

ACTSC 232 Lecture Blog Spring 2012

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I'll post brief summaries of the material we cover in lectures here.

Lecture 36 - July 25

[Diana Katherine Skrzydlo](#) - Jul 23, 2012 5:29



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This last class will be a review of the material since the second midterm. Your assignment 3 is due at the beginning!

Lecture 35 - July 23

[Diana Katherine Skrzydlo](#) - Jul 23, 2012 5:25



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Today we did some random examples about premiums and extra risk.

First we did an example of calculating premiums (and other info regarding the loss-at-issue random variable) using a select-and-ultimate table for our mortality. It works the same as usual, just remember to go across the table and then down the last column.

Then we looked at an example regarding the variance of L_0 under two different premium calculation principles.

Finally, we did a complicated example of finding the debt X to take off a $S=60,000$ whole life insurance policy for an impaired life age 45. Get notes from someone for the contract details, I'm not writing them all out here. But we didn't get to the solution, so here it is:

We calculated the premium for an unimpaired life was $P = 1722.97$

The EPV of the premiums (minus expenses) for the impaired life is $0.96P\ddot{a}_{45:20}^* - 0.25P$.

$\ddot{a}_{45:20}^* = \ddot{a}_{45:10}^* + {}_{10}E_{45}^* \ddot{a}_{55:10}^*$ since the impairment is different for the first 10 vs the next 10 years.

$\ddot{a}_{45:10}^* = \ddot{a}_{45:20}$ at an interest rate $i^* = e^{\delta+\varepsilon} - 1 = (1.05)e^{0.01} - 1 = 6.0553\%$, which is 7.4379.

${}_{10}E_{45}^* = v^{10} {}_{10}P_{45}^* = v^{10} {}_{10}p_{45} e^{0.01 \times 10} = 0.485817$

$\ddot{a}_{55:10}^* = \ddot{a}_{60:10} = 7.09373$ (due to the 5 year age rating)

So putting it all together, $\ddot{a}_{45:20}^* = 10.8842$ and hence the EPV of premiums (minus expenses) for the impaired life is 17572.28.

The EPV of the benefits for the impaired life is $(S-X)A_{4\ 5:10}^{*1} + (S-X/2){}_{10}E_{45}^* A_{5\ 5:20}^{*1} + S_{30}E_{45}^* A_{75}$

$A_{4\ 5:10}^{*1} = 1 - d\ddot{a}_{45:10}^* - {}_{10}E_{45}^* = 0.159997$ (endowment insurance minus pure endowment)

$A_{5\ 5:20}^{*1} = A_{6\ 0:20} = 0.426672$ (due to the 5 year age rating)

${}_{30}E_{45}^* = {}_{10}E_{45}^* {}_{20}E_{55}^* = 0.485817 v^{20} {}_{20}p_{60} = 0.054775$

Putting it all together, the EPV of the benefits is $24311.34 - 0.263639X$

Finally, we solve EPV premiums minus expenses = EPV benefits $\Rightarrow 17572.28 = 24311.34 - 0.263639X \Rightarrow X = \$25,562$.

Lecture 34 - July 20

[Diana Katherine Skrzydlo](#) - Jul 23, 2012 5:03



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Today we continued talking about extra risk.

An insurance company could assume the force of mortality (or the probability of death within a year) is increased by a constant multiple α . Unfortunately, neither of these assumptions gives us a nice way to express the impaired insurance functions in terms of the regular ones.

Finally, they could assume that the force of mortality is increased by adding a small amount ε . Surprisingly, this actually leads to the ability to calculate impaired life annuity and insurance functions just by changing the interest rate used to calculate them. The new interest rate i^* is $e^{\delta+\varepsilon} - 1$. We did an example of this.

Also, instead of increasing the premiums, the insurance company could reduce the benefits that an impaired life would be paid. This (the amount the benefits are reduced by) is called a debt, and we can easily calculate it using the equivalence principle and the non-impaired premium.

Lecture 33 - July 18

[Diana Katherine Skrzydlo](#) - Jul 23, 2012 4:58



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Today we talked about extra risk.

There might be cases where an insurance company wishes to insure an "impaired" life (one that has worse mortality than average due to job, health, etc) and will want to charge them higher premiums than an average life. There are several ways they can take this extra risk into account.

They can build an entirely new mortality table for that type of lives (eg, smokers)

They can use age-rating (assume the individual is older than they actually are). We did an example of this, and discussed how the amount of age rating, age of the individual, and the contract itself would impact the increase in premium.

