# Opportunistic Spectrum Access for CR-VANETs: A Game Theoretic Approach 

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#### Abstract

In this paper, we investigate the opportunistic spectrum access for cognitive radio vehicular ad hoc networks (CRVANETs). The probability distribution of the channel availability is first derived by means of a finite-state continuous-time Markov chain (CTMC), jointly considering the mobility of vehicles, and the spatial distribution and the temporal channel usage pattern of primary transmitters. Utilizing the channel availability statistics, we propose a game theoretic spectrum access scheme for vehicles to opportunistically access licensed channels in a distributed manner. Specifically, the spectrum access process is modeled as a non-cooperative congestion game. The existence of Nash equilibrium is proved and its efficiency is analyzed when employing uniform medium access control (MAC) protocol and slotted ALOHA, respectively. Furthermore, a spectrum access algorithm is devised to achieve a pure Nash equilibrium with high efficiency and fairness. Simulation results validate our analysis and demonstrate that the proposed spectrum access scheme can achieve higher utility and fairness, compared with a random access scheme.


Index Terms - VANETs, cognitive radio, opportunistic spectrum access, congestion game.

## I. Introduction

Vehicular ad hoc networks (VANETs) have been envisioned to improve road safety and efficiency, and provide Internet access on the move, by incorporating wireless communication and informatics technologies into the transportation system. VANETs can facilitate a myriad of attractive applications, which are usually divided into two main categories: safety applications (e.g., collision avoidance, safety warnings, and remote vehicle diagnostic [1], [2]) and infotainment applications (e.g., file downloading, web browsing, and video streaming [3], [4]). To support those various applications, the U.S. Federal Communication Commission (FCC) has allocated totally 75 MHz in the 5.9 GHz band for Dedicated Short Range Communications (DSRC) which basically consist of two types of communication, i.e., Vehicle-to-Vehicle (V2V) communications and Vehicle-to-Roadside (V2R) communications [5]. The dedicated bandwidth is further divided into seven channels to support safety and non-safety services simultaneously. Much research attention has been attracted

[^0]to design multi-channel access schemes based on the FCC bandwidth allocation in VANETs [6]-[9]. However, VANETs still face the problem of spectrum scarcity due to the following reasons: 1) the ever-increasing infotainment applications, such as high-quality video streaming, require a large amount of spectrum resource, and thereby the quality of service ( QoS ) is difficult to satisfy merely by the dedicated bandwidth; and 2) in urban environments, the spectrum scarcity is more severe due to high vehicle density, especially in some places where the vehicle density is much higher than normal [10], [11].

Cognitive radio is a promising approach to deal with the spectrum scarcity, which enables unlicensed users to opportunistically exploit the spectrum owned by licensed users [12], [13]. In cognitive radio networks (CRNs), licensed users and unlicensed users are typically referred to as primary users (PUs) and secondary users (SUs), respectively. Specifically, SUs perform spectrum sensing before transmission, through which they can identify and exploit spectrum opportunities without interfering with the transmissions of PUs. By means of cognitive radio, not only can CRNs provide better QoS for SUs, but also significantly improve the spectrum utilization. A natural question is that if cognitive radio can be applied to solve the problem of spectrum scarcity in VANETs. Recent researches in the literature demonstrate its feasibility [14][17]. With CR technology, VANETs have been coined as CR-VANETs, whereby vehicles can opportunistically access additional licensed spectrum owned by other systems, such as digital television (DTV) and 3G/4G cellular networks. Considering the highly dynamic mobility, vehicles are expected to exploit more spatial and temporal spectrum opportunities along the road than stationary SUs.
Although opportunistic spectrum access for CRNs has been extensively studied [18]-[20], the results may not be directly applied to CR-VANETs as the common assumption in the literature is that SUs are stationary and thus the spectrum opportunity is only affected by the spectrum usage patterns of the primary network. However, due to the mobility of vehicles, the spectrum opportunity may vary temporally and spatially, making the opportunistic spectrum access problem more challenging in CR-VANETs. In [21], Urgaonkar et al. exploited Markovian random walk model of SUs and proposed an opportunistic scheduling policy for secondary networks. The objective is to maximize the throughput of SUs by using the technique of Lyapunov optimization. In [16], Niyato et al. investigated the optimal channel access in CR-VANETs to maximize the utility of vehicles under certain QoS constraints for a grid-like urban street layout under the assumption that the
channel availability statistics are known by vehicles. In [22], Min et al. analyzed a scenario with random mobility, in which PUs are geographically independent and identically distributed (i.i.d.) and SUs move following the random waypoint model. A model for mobility-aware channel availability analysis was established by means of a continuous-time Markov chain, considering both the channel usage pattern of PUs and the mobility of SUs. They also proposed an opportunistic spectrum access strategy. However, the mobility of vehicles in urban area may lead to different channel availability.

In this paper, we first study the channel availability for CR-VANETs in urban scenarios, taking the mobility pattern of vehicles into consideration. Exploiting the statistics of the channel availability, a distributed opportunistic spectrum access scheme based on a non-cooperative congestion game is proposed for vehicles to exploit spatial and temporal access opportunities of the licensed spectrum. Specifically, we consider a grid-like urban street pattern to model the downtown area of a city. Vehicles equipped with a cognitive radio, moving in the grid, opportunistically access the spectrum of the primary network. The probability distribution of the channel availability is obtained by means of a continuoustime Markov chain (CTMC). After that, we employ a noncooperative congestion game to solve the problem of vehicles accessing multiple channels with different channel availabilities. We prove the existence of the pure Nash equilibrium (NE) and analyze the efficiency of different NEs, when applying uniform MAC and slotted ALOHA, respectively. A distributed spectrum access algorithm is then developed for vehicles to choose an access channel in a distributed manner, so that a pure NE with high efficiency and fairness is achieved. Finally, simulation results validate our analysis and demonstrate that, with the proposed spectrum access scheme, vehicles can achieve higher utility and fairness compared with the random access.

Our contributions are mainly three-fold. First, this work studies the channel availability for CR-VANETs in urban scenarios, which is crucial for devising an efficient spectrum access scheme. Second, based on the statistics of channel availability, a distributed spectrum access scheme is proposed in CR-VANETs from a game theoretic perspective, and the existence of pure NE is proved. Third, a spectrum access algorithm is introduced to achieve a pure NE with high efficiency and fairness. As the automotive industry gears for supporting high-bandwidth applications, with our proposed scheme applied, the QoS of VANETs applications can be improved by efficiently utilizing the spectrum resource of the licensed band.

The remainder of the paper is organized as follows. The detailed description of the system model is provided in Section II. In Section III, channel availability is analyzed for CRVANETs in urban scenarios. A spectrum access scheme based on game theory is presented in Section IV. Simulation results are given in Section V. Section VI concludes the paper.

## II. System Model

In urban scenarios of CR-VANETs, the transmitters of the primary network are referred to as primary transmitters (PTs),


Figure 1. Regularly distributed PTs on channel $i$.
such as TV broadcasters and cellular base stations. As SUs, vehicles equipped with a cognitive radio can opportunistically access the licensed spectrum. There is a non-empty set $\mathcal{K}$ of licensed channels that can be accessed by vehicles opportunistically. The channel usage behavior of primary users and vehicle mobility lead to intermittent channel availability for vehicles. The spectrum opportunity is characterized by the channel availability experienced by a vehicle, which is defined as the lengths of time duration in which the channel is available or unavailable for that vehicle. The availability of channel $i, i \in \mathcal{K}$, for a vehicle is determined by the spatial distribution and the temporal channel usage pattern of PTs that operate on channel $i$, as well as the mobility of the vehicle. A summary of the mathematical notations used in this paper is given in Table I.

## A. Urban Street Pattern

A grid-like street layout is considered for analyzing CRVANETs in urban environments, like the downtown area of many cities, such as Houston and Portland [23]. The network geometry comprises of a set of horizontal roads intersected with another set of vertical roads. As shown in Fig. 1, each line segment represents a road segment (the road section between any two neighboring intersections) with bi-directional vehicle traffic. In addition, all the horizontal segments have the same length $L_{H}$, and all the vertical segments have the same length $L_{V}$, leading to equal-sized street blocks of $L_{H} \times L_{V}$. For example, $L_{H}$ and $L_{V}$ are generally from 80 m to 200 m for the downtown area of Toronto [24].

## B. Spatial Distribution of PTs

We consider that PTs operating on a generic channel $i$ are regularly distributed in the grid, as shown in Fig. 1. The distance between any two neighboring PTs in the horizontal direction and vertical direction is denoted by $L_{P H, i}$ and $L_{P V, i}$, respectively. Denote by $R_{i}$ the transmission range of PTs on channel $i$. The coverage area of the PT is approximated by a square area with side length $2 R_{i}$, where $R_{i}<\min \left(L_{P H, i}, L_{P V, i}\right)$ to avoid overlapping of different coverage regions of PTs. The approximate coverage area is larger than the real coverage area to protect the primary transmission. A similar approximation of the PT coverage area can be found in [25].

