CORRESPONDENCE

Comments on "Indications of Stratified Turbulence in a Mechanistic GCM"

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1. Introduction

In a recent paper, Brune and Becker (2013, hereafter BB13) analyze idealized GCM simulations to determine whether the mesoscale dynamics are consistent with stratified turbulence theory. They investigate the horizontal wavenumber energy spectrum and spectral budget in simulations with high and low vertical resolution. With high vertical resolution, they conclude that the mesoscale part of the spectrum is consistent with the stratified turbulence cascade theory of Lindborg (2006). In this comment, we present an alternative interpretation of these results that seems to have been overlooked by BB13.

2. Stratified turbulence

The term "stratified turbulence" is frequently employed to mean different things. Most generally, it refers to turbulence in a stably stratified fluid in which buoyancy forces dominate over Coriolis effects and are at least as important as advection. This definition encompasses a wide variety of different dynamical phenomena, including nonlinearly interacting internal gravity waves (IGWs), vortical modes, thin layers of strong shear, Kelvin–Helmholtz instabilities, and more isotropic turbulence at sufficiently small scales [for a review, see Riley and Lelong (2000)]. The details of the turbulence will naturally depend on which of these phenomena happen to dominate a particular flow.

However, in recent years, stratified turbulence has come to refer more specifically to the cascade theory advanced by Lindborg (2006). Such turbulence is characterized by small horizontal Froude numbers $\mathrm{Fr}_h \equiv U/(NL_h)$, where L_h is the horizontal length scale, U is the associated horizontal velocity scale, and N is the Brunt–Väisälä frequency. By contrast, the vertical Froude number $\mathrm{Fr}_v \equiv U/(NL_v)$, where L_v is the vertical length scale, is $O(1)$. In other words, the vertical scale of the turbulence is assumed to adjust with stratification and is given by the buoyancy scale $L_b \equiv U/N$ (e.g., Billant and Chomaz 2001; Waite and Bartello 2004). Lindborg (2006) proposed a stratified turbulence cascade theory for this parameter regime, in which kinetic and potential energy are transferred through an anisotropic inertial subrange from large to small horizontal scales. The associated energy spectrum is predicted to have a $-5/3$ power law in horizontal wavenumber. It is this specific interpretation of stratified turbulence that is employed by BB13.

The assumption that $\mathrm{Fr}_v = O(1)$ has very important implications for stratified turbulence. When both horizontal and vertical Froude numbers are small, there is a natural linear decomposition of the velocity field into internal gravity waves and vortical modes, which have very different linear dynamics but may interact nonlinearly (Lelong and Riley 1991; Bartello 1995). This decomposition is based on the linear part of the potential vorticity, which dominates only when $\mathrm{Fr}_v \ll 1$ (Waite and Bartello 2006; Waite 2013). As a result, these linear modes lose their meaning when $\mathrm{Fr}_v = O(1)$, and turbulence in this regime is more accurately described as

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three-dimensional anisotropic turbulence rather than interacting waves and vortices (Lindborg 2006). Consequently, the horizontally rotational and divergent contributions to the kinetic energy, which are proxies for vortical mode and IGW energy, are expected to have similar spectral slopes and amplitudes when $\mathrm{Fr}_v \sim O(1)$ (Waite and Bartello 2004; Lindborg and Brethouwer 2007).

3. The mesoscale spectrum

There has long been interest in employing stratified turbulence as a model for the atmospheric mesoscale (e.g., Lilly 1983). Observations of the horizontal wavenumber kinetic energy spectrum frequently show a transition from a -3 power law at large scales, as expected for quasi-geostrophic turbulence, to a $-5/3$ spectrum below scales of a few hundred kilometers (e.g., Nastrom and Gage 1985). Since the Lindborg (2006) theory for stratified turbulence predicts a $-5/3$ spectrum, it suggests an intriguing potential explanation for the observed mesoscale spectrum.

A number of GCMs and NWP models have successfully reproduced the shallowing of the mesoscale energy spectrum to something resembling $-5/3$ (e.g., Koshyk and Hamilton 2001; Skamarock 2004; Hamilton et al. 2008; Augier and Lindborg 2013). BB13 investigate the mesoscale spectrum in two simulations with the Kühlungsborn Mechanistic General Circulation Model: one with coarse vertical resolution typical of many GCM studies (30 levels, which have $\Delta z \approx 1.5$ km in the upper troposphere), and one with relatively high vertical resolution (100 levels, with $\Delta z \approx 200$ m). The horizontal resolution is T330, corresponding to an effective minimum grid spacing of around 50 km. Spectra and spectral budgets are computed in the mid- and upper troposphere (5 and 11 km, respectively).