Lecture 32 - July 16

[Diana Katherine Skrzydlo](#) - Jul 23, 2012 1:17



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Today we talked about expenses.

We can explicitly account for expenses in our (gross) premium calculations relatively easily, by using the equivalence principle to set $EPV \text{ benefits} + EPV \text{ expenses} = EPV \text{ premiums}$.

We discussed the various types of expenses that can occur, and did an example.

Lecture 31 - July 13

[Diana Katherine Skrzydlo](#) - Jul 23, 2012 1:16



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Today we finished off the example by calculating the premium using the percentile principle

for a single contract as well as for 10,000 independent and identical contracts. We looked at the effect changes in the principles (number of contracts, % chance of loss allowed) would have on the premium, and also calculated the probability of a loss using the equivalence principle premium.

Lecture 30 - July 11

[Diana Katherine Skrzydlo](#) - Jul 11, 2012 5:06



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Today we talked about the different premium principles - methods to calculate premiums for a contract once we have identified the loss-at-issue RV L_0 .

The first and most common (and default, unless otherwise specified) principle is the equivalence principle. It's also the simplest, we just set P so that $E[L_0] = 0$. In other words, we set P so that EPV premiums = EPV benefits. We did that for the three examples from the last class.

We could also have a specified amount of profit per contract, or a premium that includes an extra multiple of the variance or standard deviation of L_0 (to compensate for the higher risk)

Finally, we could use a percentile principle, where we set P so that the probability of making a loss on a contract (or a portfolio of identical contracts) is less than some specified percentage, usually around 5%. We started an example which we'll continue on Friday.

Lecture 29 - July 9

[Diana Katherine Skrzydlo](#) - Jul 11, 2012 5:03



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Today we started Chapter 6 - premiums.

We briefly discussed the ways that premiums can be paid (single premium, which is rare, and the more usual level premium case.) We then defined the loss-at-issue random variable L_0 , which is equal to the PV of benefits - PV of premiums at the inception of the contract.

We looked at three different examples (of increasing complexity) of determining L_0 from a statement of the contract details. For any L_0 , we can identify the contract, and for any contract

we can identify L_0 (there is a 1-1 correspondence.)

Lecture 28 - July 6

[Diana Katherine Skrzydlo](#) - Jul 6, 2012 4:01



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Today we finished off chapter 5 by talking about variable annuities.

Like increasing insurance contracts, there are 3 variations - fully discrete, fully continuous, and discretely increasing but continuously paid. There are a couple of methods to calculate the EPV in the first two cases, but the third case needs a trick because it is a combination of integrals and sums. The notation for all three types is similar to the notation for increasing insurances.

Lecture 27 - July 4

[Diana Katherine Skrzydlo](#) - Jul 6, 2012 3:59



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Today we looked at an example of the accuracy of the different estimation methods for 1/m-thly annuities.

Then we talked about accumulations, which can be thought of as the average amount each survivor gets at the end of n years if a large number of people pay in \$1 per year while they are alive. We end up just getting a simple relationship - any "s" is the same "a" divided by ${}_nE_x$.

Lecture 26 - June 29

[Diana Katherine Skrzydlo](#) - Jun 29, 2012 12:39



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Today we talked about the relationships between the EPVs of annuities at different payment frequencies.

We applied the result for insurances that we got using UDD to the annuity EPVs and arrived at the following relationships:

$$\ddot{a}_x^{(m)} = \alpha(m)\ddot{a}_x - \beta(m)$$

$$\ddot{a}_{x:n}^{(m)} = \alpha(m)\ddot{a}_{x:n} - \beta(m)(1 - {}_nE_x)$$

$$\text{where } \alpha(m) = id/i^{(m)}d^{(m)} \text{ and } \beta(m) = (i - i^{(m)})/i^{(m)}d^{(m)}$$

We also discussed the two reasons for the difference between the EPVs. One (similar to the insurance case) is the timing of the payments within each year. Annuities due have the payments earliest, so they have the highest PV. But a second reason which only applied to annuities is that the actual amount of benefits paid in the year of death may be different for different payment frequencies. It's largest for annuities due, followed by 1/m-thly due, then continuous, then 1/m-thly immediate, then immediate. Both reasons give us the same set of inequalities:

$$\ddot{a}_x > \ddot{a}_x^{(m)} > \tilde{a}_x > a_x^{(m)} > a_x$$

Lecture 25 - June 27

[Diana Katherine Skrzydlo](#) - Jun 29, 2012 11:01



AM

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Today we looked at the last two types of annuity contracts - deferred annuities and guaranteed annuities.

