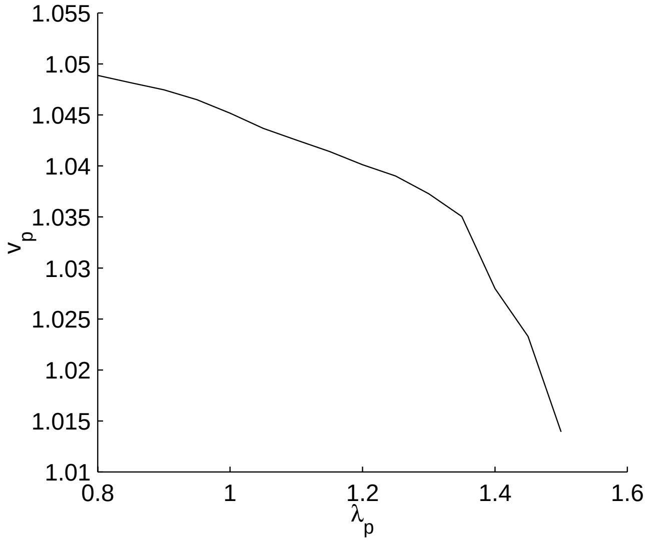

Fig. 2. Change in voltage magnitude at Bus 30 versus λ_p for the 57-bus test system.

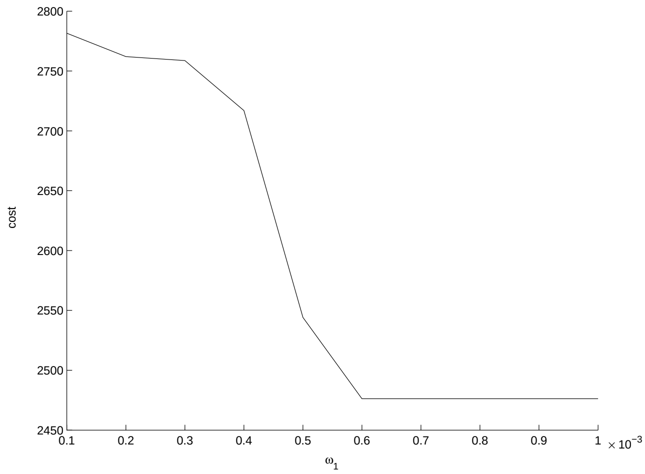

Fig. 3. Operating costs versus weighting factor ω_1 for the Linear Combination formulation applied to the 57-bus test system for $\lambda_p = 0.9$.

were used to generate the required vectors and matrices for numerical analysis; sparse matrix routines were used to manipulate and store the data. Although, this method of implementing and testing the proposed algorithms and models has limitations when dealing with large systems, as computational times significantly increased with system size, it is well suited to investigate and test the different optimization procedures proposed here, given the flexibility associated with being able to perform symbolic computations in MAPLE, while using MATLABfor all numerical computations.

A. Maximum Loading Distance

Including constraints on the current loading point p in the Maximum Loading Distance formulation resulted in different "optimum" solutions depending on the value of λ_p. The results of solving this optimization problem for the 57-bus is depicted in Fig. 1, where changes in λ_* versus λ_p are depicted. Observe that, as expected, the presence of operating limits reduces the maximum loading margin of the system ($*$ versus $\diamond$ in Fig. 1), and that the generator limits dominate over voltage limits ($\diamond$ versus $\times$ in Fig. 1).

Enforcing operating limits at the critical point $*$ results in a lower λ_*, as one would expect, since generator limits, particularly reactive power limits, are the main limiting factor ($\diamond$ versus $*$ in Fig. 1); this is consistent with the type of results that one would typically obtain in voltage stability studies. At low values of λ_p, upper limits on bus voltages become active, resulting in lower values of λ_*, with the opposite happening at higher values of λ_p. This phenomenon is clearly illustrated on Fig. 2, where the p.u. voltage magnitude at different loading levels is given for a non-generator bus (Bus 30) of the 57-bus system. At lower loading levels, the voltage tends to the upper limit, limiting the set points of generators nearby; at higher loading values, generator set points are raised, as upper voltage limits are not a problem.