With high vertical resolution, the midtroposphere kinetic energy spectrum in BB13 has a spectral slope of -3 all the way to the smallest resolved scales of around 100 km. There is no shallowing to a mesoscale $-5/3$. The spectrum is dominated by rotational kinetic energy, which has a -3 slope and accounts for almost all the kinetic energy at all scales. Interestingly, the divergent kinetic energy has a much shallower spectral slope that resembles $-5/3$, but its amplitude is too low to produce a transition in the total kinetic energy. In the upper troposphere, the slopes of the rotational and divergent spectra are similar to lower levels: the rotational is -3, and the divergent is $-5/3$. However, the amplitude of the divergent spectrum is much higher at these levels. It crosses the rotational spectrum at a horizontal scale of around 300 km, yielding a kink in the total kinetic energy spectrum. Below this scale, the rotational and divergent spectra remain -3 and $-5/3$, respectively.

BB13 present a detailed analysis of the spectral budget at both levels. They find a significant mesoscale contribution from the pressure gradient term, which they call the adiabatic conversion. [Horizontal and vertical advection are also important; Augier and Lindborg (2013) have recently presented an alternative formulation of the advection terms.] The adiabatic conversion term can be decomposed into two parts: the buoyancy flux, which represents conversion from potential to kinetic energy, and the vertical pressure flux divergence, which is associated with vertically propagating IGWs. They find that the IGW contribution dominates and yields a significant sink of mesoscale kinetic energy in the midtroposphere and a source in the upper troposphere.

These high-vertical-resolution results of BB13 are very reminiscent of the findings of Waite and Snyder (2009, hereafter WS09), who analyzed the mesoscale spectrum that develops in idealized dry baroclinic wave simulations with relatively high horizontal and vertical resolution ($\Delta x = 12.5$ km and $\Delta x \approx 60$ m, respectively). Like BB13, WS09 found the rotational and divergent kinetic energy spectral slopes to be around -3 and $-5/3$, respectively. At lower levels, the rotational energy dominates the spectrum, and no mesoscale transition occurs, as in the midtroposphere in BB13. However, at higher levels, the rotational and divergent spectra cross, yielding a kink in the total spectrum. Averaged in height over 12–14 km, the spectra cross at a horizontal scale of around 400 km. In BB13, they cross at around 300 km at a height of 11 km. As in BB13, the vertical pressure flux divergence is found to be a significant source of mesoscale kinetic energy at higher levels.

4. Is it stratified turbulence?

To determine whether the mesoscale spectrum in BB13 results from stratified turbulence of the Lindborg (2006) variety, there are three criteria that should be met, as outlined above: the mesoscale should have $\mathrm{Fr}_h \ll 1$, $\mathrm{Fr}_v \sim O(1)$, and rotational and divergent kinetic energy spectra of comparable slopes and amplitudes. The horizontal Froude number is undoubtedly small in the mesoscale; indeed, BB13 confirm that it is $O(10^{-4})$. But what about the other two criteria?

To evaluate the vertical Froude number, BB13 measure L_b (which they call ℓ_z) in their simulations as a function of horizontal scale and height. In the upper troposphere, where the kinetic energy spectrum exhibits a transition to something resembling $-5/3$, the mesoscale L_b is quite small, around 80 m in the high-vertical-resolution experiment. As a result, it is simply not possible for the mesoscale Fr_v to be $O(1)$. Given that this simulation has vertical grid spacing $\Delta z \approx 200\,\mathrm{m} \approx 2.5L_b$, and that several grid lengths are required for proper resolution, the mesoscale structures in this simulation have vertical scales that are many times larger than L_b. For example, a resolved structure with $L_v = 7\Delta z$ will have $\mathrm{Fr}_v \equiv L_b/L_v = 0.06$. By contrast, Lindborg (2006) resolved L_b with 10 times as many levels ($\Delta z = 0.25L_b$) to obtain a $-5/3$ spectrum in his idealized stratified turbulence simulations.