Deferred annuities (like deferred insurance contracts) only start paying benefits after some deferral period, and if the policyholder dies in that period, they get nothing. We can always find the EPV of any u-year deferred annuity by taking out a common factor of ${}_uE_x$, and we're left with the EPV at time u of the contract at that point. So for example, ${}_u|\ddot{a}_x = {}_uE_x\ddot{a}_{x+u}$. This also gives us an easy way to calculate term annuity EPVs, since whole life = term + deferred for any type of contract. So $\ddot{a}_{x:n} = \ddot{a}_x - {}_n|\ddot{a}_x$.

Guaranteed annuities are almost the exact opposite of term annuities. They pay until death or n years, whichever is *later* (with term, it's whichever is sooner.) So no matter what, the policyholder or their estate will receive the first n years of payments, whether they live or die. If they live longer than time n, they continue to receive payments until their death. We could derive the EPV from first principles, or notice that it is just an n year annuity certain (no conditions) plus an n-year deferred whole life annuity.

Lecture 24 - June 25

[Diana Katherine Skrzydlo](#) - Jun 29, 2012 10:51



AM

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Today we talked about term annuities, and how to calculate their EPVs three different ways just like whole life annuities. We did an example of an annual term annuity for short term, and we also looked at how the formulas behave for all of the different payment timings (annual due, annual immediate, 1/m-thly due, 1/m-thly immediate, and continuous.)

They are mostly the same as for the whole life case, but the difference between due and immediate is now a little more complicated: $a_{x:n} = \ddot{a}_{x:n} - 1 + {}_nE_x$ (we subtract off the payment at time 0, but add the possibility of a payment at time n, if the life survives to time n.)

Lecture 23 - June 22

[Diana Katherine Skrzydlo](#) - Jun 22, 2012 2:57



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Today we talked about annuities paid at each 1/m of a year. We looked at both the due and immediate cases, and the results for EPV and Variance are very similar to the annual case. We looked at a summary of all 5 types of whole life annuities that we know about.

We also discussed the recursive relationship that exists between the EPVs of annual and 1/m-thly annuities (we don't have recursions for the continuous case). It's similar to the insurance recursive relationship - we split off the benefit in the first year/period, and then the rest is the benefits from then onwards, discounted back one year/period for interest and survival.

Lecture 22 - June 20

[Diana Katherine Skrzydlo](#) - Jun 20, 2012 5:06



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Today we talked about annual annuities immediate and continuous annuities.

The great thing about life contingent immediate annuities (\$1 per year paid at the end of the year if (x) is alive) is that the only difference between them and an annuity due is the single payment of \$1 at time 0. That gives us an easy way to get the three different formulas for calculating the EPV, and also means that the variance is exactly equal in both cases.

For a continuously paid annuity, (x) receives money at a constant rate of \$1 per year, as long as they are alive. To find the EPV, we need to use integration, since Y is a function of T_x , a continuous random variable. There are again three methods to calculate the EPV, and the variance ends up having the same form as the previous cases, but with the continuous versions instead.

Finally, we looked at an example of a continuously paid annuity using the De Moivre model (and some shortcuts to evaluating the integrals from ACTSC 231!)

Lecture 21 - June 18

[Diana Katherine Skrzydlo](#) - Jun 18, 2012 4:48



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Today we finished up chapter 4 by talking about variable insurance benefits. We looked at three cases: fully continuous, discretely increasing but continuously paid, and fully discrete. The middle case is a little bit weird because it involves both sums and integrals, but we can express the EPV in a couple of clever ways to calculate it.

We then started chapter 5 - annuities by looking at a discrete whole life annuity due. This is a sequence of payments of \$1 made at the start of each year if the policyholder is alive. We looked at three different ways of calculating the expected value - using A_x , first principles, and the (amt x discount x probability) way that works for all life contingent benefits (even in ACTSC 331!). We also looked at the variance, but really only the first way is useful. We can express the variance of an annuity in terms of the variance of an insurance and find it that way.

Lecture 20 - June 15

[Diana Katherine Skrzydlo](#) - Jun 18, 2012 3:26



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Today we talked about finding the EPV of benefits payable at the end of the $1/m$ -year of death. It's all very similar to the formulas for the annual case, but m -thly instead.