It should be noted that, in Fig. 1, as λ_p is increased, the margin between the current loading point and the maximum loading point decreases. When the current loading point is eventually set equal to the maximum loading point, the loading margin $(\lambda_* - \lambda_p)$ becomes zero. If the current loading point is set higher than the maximum loading point, the problem will fail to converge.

Very similar results were obtained for the 118-bus test system.

B. Linear Combination VSC-OPF

The multi-objective Linear Combination formulation was then applied to both test systems. The effect on cost and loading margin for different values of ω_1 ($\omega_2 = 1 - \omega_1$) at a given value of λ_p for the 57-bus system are shown in Figs. 3 and 4. As the factor ω_1 is increased, more emphasis is placed on operating costs and less on loading margin, as expected.

Figures 5, 6 and 7 depict the results obtained from applying the Linear Combination formulation, the Maximum Load-

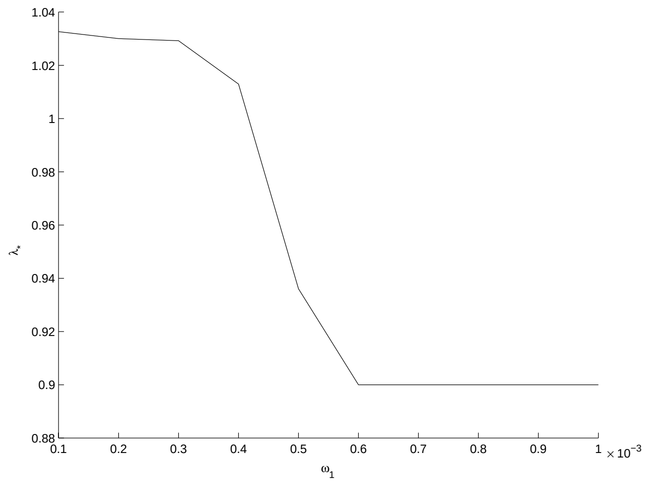

Fig. 4. Maximum loading versus weighting factor ω_1 for the Linear Combination formulation applied to the 57-bus test system for $\lambda_p = 0.9$.

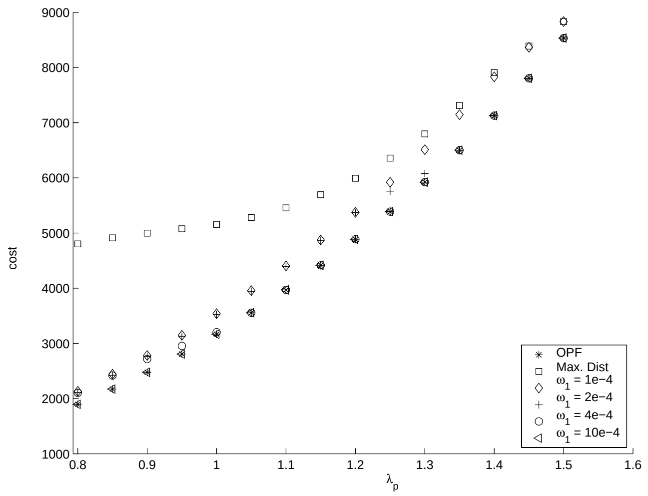

Fig. 5. Operating costs versus current operating point for the Linear Combination formulation, Maximum Loading Distance and traditional OPF for the 57-bus test system.

