Optimal Power Control in Ultra-Dense Small Cell Networks: A Game-Theoretic Approach

Jianchao Zheng, *Member, IEEE*, Yuan Wu, *Senior Member, IEEE*, Ning Zhang, *Member, IEEE*, Haibo Zhou, *Member, IEEE*, Yueming Cai, *Senior Member, IEEE*, and Xuemin Shen, *Fellow, IEEE*

***Abstract*— In this paper, we study the power control problem for interference management in the ultra-dense small cell networks, which is formulated to maximize the sum-rate of all the small cells while keeping tolerable interference to the macrocell users. We investigate the problem by proposing a novel game with dynamic pricing. Theoretically, we prove that the Nash equilibrium (NE) of the formulated game coincides with the stationary point of the original sum-rate maximization problem, which could be locally or globally optimal. Furthermore, we propose a distributed iterative power control algorithm to converge to the NE of the game with guaranteed convergence. To reduce the information exchange and computational complexity, we propose an approximation model for the original optimization problem by constructing the interfering domains, and accordingly design a local information-based iterative algorithm for updating each small cell's power strategy. Theoretic analysis shows that the local information-based power control algorithm can converge to the NE of the game, which corresponds to the stationary point of the original sum-rate maximization problem. Finally, simulation results demonstrate that the proposed approach yields a significant transmission rate gain, compared with the existing benchmark algorithms.**

***Index Terms*— Interference management, power control, ultra-dense small cell networks, game theory, distributed algorithm, local information.**

I. INTRODUCTION

FUELED by the rising popularity of smart phones and tablets, mobile wireless communication networks have been experiencing an explosive growth of data traffic, which is predicted to increase at least 1000 times over the next decade [1], [2]. Along with the explosion of mobile data traffic, it is also expected that almost 50 billion wireless devices will be connected by 2020 [3]. Therefore, great effort has been made to explore new ways to improve the network coverage and boost the network capacity. One of the most promising solutions to achieve such a goal is network densification by deploying more infrastructure nodes in the serving area [1], [2]. As a result, ultra-dense small cell networks have been proposed as a novel efficient networking paradigm by densely deploying short-range, low-power, and low-cost base stations (BSs) underlaying the legacy macro-cellular networks [4]–[6].

Ultra-dense networks offer unprecedented capacity by bringing the network close to mobile users. Theoretically, the overall capacity scales with the number of small cells deployed [1]. However, in a ultra-dense deployment, not only desired signal strength but also interference from other cells increase due to the reuse of spectrum. Thus, inter-cell interference has been considered as the key limiting factor for the system capacity in ultra-dense small cell networks. Moreover, small cells are usually opportunistically and irregularly deployed in the hotspots [7]. The ultra-dense and unplanned deployment of small cells has imposed a great challenge on interference management and radio resource optimization [8], [9].

It has been shown that efficient power control can greatly mitigate inter-cell interference and improve system capacity [10]. However, the power control problem is highly non-convex due to the presence of inter-cell interference [11]. Several optimization methods (e.g., geometric programming [12], fractional programming [13]–[15], and successive convex approximation [16], [17]) have been applied to transform the non-convex power control problem into various convex problems, which can be solved by the standard convex optimization techniques. Nevertheless, these solutions usually necessitate centralized controllers which may not work well in ultra-dense small cell networks due to the huge signaling and computational overhead for collecting information from all cells. Therefore, distributed power control schemes outperform the centralized counterparts due to less information exchange and computations [18].

As is well-known, game theory is an important mathematical tool to study and analyze the strategic interactions among individual decision makers in decentralized networks [19]. Considering that BSs could be deployed by different service providers and individual users of different interests [2], we study a non-cooperative game-theoretic setting where each BS distributively performs resource allocation to maximize its own utility. Notice that, game theory-based power control

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J. Zheng and Y. Cai are with the College of Communications Engineering, PLA University of Science and Technology, Nanjing 210007, China (e-mail: longxingren.zjc.s@163.com; caiym@vip.sina.com).

Y. Wu is with the College of Information Engineering, Zhejiang University of Technology, Hangzhou 310023, China (e-mail: iewuy@zjut.edu.cn).

N. Zhang, H. Zhou, and X. Shen are with the Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, ON N2L 3G1, Canada (e-mail: n35zhang@uwaterloo.ca; h53zhou@uwaterloo.ca; sshen@uwaterloo.ca).

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has been widely studied in the literature. For the traditional code-division multiple access (CDMA) systems, [20], [21] use game theory to design distributed power control schemes for individual users that adjust transmit power strategies to maximize their own utilities. For the orthogonal frequency division multiple access (OFDMA)-based cellular networks, [22], [23] formulate power control games to mitigate the co-channel interference and improve the system throughput.

Compared with the centralized power allocation algorithms, these game-theoretic distributed schemes require less or even no information exchange. However, the aforementioned works [20]–[23] lack a mechanism to control the amount of aggregate interference imposed on macrocells, and thus cannot guarantee the quality of service (QoS) of macrocell users. Reference [24] formulates a novel game-theoretic framework subject to the interference temperature constraints. By utilizing the equivalence between the variational inequality and constrained game theory, [24], [25] devise several distributed iterative waterfilling algorithms to find the Nash equilibrium (NE). Meanwhile, Stackelberg game has also been applied to capture the hierarchical resource competition in two-tier heterogeneous cellular networks. In [26] and [27], price-based single-leader multi-follower Stackelberg games are formulated, where the macrocell as the game leader maximizes its monetary revenue by pricing the interference from the femtocells. However, all these studies are based on the common assumption of global information, where all the parameters involved in the optimization problem can be obtained. Due to the limited coordination in large-scale dense networks, information incompleteness usually arises and the assumption of global information may not always hold in practice [28], [29].

Moreover, a number of pricing schemes have been proposed in [27] and [30]–[35] to improve the efficiency of the NE. Reference [31] studies a non-cooperative power control game in a single-cell system, and shows the NE of the game is unique and Pareto-efficient. By setting dynamic prices for users, various design goals can be achieved. However, in multicell settings where transmit powers of different cells need to be jointly optimized, inter-cell interference cannot simply be treated as noise, making the solution obtained by [31] inapplicable. Ngo *et al.* [32] propose two Pareto-optimal power control algorithms for maximizing the total utility of both macrocell and femtocell networks while robustly protecting the prescribed minimum signal-to-interference-plus-noise ratios (SINRs) of macrocell users. However, it remains unclear about the gap between these solutions and the global optima for sum-rate maximization.

In this paper, the power control problem in ultra-dense heterogeneous small cell networks is formulated to maximize the sum-rate of all the small cells while keeping tolerable interference to the macrocell users. We investigate the problem by proposing a novel game with dynamic pricing, where each small cell is modeled as a game player that independently adjusts its power strategy in a distributed manner. The main contributions of this paper are summarized as follows:

- In order to improve the efficiency of the game, we motivate and encourage the cooperation among individual game players (i.e., small cells) by properly designing the utility for each player. Specifically, the proposed utility function consists of two parts: its own transmission rate and the external effects to other players' transmission rates, which is adaptively adjusted by a dynamic price.
- Theoretically, we have proved that the NE of the formulated game coincides with the stationary point of the original sum-rate maximization problem, which could be locally or globally optimal. Moreover, we have devised a distributed iterative power control algorithm to converge to the NE of the game.
- To reduce the information exchange and computational complexity, we propose an approximation model for the original sum-rate maximization problem by constructing the interfering domains, and accordingly design a local information-based iterative algorithm for updating each small cell's power strategy. Both theoretic analysis and simulation results are provided to validate and evaluate the performance of the proposed approach.

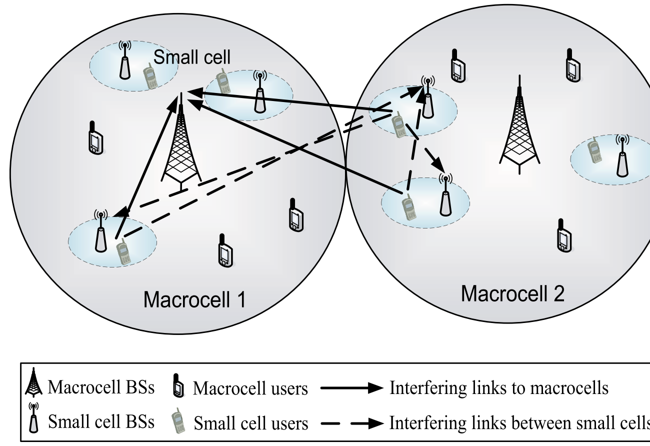

Fig. 1. System model. For clarity and simplicity, only part of the interfering links are plotted.

The remainder of this paper is organized as follows. In Section II, we present the system model and problem formulation. In Section III, we propose a game-theoretic approach for the distributed power control in ultra-dense heterogeneous small cell networks. In Section IV, we design a local information-based iterative algorithm to update each small cell's power strategy. In Section V, simulation results and discussions are presented. Concluding remarks are given in Section VI.

II. System Model and Problem Formulation

A. System Model

We consider an OFDMA-based heterogeneous cellular network, where multiple small cells (such as micro-, pico- and femtocells) underlay the macrocells, as shown in Figure 1. The sets of macrocells and small cells are denoted by $\mathcal{M} = \{m_1, m_2, \ldots, m_L\}$ and $\mathcal{N} = \{1, 2, \ldots, N\}$, respectively. In each cell, there are multiple mobile users served by a BS located in the center. This paper focuses on the uplink communications.[1] Each user and each BS are equipped with one transmit antenna and one receive antenna, respectively.

[1] Our approach can also be applied to the downlink communications with minor revisions.

TABLE I
SUMMATION OF USED NOTATIONS

Notations	Description
$\mathcal{M}$	set of macrocells
$\mathcal{N}$	set of small cells
p_n	uplink transmit power of small cell $n \in \mathcal{N}$
p_n	uplink transmit power of macrocell $m \in \mathcal{M}$
g_{ij}	link power gain from transmitter i to receiver j
η_m	interference temperature limit imposed by macrocell m
σ_n	noise power at the receiver of small cell n
R_n	transmission rate achieved by small cell n
R_{sum}	sum-rate of all the small cells
$\mathcal{P}_n$	set of available power strategy for small cell n
$\tilde{\mathcal{P}}$	convex feasible set given by (4b) and (4c)
$\mathbf{p}$	strategy profile of all the small cells
$\mathbf{p}_{-n}$	strategy profile of all the small cells excluding n
U_n^t	utility function of small cell n at time t
F_n	correspondence function for satisfaction of the constraint
$\widetilde{BR}_n$	best response of small cell n with global information
$\widehat{BR}_n$	best response of small cell n with local information

Each user communicates with its associated BS in a single-hop fashion. We study the universal frequency reuse deployment in which all macrocells/small cells share the whole spectrum to serve the mobile users.

In the OFDMA-based system, intra-cell multi-user access is orthogonal, whereas inter-cell multi-user access is simply superposed owing to full reuse of spectrum. It is the superposition of the spectral resource slots (i.e., subchannels, or frequency bands) that leads to severe inter-cell interference, which largely limits the system capacity. Thus, it is intuitive to decouple the optimization of resource allocations in different spectral slots [22], [36]. In this paper, we focus on the power control in a particular spectral slot.