Then we looked at the relationship between the discrete, continuous, and m -thly EPVs under the UDD assumption. We derived the relationship (by splitting up the integral into pieces and applying UDD to each piece) $\bar{A}_x = i/\delta A_x$. A similar result holds for term insurance and $A_x^{(m)} = i/i^{(m)} A_x$. We can see that \bar{A} is the largest because the payment is soonest (and also

because δ is the smallest), followed by $A^{(m)}$, then A .

Lecture 19 - June 13 (my birthday!)

[Diana Katherine Skrzydlo](#) - Jun 13, 2012 1:18



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Today we reviewed all the EPV (and variance) formulas and notation for all the types of contracts we know.

We discussed the relationship between term insurance, pure endowment, and endowment insurance (and showed using the covariance that the variance of an endowment insurance follows exactly the same form as other contracts)

We derived a relationship between term insurance, whole life, and deferred whole life, which enables us to calculate discrete term insurance EPVs for any integer x and integer n , if we have a life table with A_x values. (I handed out the second page of the illustrative life table in class, and it's also posted)

Finally, we started to look at how to value benefits that are paid at the end of the $1/m$ -year of death.

Lecture 18 - June 11

[Diana Katherine Skrzydlo](#) - Jun 11, 2012 1:22



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Today we talked about the last three types of insurance contracts - pure endowment, endowment insurance, and deferred contracts.

The pure endowment (EPV is called $A_{x:\overline{n}|}$ or ${}_nE_x$) is not really a contract, but can be used as a discount factor or to calculate the EPV of endowment insurance.

Endowment insurance (EPV is called $A_{x:\overline{n}|}$) is the combination of term insurance and pure endowment.

We did an example of endowment insurance, and the EPV of a 3-year endowment insurance issued to a life age 45 (using ILT@6%) is indeed 0.84025 as I wrote down. (The term

insurance part only contributes 0.011477 and the pure endowment part contributes 0.828774.)

Any deferred contract can be written as ${}_nE_x$ times the EPV of what the future benefits would be at time n .

Lecture 17 - June 8

[Diana Katherine Skrzydlo](#) - Jun 8, 2012 3:21



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Today we looked at how to use recursion to calculate values of A_x if we have a discrete life table. The recursive relationship we get is $A_x = vq_x + vp_x A_{x+1}$, and since we have one-year survival and mortality probabilities from our life table, we can start with a very high age and work backwards to our desired A_x .

Then we started to look at term insurance contracts. The EPV is a very similar expression, except the integral goes to n (or the sum goes to $k=n-1$) instead of infinity. The actuarial notation for a term insurance is A (or A with a bar on top for continuous) $A_x^{\overline{1}|n}$. We can express the variance similarly with ${}^2A_x^{\overline{1}|n}$ being evaluated at twice the force of interest.

We looked at a short term discrete insurance example using the ILT. We'll talk more about the ILT next week. The second page of it is posted.

Lecture 16 - June 6

[Diana Katherine Skrzydlo](#) - Jun 8, 2012 3:15



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Today we looked at the whole life insurance contract.

We started with the continuous case (paid immediately on death), and derived the present value random variable $Z = v^T_x = e^{-\delta T_x}$. Then we found the mean aka expected present value aka actuarial value $E[Z] = \int_0^\infty e^{-\delta t} {}_t p_x \mu_{x+t} dt$, which we call \bar{A}_x (pretend the \sim on top is just a bar, I can't make that symbol here). We can also find the second moment by just evaluating \tilde{A}_x at twice the force of interest, which we call ${}^2\tilde{A}_x$ (and hence $\text{Var}(Z) = {}^2\tilde{A}_x - \bar{A}_x^2$)

For the discrete case (payable at the end of the year of death), we get $Z = v^{K_x+1}$ and so $E[Z] =$

$\sum_{k=0}^{\infty} v^{k+1} {}_k|q_x$, which we call A_x .

Lecture 15 - June 4

[Diana Katherine Skrzydlo](#) - Jun 8, 2012 3:06



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Today we started chapter 4 - insurance.

We reviewed some interest functions (rates and annuity factors) from ACTSC 231.

We talked about the different types of insurance contracts (whole life, term life, endowment insurance, pure endowment, deferred) and the possible payment timings (immediately on death, at the end of the year of death, at the end of the 1/m-year of death).

Finally, we defined b_{T_x} , the benefit payable if the life dies at time T_x , and Z , the present value random variable of the insurance benefits.

Lecture 14 - June 1

[Diana Katherine Skrzydlo](#) - Jun 1, 2012 5:48



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Today we had a review of the first 5 weeks of material, in preparation for the test on Monday. Good luck everyone!