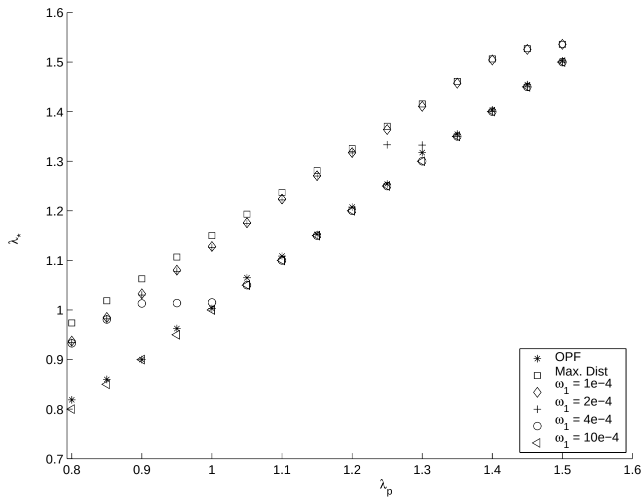

Fig. 6. Maximum loading point versus current operating point for the Linear Combination and Maximum Loading Distance formulations for the 57-bus test system.

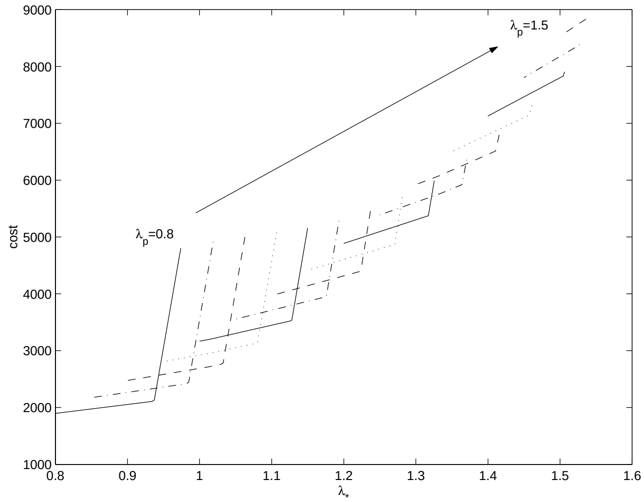

Fig. 7. Pareto-optimal set of values for the Linear Combination formulation applied to the 57-bus test system.

ing Distance and the normal OPF to the 57-bus test system. As expected, the solutions obtained from the Linear Combination formulation are *bound* by the solutions obtained from the Maximum Loading Distance and normal OPF. At lower values of ω_1, the Linear Combination solutions tend to the Maximum Loading Distance solutions, whereas at higher values of ω_1 these solutions approach the standard OPF solutions. ¿From the results obtained in these studies, it was observed that the generator powers were the variables most affected by the changes in the weighting factors, which is to be expected, as scheduling generation in an area with high loading levels will generally enhance stability but may result in increased costs. It was also observed that the generator terminal voltages tended to their maximum values as the load margin was given more weight in the optimization process, as expected.

The disadvantage of the Linear Combination formulation is illustrated in Fig. 6. Observe that there is a loading point, which varies with the values of the weighting factors, where the algorithm solution basically switches over from maximizing loading margins to minimizing costs.

The 118-bus system exhibited similar characteristics as the 57-bus system, although the computational costs were greater.

C. Fixed Loading Margin VSC-OPF

The second set of numerical analysis involves applying the Fixed Loading Margin formulation to both test systems. Recall that this method is basically an OPF where a minimum loading margin is ensured.

For both test systems, a minimum loading margin $\Delta\lambda_{min} = 0.1$ p.u. is used. In general, the algorithm found a solution that ensured this constraint; however, this resulted in higher operating costs. A comparison of the operating costs of the 118-bus system versus current loading point for the Fixed Loading Margin and traditional OPF formulations is shown in Fig. 8. Observe that costs increase as the loading increases, due to the fact that a minimum loading margin is being enforced, which becomes a dominant constraint as the system gets closer to the