Table I
The USEFUL NOTATIONS.

| Symbol |  |
| :--- | :--- |
| $\mathcal{K}$ | Description |
| $\lambda_{\text {idle }, i}, \lambda_{\text {busy }, i}$ | The set of licensed channels in the network, $\|\mathcal{K}\|=K$ |
| $L_{P H, i}, L_{P V, i}$ | The horerizontal and vertical distance between neighboring PTs on channel $i$ |
| $L_{H}, L_{V}$ | The horizontal and vertical length of the road segments |
| $R_{i}$ | The approximate transmission range of PTs on channel $i$ |
| $v$ | Vehicle speed |
| $T_{i n, i}, T_{o u t, i}$ | Time duration in which vehicle remains within/outside the coverage of PTs on channel $i$ |
| $\Psi_{i}$ | Effective channel availability (ECA) of channel $i$ |
| $\mathcal{C}$ | Set of available channels in the spectrum access game, $\mathcal{C} \in \mathcal{K},\|\mathcal{C}\|=C$ |
| $\mathcal{N}$ | Set of vehicles participating in the game, $\|\mathcal{N}\|=N$ |
| $R_{v}$ | The transmission range of the vehicle |
| $\rho_{v}$ | The vehicle density on the road |
| $U_{j}^{i}$ | The utility vehicle $j$ obtains by choosing channel $i$ |
| $r(n)$ | Resource allocation function. It corresponds to specific MAC scheme |
| $n(S)$ | Congestion vector corresponding to strategy profile $S$ |
| $\mathcal{E}_{S}$ | Efficiency of strategy profile $S$ |
| $F$ | Fairness index among vehicles |
| $\Phi$ | Channel diversity |

## C. Temporal Channel Usage Pattern of PTs

The temporal channel usage of PTs operating on a generic channel $i$ is modeled as an alternating busy (the PT is active in transmitting) and idle (the PT does not transmit) process [26] [27]. During the transmission period of a PT, vehicles in the coverage area of the PT are not permitted to use the same channel in order to avoid the interference to the primary network. The length of busy/idle period is modeled as an exponential random variable with parameters $\lambda_{\text {busy }} / \lambda_{\text {idle }}$, i.e.,

$$
T_{b u s y, i} \sim \operatorname{Exp}\left(\lambda_{b u s y, i}\right) \text { and } T_{i d l e, i} \sim \operatorname{Exp}\left(\lambda_{i d l e, i}\right),
$$

where $X \sim \operatorname{Exp}(\lambda)$ indicates that variable $X$ follows an exponential distribution with parameter $\lambda . \varpi_{i d l e, i}=$ $\frac{\lambda_{b u s y, i}}{\lambda_{\text {idle }, i}+\lambda_{b u s y, i}}$ and $\varpi_{b u s y, i}=1-\varpi_{i d l e, i}$ are the steady-state probabilities that a PT on channel $i$ is active and inactive, respectively.

## D. Mobility Model

Vehicles move in the grid at a random and slowly changing speed $v$, where $v \in\left[v_{\min }, v_{\max }\right]$. The average value of $v$ is denoted by $\bar{v}$. At each intersection, vehicles randomly select a direction to move on another road segment. Particularly, a vehicle chooses the direction of north, south, east and west with probability $P_{n}, P_{s}, P_{e}$, and $P_{w}$, respectively, as shown in Fig. 1. It holds that $P_{n}+P_{s}+P_{e}+P_{w}=1$. Once the vehicle chooses a direction at an intersection, it moves straight until it arrives at the next intersection.

## III. Channel Availability Analysis for Urban CR-VANETs

The statistics of channel availability can be utilized to design an efficient spectrum access scheme to improve the QoS of SUs and the spectrum utilization. In this section, we analyze the availability of channel $i$ for vehicles in urban scenarios, jointly considering the spatial distribution and the temporal channel usage pattern of PTs, and the mobility of vehicles. It
is assumed that PTs operating on the same channel belong to the same type of system and have the same spatial distribution and temporal channel usage pattern. A similar assumption can be seen in [22]. A continuous-time Markov chain that consists of three states is employed. Denote by $S_{\text {Idle }}, S_{\text {Busy }}$, and $S_{\bar{C}}$ the states of a vehicle in the coverage of an idle PT, in the coverage of an active PT, and outside the coverage of any PT that operates on channel $i$, respectively, as shown in Fig. 2. It can be seen that when the vehicle moves along the street, the state transits to one another. Since the channel is unavailable only when the vehicle is in $S_{\text {Busy }}$, we can further merge $S_{\text {Idle }}$ and $S_{\bar{C}}$ as one state in which the channel is available for the vehicle, which is denoted by $S_{A}$. The state of the channel being unavailable is denoted by $S_{U}$, which corresponds to the state $S_{B u s y}$.

Denote the time duration that a vehicle remains in $S_{A}$ and $S_{U}$ by $T_{A, i}$ and $T_{U, i}$, respectively. To obtain the channel availability, i.e., the probability distribution of $T_{A, i}$ and $T_{U, i}$, it is necessary to analyze the transition rates among the three states: $S_{\text {Idle }}, S_{B u s y}$, and $S_{\bar{C}}$. Denote by $T_{i n, i}$ and $T_{o u t, i}$ the time durations in which the vehicle remains within the coverage area of a PT and outside the coverage area of any PT on channel $i$, respectively. Therefore, the transition rates are closely related to the probability distribution of $T_{i n, i}$ and $T_{\text {out }, i}$. In the following, we focus on the analysis of these two time durations in urban scenarios.

## A. Analysis of $T_{i n}$ in Urban Scenarios

To analyze $T_{i n}$, we consider the case in which vehicles move within the coverage of a PT. Denote by $\Omega_{R}$ the coverage area of the PT. Recall that $\Omega_{R}$ is a square with side length $N_{R, i}$. For ease of the analysis, let $L_{V}=L_{H}=L$, and $L_{P H, i}=L_{P V, i}=L_{P, i}$. All lengths are normalized by $L$, for example, $N_{R, i}=\left\lceil\frac{2 R_{i}}{L}\right\rceil$.

In order to analyze $T_{i n}$, a two-dimensional discrete Markov chain is employed, as shown in Fig. 3. We index all the intersections within $\Omega_{R}$, and let each intersection $(b, k)$


Figure 2. The states of a vehicle w.r.t. the mobility.
be a state $C_{b, k}$. All these states form a Markov chain $\mathbb{C}_{b, k}=\left\{C_{1,1}, C_{1,2}, \ldots, C_{1, N_{R, i}}, C_{2,1}, C_{2,2}, \ldots, C_{N_{R, i}, N_{R, i}}\right\}$. It is said that a vehicle is in state $C_{b, k}$ if it is moving from intersection $(b, k)$ to a neighboring intersection $\left(b^{\prime}, k^{\prime}\right)$. When the vehicle arrives at intersection $\left(b^{\prime}, k^{\prime}\right)$, the state transits from $C_{b, k}$ to $C_{b^{\prime}, k^{\prime}}$. The states that lie on the boundary of $\Omega_{R}$ are referred to absorbing states which indicate that the vehicle moves out of the PT coverage area. Let $\mathbf{Q}_{\mathbf{A}}$ be the set of absorbing states. Denote by $M$ the number of transitions it takes before the vehicle leaves $\Omega_{R}$, i.e., transits to any state in $\mathbf{Q}_{\mathbf{A}} . T_{i n}$ can be approximated by $M * \Delta t$, where $\Delta t$ is the time that the vehicle moves through a road segment. To obtain the probability distribution of $T_{i n}$, we need to find the probability distribution of $M$. To this end, we first obtain the transition probabilities of $\mathbb{C}_{b, k}$ as follows:

$$
\begin{align*}
& \left\{\begin{array}{l}
P\left(C_{b, k-1} \mid C_{b, k}\right)=P_{l} \\
P\left(C_{b, k+1} \mid C_{b, k}\right)=P_{r} \\
P\left(C_{b-1, k} \mid C_{b, k}\right)=P_{u} \quad C_{b, k} \notin \mathbf{Q}_{\mathbf{A}} \\
P\left(C_{b+1, k} \mid C_{b, k}\right)=P_{d} \\
P\left(\text { other } \mid C_{b, k}\right)=0
\end{array}\right.  \tag{1}\\
& \left\{\begin{array}{l}
P\left(C_{b, k} \mid C_{b, k}\right)=1 \\
P\left(\text { other } \mid C_{b, k}\right)=0
\end{array} \quad C_{b, k} \in \mathbf{Q}_{\mathbf{A}},\right. \tag{2}
\end{align*}
$$