Lecture 13 - May 30

[Diana Katherine Skrzydlo](#) - May 31, 2012 2:55



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Today we finished off chapter 3 by talking about select and ultimate tables. These are a common way to display a select and ultimate model, and there are two possible formats (an excel spreadsheet is posted in Content)

We looked at using the tables to calculate probabilities, and also how to construct tables (always starting with the ultimate part of the table and then working backwards one year at a time) given the one-year probabilities.

Friday will be review for the test.

Lecture 12 - May 28

[Diana Katherine Skrzydlo](#) - May 28, 2012 4:51



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Today we started talking about select and ultimate mortality models.

The selection effect (that a policyholder who more recently purchased insurance will in general be healthier than one who purchased longer ago) can be observed in real mortality data, so we may want to include that in our models. Also, the effect tends to wear off over time and after a while the mortality only depends on age, not how long ago insurance was purchased.

To model this, we define $[x]$ as a life newly selected at age x . We can then have force of mortality and survival/death probabilities defined in terms of a life age $[x] + d$, which includes information about the age at selection and the current age.

Clearly we must have $p_{[x]+d} < p_{x+d}$ since the second life is healthier.

Also, we have a select period D and then if $d \geq D$, the force of mortality and all probabilities for $[x]+d$ are the same as for $x+d$. The time since selection no longer has any effect and the life experiences the ultimate mortality.

Lecture 11 - May 25

[Diana Katherine Skrzydlo](#) - May 28, 2012 4:46



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Today we talked about the second fractional age assumption, Constant Force of Mortality (CFM). For that, we assume that μ_{x+s} is a constant μ_x for s between 0 and 1.

In that case, we derived the result that ${}_s p_x = (p_x)^s$. Even more usefully, ${}_s p_{x+u} = (p_x)^s$ too as long

as $s+u$ is between 0 and 1.

We compared UDD and CFM in terms of accuracy - UDD is more accurate at older ages and CFM is better for younger ages where the true mortality is relatively flat.

Lecture 10 - May 23

[Diana Katherine Skrzydlo](#) - May 23, 2012 12:47



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Today we talked about the first of two fractional age assumptions, Uniform Distribution of Deaths (UDD). When we're using a life table, we need to make an assumption about the mortality between integer ages if we want to calculate non-integer age or duration probabilities.

UDD assumes that the fractional part of the last year of life, S_x , is $U(0,1)$ and independent of K_x . (Remember $T_x = K_x + S_x$)

We get three results from that, which we can use to calculate any probability:

$${}_s q_x = s \cdot q_x$$

$${}_t p_x = (1-s)_k p_x + s {}_{k+1} p_x$$

$${}_s p_x \mu_{x+s} = q_x$$

Lecture 9 - May 22

[Diana Katherine Skrzydlo](#) - May 22, 2012 9:43



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Today we started chapter 3 by defining a life table, which is a sequence of values l_x where x is an integer between some starting age x_0 and ω , and for any x , l_x represents the average number of people who survive to age x , out of an initial l_{x_0} lives age x_0 .

Then we can find any mortality or survival probability in terms of the l_x 's as follows:


$${}_t p_x = l_{x+t} / l_x$$

$${}_t q_x = (l_x - l_{x+t}) / l_x$$

$${}_{u+t} q_x = (l_{x+u} - l_{x+u+t}) / l_x$$

We can also define $d_x = l_x - l_{x+1}$, which represents the average number of lives who die

between age x and $x+1$. This simplifies some probability calculations. We looked at a simple example, and you all got a copy of the illustrative life table which we will sometimes use for examples in class or tutorial. It's also posted online in content.

 **Lecture 8 - May 18**

[Diana Katherine Skrzydlo](#) - May 18, 2012 3:47



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Today we finished up chapter 2 by talking about a new random variable, the curtate future lifetime rv $K_x = [T_x]$ (the floor of T_x). We derived the probability function, cdf, and survival function of K_x , which is a discrete random variable. We also found expressions for its mean e_x and variance.

The relationship between e_x and e_x° is approximately $e_x^\circ = e_x + 1/2$

Finally, we reviewed some common mortality models we have seen (and will see) in the course. Some are simpler mathematically, and some are more accurate but more complicated.

 **Lecture 7 - May 16**

[Diana Katherine Skrzydlo](#) - May 16, 2012 1:26



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Today we looked at moments of the random variable T_x .

We derived the mean (or complete expectation of life) $e_x^\circ = E[T_x] = \int_0^\infty t p_x dt$ (which we obtained after integrating by parts.)