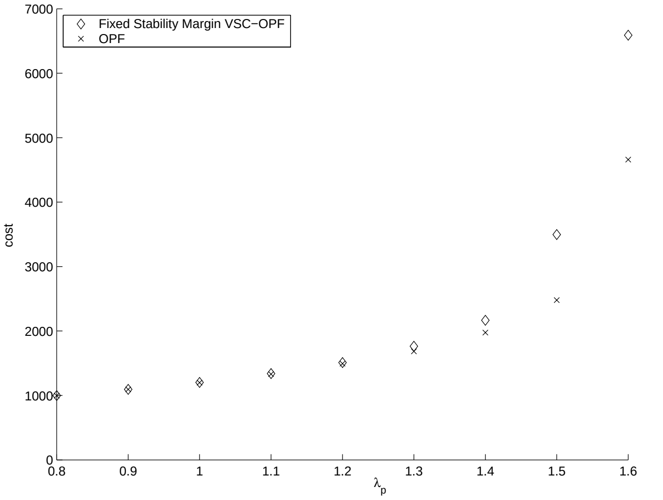

Fig. 8. Cost versus current operating point for the Fixed Loading Margin and traditional OPF formulations for the 118-bus test system.

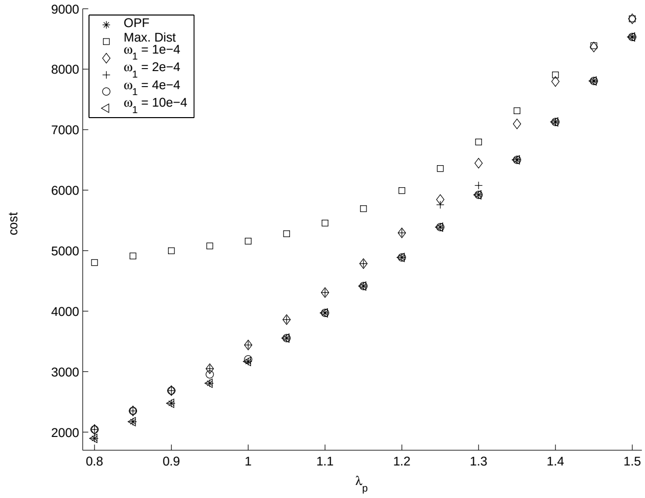

Fig. 9. Cost versus current operating point for the Goal Programming ($\omega_2 = 0.001$, $\omega_3 = 1 - \omega_1$), Maximum Loading Distance and traditional OPF formulations for the 57-bus test system.

critical point. Similar results were obtained for the 57-bus test system.

D. Modified Goal Programming VSC-OPF

The next set of numerical analysis involves applying the Goal Programming formulation to both test systems. Recall that the idea is to define a loading margin that is not a binding constraint, but that if violated increases the objective function cost. As illustrated in Fig. 9, the Goal Programming formulation shifted the importance of cost as ω_1 increased. As the loading level is increased, the cost of maintaining the desired minimum loading margin increases, and, eventually, for constant values of all weighting factors ω, the minimum loading margin is reduced to zero. When less weight is placed on cost and greater weight is placed on stability, i.e. for smaller values of ω_1, the minimum loading margin is maintained, as shown in Fig. 10. Similar observations can be made from the Pareto-optimal set of solutions depicted in Fig. 11, which is basically a combination of Figs. 9 and 10.

It was found in the numerical analysis that ω_2 does not greatly effect the solution of the problem, which is to be expected, since there is no benefit in having a loading margin greater than the desired value (this tends to also result in greater operating costs).

Similar results were obtained for the 118-bus system.