For the sake of easy presentation, we use the transmit power of small cell n (macrocell m) to refer to the transmit power of the target mobile user which is served by small cell n (macrocell m) in the considered spectral slot. Specifically, the uplink transmit power of small cell $n \in \mathcal{N}$ and macrocell $m \in \mathcal{M}$ are denoted by p_n and $\dot{p}_m$, respectively. In addition, the link power gain from the transmitter of cell i (i.e., the mobile user) to the receiver of cell j (i.e., its associated BS) is denoted by $g_{ij} = \left(d_{ij}\right)^{-\kappa} \beta_{ij}$, where d_{ij} is the distance between the transmitter of cell i and the receiver of cell j, κ is the path loss exponent, and β_{ij} is the random fading coefficient. For better reading, Table I summarizes the mainly used notations in this paper.

B. Problem Formulation

In the heterogeneous small cell network, in order to guarantee the QoS of macrocell users, we should protect them from the excessive interference caused by other cells using the same channel [7], [9], [27], [37]. Interference temperature is used to limit such a suffered interference from other cells, which is given by:

$$\sum_{i \in \mathcal{M} \setminus \{m\}} \dot{p}_i g_{i,m} + \sum_{j \in \mathcal{N}} p_j g_{j,m} \leq \eta_m, \quad \forall m \in \mathcal{M}. \tag{1}$$

Here, $\sum_{i \in \mathcal{M} \setminus \{m\}} \dot{p}_i g_{i,m}$ represents the aggregated interference from other macrocells, $\sum_{j \in \mathcal{N}} p_j g_{j,m}$ denotes the interference from small cells, and η_m denotes the interference temperature limit imposed by macrocell m.

Each small cell $n \in \mathcal{N}$ experiences interference from both the macrocells and the other small cells, and thus the signal-to-interference-plus-noise ratio (SINR) at small cell n can be expressed as:

$$\text{SINR}_n = \frac{p_n g_{n,n}}{\sum_{m \in \mathcal{M}} \dot{p}_m g_{m,n} + \sum_{j \in \mathcal{N} \setminus \{n\}} p_j g_{j,n} + \sigma_n}, \tag{2}$$

where σ_n denotes the noise power at the receiver of small cell n. As a result, the corresponding transmission rate achieved by small cell n can be given by the following Shannon's formula:

$$R_n = \log_2 \left(1 + \text{SINR}_n\right). \tag{3}$$

In this paper, we aim to jointly optimize the transmit power vector of the small cells under the constraint of not exceeding the interference temperature limit of macrocell users. The optimization vector is defined as follows.

Definition 1: A ***transmit power vector*** characterizes the set of uplink transmit power used by each small cell: $\mathbf{p} = (p_1, \ldots, p_n, \ldots, p_N)$, where $\left[\mathbf{p}\right]_n = p_n$ is the uplink transmit power of small cell n. The maximal transmit power of small cell n is given by $P_n^{\max}$.

Then, the optimization problem can be formulated as:

$$(\text{P1}): \quad \max_{\mathbf{p}} R_{\text{sum}}(\mathbf{p}) = \sum_{n \in \mathcal{N}} R_n(\mathbf{p}), \tag{4a}$$

$$s.t.\ 0 < p_n < P_n^{\max}, \quad \forall n \in \mathcal{N}, \tag{4b}$$

$$\sum_{i \in \mathcal{M} \setminus \{m\}} \dot{p}_i g_{i,m} + \sum_{j \in \mathcal{N}} p_j g_{j,m} \leq \eta_m, \quad \forall m \in \mathcal{M}. \tag{4c}$$

Due to the complicated interference relationship, the above optimization problem (P1) is non-convex [11]. Thus, standard optimization techniques cannot be directly applied to compute the globally optimal solution. Moreover, even if computational issues could be resolved, to execute the computation necessitates a central controller to collect instantaneous channel state information, which results in enormous signaling overhead, especially in ultra-dense small cell networks. Therefore, designing a local information-based distributed scheme to find the optimal solution is urgently needed.

III. Power Control Game With Global Information

In this section, we discuss about the distributed optimization of the above problem (P1) by using game theory [19], which is a powerful tool to analyze the interactions among distributed decision makers and improve the performance of decentralized networks. We will first study the case of global information in this section, and then discuss the case of local information in the next section.

A. Game Model

In consideration of the self-organizing nature of small cells, we model each small cell as a self-interested game player. Specifically, the distributed power control across different small cells is formulated as a game, formally denoted by $\mathcal{G} = \left[\mathcal{N}, \{\mathcal{P}_n\}_{n\in\mathcal{N}}, \left\{U_n^t\right\}_{n\in\mathcal{N}}, \{F_n\}_{n\in\mathcal{N}}\right]$. Symbol $\mathcal{N}$ is the set of game players (i.e., small cells), $\mathcal{P}_n = \{p_n \mid 0 < p_n \le P_n^{\max}\}$ is the set of available power strategy for player n, U_n^t is the utility function of player n at time t, and F_n represents a correspondence function for satisfaction of the constraint [38]. A strategy profile of all the players is a vector, denoted by $\mathbf{p} = (p_1, p_2, \ldots, p_N) \in \mathcal{P}$, where $\mathcal{P} = \mathcal{P}_1 \otimes \mathcal{P}_2 \otimes \cdots \otimes \mathcal{P}_N$ represents the joint strategy space for all the players. Besides, the strategy profile of all the players excluding n is denoted by $\mathbf{p}_{-n} = (p_1, \ldots, p_{n-1}, p_{n+1} \ldots, p_N) \in \mathcal{P}_{-n}$, where $\mathcal{P}_{-n} = \mathcal{P}_1 \otimes \cdots \otimes \mathcal{P}_{n-1} \otimes \mathcal{P}_{n+1} \otimes \cdots \otimes \mathcal{P}_N$.

In our game model, $F_n : \mathcal{P}_{-n} \to \mathcal{S}_n$ is a mapping which determines the set of player n's satisfied actions given other players' actions. In order to guarantee the QoS of macrocell users, we define the correspondence by $F_n(\mathbf{p}_{-n}) = \left\{p_n \in \mathcal{P}_n : \sum_{i\in\mathcal{M}\setminus\{m\}} \dot{p}_i g_{i,m} + \sum_{j\in\mathcal{N}} p_j g_{j,m} \le \eta_m, \forall m \in \mathcal{M}\right\}$. Obviously, $\mathcal{S}_n = F_n(\mathcal{P}_{-n})$ is a subset of the set $\mathcal{P}_n$, i.e., $\mathcal{S}_n \subseteq \mathcal{P}_n$.

In order to improve the efficiency of the game, we design each player's utility function at time t as:

$$U_n^t(p_n, \mathbf{p}_{-n}^t) = R_n(p_n, \mathbf{p}_{-n}^t) + \lambda_n^t p_n, \tag{5}$$

where $\lambda_n^t = \sum_{i\in\mathcal{N}\setminus\{n\}} \frac{\partial}{\partial p_n^t} R_i\left(\mathbf{p}^t\right)$ can be regarded as a dynamic price for the transmit power, and the superscript t is added to specify the time t. In particular, we notice that this used dynamic pricing scheme is similar to the pigouvian tax in microeconomic theory [39], which is an effective approach to quantify and correct the negative externality due to a player's individual decision in a social welfare environment. In our scenario, the key purpose of the proposed dynamic pricing is to correct the negative externality of each small cell's power strategy to other small cells. Thus, by incorporating such a pricing scheme, each small cell carefully determines its transmit power by taking into account its impact on other cells. Such a reciprocal utility design can encourage the cooperation among different cells, which will yield a locally or globally optimal solution to the sum-rate maximization problem by playing a non-cooperative game.

With the utility function designed in (5), the power control game is expressed as:

$$(\mathcal{G}) : \max_{p_n\in F_n\left(\mathbf{p}_{-n}^t\right)} U_n^t\left(p_n, \mathbf{p}_{-n}^t\right), \quad \forall n \in \mathcal{N},\ \forall t. \tag{6}$$

Obviously, $U_n^t(p_n, \mathbf{p}_{-n}^t)$ is a convex function with respect to the variable p_n. Thus, by solving the standard convex optimization, we can achieve the (unique) optimal strategy for small cell n as follows:

$$\widetilde{BR}_n\left(\mathbf{p}_{-n}^t\right) \triangleq \arg\max_{p_n\in F_n\left(\mathbf{p}_{-n}^t\right)} U_n^t\left(p_n, \mathbf{p}_{-n}^t\right), \tag{7}$$

which is known as the best response of small cell n.

B. Analysis of NE

Definition 2 (Nash Equilibrium (NE)): A power allocation profile $\mathbf{p}^t = \left(p_1^t, p_2^t, \ldots, p_N^t\right) \in \mathcal{P}$ is a pure-strategy NE of the game $\mathcal{G}$, if and only if every player's strategy is the best-response correspondence to others' strategies, i.e.,

$$p_n^t = \widetilde{BR}_n\left(\mathbf{p}_{-n}^t\right), \quad \forall n \in \mathcal{N}. \tag{8}$$

The above definition guarantees that no single player has an incentive to deviate from NE unilaterally.

Let $\widetilde{BR}\left(\mathbf{p}^t\right) = \left(\widetilde{BR}_1\left(\mathbf{p}_{-1}^t\right), \ldots, \widetilde{BR}_N\left(\mathbf{p}_{-N}^t\right)\right)$ denote the best-response vector of all players. According to (7), we can derive

$$\widetilde{BR}\left(\mathbf{p}^t\right) = \arg\max_{\mathbf{p}\in\tilde{\mathcal{P}}} \tilde{R}_{\text{sum}}\left(\mathbf{p}; \mathbf{p}^t\right), \tag{9}$$

where $\tilde{\mathcal{P}}$ denotes the convex feasible set given by (4b) and (4c), and

$$\tilde{R}_{\text{sum}}\left(\mathbf{p}; \mathbf{p}^t\right) \triangleq \sum_{n=1}^{N} U_n^t(p_n, \mathbf{p}_{-n}^t), \tag{10}$$

which can be seen as a convex approximation of the system sum-rate R_{sum} at $\mathbf{p}^t$.

Definition 3 (Stationary Point [40], [43]): A point $\mathbf{p}^t$ is a stationary point of the constrained optimization problem (P1) if the following condition is satisfied:

$$\nabla R_{\text{sum}}\left(\mathbf{p}^t\right)^T \left(\mathbf{p}^t - \mathbf{p}\right) \ge 0, \quad \forall \mathbf{p} \in \tilde{\mathcal{P}}, \tag{11}$$

where ∇ denotes the gradient operation with respect to the variable $\mathbf{p}$.

Condition (11) is the necessary condition for $\mathbf{p}^t$ to be locally (and also globally) optimal. Every locally or globally optimal solution must be stationary. For convex problems, the stationary point coincides with the globally optimal solution, and condition (11) is also sufficient for global optimality of $\mathbf{p}^t$. For non-convex problems like (P1), it is challenging to achieve the global optimality. Thus, the stationary points, which are computational efficient to achieve, are generally desired.

Theorem 1: Each stationary point of the optimization problem (P1) is an NE of the game $\mathcal{G}$, and each NE of the game $\mathcal{G}$ is also a stationary point of the optimization problem (P1).

