Definition X1.1 (Implication $A \Rightarrow B$)**.** The truth value for is defined by the truth table

A	B	$A \Rightarrow B$
T	T	
T	F	
F	T	
F	F	

We refer to A as the and B as the

Example X1.2 (Implication)**.** If you are in Ontario, then you are in Canada.

Question: What truth values for A, B are inconsistent with $A \Rightarrow B$?

Answer:

Statement X1.3. If x is a positive number, then $x + 10$ is a positive number.

Question: What is the hypothesis of this statement?
Answer:
Question: What is the conclusion?
Answer:
Question: Consider a negative number x. Does Statement X1.3 tell us whether the value of $x + 10$ is positive or negative?
Answer:

Example X1.4. Both the implications and are

Note: see Remarks on text p 28.

Negating Implications
Question: What is the negation of $A \Rightarrow B$?

A	B	$A \Rightarrow B$	$\neg(A \Rightarrow B)$
T	T	T	
T	F	F	
F	T	T	
F	F	T	

Answer:

Try Exercise X1.5. Write down the negation of the following:
(a) If 7 is odd and $5 \leq 6$, then 24 is a perfect square.
(b) If 7 is odd, then $5 \leq 6$ or 24 is a perfect square.

(a)

(b)

Example X1.6. Complete the truth table to determine whether
$((A \Rightarrow C) \vee (B \Rightarrow C)) \equiv ((A \wedge B) \Rightarrow C)$.

A	B	C	$A \Rightarrow C$	$B \Rightarrow C$	$(A \Rightarrow C) \vee (B \Rightarrow C)$	$A \wedge B$	$((A \wedge B) \Rightarrow C)$
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

Try After Class

Try Exercise X1.7. Construct a truth table for $A \Rightarrow (B \wedge C)$.

Definition X1.8 (Converse).

Try Exercise X1.9 (Converse)**.** Write the converse of the following statement.
Statement C: For all $x \in \mathbb{R}$, if $x > 0$ then $x^2 > 0$.

Converse of C:

Question: Is Statement C true or false?

Answer:

Question: Is the converse of C true or false?

Answer:

Example X1.10 ($A \Rightarrow B$ does not tell us whether $B \Rightarrow A$)**.**

Definition X1.11 (Contrapositive)**.** The contrapositive of $A \Rightarrow B$ is

Try Exercise X1.12 (Contrapositive)**.** Write the contrapositive of "If you are in Ontario, then you are in Canada".
Contrapositive:

Example X1.13 (Implication equivalent to its Contrapositive)**.** Show that $(A \Rightarrow B) \equiv (\neg B \Rightarrow \neg A)$.

Definition X1.14 ($A \Leftrightarrow B$)**.** The definition of $\underline{A \text{ if and only if } B}$, written ______ or ______ is

A	B	$A \Leftrightarrow B$
T	T	
T	F	
F	T	
F	F	

Note that ______

Statement X1.15. For all integers x, x is even if and only if $x^2 + 5x + 6$.

Try Exercise X1.16. Is Statement X1.15 is true or false? Convince me.

Try Exercise X1.17.

Determine whether the statement $(P \wedge \neg Q) \vee (R \Leftrightarrow P)$ is equivalent to $Q \Leftrightarrow R$.