Furthermore, the rotational and divergent kinetic energy spectra in BB13 do not agree with the Lindborg (2006) theory. With high vertical resolution, these spectra in BB13 have distinct spectral slopes of -3 and $-5/3$, respectively, right through the mesoscale. With low vertical resolution, there is some shallowing of the rotational spectrum toward $-5/3$; however, this behavior appears to be an artifact of the coarse vertical grid, since it does not occur with higher resolution. These findings are in contrast with stratified turbulence simulations, in which both spectra have an approximately $-5/3$ slope when L_b is resolved in the vertical (Waite and Bartello 2004; Lindborg and Brethouwer 2007). As discussed above, the similarity of the rotational and divergent spectra is to be expected when $\mathrm{Fr}_v \sim O(1)$. The fact that these spectra have different slopes in BB13 is consistent with the small vertical Froude number.

5. An alternative interpretation

Given that two of the three above criteria are not met, it seems doubtful that the mesoscale dynamics in BB13 can be interpreted as stratified turbulence in the specific Lindborg (2006) sense. In fact, an alternative explanation was presented in WS09 to account for a very similar set of results in idealized dry baroclinic wave simulations. WS09 outlined three interacting phenomena that contribute to the development of mesoscale spectra: generation of mesoscale IGWs by spontaneous emission from the synoptic-scale flow, transfer of IGW energy to small scales via straining wave–vortex interactions, and vertical propagation.

IGWs are well known to be excited through spontaneous imbalance of the large-scale flow. Different generation mechanisms are responsible for these waves, including fronts (Snyder et al. 1993) and jets (e.g., Zhang 2004; Plougonven and Snyder 2007). WS09 observed three distinct packets of IGWs at the early stage of the baroclinic wave evolution. While these particular mechanisms may be obscured in more forced–dissipative GCM simulations like BB13, the underlying physical mechanisms of baroclinic instability and excitation of IGWs are expected to be present. Furthermore, moist physics and latent heating, which are crudely parameterized in BB13, provide an even stronger source of mesoscale IGWs (Waite and Snyder 2013).

Once generated, IGWs propagate through the velocity field of the synoptic-scale vortical flow. The associated large-scale strain tends to contract the horizontal wavelengths of the waves, thereby transferring IGW energy to smaller horizontal scales (Bühler and McIntyre 2005; Plougonven and Snyder 2005). This process appears to be the physical space manifestation of the catalytic wave–vortex triad interaction, by which vortical modes facilitate the transfer of IGW energy to small scales (Lelong and Riley 1991; Bartello 1995). Indeed, Bartello (1995) showed that such interactions may dominate the wave energy budget, and that the resulting IGW energy spectrum is much shallower than -3. This mechanism seems consistent with the findings of WS09 and BB13 that the rotational and divergent kinetic energy spectra have distinct slopes right through the mesoscale.

In addition to a downscale cascade, the spectral budgets in WS09 and BB13 are strongly influenced by the vertical pressure flux divergence, which would result from vertical IGW propagation. At upper levels, where the shallowing of the kinetic energy spectrum is most pronounced, this term provides an injection of mesoscale kinetic energy. This process is not accounted for in the stratified turbulence theory of Lindborg (2006), which assumes homogeneous statistics. Indeed, at a particular height, this source of kinetic energy resembles a forcing in the equation of the kinetic energy spectrum and is not consistent with a conservative cascade of energy from large to small horizontal scales. Interestingly, vertical propagation does account for the similarity in the divergent kinetic energy spectra at different heights, given that the IGW sources may be primarily at levels lower than 11 km.

6. Conclusions

The mesoscale spectra in BB13 can be understood as resulting from wave–vortex interactions: IGWs cascading downscale with a $-5/3$ spectrum through a vortical flow with a -3 spectrum. This regime may be broadly described as stratified turbulence, but not in the specific sense employed by Lindborg (2006), contrary to the suggestion in BB13. It is possible that, with even higher vertical resolution, numerical simulations such as those in WS09 and BB13 will transition to a regime that more closely resembles Lindborg (2006) stratified turbulence, but this is still an open question. Indeed, Lindborg (2007) found $-5/3$ spectra of rotational and divergent kinetic energy with comparable amplitudes only below scales of 100 km, which are subgrid scale in BB13. Based on BB13's estimate of $L_b \approx 80\,\mathrm{m}$, such a transition would require extremely fine vertical grids with $\Delta z \sim O(10\,\mathrm{m})$. It seems clear that the simulations in BB13 are not in this regime.

It is interesting that the high-vertical-resolution simulations in BB13 so closely resemble the results of WS09, given that both studies used much higher vertical resolution than is typically employed. Lower vertical resolution—as in the 30-level case of BB13 and in other studies (Koshyk and Hamilton 2001; Hamilton et al. 2008)—yields increased energy at the mesoscale and leads to a shallowing of the rotational kinetic energy spectrum. By comparing high- and low-vertical-resolution cases, BB13 have made an important contribution by showing that such coarse Δz results are probably spurious. It is peculiar, however, that these low-resolution artifacts lead to mesoscale spectra that are in better agreement with observations. This discrepancy is still not well understood and needs further study.