Proof: *i)* Assume $\mathbf{p}^t$ is a stationary point of (P1) that satisfies the first-order optimality condition: $\nabla R_{\text{sum}}\left(\mathbf{p}^t\right)^T \left(\mathbf{p}^t - \mathbf{p}\right) \ge 0, \forall \mathbf{p} \in \tilde{\mathcal{P}}$. According to (5) and (10), it is easy to know

$$\nabla \tilde{R}_{\text{sum}}\left(\mathbf{p}^t; \mathbf{p}^t\right) = \nabla R_{\text{sum}}\left(\mathbf{p}^t\right),$$

thus, we have

$$\nabla \tilde{R}_{\text{sum}}\left(\mathbf{p}^t; \mathbf{p}^t\right)^T \left(\mathbf{p}^t - \mathbf{p}\right) \ge 0, \quad \forall \mathbf{p} \in \tilde{\mathcal{P}}. \tag{12}$$

Since $\nabla \tilde{R}_{\text{sum}}\left(; \mathbf{p}^t\right)$ is convex, (12) indicates

$$\nabla \tilde{R}_{\text{sum}}\left(\mathbf{p}; \mathbf{p}^t\right) \le \nabla \tilde{R}_{\text{sum}}\left(\mathbf{p}^t; \mathbf{p}^t\right), \quad \forall \mathbf{p} \in \tilde{\mathcal{P}}.$$

Therefore, $\tilde{R}_{\text{sum}}\left(\mathbf{p}^t; \mathbf{p}^t\right) = \max_{\mathbf{p}\in\tilde{\mathcal{P}}} \tilde{R}_{\text{sum}}\left(\mathbf{p}; \mathbf{p}^t\right)$, and $\mathbf{p}^t = \widetilde{BR}\left(\mathbf{p}^t\right)$.

ii) Suppose that $\mathbf{p}^t = \widetilde{BR}\left(\mathbf{p}^t\right)$. We readily get

$$\nabla \tilde{R}_{\text{sum}}\left(\mathbf{p}^t;\mathbf{p}^t\right)^T\left(\mathbf{p}^t - \mathbf{p}\right) \geq 0, \quad \forall \mathbf{p} \in \tilde{\mathcal{P}}.$$

Thus, $\nabla R_{\text{sum}}\left(\mathbf{p}^t\right) = \nabla \tilde{R}_{\text{sum}}\left(\mathbf{p}^t;\mathbf{p}^t\right)$ leads to

$$\nabla R_{\text{sum}}\left(\mathbf{p}^t\right)^T\left(\mathbf{p}^t - \mathbf{p}\right) \geq 0, \quad \forall \mathbf{p} \in \tilde{\mathcal{P}},$$

which proves $\mathbf{p}^t$ is a stationary point of (P1).

Therefore, Theorem 1 is proved. ■

According to the above analysis, the NE points exhibit very desirable and attractive properties, and we know the globally optimal solution to the non-convex optimization problem (P1) is an NE point. Next, we will propose a distributed algorithm to find the NE of the game, namely, the stationary point of the optimization problem (P1).

C. Algorithm Design

The detailed algorithm process is illustrated as follows.

Lemma 1: If $\mathbf{p}^t$ is not the NE of the game $\mathcal{G}$, $\widetilde{BR}\left(\mathbf{p}^t\right) - \mathbf{p}^t$ is an ascent direction of the system sum-rate $R_{sum}(\mathbf{p})$, i.e.,

$$\nabla R_{sum}\left(\mathbf{p}^t\right)^T\left(\widetilde{BR}\left(\mathbf{p}^t\right) - \mathbf{p}^t\right) > 0. \tag{13}$$

Proof: Based on the definition of $\widetilde{BR}\left(\mathbf{p}^t\right)$,

$$\tilde{R}_{\text{sum}}\left(\mathbf{p}^t;\mathbf{p}^t\right) \leq \tilde{R}_{\text{sum}}\left(\widetilde{BR}\left(\mathbf{p}^t\right);\mathbf{p}^t\right). \tag{14}$$

If $\tilde{R}_{\text{sum}}\left(\mathbf{p}^t;\mathbf{p}^t\right) = \tilde{R}_{\text{sum}}\left(\widetilde{BR}\left(\mathbf{p}^t\right);\mathbf{p}^t\right)$, we have $\mathbf{p}^t = \widetilde{BR}\left(\mathbf{p}^t\right)$, and thus $\mathbf{p}^t$ is a stationary point of (P1) and the NE of the game, which contradicts with the proposition. Therefore, $\tilde{R}_{\text{sum}}\left(\mathbf{p}^t;\mathbf{p}^t\right) \neq \tilde{R}_{\text{sum}}\left(\widetilde{BR}\left(\mathbf{p}^t\right);\mathbf{p}^t\right)$, which with (14) yields

$$\tilde{R}_{\text{sum}}\left(\mathbf{p}^t;\mathbf{p}^t\right) < \tilde{R}_{\text{sum}}\left(\widetilde{BR}\left(\mathbf{p}^t\right);\mathbf{p}^t\right).$$

Due to the convexity of $\nabla \tilde{R}_{\text{sum}}\left(\mathbf{p};\mathbf{p}^t\right)$,

$$\nabla \tilde{R}_{\text{sum}}\left(\mathbf{p}^t;\mathbf{p}^t\right)^T\left(\widetilde{BR}\left(\mathbf{p}^t\right) - \mathbf{p}^t\right) > 0.$$

Thus, $\nabla R_{\text{sum}}\left(\mathbf{p}^t\right)^T\left(\widetilde{BR}\left(\mathbf{p}^t\right) - \mathbf{p}^t\right) > 0$.

The proof is completed. ■

Based on Lemma 1, we can immediately derive the following theorem by following the similar proof in [16].

Theorem 2: The iterative process of Algorithm 1 converges to an NE of the game, namely, a stationary point of the sum-rate maximization problem (P1).

In the proposed algorithm, each small cell needs global information to compute the optimal power strategy $\widetilde{BR}_n\left(\mathbf{p}_{-n}^t\right)$, since each small cell's utility function (5) depends on global information, e.g., all the small cells' current power strategies and the global channel state information. Considering that collecting global information would cause enormous signaling overhead in ultra-dense small cell networks, we next design a local information-based distributed algorithm to compute the optimal power strategy $\widetilde{BR}_n\left(\mathbf{p}_{-n}^t\right)$.

IV. Achieving Optimal Power Strategy With Local Information

In this section, we first propose an approximation model for the optimization problem, and then design a low-complexity, distributed iterative algorithm to compute the optimal power strategy for each small cell based on local information.

Algorithm 1 Iterative Power Control for Achieving NE

Initialization: At the initial time $t = 0$, set the initial power strategy p_n^0, for all $n \in \mathcal{N}$.

1) Given the power allocation vector $\mathbf{p}^{t-1} = \left(p_1^{t-1}, \ldots, p_N^{t-1}\right)$, each small cell $n \in \mathcal{N}$ independently computes its optimal power strategy $\widetilde{BR}_n\left(\mathbf{p}_{-n}^t\right)$ by (7). Thus, the optimal power allocation vector $\widetilde{BR}\left(\mathbf{p}^{t-1}\right)$ can be achieved;
2) Update the power vector by
$$\mathbf{p}^t = \mathbf{p}^{t-1} + \alpha^t\left(\widetilde{BR}\left(\mathbf{p}^{t-1}\right) - \mathbf{p}^{t-1}\right). \tag{15}$$
Here, $\alpha^t \in (0, 1]$ is the updating step size generated by the decreasing rule proposed in [16], specifically,
$$\alpha^t = \alpha^{t-1}(1 - \sigma\alpha^{t-1}), \tag{16}$$
where $\sigma \in (0, 1)$ is a constant;
3) Stop the algorithm if the termination condition is satisfied, i.e., $|R_{\text{sum}}(\mathbf{p}^{t+1}) - R_{\text{sum}}(\mathbf{p}^t)| < \delta$, where δ is a predefined sufficiently small constant. Otherwise, set $t \leftarrow t + 1$, and go back to step 1).

A. Approximation Model

Due to the path loss, the signal transmitted by a given cell causes a significant co-channel interference only to the surrounding cells [36]. Thus, by using D_I to represent the interfering distance, we define two interfering domains[2] for each cell $i \in \mathcal{N} \cup \mathcal{M}$:

- Interfering domain in the small cell tier:
$$\mathcal{B}_i = \{n \in \mathcal{N} : n \neq i, d_{ni} \leq D_I\}. \tag{17}$$
- Interfering domain in the macrocell tier:
$$\mathcal{C}_i = \{m \in \mathcal{M} : m \neq i, d_{mi} \leq D_I\}. \tag{18}$$

Based on the defined interfering domains, the transmission rate of small cell $n \in \mathcal{N}$ can be approximated by:

$$\hat{R}_n = \log_2\left(1 + \frac{p_n g_{n,n}}{\sum\limits_{i\in\mathcal{B}_n} p_i g_{i,n} + \sum\limits_{m\in\mathcal{C}_n} \dot{p}_m g_{m,n} + \sigma_n}\right). \tag{19}$$

Then, the optimization problem (P1) can be transformed into:

$$\text{(P2)}: \quad \max_{\mathbf{p}} \sum_{n\in\mathcal{N}} \hat{R}_n(\mathbf{p}), \tag{20a}$$

$$s.t.\ 0 < p_n < P_n^{\max}, \quad \forall n \in \mathcal{N}, \tag{20b}$$

$$\sum_{j\in\mathcal{B}_m} p_j g_{j,m} + \sum_{i\in\mathcal{C}_m} \dot{p}_i g_{i,m} \leq \eta_m, \quad \forall m \in \mathcal{M}. \tag{20c}$$

[2]It is worth noting that other metrics in [22] and [36] can also be used to decide the interfering domains (e.g., based on the received power, or further considering the traffic load, etc.). However, the optimal construction of the interference domains is not the focus of this work, and it would not cause large variations in the following game model and the main conclusions.

Accordingly, the pricing factor λ_n^t in (5) is approximated by:

$$\hat{\lambda}_n^t = \sum_{i\in\mathcal{H}_n} \frac{\partial}{\partial p_n^t} \hat{R}_i\left(\mathbf{p}^t\right) = -\sum_{i\in\mathcal{H}_n} \frac{Q_i^t g_{n,i}}{\ln 2\left(Q_i^t + W_i^t\right) W_i^t}, \quad (21)$$

where $Q_i^t = p_i^t g_{i,i}$ denotes the received power of small cell i, and $W_i^t = \sum_{j\in\mathcal{B}_i} p_j^t g_{j,i} + \sum_{m\in\mathcal{C}_i} \dot{p}_m g_{m,i} + \sigma_i$ represents the interference-plus-noise experienced by small cell i. Set $\mathcal{H}_n$ represents the set of interfered small cells by small cell n. Notably, there always exists $n \in \mathcal{B}_i \Leftrightarrow i \in \mathcal{H}_n$.