which infers that the transition matrix $\mathbf{P}$ is sparse. Denote by $\pi^{(m)}$ the probability distribution of the states after $m$ transitions. Specifically, $\pi^{(0)}$ is the probability distribution of the initial states. It holds that $\pi^{(m)}=\pi^{(0)} \mathbf{P}^{m}$. At initial time $t_{0}$, it is possible for the vehicle to be in any state in $\Omega_{R}$ except those in $\mathbf{Q}_{\mathbf{A}}$. Denote by $\mathbf{Q}_{\mathbf{I}}$ the set of these possible initial states. Then the cardinality of $\mathbf{Q}_{\mathbf{I}}$, denoted by $C_{I}$, can be calculated by $C_{I}=\left|\mathbf{Q}_{\mathbf{I}}\right|=\left(N_{R, i}-2\right)^{2}$. All possible initial states are considered to be with equal probability, and thus $\pi^{(0)}$ can be obtained as follows:

$$
\pi_{(b, k)}^{(0)}= \begin{cases}p_{I}=\frac{1}{C_{I}}=\frac{1}{\left(N_{R, i}-2\right)^{2}} & C_{b, k} \in \mathbf{Q}_{\mathbf{I}} \\ 0 & \text { otherwise }\end{cases}
$$

where $p_{I}$ is the probability of each possible initial state. The probability of the event that $M$ is no more than $m$ is given by

$$
\operatorname{Pr}(M \leq m)=\sum_{C_{b, k} \in \mathbf{Q}_{\mathbf{A}}} \pi^{(m)}
$$



Figure 3. Analysis of $T_{i n}$ : A two-dimensional Markov chain.


Figure 4. Analysis of $T_{\text {out }}$ : A two-dimensional Markov chain.

Therefore, the probability mass function of $M$ is

$$
\begin{align*}
\operatorname{Pr}(M=m) & =\operatorname{Pr}(M \leq m)-\mathbf{P r}(M \leq m-1) \\
& =\sum_{C_{b, k} \in \mathbf{Q}_{\mathbf{A}}} \pi^{(m)}-\sum_{C_{b, k} \in \mathbf{Q}_{\mathbf{A}}} \pi^{(m-1)} . \tag{3}
\end{align*}
$$

On the other hand, if all the states in $\mathbf{Q}_{\mathbf{A}}$ are considered as one state $S_{E n d}$ and all other states as another state $S_{\text {Begin }}$, the two-dimensional Markov chain $\mathbb{C}_{b, k}$ can be reduced to a twostate Markov chain $\left\{S_{B e g i n}, S_{E n d}\right\}$. The vehicle is in $S_{\text {Begin }}$ at the beginning. In each transition, it either transits to $S_{\text {End }}$ with probability $p_{0}$ or remains in $S_{\text {Begin }}$ with probability $1-$ $p_{0}$, where $p_{0}$ is as follows:

$$
p_{0}=\frac{1}{C_{I}} \sum_{C_{b, k} \in \mathbf{Q}_{\mathbf{I}}} \sum_{C_{b^{\prime}, k^{\prime}} \in \mathbf{Q}_{\mathbf{A}}} P\left(C_{b^{\prime}, k^{\prime}} \mid C_{b, k}\right)
$$

The vehicle does not stop moving until the state transits to $S_{E n d}$. Thus, the number of transitions before the vehicle leaves the PT coverage area can be considered to follow a geometric distribution with $p=p_{0}$. From this perspective, $T_{i n}$ may be approximated by an exponential distribution, which will be discussed later.

## B. Analysis of $T_{\text {out }}$ in Urban Scenarios

A two-dimensional Markov chain is also employed to analyze $T_{\text {out }}$. Since PTs operating on the same channel are regularly deployed, we can just take the area around one PT to analyze, as shown in Fig. 4. Denote this square area by $\Omega_{D}$, with side length $N_{D, i}=\left\lceil\frac{L_{P, i}}{L}\right\rceil$. We consider that $\Omega_{D}$ is a torus: when a vehicle leaves the boundary of $\Omega_{D}$, it moves into $\Omega_{D}$ on the same road from the opposite side of the area. In this situation, the intersections that lie on the boundary of $\Omega_{R}$ are referred to as absorbing states indicating that vehicles in these states move into the coverage of a PT. We can get the transition matrix $\mathbf{P}$, which is similar to (1) and (2) except

$$
\left\{\begin{array}{l}
P\left(C_{b, N_{D}} \mid C_{b, 1}\right)=P_{l}  \tag{4}\\
P\left(C_{b, 1} \mid C_{b, N_{D}}\right)=P_{r} \\
P\left(C_{N_{D}, k} \mid C_{1, k}\right)=P_{u} \\
P\left(C_{1, k} \mid C_{N_{D}, k}\right)=P_{d}
\end{array}\right.
$$

States which are within $\Omega_{D}$ but outside $\Omega_{R}$ are initial states. Similar to the analysis of $T_{i n}$, the possible initial states are with equal probability. Denote by $\mathbf{Q}_{\mathbf{I}}$ the set of possible initial states, and $\left|\mathbf{Q}_{\mathbf{I}}\right|=N_{D, i}^{2}-N_{R, i}^{2}$. Then the probability distribution of initial states, denoted by $\pi^{(0)}$, is as follows:

$$
\pi_{(b, k)}^{(0)}= \begin{cases}p_{I}=\frac{1}{N_{D, i}^{2}-N_{R, i}^{2}} & C_{b, k} \in \mathbf{Q}_{\mathbf{I}} \\ 0 & \text { otherwise }\end{cases}
$$

$M^{\prime}$ denotes the number of transitions before a vehicle moves into the coverage area of a PT. Similar to (3), we can get the probability mass function of $M^{\prime}$ as follows:

$$
\begin{align*}
\operatorname{Pr}\left(M^{\prime}=m\right) & =\operatorname{Pr}\left(M^{\prime} \leq m\right)-\operatorname{Pr}\left(M^{\prime} \leq m-1\right) \\
& =\sum_{C_{b, k} \in \mathbf{Q}_{\mathbf{A}}} \pi^{(m)}-\sum_{C_{b, k} \in \mathbf{Q}_{\mathbf{A}}} \pi^{(m-1)} \tag{5}
\end{align*}
$$

## C. Estimation of $\lambda_{\text {in }}$ and $\lambda_{\text {out }}$

The probability density function (PDF) of $T_{\text {in }}$ and $T_{\text {out }}$ from (3) and (5) are shown in Fig. 5. It can be seen that both $T_{i n}$ and $T_{\text {out }}$ can be approximated by an exponential distribution. Furthermore, the parameter of the distribution can be estimated by using Maximum Likelihood Estimation (MLE) as follows:

$$
\lambda=\frac{1}{\bar{x}}
$$

where $\bar{x}$ is the sample mean, and $\lambda$ is the estimated parameter of the exponential distribution. More interestingly, the expected value of $T_{\text {in }}$ and $T_{\text {out }}$ (denoted by $\bar{T}_{\text {in }}$ and $\bar{T}_{\text {out }}$, respectively) change with vehicle speed $\bar{v}$, and the spatial parameters of PTs, i.e., $R_{i}$ and $L_{P, i}$. Specifically, the parameters of the two exponential distributions can be approximated by

$$
\lambda_{\text {in }} \approx \frac{\bar{v}}{R_{i}} \text { and } \lambda_{\text {out }} \approx \frac{\bar{v}}{f\left(L_{P, i}-R_{i}\right)}
$$

When $L_{p, i}-R_{i}<5 L, f(\cdot)$ is linear, and we have

$$
\lambda_{o u t} \approx \frac{\bar{v}}{17.4\left(L_{P, i}-R_{i}\right)-16.4}
$$



Figure 5. PDF of $T_{\text {in }}$ and $T_{\text {out }}(L=100 \mathrm{~m})$.

Fig. 6 shows the comparison between analytical and approximate cumulative distribution function (CDF) of $T_{\text {in }}$ and $T_{\text {out }}$. It can be seen that they closely match each other, which validates the accuracy of the estimation. The effect of $R_{i}$ ( $N_{R, i}$ ) and $L_{P, i}\left(N_{D, i}\right)$ on $\bar{T}_{i n}$ and $\bar{T}_{\text {out }}$ is shown in Fig. 7(a) and 7(b), respectively. A larger value of $R_{i}$ leads to a larger value of $\bar{T}_{i n}$, while a larger value of $L_{P, i}$ leads to a larger value of $\bar{T}_{\text {out }}$, which is consistent with our expectation.