E. VSC-OPF with Reactive Power Pricing

The final set of numerical analysis involves applying the Goal Programming formulation considering reactive power "costs" to both test systems. In the tests cases discussed here, a penalty is added to the objective function if all of the generators are not operated at the desired power factor. Figures 12, 13 and 14 depict the results obtained for operating cost and maximum loading point, and are somewhat similar to the results obtained when applying the Goal Programming formulation without including reactive power costs. Thus, costs increase with loading and larger weighting on the loading margin, and, as the system is

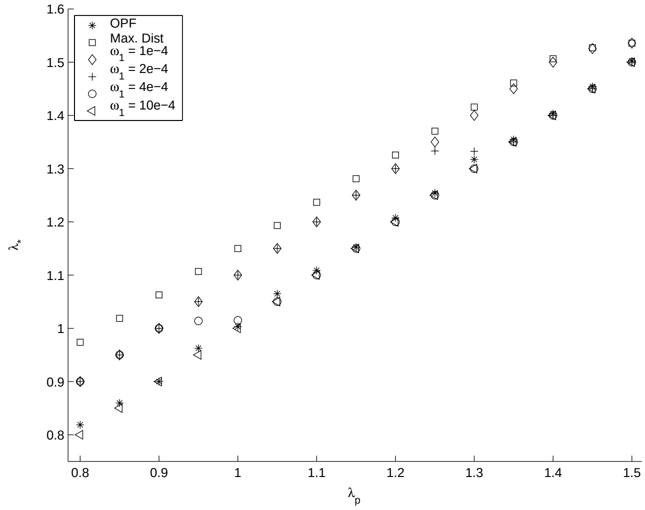

Fig. 10. Maximum loading point versus current operating point for the Goal Programming ($\omega_2 = 0.001$, $\omega_3 = 1 - \omega_1$) and Maximum Loading Distance formulations for the 57-bus test system.

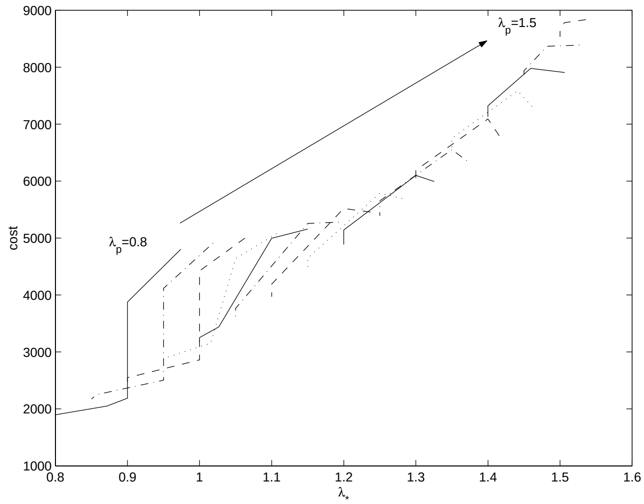

Fig. 11. Pareto-optimal set of solutions for the Goal Programming formulation ($\omega_2 = 0.001$, $\omega_3 = 1 - \omega_1$) applied to the 57-bus test system.

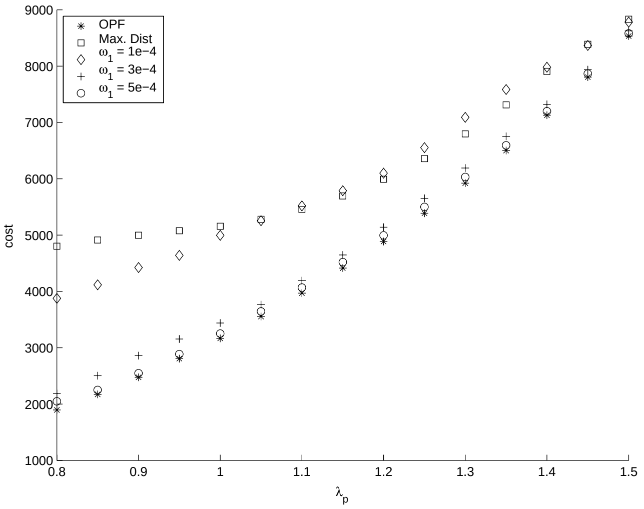

Fig. 12. Cost versus current operating point for the Goal Programming formulation considering reactive power costs ($\omega_2 = 0.001$, $\omega_3 = 1 - \omega_1$, $\omega_4 = \omega_5 = 0.3$), Maximum Loading Distance and traditional OPF formulation for the 57-bus test system.