Thus, the utility function of each small cell $n \in \mathcal{N}$ can be approximated by:

$$\hat{U}_n^t\left(p_n, \mathbf{p}_{-n}^t\right) = \hat{R}_n(p_n, \mathbf{p}_{-n}^t) + \hat{\lambda}_n^t p_n. \quad (22)$$

Since $\hat{U}_n^t(p_n, \mathbf{p}_{-n})$ is also a convex function with respect to the variable p_n, we can achieve its unique optimal (best-response) strategy, denoted by:

$$\widehat{BR}_n\left(\mathbf{p}_{-n}^t\right) \triangleq \underset{p_n\in\hat{F}_n\left(\mathbf{p}_{-n}^t\right)}{\arg\max}\ \hat{U}_n^t\left(p_n, \mathbf{p}_{-n}^t\right), \quad (23)$$

where the correspondence function $\hat{F}_n\left(\mathbf{p}_{-n}\right)$ is defined based on the interfering domains, specifically $\hat{F}_n\left(\mathbf{p}_{-n}\right) = \left\{p_n \in \mathcal{P}_n : \sum_{j\in\mathcal{B}_m} p_j g_{j,m} + \sum_{i\in\mathcal{C}_m} \dot{p}_i g_{i,m} \le \eta_m, \forall m \in \mathcal{M}\right\}$.

Let $\widehat{BR}\left(\mathbf{p}^t\right) = (\widehat{BR}_1\left(\mathbf{p}_{-1}^t\right), \ldots, \widehat{BR}_N\left(\mathbf{p}_{-N}^t\right))$ denote the best-response vector of all the small cells. According to (23), we can derive

$$\widehat{BR}\left(\mathbf{p}^t\right) = \underset{\mathbf{p}\in\hat{\mathcal{P}}}{\arg\max}\ \hat{R}_{\text{sum}}\left(\mathbf{p}; \mathbf{p}^t\right), \quad (24)$$

where $\hat{\mathcal{P}}$ denotes the convex feasible set given by (20b) and (20c), and

$$\hat{R}_{\text{sum}}\left(\mathbf{p}; \mathbf{p}^t\right) \triangleq \sum_{n=1}^{N} \hat{U}_n^t(p_n, \mathbf{p}_{-n}^t). \quad (25)$$

B. Algorithm Design

Since the optimization objective $\hat{R}_{\text{sum}}\left(\mathbf{p}; \mathbf{p}^t\right)$ and the constraint set $\hat{\mathcal{P}}$ are convex, we can adopt the standard convex optimization techniques to achieve the best-response power allocation vector $\widehat{BR}\left(\mathbf{p}^t\right)$.

The Lagrangian dual function associated with optimization objective $\hat{R}_{\text{sum}}\left(\mathbf{p}; \mathbf{p}^t\right)$ and constraint $\hat{\mathcal{P}}$ is given by:

$$L\left(\mu; \mathbf{p}^t\right) \triangleq \max_{\mathbf{p}\in\mathcal{P}} \left\{\hat{R}_{\text{sum}}\left(\mathbf{p}; \mathbf{p}^t\right) - \sum_{m\in\mathcal{M}} \mu_m \varphi_m\left(\mathbf{p}\right)\right\}, \quad (26)$$

where $\varphi_m\left(\mathbf{p}\right) = \sum_{i\in\mathcal{C}_m} \dot{p}_i g_{i,m} + \sum_{j\in\mathcal{B}_m} p_j g_{j,m} - \eta_m$, and $\mu = (\mu_1, \ldots, \mu_M)$ denotes the dual vector.

Since the approximate sum-rate function $\hat{R}_{\text{sum}}$ is separable in the individual's variable p_n, the computation of (26) can be decoupled across different small cells. The optimal solution of (26) can be denoted by $\Lambda\left(\mu; \mathbf{p}^t\right) \triangleq \left(\Lambda_n\left(\mu; \mathbf{p}^t\right)\right)_{n=1}^N$, where $\Lambda_n\left(\mu; \mathbf{p}^t\right)$ represents small cell n's unique optimal transmit power given by:

$$\Lambda_n\left(\mu; \mathbf{p}^t\right) \triangleq \underset{p_n\in\mathcal{P}_n}{\arg\max} \left\{\hat{U}_n^t\left(p_n; \mathbf{p}^t\right) - \sum_{m\in\mathcal{D}_n} \mu_m p_n g_{n,m}\right\}. \quad (27)$$

Set $\mathcal{D}_n$ represents the set of interfered macrocells by small cell n. Notice that there always exists $n \in \mathcal{B}_m \Leftrightarrow m \in \mathcal{D}_n$.

Thus, based on the dual-decomposition, the optimal power allocation vector can be computed in a distributed manner, as shown in Algorithm 2, which belongs to the class of well-known gradient algorithms [41]. Each small cell autonomously computes its optimal transmit power in a distributed manner, which only involves local information exchange among neighboring cells. Specifically, small cell n only needs the ***local information*** including:

- Its own power gain $g_{n,n}$ and the experienced interference-plus-noise W_n^t.
- The received power Q_i^t and the interference-plus-noise W_i^t experienced by n's interfered neighboring small cells $i \in \mathcal{H}_n$, and the power gains $g_{n,i}$ from n to $i \in \mathcal{H}_n$.
- The power gains $g_{n,m}$ from n to n's interfered neighboring macrocells $m \in \mathcal{D}_n$, and the price factor μ_m set by n's interfered neighboring macrocells.

Algorithm 2 Distributed Computation of Optimal Power Allocation Vector Based on Local Information

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Input: $\mathbf{p}^t$, $\{\varepsilon^\upsilon\} > 0$, $\mu_m^0 \ge 0$ for all $m \in \mathcal{M}$.

Loop for $\upsilon = 0, 1, 2, \ldots$

1) **Distributed power computation:** Each small cell $n \in \mathcal{N}$ distributedly computes its optimal transmit power $\Lambda_n\left(\mu^\upsilon; \mathbf{p}^t\right)$ by (27), and reports its calculated transmit power to neighboring cells.
2) **Updating price factor:** Each macrocell $m \in \mathcal{M}$ distributedly updates its price factor μ_m^υ according to

$$\varphi_m^\upsilon = \sum_{i\in\mathcal{C}_m} \dot{p}_i g_{i,m}^t + \sum_{n\in\mathcal{B}_m} \Lambda_n\left(\mu^\upsilon; \mathbf{p}^t\right) g_{n,m}^t - \eta_m, \quad (28)$$

$$\mu_m^{\upsilon+1} = \left[\ \mu_m^\upsilon + \varepsilon^\upsilon \varphi_m^\upsilon\ \right]^+, \quad (29)$$

where ε^υ is the step size for updating the price factor, $[\cdot]^+$ denotes the Euclidean projection onto $\mathbb{R}^+$, i.e., $[x]^+ \triangleq \max\{0, x\}$.

End loop until the maximum number of iterations is reached.

Output: The optimal power allocation vector $\widehat{BR}\left(\mathbf{p}^t\right) \triangleq \left(\widehat{BR}_n\left(\mathbf{p}^t\right)\right)_{n=1}^N$, where $\widehat{BR}_n\left(\mathbf{p}^t\right) = \Lambda_n\left(\mu^\upsilon; \mathbf{p}^t\right)$.

Besides, it is easy to know each small cell n's computational complexity for distributed power computation in Step 1) is $O(|\mathcal{H}_n| + |\mathcal{D}_n|)$, and each macrocell m's computational complexity for updating price factor in Step 2) is $O(|\mathcal{B}_m| + |\mathcal{C}_m|)$. Thus, the total computational complexity for each iteration is $O\left(\sum_{n\in\mathcal{N}}(|\mathcal{H}_n| + |\mathcal{D}_n|) + \sum_{m\in\mathcal{M}}(|\mathcal{B}_m| + |\mathcal{C}_m|)\right)$, which depends on the scale of the interfering domains. In contrast, if we adopt the global interference model (i.e., each cell's transmission signal causes interference to all the other cells), the interfering domains will be $|\mathcal{H}_n| = N-1$, $|\mathcal{D}_n| = M$, $|\mathcal{B}_m| = N$, and $|\mathcal{C}_m| = M-1$, leading to a larger computational complexity as $O\left(N^2+NM+M^2\right)$.

Theorem 3: If the step size $\{\varepsilon^{\upsilon}\}$ is chosen such that $\varepsilon^{\upsilon} > 0$, $\varepsilon^{\upsilon} \to 0$, $\sum_{\upsilon}\varepsilon^{\upsilon} = \infty$, and $\sum_{\upsilon}(\varepsilon^{\upsilon})^2 < \infty$, the sequence of price factor $\{\mu^{\upsilon}\}$ generated by Algorithm 2 converges to a solution to the dual problem $\min_{\mu\geq 0} L\left(\mu;\mathbf{p}^t\right)$, and the sequence $\left\{\Lambda\left((\mu^{\upsilon}),\mathbf{p}^t\right)\right\}$ converges to the unique optimal power allocation solution $\widehat{BR}\left(\mathbf{p}^t\right)$, defined in (24).

The proof of the theorem follows the standard convergence results of gradient projection algorithms [41], which is omitted here. To guarantee convergence, the step-size sequence $\{\varepsilon^t\}$ is generated by the decreasing rule

$$\varepsilon^t = \varepsilon^{t-1}(1-\sigma\varepsilon^{t-1}), \tag{30}$$

where $\sigma \in (0,1)$ is a constant. For more proper selection rules of the step size sequence $\{\varepsilon^{\upsilon}\}$, interested readers are referred to [16] and [17] for more details.

By constructing the interfering domain, we present another approximate function for the network sum-rate, and then use the Lagrangian dual-decomposition technique to design a local information-based distributed algorithm (i.e., Algorithm 2) to compute the optimal power allocation vector $\widehat{BR}\left(\mathbf{p}^t\right)$. As analyzed before, the computation of $\widehat{BR}\left(\mathbf{p}^t\right)$ relies only on local information and smaller computational complexity. In contrast, the computation of $\widetilde{BR}\left(\mathbf{p}^t\right)$ in the previous Section requires global information and larger complexity, but it could guarantee the convergence to a stationary solution, as proved in Theorems 1 and 2 before. Thus, the most important problem here is to prove whether the proposed local information-based approximation approach can guarantee the optimality of the solution or not. In what follows, we will provide an answer for this question.

C. Convergence Analysis

Notably, the construction of the interfering domains in (17) and (18) ignores the very weak interference from remote BSs, which inevitably brings computation inaccuracy. In contrast, the computation of $\widetilde{BR}\left(\mathbf{p}^t\right)$ is based on the exact interference relationship (including the very weak interference). For the convenience of analysis, we model the computation inaccuracy by $\left\|\widehat{BR}\left(\mathbf{p}^t\right)-\widetilde{BR}\left(\mathbf{p}^t\right)\right\| \leq \xi^t$.

Theorem 4: Suppose that the following conditions are satisfied: i) $\sum_t \alpha^t\xi^t < \infty$; and ii) $\alpha^t \to 0$. Then, $\left\{R_{sum}\left(\mathbf{p}^t\right)\right\}$ converges to a stationary solution which is either locally or globally optimal.

Proof: For clear presentation, the proof is divided into two steps.