## D. Derivation of Channel Availability

From the above analysis, the transition rates among the states in the Markov chain shown in Fig. 2 can be obtained, as listed in Table II. Denote by $\zeta_{i}$ the average fraction of the area of PT coverage on channel $i$, and $\zeta_{i}=\frac{4 R_{i}^{2}}{L_{p, i}^{2}}$. Thus, the average fraction of areas where channel $i$ is available at any given time, denoted by $\delta_{i}$, can be given by:

$$
\begin{equation*}
\delta_{i}=\left(1-\zeta_{i}\right)+\zeta_{i} \varpi_{i d l e, i}=1-\zeta_{i} \varpi_{b u s y, i} \tag{6}
\end{equation*}
$$

The state $S_{U, i}$ ends up when the vehicle moves out of the coverage of the PT or the PT stops transmission. Therefore, $T_{U, i}$ follows an exponential distribution with parameter $\lambda_{U, i}$ where:

$$
\lambda_{U, i}=\lambda_{b u s y, i}+\frac{\bar{v}}{f\left(L_{p, i}-R_{i}\right)} \varpi_{b u s y, i}
$$


(a) CDF of $T_{i n}$.

(b) CDF of $T_{\text {out }}$.

Figure 6. Comparison of analytical and approximate results.

Based on the balance condition $\varpi_{A, i} \lambda_{A, i}=\varpi_{U, i} \lambda_{U, i}$, where $\varpi_{A, i}=\delta_{i}$ and $\varpi_{U, i}=1-\delta_{i}$, we can get the $S_{A, i} \rightarrow S_{U, i}$ transition rate $\lambda_{A, i}$ as follows:

$$
\begin{align*}
\lambda_{A, i} & =\frac{\delta_{i}}{1-\delta_{i}} \lambda_{U, i} \\
& =\frac{\zeta_{i} \varpi_{b u s y, i}}{1-\zeta_{i} \varpi_{b u s y, i}}\left(\lambda_{b u s y, i}+\frac{\bar{v} \cdot \varpi_{b u s y, i}}{f\left(L_{p, i}-R_{i}\right)}\right) \tag{7}
\end{align*}
$$

and thus, $T_{A, i} \sim \operatorname{Exp}\left(\lambda_{A, i}\right)$. Fig. 8 shows the comparison between analytical and simulation results. The two curves closely match to each other for both $T_{A, i}$ and $T_{U, i}$, which demonstrates the accuracy of the analysis.

Define the effective channel availability (ECA) $\Psi$ of channel $i$ as the average time duration in which channel $i$ is available for a vehicle to access, which can be calculated as follows:

$$
\begin{equation*}
\Psi_{i}=\eta_{i} \cdot \bar{T}_{A, i}=\frac{\eta_{i}}{\lambda_{A, i}} \tag{8}
\end{equation*}
$$

where $\eta_{i} \in(0,1)$ is the interference factor representing the tolerance level of interference of primary network. Note that a larger value of $\eta$ brings more spectrum opportunities, but at the same time results in more interference to the primary network.

Considering a real-world road map can facilitate a more precise analysis of channel availability. This, however, introduces cumbersome challenges. Our approach is based on a simple

(a) Impact of $N_{R, i}$ on $\bar{T}_{i n}$.

(b) Impact of $N_{D, i}$ on $\bar{T}_{\text {out }}$.

Figure 7. Impact of $N_{R, i}$ and $N_{D, i}$ on $\bar{T}_{\text {in }}$ and $\bar{T}_{\text {out }}$.
Table II
State Transition Rates of a Vehicle for Channel $i$

| State Transition | Rate |
| :---: | :---: |
| $S_{\text {idle }} \rightarrow S_{\text {busy }}$ | $\lambda_{\text {idle }, i}$ |
| $S_{\text {busy }} \rightarrow S_{\text {idle }}$ | $\lambda_{\text {busy }, i}$ |
| $S_{\text {busy }} \rightarrow S_{\bar{C}}$ | $\frac{\bar{v}}{R_{i}}$ |
| $S_{\bar{C}} \rightarrow S_{\text {busy }}$ | $\frac{\bar{v}}{f\left(L_{p, i}-R_{i}\right)} \varpi_{\text {busy }, i}$ |
| $S_{\text {idle }} \rightarrow S_{\bar{C}}$ | $\frac{\overline{R_{i}}}{}$ |
| $S_{\bar{C}} \rightarrow S_{\text {idle }}$ | $\frac{\bar{v}}{f\left(L_{p, i}-R_{i}\right)} \varpi_{\text {idle }, i}$ |

regular road pattern, which offers a workable approximation.

## IV. Game Theoretic Spectrum Access Scheme

From the previous section, the channel availability statistics, i.e., ECA of each channel, are obtained. Assume that vehicles are aware of the spatial distribution and temporal channel usage pattern of PTs of each channel, i.e., $L_{P H, i}, L_{P V, i}$, $R_{i}, \lambda_{b u s y, i}$ and $\lambda_{\text {idle }, i}$, since the information of primary networks can be obtained from network operators. With such information, vehicles can obtain the ECA of each channel, i.e., $\Psi_{i}, i \in \mathcal{K}$, based on their speed, by using (7) and (8). Before transmitting, vehicles conduct spectrum sensing, which is assumed to be accurate in this work. Since the channel availability follows the exponential distribution which


Figure 8. The analytical and simulation results of $T_{A}$ and $T_{U}$.
is memoryless, the channel availability is independent of the situation before the spectrum sensing. Notably, if a channel is unavailable due to primary transmissions, its ECA is zero. We assume that the bandwidth of each channel is identical. Vehicles on the road often maintain relatively stable topology since they follow the same direction and similar speed, and thus they usually form clusters where vehicles within a cluster can communicate with each other [16] [28]. To utilize the licensed spectrum, vehicles should opportunistically access the channels with different ECAs in a distributed manner. Game theory [29] is a well-known tool to analyze the behavior of distributed players who are considered to be selfish and rational. We apply a non-cooperative congestion game to model the spectrum access process in CR-VANETs, in which vehicles in a cluster choose channels to access in a distributed manner, trying to maximize their own utilities. Then, we analyze the existence and the efficiency of NE using uniform MAC and slotted ALOHA, respectively, and devise a spectrum access algorithm for each vehicle to decide the access channel to achieve an NE with high efficiency.

## A. Formulation of Spectrum Access Game

The spectrum access problem is modeled as a congestion game, where there are multiple players and resources, and the payoff of each player by selecting one resource is related to
the number of other players selecting the same resource [30]. In this paper, the spectrum access congestion game is defined as $\Gamma=\left\{\mathcal{N}, \mathcal{C},\left\{S_{j}\right\}_{j \in \mathcal{N}},\left\{U_{j}\right\}_{j \in \mathcal{N}}\right\}$, where $\mathcal{N}=\{1, \ldots, N\}$ is the finite set of players, i.e., vehicles in a cluster. $N$ is related to the vehicle density, which is denoted by $\rho_{v} ; \mathcal{C}=$ $\{1, \ldots, C\}$ is the set of available channels, where "available" means that they are sensed to be idle, and $\mathcal{C} \subseteq \mathcal{K} ; S_{j}$ is the set of pure strategies associated with vehicle $j$; and $U_{j}$ is the utility function of vehicle $j$. Vehicles in the game are aware of the ECA of all channels $\left(\Psi_{i}\right)$ and the number of vehicles in the game $(N)$. The bandwidth (resource) of each channel is identical, and thus we set the bandwidth to one unit.

Since each vehicle is equipped with only one cognitive radio, it can access at most one channel at a time, and thus $S_{j}=\mathcal{C}$ for all $j \in \mathcal{N}$. In this case, denote by $U_{j}^{i}$ the utility of vehicle $j$ by choosing channel $i$. Note that $U_{j}^{i}$ is a function of both $s_{j}$ and $s_{-j}$, which are the strategies selected by vehicle $j$ and all of its opponents, respectively. In this game, we define the utility $U_{j}^{i}$ as the average total channel resource vehicle $j$ obtains by choosing channel $i$, before this channel is reoccupied by PTs, i.e.,

$$
\begin{equation*}
U_{j}^{i}=\Psi_{i} r\left(n_{i}\right) . \tag{9}
\end{equation*}
$$

$n_{i}$ is the total number of vehicles choosing channel $i$ simultaneously, including vehicle $j$. Resource allocation function $r\left(n_{i}\right)$ indicates the share of channel $i$ obtained by each of the $n_{i}$ vehicles. For an arbitrary vehicle, $\Psi_{i}$ is used to measure the average time duration in which channel $i$ is available. Thus, $U_{j}^{i}=\Psi_{i} r\left(n_{i}\right)$ shows the average total amount of channel resource vehicle $j$ can obtain before it must cease transmitting due to the appearance of active PTs. The channel with higher $\Psi_{i}$ is preferred because choosing it can reduce spectrum sensing and unpredictable channel switching. The form of $r(\cdot)$ is related to the specific MAC scheme. However, based on [31], $r(\cdot)$ should satisfy the following conditions:

- $r(1)=1$, which means that a user can get all the resource of a channel if it is the only one choosing that channel.
- $r(n)$ is a decreasing function of $n$.
- Define $f(n)=n r(n) . f(n)$ decreases with $n$ and should be convex, i.e., $f^{\prime}(n)<0$ and $f^{\prime \prime}(n)>0$.
- $n_{i} r\left(n_{i}\right) \leq 1$. Resource waste may happen when multiple users share the same resource due to contention or collision.