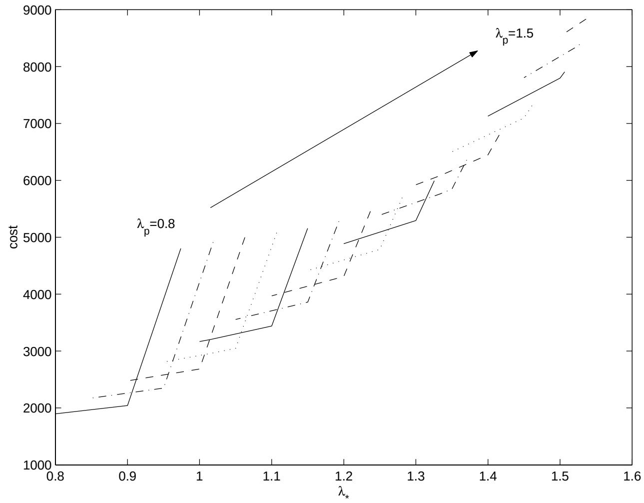

Fig. 14. Pareto-optimal set of solutions for the Goal Programming formulation considering reactive power costs ($\omega_2 = 0.001, \omega_3 = 1-\omega_1, \omega_4 = \omega_5 = 0.3$) applied to the 57-bus test system.

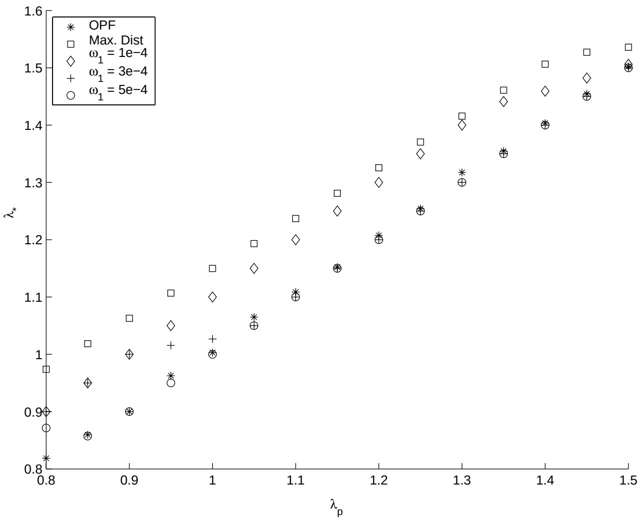

Fig. 13. Maximum loading point versus current operating point for Goal Programming formulation considering reactive power costs ($\omega_2 = 0.001$, $\omega_3 = 1 - \omega_1$, $\omega_4 = \omega_5 = 0.3$) and Maximum Loading Distance formulations for the 57-bus test system.

loaded, the formulation puts more emphasis on cost minimization than on maintaining a given loading margin, as enforcing this margin becomes more expensive. However, observe that when the reactive power costs become dominant with respect to the other two terms in the objective function, i.e. for smaller values of ω_1, it leads to higher operating costs while increasing the loading margin.

Similar results were obtained for the 118-bus system.

V. CONCLUSIONS

This paper demonstrates that voltage stability and OPF studies can be performed concurrently, proposing and comparing a variety of methodologies to allow operators to carry on this task in an EMS environment. It is shown that incorporating voltage stability into a traditional OPF problem can result in higher operating costs. The results show the importance of including the current loading point in optimization procedures used for voltage stability analyses, as limits on this point significantly influence the results obtained in these types of studies. Finally, the paper proposes a feasible way to include reactive power in an OPF objective, which could be a very useful tool in the operation of competitive electricity markets.

As the proposed OPF formulations include stability constraints, a possible enhancement to these techniques would be to improve the steady state system models used, so that accuracy can be improved at higher loading conditions.

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