1) By definition (7), for a given $\mathbf{p}^t \in \tilde{\mathcal{P}}$, $\widetilde{BR}\left(\mathbf{p}^t\right)$ satisfies the maximum principle: $\forall\mathbf{p} \triangleq (p_n)_{n=1}^N \in \mathbf{\Omega}$,

$$\left(\widetilde{BR}\left(\mathbf{p}^t\right)-\mathbf{p}\right)^T\nabla\tilde{R}_{\text{sum}}\left(\widetilde{BR}\left(\mathbf{p}^t\right);\mathbf{p}^t\right) \geq 0.$$

That is,

$$\begin{aligned}\sum_{n=1}^N &\left(\widetilde{BR}_n\left(\mathbf{p}^t\right)-p_n\right)\Big[\nabla_{p_n}f_n\left(\widetilde{BR}_n\left(\mathbf{p}^t\right),\mathbf{p}_{-n}^t\right)\\ &+\sum_{i\neq n}\nabla_{p_n}f_i\left(\mathbf{p}^t\right)-\sum_{i=1}^N\nabla_{p_n}h_i\left(\mathbf{p}^t\right)-\tau_n\left(\widetilde{BR}_n\left(\mathbf{p}^t\right)-p_n^t\right)\Big]\\ &\geq 0,\end{aligned} \tag{31}$$

where we used the equation $\widetilde{BR}\left(\mathbf{p}^t\right) \triangleq \left(\widetilde{BR}_n\left(\mathbf{p}^t\right)\right)_{n=1}^N$.

Furthermore, by letting $p_n = p_n^t$, and adding and subtracting $\nabla_{p_n}f_n\left(\mathbf{p}^t\right)$ for each term n of the sum (31), we derive

$$\begin{aligned}\sum_{n=1}^N \left(\widetilde{BR}_n\left(\mathbf{p}^t\right)-p_n^t\right)&\Big[\nabla_{p_n}f_n\left(\widetilde{BR}_n\left(\mathbf{p}^t\right),\mathbf{p}_{-n}^t\right)+\nabla R_{\text{sum}}\left(\mathbf{p}^t\right)\\ &-\nabla_{p_n}f_n\left(\mathbf{x}\right)-\tau_n\left(\widetilde{BR}_n\left(\mathbf{p}^t\right)-p_n^t\right)\Big] \geq 0,\end{aligned}$$

and thus

$$\begin{aligned}&\left(\widetilde{BR}\left(\mathbf{p}^t\right)-\mathbf{p}^t\right)^T\nabla R_{\text{sum}}\left(\mathbf{p}^t\right)\\ &\geq \sum_{n=1}^N\left(p_n^t-\widetilde{BR}_n\left(\mathbf{p}^t\right)\right)\cdot\Big[\nabla_{p_n}f_n\left(\widetilde{BR}_n\left(\mathbf{p}^t\right),\mathbf{p}_{-n}^t\right)-\nabla_{p_n}f_n\left(\mathbf{p}^t\right)\\ &\qquad-\tau_n\left(\widetilde{BR}_n\left(\mathbf{p}^t\right)-p_n^t\right)\Big].\end{aligned}$$

Since f_n is a concave function, we have the following result:

$$\begin{aligned}\sum_{n=1}^N &\left(p_n^t-\widetilde{BR}_n\left(\mathbf{p}^t\right)\right)\left[\nabla_{p_n}f_n\left(\widetilde{BR}_n\left(\mathbf{p}^t\right),\mathbf{p}_{-n}^t\right)-\nabla_{p_n}f_n\left(\mathbf{p}^t\right)\right]\\ &\geq 0,\end{aligned}$$

then we have

$$\left(\widetilde{BR}\left(\mathbf{p}^t\right)-\mathbf{p}^t\right)^T\nabla R_{\text{sum}}\left(\mathbf{p}^t\right) \geq \sum_{n=1}^N\tau_n\left(\widetilde{BR}_n\left(\mathbf{p}^t\right)-p_n^t\right)^2.$$

By defining $c_\tau = \min_{n\in\mathcal{N}}\{\tau_n\} \geq 0$, we can get

$$\left(\widetilde{BR}\left(\mathbf{p}^t\right)-\mathbf{p}^t\right)^T\nabla R_{\text{sum}}\left(\mathbf{p}^t\right) \geq c_\tau\left\|\widetilde{BR}\left(\mathbf{p}^t\right)-\mathbf{p}^t\right\|^2. \tag{32}$$

2) According to the Descent Lemma in Appendix and letting $\Phi\left(\mathbf{x}\right) = -R_{\text{sum}}\left(\mathbf{x}\right)$ (obviously $L_{\nabla\Phi} = L_{\nabla R}$), we have

$$\begin{aligned}R_{\text{sum}}\left(\mathbf{x}_1\right)-R_{\text{sum}}\left(\mathbf{x}_2\right) \leq &-\nabla R_{\text{sum}}(\mathbf{x}_1)^T\left(\mathbf{x}_2-\mathbf{x}_1\right)\\ &+\frac{L_{\nabla R}}{2}\|\mathbf{x}_2-\mathbf{x}_1\|^2.\end{aligned}$$

Thus, we obtain

$$\begin{aligned}R_{\text{sum}}\left(\mathbf{x}_2\right)-R_{\text{sum}}\left(\mathbf{x}_1\right) \geq &\nabla R_{\text{sum}}(\mathbf{x}_1)^T\left(\mathbf{x}_2-\mathbf{x}_1\right)\\ &-\frac{L_{\nabla R}}{2}\|\mathbf{x}_2-\mathbf{x}_1\|^2.\end{aligned}$$

For any given $t \geq 0$, there exists

$$\begin{aligned}&R_{\text{sum}}\left(\mathbf{p}^{t+1}\right)-R_{\text{sum}}\left(\mathbf{p}^t\right)\\ &\geq \nabla R_{\text{sum}}\left(\mathbf{p}^t\right)^T\left(\mathbf{p}^{t+1}-\mathbf{p}^t\right)-\frac{L_{\nabla R}}{2}\left\|\mathbf{p}^{t+1}-\mathbf{p}^t\right\|^2\\ &= \alpha^t\nabla R_{\text{sum}}\left(\mathbf{p}^t\right)^T\left(\widehat{BR}\left(\mathbf{p}^t\right)-\mathbf{p}^t\right)\\ &\quad-\frac{L_{\nabla R}}{2}\left(\alpha^t\right)^2\left\|\widehat{BR}\left(\mathbf{p}^t\right)-\mathbf{p}^t\right\|^2,\end{aligned} \tag{33}$$

where A^T denotes the transpose of the matrix A. Notice that, in the last equation in (33), we have used the iterative updating rule $\mathbf{p}^{t+1} = \mathbf{p}^t + \alpha^t \left(\widetilde{BR}\left(\mathbf{p}^t\right) - \mathbf{p}^t\right)$.

Notably, the following inequality holds:

$$\begin{aligned} &\left\|\widehat{BR}\left(\mathbf{p}^t\right) - \widetilde{BR}\left(\mathbf{p}^t\right)\right\|^2 + \left\|\widetilde{BR}\left(\mathbf{p}^t\right) - \mathbf{p}^t\right\|^2 \\ &\geq \frac{1}{2}\left\|\widehat{BR}\left(\mathbf{p}^t\right) - \mathbf{p}^t\right\|^2, \end{aligned} \tag{34}$$

which consequently leads to

$$\left\|\widehat{BR}\left(\mathbf{p}^t\right) - \mathbf{p}^t\right\|^2 \leq 2\left\|\widetilde{BR}\left(\mathbf{p}^t\right) - \mathbf{p}^t\right\|^2 + 2\left(\xi^t\right)^2.$$

Moreover, we have

$$\begin{aligned} &\nabla R_{\text{sum}}\left(\mathbf{p}^t\right)^T \left(\widehat{BR}\left(\mathbf{p}^t\right) - \mathbf{p}^t\right) \\ &= \nabla R_{\text{sum}}\left(\mathbf{p}^t\right)^T \left(\widehat{BR}\left(\mathbf{p}^t\right) - \widetilde{BR}\left(\mathbf{p}^t\right)\right) \\ &\quad + \nabla R_{\text{sum}}\left(\mathbf{p}^t\right)^T \left(\widetilde{BR}\left(\mathbf{p}^t\right) - \mathbf{p}^t\right). \end{aligned} \tag{35}$$

Since

$$\begin{aligned} &\nabla R_{\text{sum}}\left(\mathbf{p}^t\right)^T \left(\widetilde{BR}\left(\mathbf{p}^t\right) - \widehat{BR}\left(\mathbf{p}^t\right)\right) \\ &\leq \left|\nabla R_{\text{sum}}\left(\mathbf{p}^t\right)^T \left(\widetilde{BR}\left(\mathbf{p}^t\right) - \widehat{BR}\left(\mathbf{p}^t\right)\right)\right| \\ &\leq \left\|\nabla R_{\text{sum}}\left(\mathbf{p}^t\right)^T\right\| \left\|\widetilde{BR}\left(\mathbf{p}^t\right) - \widehat{BR}\left(\mathbf{p}^t\right)\right\| \leq \xi^t \left\|\nabla R_{\text{sum}}\left(\mathbf{p}^t\right)\right\|, \end{aligned}$$

we can derive the following result:

$$\nabla R_{\text{sum}}\left(\mathbf{p}^t\right)^T \left(\widehat{BR}\left(\mathbf{p}^t\right) - \widetilde{BR}\left(\mathbf{p}^t\right)\right) \geq -\xi^t \left\|\nabla R_{\text{sum}}\left(\mathbf{p}^t\right)\right\|. \tag{36}$$

Then, (32), (35), and (36) lead to

$$\begin{aligned} &\nabla R_{\text{sum}}\left(\mathbf{p}^t\right)^T \left(\widetilde{BR}\left(\mathbf{p}^t\right) - \mathbf{p}^t\right) \\ &\geq c_\tau \left\|\widetilde{BR}\left(\mathbf{p}^t\right) - \mathbf{p}^t\right\| - \xi^t \left\|\nabla R_{\text{sum}}\left(\mathbf{p}^t\right)\right\|. \end{aligned} \tag{37}$$

Therefore, based on (33), (34), and (37), we can get

$$\begin{aligned} &R_{\text{sum}}\left(\mathbf{p}^{t+1}\right) - R_{\text{sum}}\left(\mathbf{p}^t\right) \\ &\geq \alpha^t \nabla R_{\text{sum}}\left(\mathbf{p}^t\right)^T \left(\widetilde{BR}\left(\mathbf{p}^t\right) - \mathbf{p}^t\right) - \frac{L_{\nabla R}}{2}\left(\alpha^t\right)^2 \left\|\widehat{BR}\left(\mathbf{p}^t\right) - \mathbf{p}^t\right\|^2 \\ &\geq \alpha^t c_\tau \left\|\widetilde{BR}\left(\mathbf{p}^t\right) - \mathbf{p}^t\right\| - \alpha^t \xi^t \left\|\nabla R_{\text{sum}}\left(\mathbf{p}^t\right)\right\| \\ &\quad - L_{\nabla R}\left(\alpha^t\right)^2 \left(\left\|\widetilde{BR}\left(\mathbf{p}^t\right) - \mathbf{p}^t\right\|^2 + \left(\xi^t\right)^2\right) \\ &= \left(\alpha^t c_\tau - L_{\nabla R}\left(\alpha^t\right)^2\right) \left\|\widetilde{BR}\left(\mathbf{p}^t\right) - \mathbf{p}^t\right\| - \theta^t, \end{aligned} \tag{38}$$

where $\theta^t = \alpha^t \xi^t \left\|\nabla R_{\text{sum}}\left(\mathbf{p}^t\right)\right\| + L_{\nabla R}\left(\alpha^t \xi^t\right)^2$. Under the assumptions of the theorem, there exists $\sum_{t=0}^{\infty} \theta^t < \infty$. Moreover, since $\alpha^t \to 0$, we have $\alpha^t c_\tau - L_{\nabla R}\left(\alpha^t\right)^2 > 0$ when t is sufficiently large.