Since vehicles are rational and selfish, they prefer the strategy that can maximize their utilities. To analyze this game, we focus on NE. We will analyze the existence, condition and efficiency ratio (ER) of the pure NE, using uniform MAC and slotted ALOHA, respectively. After that, a spectrum access algorithm to achieve the pure NE with high ER is derived.

## B. Nash Equilibrium in Channel Access Game

Nash equilibrium is a well-known concept to analyze the outcome of the game, which states that in the equilibrium every user can select a utility-maximizing strategy given the strategies of other users.

Definition 1: A strategy profile for the players $S^{*}=$ $\left(s_{1}^{*}, s_{2}^{*}, \ldots, s_{N}^{*}\right)$ is an NE if and only if

$$
\begin{equation*}
U_{j}\left(s_{j}^{*}, s_{-j}^{*}\right) \geq U_{j}\left(s_{j}^{\prime}, s_{-j}^{*}\right), \forall j \in \mathcal{N}, s_{j}^{\prime} \in S_{j}, \tag{10}
\end{equation*}
$$

which means that no one can increase its utility alone by changing its own strategy, given strategies of the other users. If the strategy profile $S$ in (10) is deterministic, it is called a pure NE. In this paper, we consider pure NE only, so we use the term NE and pure NE interchangeably.

In the spectrum access game $\Gamma$, given a strategy profile, if no vehicle can improve its utility by shifting to another channel alone, the strategy profile is referred to as a pure NE. Denote by $S=\left(s_{1}, s_{2}, \ldots, s_{N}\right)$ the strategy profile of all vehicles, where $s_{i}$ is a specific channel. Denote by $\mathbf{n}(S)=\left(n_{1}, n_{2}, \ldots, n_{C}\right)$ the congestion vector, which shows the number of vehicles choosing each channel, corresponding to the strategy profile $S$. According to Definition 1, the spectrum access game $\Gamma$ has pure $\mathrm{NE}(\mathrm{s})$ if and only if for each player $j \in \mathcal{N}$,

$$
\begin{equation*}
\Psi_{s_{j}} r\left(n_{s_{j}}\right) \geq \Psi_{k} r\left(n_{k}+1\right), \forall k \in \mathcal{C}, k \neq s_{j} . \tag{11}
\end{equation*}
$$

Note that there are typically multiple strategy profiles that correspond to one congestion vector. If a strategy profile $S$ corresponding to congestion vector $\mathbf{n}^{*}$ is a pure NE, then all strategy profiles corresponding to $\mathbf{n}^{*}$ are pure NEs according to (11). Denote by NE-set $(\mathbf{n})$ the set of pure NEs corresponding to congestion vector $\mathbf{n}$. The NEs in NE-set ( $\mathbf{n}$ ) may yield different utilities for each player. However, they yield the same total utility, which is defined as the summation of the utilities of all vehicles, and is given by

$$
\begin{equation*}
U_{t o t a l, \mathbf{n}}=\sum_{i=1}^{C} \Psi_{i} n_{i} r\left(n_{i}\right)=\sum_{i=1}^{C} \Psi_{i} f\left(n_{i}\right), \tag{12}
\end{equation*}
$$

where $\mathbf{n}=\left(n_{1}, n_{2}, \ldots, n_{C}\right)$. In the following, the NE of the spectrum access game is analyzed using uniform MAC and slotted ALOHA, respectively.

## C. Uniform MAC

The simplest way to share the channel among multiple users is to make each of them access the channel equally likely, which is referred as to uniform MAC [31]. Each vehicle starts a back-off with the back-off time randomly chosen from a fixed window. If one vehicle finds that its back-off expires and the channel is idle, it can capture the channel during the whole time slot, while others should keep silent. In uniform MAC, the resource allocation function $r(n)=\frac{1}{n}$, thus the utility function:

$$
U_{j\{\mathbf{u n i}\}}^{i}=\frac{\Psi_{i}}{n_{i}} .
$$

Note that $f_{\text {uni }}(n)=1$. It is shown in [31] that such a game using uniform MAC does have the pure NE. In proposition 1, we obtain the condition of the pure NE when uniform MAC is employed, and show that there may exist multiple NE-sets.

Proposition 1: For the spectrum access game $\Gamma$ using uniform MAC, if a congestion vector $\mathbf{n}=\left(n_{1}, n_{2}, \ldots, n_{C}\right)$ yields NE-set $(\mathbf{n})$, the following condition should be satisfied:

$$
\left\{\begin{array}{l}
n_{i}=\left\lceil\frac{\Psi_{i} N-\sum_{k \neq i, k \in \mathcal{C}} \Psi_{k}}{\sum_{k \in \mathcal{C}} \Psi_{k}}\right\rceil+W_{0}, i=1,2, \ldots, C  \tag{13}\\
\sum_{i=1}^{C} n_{i}=N
\end{array}\right.
$$

where $W_{0} \in\left\{0,1,2, \ldots,\left\lceil\frac{\Psi_{i}|N|+\Psi_{i}(|\mathcal{C}|-1)}{\sum_{k \in \mathcal{C}} \Psi_{k}}\right\rceil \quad-\right.$ $\left.\left\lceil\frac{\Psi_{i}|N|-\sum_{k \neq i, k \in \mathcal{C}} \Psi_{k}}{\sum_{k \in \mathcal{C}} \Psi_{k}}\right\rceil-1\right\}$. See the proof in Appendix A. From (13), it can be seen that there may exist more than one NE-set.

## D. Slotted ALOHA

Compared with uniform MAC, slotted ALOHA is a more typical MAC used in ad hoc networks, including VANETs. In slotted ALOHA, vehicles access the channel with probability $p$, and the throughput of each vehicle is $t h(p)=p(1-p)^{n-1}$. To maximize the throughput, let $t h^{\prime}(p)=0$. Then we get $p=$ $\frac{1}{n}$, and the resource allocation function using slotted ALOHA is:

$$
r_{\mathbf{S A}}(n)=\frac{1}{n}\left(1-\frac{1}{n}\right)^{n-1}
$$

It can be shown that for slotted ALOHA, $f_{\mathbf{S A}}(n)=(1-$ $\left.\frac{1}{n}\right)^{n-1}$, with $f_{\mathbf{S A}}^{\prime}(n)<0$ and $f_{\mathbf{S A}}^{\prime \prime}(n)>0$. (See the proof in [31]). Moreover, if $n$ goes to infinity, the total throughput of slotted ALOHA:

$$
\begin{equation*}
\lim _{n \rightarrow \infty} f_{\mathbf{S A}}(n)=\frac{1}{e} \tag{14}
\end{equation*}
$$

The utility of vehicle $j$ choosing channel $i$ using slotted ALOHA is given by:

$$
U_{j\{\mathbf{S A}\}}^{i}=\Psi_{i} \frac{1}{n_{i}}\left(1-\frac{1}{n_{i}}\right)^{n_{i}-1} .
$$

Different from uniform MAC, it is more difficult to derive the explicit condition of pure NE using slotted ALOHA. However, we show the existence of the pure NE and propose a scheme to achieve it.

Proposition 2: In the spectrum access game with vehicle set $\mathcal{N}$ and channel set $\mathcal{C}$, each vehicle sequentially chooses the access channel one by one. In each round, one vehicle chooses the best response to the strategies of the vehicles before it as the channel to access, i.e., its strategy in this game. Then, in each round, the strategy profile of the vehicles who have already made the decision is a pure NE.

The proof is given in Appendix B. Proposition 2 shows the existence of the pure NE in the spectrum access game $\Gamma$ when using slotted ALOHA and provides a simple way to achieve a pure NE. However, to better understand the utilization of the channel resource, the efficiency of different NEs should be analyzed.