The median and the mode of T_x are not as useful but can be defined the usual way for a random variable.

Finally, we derived the second moment $E[T_x^2] = 2 \int_0^\infty t^2 p_x dt$ (again obtained through integration by parts) which gives us the variance of T_x , which is extremely important in terms of risk management.

 **Lecture 6 - May 14**

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[Diana Katherine Skrzydlo](#) - May 16, 2012 1:22

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Today we learned about the first (of many!) pieces of actuarial notation. We have ${}_t p_x = P(T_x > t)$, ${}_t q_x = P(T_x \leq t)$, and ${}_{t|u} q_x = P(t < T_x \leq t+u)$.

We re-expressed several of our previous results in terms of this new notation, keeping in mind that ${}_t p_x = S_x(t)$.

We also looked at the relationship between the cdf ${}_t q_x$ and pdf ${}_t p_x \mu_{x+t}$ of the random variable T_x , and how to logically explain it.

Finally, we compared q_x (the probability of dying between x and $x+1$) with $\mu_x dx$ (the approximate probability of dying between x and $x+dx$). If the force of mortality is level, letting $dx=1$ gives us the approximation $q_x \approx \mu_x$.

Lecture 5 - May 11

[Diana Katherine Skrzydlo](#) - May 11, 2012 11:37

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Today we looked at a couple of simple models which we will see many times during the term - the exponential model and the de moivre model. Both are nice mathematically but not that realistic in terms of human lives, although the SOA loves to ask questions about them. *

We derived the reverse relationship from the previous class - how to get the survival function if we have the force of mortality, and the result is $S_x(t) = e^{-\int_0^t \mu_{x+r} dr}$.

We then looked at two examples using this new result, including the modified de moivre model which we used two classes ago, and the Gompertz Law of mortality (which we will also see again) where we define the model in terms of $\mu_x = Bc^x$. A natural extension of the Gompertz Law is the Makeham Law which has $\mu_x = A + Bc^x$, and you'll see this model a lot in the textbook and on your first assignment! :)

* I want to apologize for snapping at the question "what section of the book is this in" - I think I misinterpreted the intent of the question, but in any case I shouldn't have been so rude. Please accept my apology.

Lecture 4 - May 9

[Diana Katherine Skrzydlo](#) - May 9, 2012 2:58



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Today we talked about the force of mortality, μ_x , which is yet another function that completely defines the survival model.

μ_x is defined as the limit as $dx \rightarrow 0$ of $P(T_0 \leq x+dx | T_0 > x)/dx$, and can be thought of as an annual rate of death. If we multiply it by a small interval length, we get the approximate probability of an individual dying within that interval of age x .

We showed that we can calculate $\mu_x = f_0(x)/S_0(x)$ and similarly $\mu_{x+t} = f_x(t)/S_x(t)$, and also that it is the derivative of the log of the survival function.

Lecture 3 - May 7

[Diana Katherine Skrzydlo](#) - May 7, 2012 1:00



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Today we looked at the conditions for survival models.

We need

$S_x(0) = 1$ (the life is alive right now at time 0)

$S_x(t) \rightarrow 0$ as $t \rightarrow \infty$ (no one lives forever)

$S_x(t)$ is non-increasing

plus for practicality it's useful to have

$S_x(t)$ is differentiable

$tS_x(t) \rightarrow 0$ as $t \rightarrow \infty$

$t^2S_x(t) \rightarrow 0$ as $t \rightarrow \infty$

We looked at an example and calculated several probabilities, found the pdf and limiting age, and even saw that the limiting age is consistent for older lives.

Lecture 2 - May 4

[Diana Katherine Skrzydlo](#) - May 7, 2012 12:56



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We started chapter 2 - survival models.

We defined the future lifetime random variable T_0 (for a life age 0) and T_x (for a life age x) and the functions associated with them - the cdf F , survival function S , and pdf f .

We derived a useful relationship between the survival function for a life age 0 and a life age x as follows: $S_x(t) = S_0(x+t)/S_0(x)$. This also leads to the very logical relationship that $S_0(x+t) = S_0(x) * S_x(t)$ - for a life age 0 to survive to age $x+t$, first they have to survive to age x , then they (age x now) have to survive t more years.

 **Lecture 1 - May 2**

[Diana Katherine Skrzydlo](#) - May 7, 2012 12:53



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We went over the course syllabus.

We briefly discussed reasons for insurance, and looked at the premium payment and benefit structure of common life insurance contracts.