Thus, on the basis of Lemma 3 in Appendix, we have either $\left\{R_{\text{sum}}\left(\mathbf{p}^t\right)\right\} \to \infty$ or $\left\{R_{\text{sum}}\left(\mathbf{p}^t\right)\right\}$ converges to a finite value. Due to the limitation of system capacity, $\left\{R_{\text{sum}}\left(\mathbf{p}^t\right)\right\}$ is a bounded function. Thus, we know $\left\{R_{\text{sum}}\left(\mathbf{p}^t\right)\right\}$ converges to a finite value. That implies the sequence $\left\{\mathbf{p}^t\right\}$ converges to a fixed point (i.e., NE) of the game. According to Lemma 1, we know $\left\{R_{\text{sum}}\left(\mathbf{p}^t\right)\right\}$ converges to a stationary solution which is locally or globally optimal. ■

According to Theorem 4, convergence is guaranteed if $\sum_t \alpha^t \xi^t < \infty$ and $\alpha^t \to 0$. We generate the step size sequence $\{\alpha^t\}$ by the decreasing rule (30), which satisfies $\sum_t \alpha^t < \infty$ and $\alpha^t \to 0$. Thus, only if the computation error sequence $\{\xi^t\}$ is upper bounded, the convergence conditions can be satisfied. In the approximation model, the construction of the interfering domains in (17) and (18) only ignores the very weak interference from remote BSs, which would not cause large computation error. Therefore, the proposed local information-based distributed power control approach can converge to a stationary point of the sum-rate maximization problem, which could be locally or globally optimal. It is worth noting that devising distributed algorithms that definitely converge to the globally optimal solution of the system-wide optimization problem is up to date an open problem that is worth to be investigated [24].

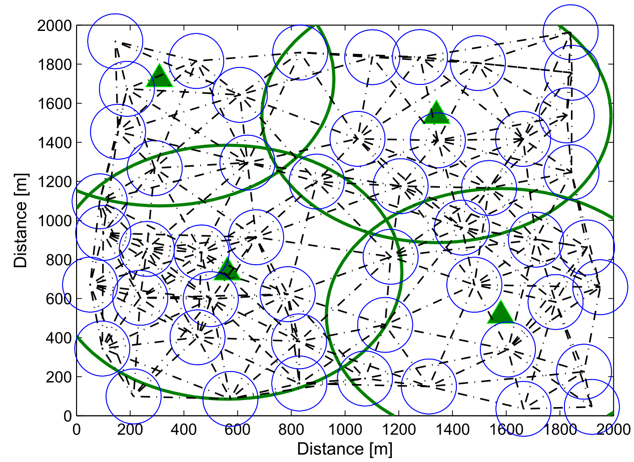


Fig. 2. Network topology with 4 macrocells and 50 small cells. The triangles denote the macrocell BSs, and their coverage is plotted by large green circles. Besides, the coverage of small cells is plotted by small blue circles, and the edge connecting two small cells represents their interference relationship. We assume that the interference exists only when the distance of two small cells are smaller than the interfering distance $D_I = 600$ m.

V. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed distributed power control algorithm in ultra-dense heterogeneous small cell networks.

A. Simulation Settings

In the simulation, we study a densely deployed heterogeneous cellular network with 4 macrocells and 50 small cells in a 2000m-by-2000m square area, as shown in Figure 2. Each BS is located at the center of its serving cell, and mobile users are randomly generated in each cell according to a uniform distribution. Rayleigh fading model is considered in the simulation, and the random fading coefficient β is exponentially distributed with unit-mean. The interference temperature limit is set to be $\eta = 10^{-6}$ W. Further simulation parameters are summarized in Table II.

The average transmission rate achieved by each small cell (the network sum-rate divided by the number of small cells) is adopted as the main performance metric. For the proposed distributed power control algorithm, the step size sequences $\{\alpha^t\}$ and $\{\xi^t\}$ are both generated by the decreasing rule, and the parameters are selected to be $\alpha^0 = \xi^0 = 1$, and $\sigma = 10^{-2}$ [16]. The proximal parameter τ_n is set to 10 for all

TABLE II
PARAMETERS SETTING

Macrocell radius	650 m
Small cell radius	25 m
Number of users in each macrocell	16
Number of users in each small cell	4
Maximal transmit power of each user	100 mW
Number of subchannels	16
Subchannel bandwidth	200 KHz
Pass loss exponent κ	3.7
Thermal noise density	-174 dBm/Hz

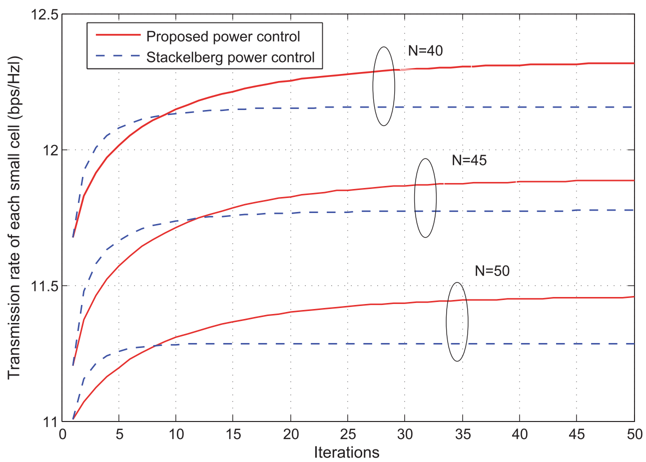


Fig. 3. The evolution of average transmission rate of each small cell for different power control schemes.

small cells. As for performance comparison, we also conduct simulations for the Stackelberg power control scheme in [27].

B. Convergence Analysis

For convergence evaluation of the proposed power control scheme, Figure 3 shows the evolution of system transmission rate achieved by each small cell for different numbers of small cells. The system transmission rates achieved by both the proposed power control scheme and the Stackelberg power control scheme are updated in each iteration and significantly improved at the convergence time. It can be seen that the Stackelberg power control scheme has a faster convergence speed, but it converges to a relatively inferior stable solution. Although the convergence time for the proposed scheme can even get a little longer when the number of small cells increases, the convergence can all be achieved within 50 iterations. In addition, when the number of small cells increases (from 40 to 50), the system transmission rate achieved by each small cell is reduced due to more severe interference.

Figure 4 shows the interference received by the 4 macrocells updating versus the number of iterations. Specifically, the received interference of each macrocell updates with the iteration step and remains unchanged after about 50 iterations, thus validating the convergence of the proposed algorithm. In order to improve the transmission rate, the small cells increase the transmit power, which leads to the increase in the received interference at macrocells. However, we can see that after convergence, the received interference at each macrocell

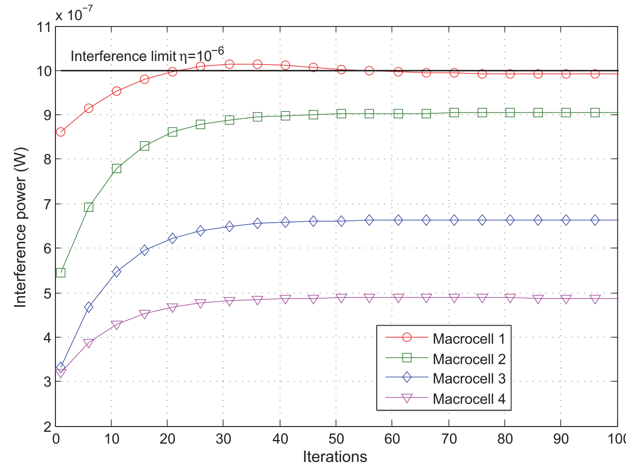


Fig. 4. The evolution of received interference by macrocells when the number of small cells is 50.

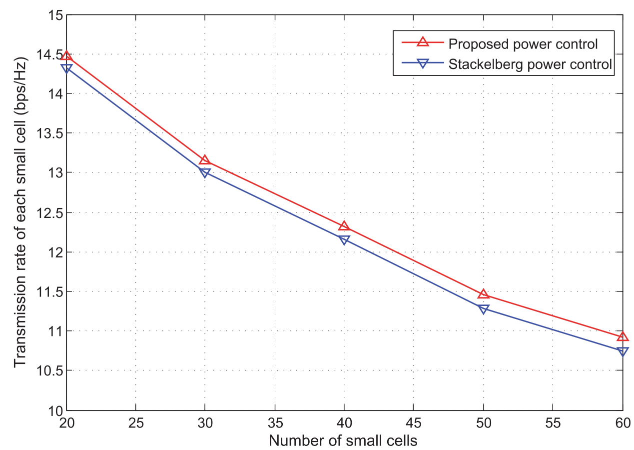


Fig. 5. The performance evaluation of average transmission rate for different solutions versus the number of small cells.

is below the interference limit $\eta = 10^{-6}$ W. The reason can be explained as follows: if a macrocell m receives interference larger than the interference limit, it will increase its pricing factor μ_m which leads to the decrease of the transmit power of its neighboring small cells.

C. Performance Evaluation

Figure 5 shows performance comparison results for the different solutions versus the number of small cells. The presented results are obtained by simulating 100 independent trials and then taking the average value. As shown in Figure 5, the average transmission rate achieved by each small cell decreases with the increase of the number of small cells due to more severe interference. Moreover, it can be seen that the proposed power control scheme outperforms the Stackelberg power control scheme in [27].

Figure 6 shows the performance of the proposed local information-based distributed power control algorithm with respect to the interfering distance D_I. The iterative power control algorithm (illustrated in Algorithm 1) is considered for comparison, which is irrespective of the defined interfering distance, since it collects global information for computation.

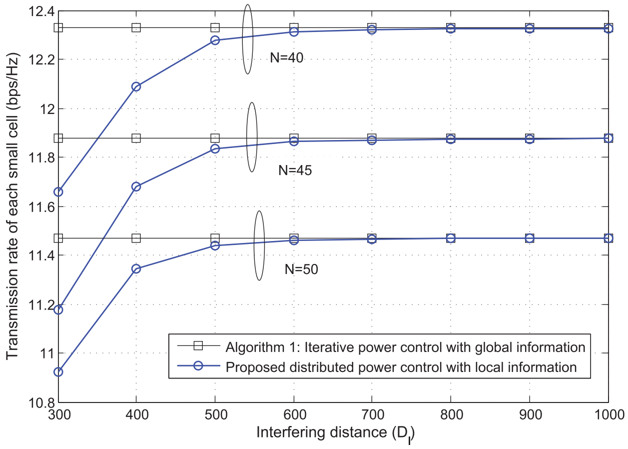


Fig. 6. The performance evaluation of average transmission rate for different solutions versus the scale of interfering domain.

