BQP and QMA

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BQP and QMA

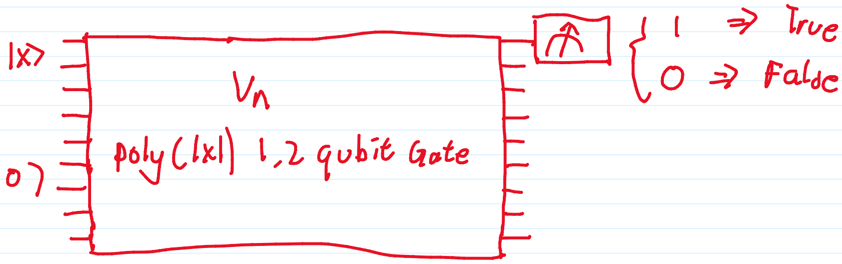
Classical	Quantum
P	BQP
NP	QMA
SAT problem	local Hamiltonian problem

What is similar with Shor, Grover and Random walk search? (Besides Quantum).

They are probabilistic! Hence we need to modify the definition a bit

BQP (Bounded Quantum Polynomial): $L \in BQP$ if $\exists$ a poly size Quantum circuit V_n such that for $x \in \{0,1\}^*$, $|x| = n$

1. $x \in L \Rightarrow V_n |x\rangle |0\rangle^{\otimes aux} = 1$ w/p $> 2/3$ (Completeness)
2. $x \notin L \Rightarrow V_n |x\rangle |0\rangle^{\otimes aux} = 1$ w/p $< 1/3$ (soundness)



Why 2/3? it's a constant $c > 1/2$, can run the circuit multiple time to amplity success probability

In fact, can run V_n poly(n) times, to obtain successrate to $\langle 1 - e^{-n^g}, \geq e^{-n^g}$, $g = O(1)$. (Using chernoff bound)

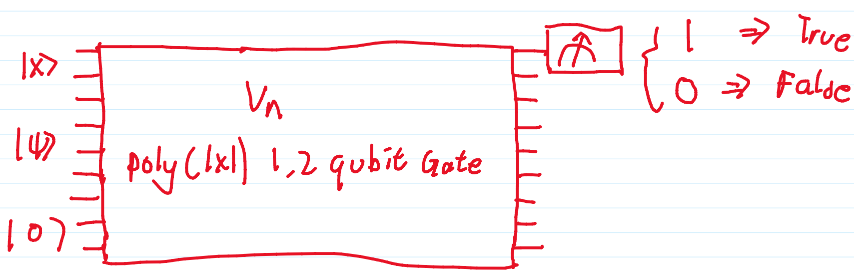
QMA (Quantum Merlin-Arthur): $L \in QMA$ if $\exists$ a poly size Quantum circuit V_n such that for $x \in \{0,1\}^*$, $|x| = n$:

1. $x \in L \Rightarrow \exists$ poly(n)-qubit state $|\psi\rangle$. S.t

   $V_n |x\rangle |\psi\rangle |0\rangle = 0$ w/p $< 2/3$ (Completeness)

2. $x \notin L \Rightarrow \forall$ poly(n)-qubit state $|\psi\rangle$. we have

   $V_n |x\rangle |\psi\rangle |0\rangle = 0$ w/p $> 1/3$ (soundness)



ℓ- local - Hamiltonian problem

Recall:

Hamiltonian: $H^* = H$ (A hermitian matrix)

H: Hermitian $\Rightarrow \sigma(H) \subseteq \mathbb{R}$

$\Rightarrow H = \sum_i \lambda_i |\psi_i\rangle\langle\psi_i|$ for $\lambda_1 \le \cdots \le \lambda_N$

k - local Hamiltonian: let $k = o(1)$

$H = \sum_{j=1}^{m} h_j$. where each h_j is a Hamiltonian and acts nontrivially on $\le k$ qubits and as identity on all other qubit

k - local Hamiltonian problem:

Given $(H_1, \cdots, H_m)$ k - local Hamiltonian acting on n - qubit. such that $0 \le \lambda_1 \le \cdots \le \lambda_N \le 1$ for all H_i $1 \le i \le m$

Given $0 \le a \le b \le 1$ s.t $b - a \le O(m^{-\alpha})$ (α constant)

Let $H = \sum_i H_i$

decide whether.

(Yes case): $\lambda_{min}(H) \le a$

(No case): $\lambda_{min} \ge b$

Aside: since ℓ is a constant, each H_i can be encode (classically) in $O(poly\, m)$ space

Classical	Quantum
P	BQP
NP	QMA
SAT problem	local Hamiltonian problem
Variables	qubits
Clause	Local Hamiltonians
Formula	$H = \sum_i H_i$
Assignment for the variable	n- qubit state $\|\psi\rangle$
Solution Quality (# of Clause satisfy)	$\langle\psi\|H\|\psi\rangle = \sum_i \langle\psi\|H_i\|\psi\rangle$
Optimal assignment	Eigenstate of $\lambda_{min}(H)$ "Ground state"

"Cost" of the state for H_i being away from the smaller eigenspace.

Too see this, for Ground state $|\psi\rangle$, note

$$\lambda_{min}(H) = \langle\psi|H|\psi\rangle = \sum_i \langle\psi|H_i|\psi\rangle$$

Hence, if each of the $\langle\psi|H_i|\psi\rangle$ is low $\Rightarrow \lambda_{min}(H)$ is small

Feumann - kitaev: 5 - Local Hamiltonian is QMA - complete

Hence, if each of the $\langle\psi|H_i|\psi\rangle$ is low $\Rightarrow$ $\lambda_{min}(H)$ is small

Feymann-kitaev : 5 - Local Hamiltonian is QMA - complete

In fact, this quantity can be reduce to 2-local [KKR04], but both line of work is build upon the 5-local case

Aside, QMA $\subseteq$ PSPACE

Reference

[KSV02]: Classical and Quantum Computation.

[KKR04]: The complexity of Local Hamiltonian Problem

[Yue 20]: The Complexity of Entanglement. Lecture 2, 3, 4