## E. Efficiency Analysis

In the previous subsection, we prove the existence of pure $\mathrm{NE}(\mathrm{s})$ in the spectrum access game $\Gamma$. Generally speaking, an NE does not achieve global optimality due to the selfish behavior of the players. The efficiency of an NE is analyzed to evaluate the utilization of resources, which is defined as the total utility of all players under this NE. According to (12),
in the spectrum access game $\Gamma$, the efficiency of a pure NE is defined as:

$$
\mathcal{E}_{S}=\sum_{j=1}^{N} U_{j}^{i}=\sum_{j=1}^{N} \Psi_{s_{j}} r\left(n_{s_{j}}\right)=\sum_{i=1}^{C} \Psi_{i} n_{i} r\left(n_{i}\right),
$$

where $S$ is a strategy profile that is a pure NE.
The social optimality is defined as the maximum total utility of all player among all possible strategy profiles. For a specific game, the social optimality is fixed. It is proved in [31] that the social optimality in $\Gamma$ is:
$o p t_{\Gamma}=\left\{\begin{array}{lr}\sum_{i=1}^{N} \Psi_{i}, & \text { if } N \leq C ; \\ \sum_{i=1}^{C-1} \Psi_{i}+\sum_{i=C}^{N} \Psi_{C} r(N-C+1), & \text { if } N>C,\end{array}\right.$
where $\Psi_{i}$ is ordered such that $\Psi_{1} \geq \Psi_{2} \geq \cdots \geq \Psi_{C}$. Thus, to evaluate the efficiency of an NE, we define efficiency ratio (ER) of an NE as the ratio between the efficiency and the social optimality:

$$
E R_{S}=\frac{\mathcal{E}_{S}}{o p t_{\Gamma}}
$$

Different NE-sets may achieve different ERs. For example, in a game using uniform MAC with two channels $\left(\Psi_{1}=\right.$ 30 and $\Psi_{2}=10$ ) and three vehicles, there are two NE-sets, as shown in Table III, as well as their efficiency ratios. Obviously, NE-set ${ }_{2}$ is better than NE-set ${ }_{1}$ because it achieves a higher ER. In the following, the ER of the pure NE using uniform MAC and slotted ALOHA is discussed, respectively.

1) Uniform MAC: In uniform MAC, $f(n)=1$. Among multiple NE-sets with different congestion vectors, we can easily draw the following conclusions:

- A pure NE in which each channel is chosen by at least one vehicle has $E R=1$.
- For any two different NE-sets NE-set $1_{1}$ and NE-set ${ }_{2}$ in which not all channels are chosen, if

$$
\sum_{i=1}^{C} \Psi_{i} I_{i}^{1} \geq \sum_{i=1}^{C} \Psi_{i} I_{i}^{2}
$$

where $I_{i}^{j}$ is the indicator of whether channel $i$ is chosen in NE-set ${ }_{j}$, then $\mathrm{ER}_{1} \geq \mathrm{ER}_{2}$.
The proof is straightforward. When uniform MAC is employed, the efficiency equals the summation of the ESA of all channels that are selected, i.e., $\mathcal{E}_{S}=\sum_{i=1}^{C} \Psi_{i} I_{i}^{S}$. When all channels are selected, all resource is fully utilized, and therefore, $\mathrm{ER}=1$. Otherwise, the higher efficiency yields higher efficiency ratio since the social optimality is fixed for a specific game.
2) Slotted ALOHA: In slotted ALOHA, although there is no explicit relation between the congestion vector and ER, Corollary 1 can help to lead a pure NE with high ER.

Corollary 1: When Slotted ALOHA is used, in the process of composing a pure NE described in Proposition 2, the following rules can yield an NE with the highest efficiency ratio.

If in a round the new vehicle has two best responses $\left(\mathrm{BE}_{1}\right.$ and $\mathrm{BE}_{2}$ ),

- when $\mathrm{BE}_{1}$ corresponds to a vacant channel (no vehicle

Table III
Multiple NE-sets in A Game

|  | n1 | n2 | ER |
| :--- | :---: | :---: | :---: |
| NE-set $_{1}$ | 3 | 0 | 0.75 |
| NE-set $_{2}$ | 2 | 1 | 1 |

chooses it) and $\mathrm{BE}_{2}$ corresponds to a channel that has been already chosen, then $\mathrm{BE}_{1}$ is preferred;

- when each channel has been selected by at least one vehicle, the channel with higher ECA is preferred.
See the proof in Appendix C.


## F. Distributed Algorithms to Achieve NE with High ER

After spectrum sensing, each vehicle has the knowledge of the available channels $i \in \mathcal{C}$, and the ECA of each channel, i.e., $\Psi_{i}$. Vehicles maintain a sorted list of the channels in $\mathcal{C}$ in a decreasing order of $\Psi$. Then they participate in the distributed spectrum access game $\Gamma$. Since vehicles behave in a distributed manner in CR-VANETs, the best solution to the game is a pure NE in which each vehicle has no incentive to change its current choice of the access channel unilaterally. According to Proposition 2, the pure NE can be achieved by each vehicle choosing the best response sequentially. Moreover, based on the analysis of Section IV-E and Corollary 1, a pure NE with high ER can be achieved.

However, in such a process to achieve the NE, the vehicles who choose their strategy before others usually benefit more. For instance, in a game $\Gamma$ with two channels and two vehicles, and $\Psi_{1}=15$ and $\Psi_{2}=10$, the one making decision first could obtain utility of 15 while the other could only get 10 . To solve the problem, and achieve a pure NE with high ER in a distributed manner, we design a distributed cognitive spectrum access algorithm, as shown in Algorithm 1. Each vehicle will randomly select a back-off time and start the back-off. When the back-off timer expires, the vehicle chooses one channel to access according to the best response to the strategies of vehicles that have already chosen the channel. Then the vehicle broadcasts its decision in order for other vehicles to derive their strategies. Since the selection of the back-off time is random, the proposed algorithm is fair for each vehicle.

## V. Performance Evaluation

In this section, we evaluate the performance of the proposed congestion game based opportunistic spectrum access scheme. We consider an urban scenario with $10 \mathrm{~km} \times 10 \mathrm{~km}$, where PTs and vehicles coexist. There are five licensed channels each with bandwidth of 1 MHz which vehicles can access in an opportunistic manner. PTs operating on different channels are associated with different parameters, i.e., $R, L_{P}, \lambda_{i d l e}$, and $\lambda_{\text {busy }}$. The length of road segment $L$ is set to 100 m . Vehicles move in the area with a constant speed $v \in[10,30] \mathrm{m} / \mathrm{s}$. The probabilities of vehicles selecting a direction at the intersection are given by $P_{n}=P_{s}=P_{e}=P_{w}=0.25$. Denote by $T h_{j}$ the utility of vehicle $j$ by accessing the selected channel before the

```
Algorithm 1 Distributed Cognitive Spectrum Access Algo-
rithm
    // Initialization
    Get available channels }\mathcal{C}\mathrm{ by sensing.
    Update and order the channel availability
    [\Psi , , \Psi2,\ldots, \Psi \Psi ] decreasingly using (7) and (8).
    Consider the current time is }\mp@subsup{t}{s}{}\mathrm{ .
    Each vehicle that seeks for transmission opportunity picks
    a random back-off time tb from ( }0,\mp@subsup{t}{\mp@subsup{b}{_}{\prime}\mathrm{ max }}{}]\mathrm{ , and starts the
    back-off.
    while current time\leq ( }\mp@subsup{t}{s}{}+\mp@subsup{t}{\mp@subsup{b}{_}{\prime}\operatorname{max}}{})\mathrm{ do
        if The back-off timer of vehicle }i\mathrm{ expires then
            if uniform MAC then
                Select the best response with free channel consid-
                ering the strategies that it receives. If all channels
                have been already chosen, then select any best
                response.
            end if
            if Slotted ALOHA then
                Select the channel according to Corollary 1.
            end if
            Broadcast the channel sequence number that it
            chooses.
        end if
    end while
    Each vehicle tunes its radio to its strategic channel, and
    starts transmission using specific MAC.
    return
```

channel becomes unavailable, and the fairness index is defined as:

$$
F=\frac{\left(\sum_{j} T h_{j}\right)^{2}}{N \sum_{j} T h_{j}^{2}}
$$

which is used to evaluate the fairness among vehicles [32]. Specifically, we compare the proposed spectrum access (denoted by 'NE') with a random channel access (denoted by 'random') in which vehicles uniformly choose a channel from $\mathcal{C}$ to access.

Fig. 9 shows the impact of vehicle density on the road $\left(\rho_{v}\right)$ on the NE of the game, when uniform MAC is used. $\rho_{v}$ captures the average number of vehicles on the road with unit length. Define the channel selection indicator of channel $i$ as the ratio between the number of vehicles choosing channel $i$ and the total number of vehicles, i.e., $n_{i} / N$, which reflects the popularity of the channel. When the value of $\rho_{v}$ is small, some channels may not be chosen by any vehicle (e.g., channel 2 in Fig. 9 when $\rho_{v}$ is $20 / \mathrm{km}$ ). When the density of vehicles becomes higher, all channels are selected by at least one vehicle and the selection indicator of each channel also changes to satisfy the NE condition (Proposition 1).

Fig. 10 shows the performance of the proposed spectrum access scheme with respect to the vehicle speed $v$, when $\rho$ is set to $20 / \mathrm{km}$. From Fig. 10, it can be seen that 'NE' outperforms 'random' on average utility when using either uniform MAC or slotted ALOHA. This is because for uniform MAC, in 'NE', vehicles access the channels based on a pure


Figure 9. Impact of $\rho_{v}$ on Nash Equilibrium.