REFERENCES

Augier, P., and E. Lindborg, 2013: A new formulation of the spectral energy budget of the atmosphere, with applications to two high-resolution general circulation models. *J. Atmos. Sci.,* **70,** 2293–2308.

Bartello, P., 1995: Geostrophic adjustment and inverse cascades in rotating stratified turbulence. *J. Atmos. Sci.,* **52,** 4410–4428.

Billant, P., and J.-M. Chomaz, 2001: Self-similarity of strongly stratified inviscid flows. *Phys. Fluids,* **13,** 1645–1651.

Brune, S., and E. Becker, 2013: Indications of stratified turbulence in a mechanistic GCM. *J. Atmos. Sci.,* **70,** 231–247.

Bühler, O., and M. E. McIntyre, 2005: Wave capture and wave-vortex duality. *J. Fluid Mech.,* **534,** 67–95.

Hamilton, K., Y. O. Takahashi, and W. Ohfuchi, 2008: The mesoscale spectrum of atmospheric motions investigated in a very fine resolution global general circulation model. *J. Geophys. Res.,* **113,** D18110, doi:10.1029/2008JD009785.

Koshyk, J. N., and K. Hamilton, 2001: The horizontal kinetic energy spectrum and spectral budget simulated by a high-resolution troposphere–stratosphere–mesosphere GCM. *J. Atmos. Sci.,* **58,** 329–348.

Lelong, M.-P., and J. J. Riley, 1991: Internal wave-vortical mode interactions in strongly stratified flows. *J. Fluid Mech.,* **232,** 1–19.

Lilly, D. K., 1983: Stratified turbulence and the mesoscale variability of the atmosphere. *J. Atmos. Sci.,* **40,** 749–761.

Lindborg, E., 2006: The energy cascade in a strongly stratified fluid. *J. Fluid Mech.,* **550,** 207–242.

——, 2007: Horizontal wavenumber spectra of vertical vorticity and horizontal divergence in the upper troposphere and lower stratosphere. *J. Atmos. Sci.,* **64,** 1017–1025.

——, and G. Brethouwer, 2007: Stratified turbulence forced in rotational and divergent modes. *J. Fluid Mech.,* **586,** 83–108.

Nastrom, G. D., and K. S. Gage, 1985: A climatology of atmospheric wavenumber spectra observed by commercial aircraft. *J. Atmos. Sci.,* **42,** 950–960.

Plougonven, R., and C. Snyder, 2005: Gravity waves excited by jets: Propagation versus generation. *Geophys. Res. Lett.,* **32,** L18802, doi:10.1029/2005GL023730.

——, and ——, 2007: Inertia–gravity waves spontaneously generated by jets and fronts. Part I: Different baroclinic life cycles. *J. Atmos. Sci.,* **64,** 2502–2520.

Riley, J. J., and M.-P. Lelong, 2000: Fluid motions in the presence of strong stable stratification. *Annu. Rev. Fluid Mech.,* **32,** 613–657.

Skamarock, W. C., 2004: Evaluating mesoscale NWP models using kinetic energy spectra. *Mon. Wea. Rev.,* **132,** 3019–3032.

Snyder, C., W. C. Skamarock, and R. Rotunno, 1993: Frontal dynamics near and following frontal collapse. *J. Atmos. Sci.,* **50,** 3194–3211.

Waite, M. L., 2013: Potential enstrophy in stratified turbulence. *J. Fluid Mech.,* **722,** R4, doi:10.1017/jfm.2013.150.

——, and P. Bartello, 2004: Stratified turbulence dominated by vortical motion. *J. Fluid Mech.,* **517,** 281–308.

——, and ——, 2006: The transition from geostrophic to stratified turbulence. *J. Fluid Mech.,* **568,** 89–108.

——, and C. Snyder, 2009: The mesoscale kinetic energy spectrum of a baroclinic life cycle. *J. Atmos. Sci.,* **66,** 883–901.

——, and ——, 2013: Mesoscale energy spectra of moist baroclinic waves. *J. Atmos. Sci.,* **70,** 1242–1256.

Zhang, F., 2004: Generation of mesoscale gravity waves in upper-tropospheric jet-front systems. *J. Atmos. Sci.,* **61,** 440–457.