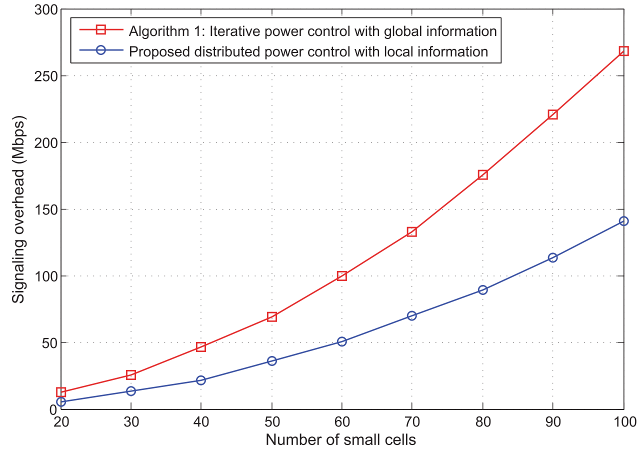


Fig. 7. Signaling overhead analysis.

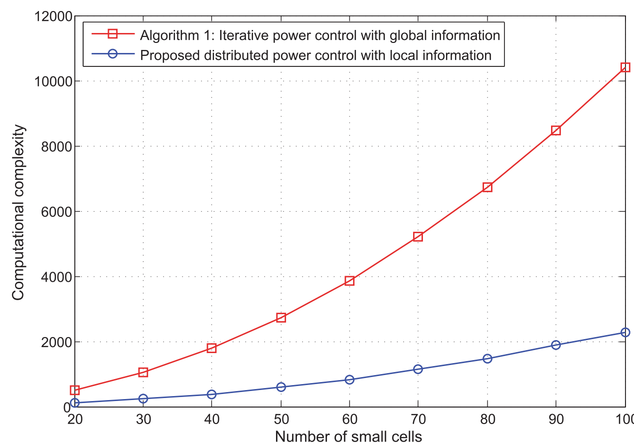


Fig. 8. Computational complexity analysis.

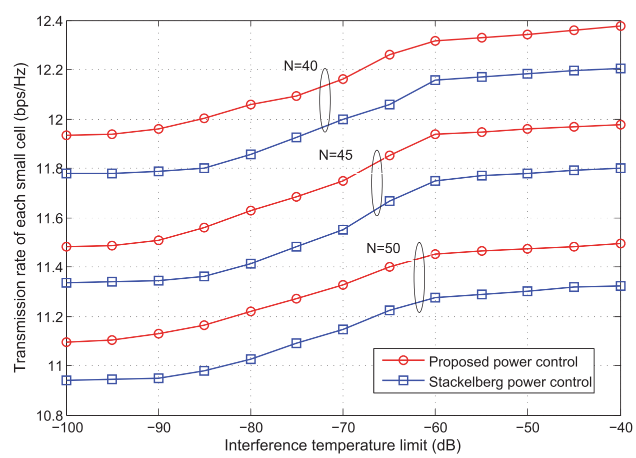


Fig. 9. The performance evaluation of average transmission rate for different solutions versus the interference temperature limit.

As shown in Figure 6, the achieved average transmission rate increases with the interfering distance. That is because a larger scale of interfering domain can approximate the real interference relationship more accurately. Thus, the obtained solution is closer to the optimal solution with minor computing error, as proved in Theorem 4. However, when the interfering distance increases, more algorithm cost (including information exchange overhead and computational complexity) would be incurred, which is consistent with our analysis in Section IV. Therefore, there is a tradeoff between the signaling and computational cost and the performance gain of proposed solution. Besides, we can see that when the interfering distance is set to be larger than 600 m, the performance of the low-complexity distributed algorithm with local information exchange is quite close to the optimal solution obtained by the global information-based power control algorithm. Therefore, in consideration of both system performance and algorithm cost, it is quite cost-effective to set the interfering distance to be 600 m, which achieves a near-optimal solution but with lower computational complexity and only a small amount of local information exchange.

In order to better understand the impact of ultra-dense on the problem and the advantage of our proposed optimization scheme, we plot the signaling overhead[3] and computational complexity for different optimization schemes in Figure 7 and Figure 8, respectively. We can see from Figure 7 that as the number of small cells increases (i.e., the small cells are more densely deployed), the signaling overhead of the global information-based optimization scheme increases significantly, while the signaling overhead of the proposed local information-based power control scheme increases slowly. Figure 8 also clearly shows the advantage of our proposed local information-based optimization scheme in terms of effectively reducing the computational complexity.

Figure 9 shows the average transmission rate of each small cell versus the interference temperature limit η_m. Intuitively, the performance of both schemes is improved with the increased interference temperature limit. However, the increase of the interference temperature limit would lead to the increase in the interference received by macrocell users, and thus reduce the macrocell transmission rate. Therefore, there is a tradeoff between the small-cell transmission rate and macrocell transmission rate, which can be adjusted by properly setting the interference temperature limit for the macrocell

[3]In the simulations, we assume that power control is executed once every 5 ms (the WiMAX frame duration) and transmitting a binary number (and a floating number) needs 1 bit (and 16 bits).

users. Moreover, Figure 9 also demonstrates that our proposed distributed power control scheme outperforms the Stackelberg power control scheme.

VI. Conclusion

In this paper, we have proposed a novel game to study the optimal power control for interference management in ultra-dense small cell networks. By properly designing the utility function with dynamic pricing, different small cells are motivated to cooperate with each other, leading to desirable game outcomes. We have proved that the NE of the game coincides with the stationary point of the sum-rate maximization problem, which could be locally or globally optimal. Furthermore, a low-complexity distributed power control algorithm, which involves only local information exchange, has been proposed with guaranteed convergence performance. Simulation results have demonstrated the effectiveness of our proposed algorithms. For the future work, we will study the optimal power control in dynamic environments, e.g., dynamic BS switching on/off and time-varying channels.

Appendix

Lemma 2 (Descent Lemma [40]): For a continuously differentiable function $\Phi : \mathbb{R}^N \to \mathbb{R}$, if $\nabla\Phi$ is Lipschitz continuous with constant $L_{\nabla\Phi}$, then $\forall \mathbf{x}_1, \mathbf{x}_2 \in \mathbb{R}^N$,

$$\Phi(\mathbf{x}_2) - \Phi(\mathbf{x}_1) \leq \nabla\Phi(\mathbf{x}_1)^T (\mathbf{x}_2 - \mathbf{x}_1) + \frac{L_{\nabla\Phi}}{2}\|\mathbf{x}_2 - \mathbf{x}_1\|^2. \tag{39}$$

Lemma 3 (Robbins-Siegmund Lemma [42]): For three sequences of numbers $\{X^t\}$, $\{Y^t\}$, and $\{Z^t\}$, if $Y^t \geq 0$, $X^{t+1} \geq X^t + Y^t - Z^t$, $\forall t = 0, 1, \ldots$, and $\sum_t Z^t < \infty$, then either $X^t \to \infty$ or else $\{X^t\}$ converges to a finite value and $\sum_t Y^t < \infty$.

References

[1] I. Hwang, B. Song, and S. S. Soliman, "A holistic view on hyper-dense heterogeneous and small cell networks," *IEEE Commun. Mag.*, vol. 51, no. 6, pp. 20–27, Jun. 2013.

[2] N. Bhushan *et al.*, "Network densification: The dominant theme for wireless evolution into 5G," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 82–89, Feb. 2014.

[3] Z. Zhou, M. Dong, K. Ota, and Z. Chang, "Energy-efficient context-aware matching for resource allocation in ultra-dense small cells," *IEEE Access*, vol. 3, pp. 1849–1860, 2015.

[4] S. Zhang, N. Zhang, S. Zhou, J. Gong, Z. Niu, and X. Shen, "Energy-aware traffic offloading for green heterogeneous networks," *IEEE J. Sel. Areas Commun.*, vol. 34, no. 5, pp. 1116–1129, May 2016.

[5] A. Abdelnasser, E. Hossain, and D. I. Kim, "Tier-aware resource allocation in OFDMA macrocell-small cell networks," *IEEE Trans. Commun.*, vol. 63, no. 3, pp. 695–710, Mar. 2015.

[6] H. Zhang, Y. Dong, J. Cheng, M. J. Hossain, and V. C. M. Leung, "Fronthauling for 5G LTE-U ultra dense cloud small cell networks," *IEEE Wireless Commun.*, vol. 23, no. 6, pp. 48–53, Dec. 2016.

[7] H. Elsawy, E. Hossain, and D. I. Kim, "HetNets with cognitive small cells: User offloading and distributed channel access techniques," *IEEE Commun. Mag.*, vol. 51, no. 6, pp. 28–36, Jun. 2013.

[8] J. Xu *et al.*, "Cooperative distributed optimization for the hyper-dense small cell deployment," *IEEE Commun. Mag.*, vol. 52, no. 5, pp. 61–67, May 2014.

[9] J. Zheng, Y. Cai, and A. Anpalagan, "A stochastic game-theoretic approach for interference mitigation in small cell networks," *IEEE Commun. Lett.*, vol. 19, no. 2, pp. 251–254, Feb. 2015.

[10] V. Chandrasekhar, J. G. Andrews, T. Muharemovic, Z. Shen, and A. Gatherer, "Power control in two-tier femtocell networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, pp. 4316–4328, Aug. 2009.

[11] Z.-Q. Luo and S. Zhang, "Dynamic spectrum management: Complexity and duality," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 57–73, Feb. 2008.

[12] K. Yang, W. Xu, S. Li, and J. Lin, "Distributed multicast resource allocation in OFDM-based cognitive radio networks," in *Proc. 8th Int. ICST Conf. Commun. Netw. China (CHINACOM)*, Guilin, China, Aug. 2013, pp. 57–62.

[13] L. P. Qian, Y. J. Zhang, and J. Huang, "MAPEL: Achieving global optimality for a non-convex wireless power control problem," *IEEE Trans. Wireless Commun.*, vol. 8, no. 3, pp. 1553–1563, Mar. 2009.

[14] Z. Zhou, M. Dong, K. Ota, G. Wang, and L. T. Yang, "Energy-efficient resource allocation for D2D communications underlaying cloud-RAN-based LTE-A networks," *IEEE Internet Things J.*, vol. 3, no. 3, pp. 428–438, Jun. 2016.

[15] Z. Zhou, M. Dong, K. Ota, R. Shi, Z. Liu, and T. Sato, "Game-theoretic approach to energy-efficient resource allocation in device-to-device underlay communications," *IET Commun.*, vol. 9, no. 3, pp. 375–385, Feb. 2015.

[16] G. Scutari, F. Facchinei, P. Song, D. P. Palomar, and J.-S. Pang, "Decomposition by partial linearization: Parallel optimization of multi-agent systems," *IEEE Trans. Signal Process.*, vol. 62, no. 3, pp. 641–656, Feb. 2014.

[17] A. Alvarado, G. Scutari, and J.-S. Pang, "A new decomposition method for multiuser DC-programming and its applications," *IEEE Trans. Signal Process.*, vol. 62, no. 11, pp. 2984–2998, Jun. 2014.

[18] K. Ahuja, Y. Xiao, and M. Schaar, "Distributed interference management policies for heterogeneous small cell networks," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 6, pp. 1112–1126, Jun. 2015.