Figure 10. Performance w.r.t. speed. $\rho_{v}=20 / \mathrm{km}$.

NE while in 'random', each vehicle chooses a channel in random manner, which may result in lower average utility since there may exist some channels which are not selected by any vehicle. However, for slotted ALOHA, channels with larger ECA are chosen by more vehicles, resulting in more collisions. When vehicle density is $20 / \mathrm{km}$, the decrease of resource utilization caused by collisions is less than that caused by random access in which some channels are not utilized, which is the reason for that ' NE ' outperforms 'random' on average utility when using slotted ALOHA. The utility of both 'NE' and 'random' decreases with the increase of vehicle speed, because a higher speed leads to a smaller channel availability $\Psi$, and thus a smaller average utility.

Fig. 11 shows the performance of the proposed spectrum access scheme with respect to the vehicle density. From Fig. 11(a), it can be seen that the average utility decreases with the vehicle density. This is straightforward since the total channel resource is fixed and the resource allocation function $r(n)$ is decreasing with $n$. For uniform MAC, the reason that ' NE ' achieves higher average utility than 'random' is that in ' NE ', vehicles always choose channels with higher ECA while in 'random' vehicles randomly choose the access channel. When $\rho_{v}$ increases, the probability that all channels are chosen
by at least one channel increases. Note that if all channels are selected by at least one vehicle, the average utility of 'NE' and 'random' is the same. This fact explains the reason that the difference of average utility between ' $\mathrm{NE}_{\text {uni }}$ ' and 'random ${ }_{\text {uni }}$ ' becomes smaller when vehicle density increases. For slotted ALOHA, when $\rho_{v}$ increases, the average utility of 'NE' and 'random' also becomes closer, and the average utility of 'random' is slightly higher than that of 'NE' $\left(\rho_{v}>20\right.$ $/ \mathrm{km}$ ) because in ' NE ', the preference to choose channels with higher ECA results in more collisions.

Fig. 11(b) shows the fairness index of 'NE' and 'random' in terms of vehicle density. It can be seen that NE outperforms random access in terms of fairness in both uniform MAC and slotted ALOHA. This is because in a pure NE, the selfish property of vehicles leads to an even share of the spectrum resources. On the other hand, in random access, each vehicle randomly chooses a channel to access, which results in different utilities among vehicles. For ' $\mathrm{NE}_{\mathrm{sa}}$ ', when $\rho_{v}$ is low, the increase of $\rho_{v}$ may make vehicles choose channels with different ECA to achieve NE, resulting in the decrease of fairness. For example, two vehicles may both choose the channel with largest ECA, and the fairness index is 1 . When $\rho_{v}$ increases, a third vehicle may choose another channel, which makes the fairness index decrease. However, when $\rho_{v}$ is high, with all channels selected, the increase of the number of vehicles will make utilities among vehicles closer based on the NE condition. If the density is extremely high, from (14), the game using slotted ALOHA turns into a game using uniform MAC, with the channel bandwidth $\frac{1}{e}$ of the original bandwidth.

Fig. 12 shows the efficiency ratio of the obtained NE in the proposed spectrum access game $\Gamma$. We introduce the channel diversity, which is defined as

$$
\Phi=\sum_{i=1}^{C}\left(\Psi_{i}-\bar{\Psi}\right)^{2}
$$

where $\bar{\Psi}$ is the mean ECA of all channels. Channel diversity shows the variance among primary channels due to the properties of PTs, such as the spatial distribution and temporal channel usage pattern. A smaller value of $\Phi$ indicates that on average, the channels have relatively similar ECAs, and vice versa. Fig. 12(a) shows the ER with respect to vehicle density. It can be seen that 'NE' achieves a higher ER than 'random' by utilizing either uniform MAC or slotted ALOHA, because the total utility of 'NE' is higher, as shown in Fig. 11(a). The decrease of ER using random access when $\rho_{v}$ is low is because with the increase of $\rho_{v}$, the social optimality also increases. However, for 'random', more vehicles will not lead to as much increase in total utility as in social optimality. The reason for the increase of the ER of 'random uni ' when vehicle density becomes higher ( $\rho_{v} \geq 25 / \mathrm{km}$ ) is that the social optimality will not change (all channels are selected) with the increase of $\rho_{v}$, while the total utility increases due to that in expectation, more channels are chosen. In fact, it approaches to the ER of ' $\mathrm{NE}_{\text {uni }}$ ', which is not shown in the figure. The ER of 'random ${ }_{\text {sa }}$ ' changes slightly when vehicle density is high. This is because when $\rho_{v}$ is high, $f_{\mathbf{S A}}(n)$ changes very little


Figure 11. Performance w.r.t. vehicle density. Vehicle speed $v=10 \mathrm{~m} / \mathrm{s}$.
with $\rho_{v}$, and thus the total utility changes little due to (12). And with (14), when vehicle density is extremely high, we have

$$
\begin{equation*}
E R_{\mathbf{S A}} \rightarrow \frac{\frac{1}{e} \sum_{i=1}^{C} \Psi_{i}}{\sum_{i=1}^{C-1} \Psi_{i}+\frac{1}{e} \Psi_{C}} \tag{15}
\end{equation*}
$$

where (15) is the lower bound of $E R_{\mathbf{S A}}$ when vehicle density increases.

Fig. 12(b) shows the relationship between ER and the channel diversity $\Phi$. The ER of uniform MAC remains stable when $\Phi$ increases, because although the channels with smaller ECA are chosen less often, they have little impact on the ER since their ECA are small. However, for slotted ALOHA, the reason for the decrease of ER is two-fold: first, the channels with smaller ECA are rarely chosen; second, more vehicles choose channels with higher ECA, which results in more contentions and collisions. When $\Phi$ increases, more vehicles contend for the channels with high ECA, resulting in more collisions, and smaller value of ER.

## VI. CONCLUSION

In this paper, we have analyzed the channel availability for vehicles in urban CR-VANETs, jointly considering the mobil-


Figure 12. Efficiency ratio. $\bar{\Psi}=20$.
ity of vehicles, and the spatial distribution and the temporal channel usage pattern of PTs. We have then proposed a game theoretic spectrum access scheme to better exploit the spatial and temporal spectrum resource. Simulation results have demonstrated that the proposed scheme can achieve higher average utility and fairness than the random access scheme, when applying either uniform MAC or slotted ALOHA. The impact of vehicle density and channel diversity on the performance of the proposed scheme has also been studied. The research results can be applied for designing efficient spectrum sensing and access schemes in CR-VANETs. For future works, we will study the effect of sensing errors, and consider other usage patterns of PTs.

## Appendix

## A. NE condition for uniform MAC

First, the situation for two channels is considered. Since $C=2$ and $N \in \mathbb{Z}^{+}$, we have

$$
\frac{\Psi_{1}}{n_{1}} \geq \frac{\Psi_{2}}{n_{2}+1} \text { and } \frac{\Psi_{2}}{n_{2}} \geq \frac{\Psi_{1}}{n_{1}+1}
$$

which can be rewritten as follows:

$$
\begin{equation*}
\frac{\Psi_{1}}{\Psi_{2}} n_{2}-1 \leq n_{1} \leq \frac{\Psi_{1}}{\Psi_{2}} n_{2}+\frac{\Psi_{1}}{\Psi_{2}} \tag{16}
\end{equation*}
$$

Substitute $n_{2}=N-n_{1}$ into (16), we obtain

$$
\begin{equation*}
\frac{\Psi_{1} N-\Psi_{2}}{\Psi_{1}+\Psi_{2}} \leq n_{1} \leq \frac{\Psi_{1} N+\Psi_{1}}{\Psi_{1}+\Psi_{2}} \tag{17}
\end{equation*}
$$

Since

$$
\begin{equation*}
\frac{\Psi_{1} N+\Psi_{1}}{\Psi_{1}+\Psi_{2}}-\frac{\Psi_{1} N-\Psi_{2}}{\Psi_{1}+\Psi_{2}}=1 \tag{18}
\end{equation*}
$$

and

$$
\begin{equation*}
-1<\frac{\Psi_{1} N-\Psi_{2}}{\Psi_{1}+\Psi_{2}}<N \tag{19}
\end{equation*}
$$

$\Gamma$ has at least one pure NE, in which

$$
\begin{equation*}
n_{1}=\left\lceil\frac{\Psi_{1} N-\Psi_{2}}{\Psi_{1}+\Psi_{2}}\right\rceil \text { and } n_{2}=N-n_{1} \tag{20}
\end{equation*}
$$