[19] R. B. Myerson, *Game Theory: Analysis of Conflict*. Cambridge, MA, USA: Harvard Univ. Press, Sep. 1991.

[20] H. Ji and C.-Y. Huang, "Non-cooperative uplink power control in cellular radio systems," *Wireless Netw.*, vol. 4, no. 3, pp. 233–240, 1998.

[21] C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Trans. Commun.*, vol. 50, no. 2, pp. 291–303, Feb. 2002.

[22] J. Zheng, Y. Cai, Y. Liu, Y. Xu, B. Duan, and X. Shen, "Optimal power allocation and user scheduling in multicell networks: Base station cooperation using a game-theoretic approach," *IEEE Trans. Wireless Commun.*, vol. 13, no. 12, pp. 6928–6942, Dec. 2014.

[23] Z. Han, Z. Ji, and K. J. R. Liu, "Non-cooperative resource competition game by virtual referee in multi-cell OFDMA networks," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 6, pp. 1079–1090, Aug. 2007.

[24] J.-S. Pang, G. Scutari, D. P. Palomar, and F. Facchinei, "Design of cognitive radio systems under temperature-interference constraints: A variational inequality approach," *IEEE Trans. Signal Process.*, vol. 58, no. 6, pp. 3251–3271, Jun. 2010.

[25] I. Stupia, L. Sanguinetti, G. Bacci, and L. Vandendorpe, "Power control in networks with heterogeneous users: A quasi-variational inequality approach," *IEEE Trans. Signal Process.*, vol. 63, no. 21, pp. 5691–5705, Nov. 2015.

[26] P. Li and Y. Zhu, "Price-based power control of femtocell networks: A Stackelberg game approach," in *Proc. IEEE PIMRC*, Sep. 2012, pp. 1185–1191.

[27] X. Kang, R. Zhang, and M. Motani, "Price-based resource allocation for spectrum-sharing femtocell networks: A Stackelberg game approach," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 3, pp. 538–549, Apr. 2012.

[28] N. D. Duong, A. S. Madhukumar, and D. Niyato, "Stackelberg Bayesian game for power allocation in two-tier networks," *IEEE Trans. Veh. Technol.*, vol. 65, no. 4, pp. 2341–2354, Apr. 2016.

[29] K. Zhu, E. Hossain, and A. Anpalagan, "Downlink power control in two-tier cellular OFDMA networks under uncertainties: A robust Stackelberg game," *IEEE Trans. Commun.*, vol. 63, no. 2, pp. 520–535, Feb. 2015.

[30] H. Wang, J. Wang, and Z. Ding, "Distributed power control in a two-tier heterogeneous network," *IEEE Trans. Wireless Commun.*, vol. 14, no. 12, pp. 6509–6523, Dec. 2015.

[31] M. Rasti, A. R. Sharafat, and B. Seyfe, "Pareto-efficient and goal-driven power control in wireless networks: A game-theoretic approach with a novel pricing scheme," *IEEE/ACM Trans. Netw.*, vol. 17, no. 2, pp. 556–569, Apr. 2009.

[32] D. T. Ngo, L. B. Le, and T. Le-Ngoc, "Distributed Pareto-optimal power control for utility maximization in femtocell networks," *IEEE Trans. Wireless Commun.*, vol. 11, no. 10, pp. 3434–3446, Oct. 2012.

[33] J. Huang, R. A. Berry, and M. L. Honig, "Distributed interference compensation for wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 5, pp. 1074–1084, May 2006.

[34] H. Zhang, C. Jiang, N. C. Beaulieu, X. Chu, X. Wang, and T. Q. S. Quek, "Resource allocation for cognitive small cell networks: A cooperative bargaining game theoretic approach," *IEEE Trans. Wireless Commun.*, vol. 14, no. 6, pp. 3481–3493, Jun. 2015.

[35] H. Zhang, X. Chu, W. Ma, W. Zheng, and X. Wen, "Resource allocation with interference mitigation in OFDMA femtocells for co-channel deployment," *EURASIP J. Wireless Commun. Netw.*, vol. 2012, no. 1, pp. 1–9, Dec. 2012.

[36] H. Zhang, L. Venturino, N. Prasad, P. Li, S. Rangarajan, and X. Wang, "Weighted sum-rate maximization in multi-cell networks via coordinated scheduling and discrete power control," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 6, pp. 1214–1224, Jun. 2011.

[37] M. Bennis, S. M. Perlaza, P. Blasco, Z. Han, and H. V. Poor, "Self-organization in small cell networks: A reinforcement learning approach," *IEEE Trans. Wireless Commun.*, vol. 12, no. 7, pp. 3202–3212, Jul. 2013.

[38] S. M. Perlaza, H. Tembine, S. Lasaulce, and M. Debbah, "Quality-of-service provisioning in decentralized networks: A satisfaction equilibrium approach," *IEEE J. Sel. Topics Signal Process.*, vol. 6, no. 2, pp. 104–116, Apr. 2012.

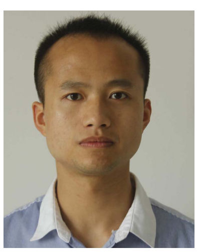
[39] A. Mas-Colell, M. D. Whinston, and J. R. Green, *Microeconomic Theory*. London, U.K.: Oxford Univ. Press, 1995.

[40] D. P. Bertsekas, *Nonlinear Programming*, 2nd ed. Belmont, MA, USA: Athena Scientific, 1999.

[41] D. P. Bertsekas, A. Nedic, and A. E. Ozdaglar, *Convex Analysis and Optimization*. Belmont, MA, USA: Athena Scientific, 2003.

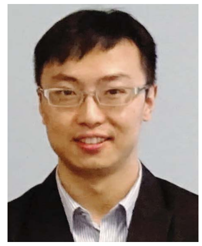
[42] B. T. Polyak, *Introduction to Optimization*. New York, NY, USA: Optimization Software, Inc., 1987.

[43] Y. Yang and M. Pesavento, "A unified successive pseudo-convex approximation framework," *IEEE Trans. Signal Process.*, vol. 65, no. 13, pp. 3313–3328, Jul. 2017.



Jianchao Zheng (M'16) received the B.S. degree in communications engineering and the Ph.D. degree in communications and information systems from the College of Communications Engineering, PLA University of Science and Technology, Nanjing, China, in 2010 and 2016, respectively. From 2015 to 2016, he was a Visiting Scholar with the Broadband Communications Research Group, Department of Electrical and Computer Engineering, University of Waterloo, Canada. He is currently an Assistant Professor with the College of Communications Engineering, PLA University of Science and Technology. He has authored or co-authored several papers in international conferences and reputed journals in his research area. His research interests focus on interference mitigation techniques, green communications and computing networks, game theory, learning theory, and optimization techniques.

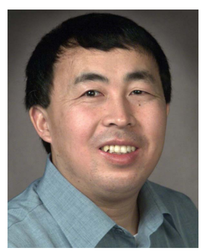
Yuan Wu (M'10–SM'16) received the Ph.D. degree in electronic and computer engineering from the Hong Kong University of Science and Technology, Hong Kong, in 2010. From 2010 to 2011, he was the Post-Doctoral Research Associate with the Hong Kong University of Science and Technology. From 2016 to 2017, he is the Visiting Scholar with the Broadband Communications Research Group, Department of Electrical and Computer Engineering, University of Waterloo, Canada. He is currently an Associate Professor with the College of Information Engineering, Zhejiang University of Technology, Hangzhou, China. His research interests focus on radio resource allocations for wireless communications and networks, and smart grid. He was a recipient of the Best Paper Award at the IEEE International Conference on Communications in 2016.



Ning Zhang (M'16) received the B.Sc. degree from Beijing Jiaotong University, Beijing, China, in 2007, the M.Sc. degree from the Beijing University of Posts and Telecommunications, Beijing, in 2010, and the Ph.D. degree from the University of Waterloo in 2015. From 2015 to 2016, he was a Post-Doctoral Research Fellow with the Broadband Communications Research Group Laboratory, University of Waterloo. His current research interests include next generation wireless networks, software defined networking, green communication, and physical layer security.

Haibo Zhou (M'14) received the Ph.D. degree in information and communication engineering from Shanghai Jiaotong University, Shanghai, China, in 2014. He is currently a Post-Doctoral Fellow with the Broadband Communications Research Group, University of Waterloo. His current research interests include resource management and protocol design in cognitive radio networks and vehicular networks.

Yueming Cai (M'05–SM'12) received the B.S. degree in physics from Xiamen University, Xiamen, China, in 1982, and the M.S. degree in micro-electronics engineering and the Ph.D. degree in communications and information systems from Southeast University, Nanjing, China, in 1988 and 1996, respectively. His current research interest includes cooperative communications, signal processing in communications, wireless sensor networks, and physical layer security.



Xuemin (Sherman) Shen (M'97–SM'02–F'09) received the B.Sc. degree from Dalian Maritime University, China, in 1982, and the M.Sc. and Ph.D. degrees from Rutgers University, New Brunswick, NJ, USA, in 1987 and 1990, respectively, all in electrical engineering. He is currently a Professor and the University Research Chair with the Department of Electrical and Computer Engineering, University of Waterloo, Canada. He is also an Associate Chair for Graduate Studies. His research focuses on resource management in interconnected wireless/wired networks, wireless network security, social networks, smart grid, and vehicular ad hoc and sensor networks. He is an elected member of the IEEE ComSoc Board of Governors and the Chair of Distinguished Lecturers Selection Committee. He served as the Technical Program Committee Chair/Co-Chair of the IEEE Globecom'16, the Infocom'14, the IEEE VTC'10 Fall, and the Globecom'07; the Symposia Chair at the IEEE ICC'10; the Tutorial Chair at the IEEE VTC'11 Spring and the IEEE ICC'08; the General Co-Chair at the ACM Mobihoc'15, the Chinacom'07, and the QShine'06; and the Chair of the IEEE Communications Society Technical Committee on Wireless Communications and the P2P Communications and Networking. He also serves/served as the Editor-in- Chief of the IEEE Network, *Peer-to-Peer Networking and Application*, and *IET Communications*; an Associate Editor-in-Chief of the IEEE Internet of Things Journal; a Founding Area Editor of the IEEE Transactions on Wireless Communications; an Associate Editor of the IEEE Transactions on Vehicular Technology, *Computer Networks*, and *ACM/Wireless Networks*; and Guest Editor of the IEEE Journal on Selected Areas in Communications, the IEEE Wireless Communications, the *IEEE Communications Magazine*, and *ACM Mobile Networks and Applications*. He received the Excellent Graduate Supervision Award in 2006 and the Outstanding Performance Award in 2004, 2007, 2010, and 2014, respectively, from the University of Waterloo, the Premiers Research Excellence Award in 2003 from the Province of Ontario, Canada, and the Distinguished Performance Award in 2002 and 2007, respectively, from the Faculty of Engineering, University of Waterloo. He is a registered Professional Engineer of Ontario, Canada, an Engineering Institute of Canada Fellow, a Canadian Academy of Engineering Fellow, a Royal Society of Canada Fellow, and a Distinguished Lecturer of the IEEE Vehicular Technology Society and Communications Society.