Next, we extend this conclusion to the situation where more than two channels are available, i.e., $C>2$. When $C>2$, any two arbitrary channels $i$ and $k, i, k \in \mathcal{C}$ should satisfy (16) to constitute an NE. Thus,

$$
\begin{equation*}
\frac{\Psi_{k}}{\Psi_{i}} n_{i}-1 \leq n_{k} \leq \frac{\Psi_{k}}{\Psi_{i}} n_{i}+\frac{\Psi_{k}}{\Psi_{i}} \tag{21}
\end{equation*}
$$

Define $F_{L, k i}$ and $F_{U, k i}$ as

$$
F_{L, k i}=\frac{\Psi_{k}}{\Psi_{i}} n_{i}-1 \text { and } F_{U, k i}=\frac{\Psi_{k}}{\Psi_{i}} n_{i}+\frac{\Psi_{k}}{\Psi_{i}}
$$

Then, for channels $i$ and $\forall k \neq i, i, k \in \mathcal{C}$, we have

$$
F_{L, k i} \leq n_{k} \leq F_{U, k i}
$$

It holds that

$$
\begin{equation*}
\sum_{k \neq i, k \in \mathcal{C}} F_{L, k i} \leq \sum_{k \neq i, k \in \mathcal{C}} n_{k} \leq \sum_{k \neq i, k \in \mathcal{C}} F_{U, k i} \tag{22}
\end{equation*}
$$

By substituting $\sum_{k \neq i, k \in \mathcal{C}} n_{k}=N-n_{i}$ into (22), we have

$$
\begin{equation*}
\frac{\Psi_{i} N-\sum_{k \neq i, k \in \mathcal{C}} \Psi_{k}}{\sum_{k \in \mathcal{C}} \Psi_{k}} \leq n_{i} \leq \frac{\Psi_{i} N+\Psi_{i}(C-1)}{\sum_{k \in \mathcal{C}} \Psi_{k}} \tag{23}
\end{equation*}
$$

Similar to (18) and (19), it can be proved that

$$
\frac{\Psi_{i} N+\Psi_{i}(C-1)}{\sum_{k \in \mathcal{C}} \Psi_{k}}-\frac{\Psi_{i} N-\sum_{k \neq i, k \in \mathcal{C}} \Psi_{k}}{\sum_{k \in \mathcal{C}} \Psi_{k}}>1
$$

and

$$
-1<\frac{\Psi_{i} N-\sum_{k \neq i, k \in \mathcal{C}} \Psi_{k}}{\sum_{k \in \mathcal{C}} \Psi_{k}}<N .
$$

Then, for any $\mathcal{C}$ and $\mathcal{N}$, (8) has at least one solution, which is

$$
n_{i}=\left\lceil\frac{\Psi_{i} N-\sum_{k \neq i, k \in \mathcal{C}} \Psi_{k}}{\sum_{k \in \mathcal{C}} \Psi_{k}}\right\rceil+W_{0}
$$

where $W_{0} \in\left\{0,1,2, \ldots,\left\lceil\frac{\Psi_{i} N+\Psi_{i}(C-1)}{\sum_{k \in \mathcal{C}} \Psi_{k}}\right\rceil \quad-\right.$ $\left.\left\lceil\frac{\Psi_{i} N-\sum_{k \neq i, k \in \mathcal{C}} \Psi_{k}}{\sum_{k \in \mathcal{C}} \Psi_{k}}\right\rceil-1\right\}$. With $\sum_{i \in \mathcal{C}} n_{i}=N$, we have (13). Thus, the game $\Gamma$ has at least one pure NE. (13) is called NE condition of the spectrum access game $\Gamma$ when
uniform MAC is used.

## B. Proposition 2

Assume that for a given round $R_{t}$, the congestion vector $\mathbf{n}\left(S_{t}\right)=\left\{n_{1}, n_{2}, \ldots, n_{C}\right\}$ composes a pure NE. According to (8), for each channel $i \in \mathcal{C}$,

$$
\Psi_{i} r\left(n_{i}\right) \geq \Psi_{k} r\left(n_{k}+1\right), \quad \forall k \in \mathcal{C}, k \neq i
$$

Then for a new round $R_{t+1}$, a new vehicle joins the game and chooses its best response according to the existing strategy profile, i.e., $\mathbf{n}\left(S_{t}\right)$. Consider its best response is channel $m$, and thus the new congestion vector is $\mathbf{n}\left(S_{t+1}\right)=$ $\left\{n_{1}, \ldots, n_{m}+1, \ldots, n_{C}\right\}$. For the new congestion vector, we have the following observations:

1. For each channel $i \in \mathcal{C}, i \neq m, \Psi_{i} r\left(n_{i}\right) \geq \Psi_{k} r\left(n_{k}+\right.$ 1), $\forall k \in \mathcal{C} \backslash\{i, m\}$ holds because the number of vehicles that choose the channels other than channel $m$ does not change, and $r\left(n_{i}\right), i \neq m$ remains unchanged.
2. $\Psi_{m} r\left(n_{m}+1\right) \geq \Psi_{k} r\left(n_{k}+1\right), \forall k \in \mathcal{C}, k \neq m$. This statement holds due to that channel $m$ is the best response for the new vehicle.
3. $\Psi_{k} r\left(n_{k}\right) \geq \Psi_{m} r\left(n_{m}+1+1\right), \forall k \in \mathcal{C}, k \neq m$. Remember in round $t, \Psi_{k} r\left(n_{k}\right) \geq \Psi_{m} r\left(n_{m}+1\right) . r(n)$ is a nonincreasing function, and thus $r\left(n_{m}+1\right) \geq r\left(n_{m}+1+1\right)$.
Therefore, $\mathbf{n}\left(S_{t+1}\right)$ also constitutes a pure NE. For a specific game, the first vehicle chooses the channel with largest ECA and of course composes a pure NE. Then, for each round, the strategies of vehicles which have participated in the game constitute a new pure NE, until all vehicles have chosen their strategies.

## C. Corollary 1

For any round in proposition 2, assume that the congestion vector $\mathbf{n}(S)=\left\{n_{1}, n_{2}, \ldots, n_{C}\right\}$ constitutes a pure NE and $\Psi_{i}$ is sorted so that $\Psi_{1} \geq \Psi_{2} \geq \cdots \geq \Psi_{C}$. The efficiency of the NE is

$$
\mathcal{E}_{S}=\sum_{i=1}^{C} f\left(n_{i}\right)
$$

Remember that in slotted ALOHA, $f(n)=\left(1-\frac{1}{n}\right)^{n-1}$. A new vehicle comes and finds there are more than one best response (BR).

1) If $\mathrm{BR}_{1}$ corresponds to a free channel $i$ when $\mathrm{BR}_{2}$ corresponds to channel $j$ that has been selected by at least one vehicle, then $\mathrm{BR}_{1}$ leads to a NE with efficiency:

$$
\mathcal{E}_{S 1}=\mathcal{E}_{S}+\Psi_{i}>\mathcal{E}_{S}
$$

$\mathrm{BR}_{2}$ leads to a NE with efficiency:

$$
\mathcal{E}_{S 2}=\mathcal{E}_{S}-\Delta<\mathcal{E}_{S} .
$$

where $\Delta$ is the loss of $f\left(n_{j}\right)$ since $f(n)$ decreases with $n$. Obviously, $\mathcal{E}_{S 1}>\mathcal{E}_{S 2}$.
2) Consider that $\mathrm{BR}_{1}$ and $\mathrm{BR}_{2}$ correspond to channel $i$ and $j$ with $n_{i} \geq 1$ and $n_{j} \geq 1$, respectively. Without loss of generality, consider $\Psi_{i}>\Psi_{j}$. Under this condition, it is clear that $n_{i}>n_{j}$, or else channel $i$ and $j$ cannot be the
best response simultaneously. Consider only the total utility of users choosing channel $i$ and $j$ since other channels are not affected in this round. $\mathrm{BR}_{1}$ will lead to an NE with utility $\mathcal{E}_{1}=\Psi_{i} f\left(n_{i}+1\right)+\Psi_{j} f\left(n_{j}\right)$, while $\mathrm{BE}_{2}$ will lead to an NE with utility $\mathcal{E}_{2}=\Psi_{i} f\left(n_{i}\right)+\Psi_{j} f\left(n_{j}+1\right)$. Using the property of the pure NE, we have $\Psi_{i} r\left(n_{i}+1\right) \geq \Psi_{j} r\left(n_{j}+1\right)$ and $\Psi_{j} r\left(n_{j}+1\right) \geq \Psi_{i} r\left(n_{i}+1\right)$, and thus $\Psi_{i} r\left(n_{i}+1\right)=$ $\Psi_{j} r\left(n_{j}+1\right)$, i.e., $\Psi_{i} \frac{f\left(n_{i}+1\right)}{n_{i}+1}=\Psi_{j} \frac{f\left(n_{j}+1\right)}{n_{j}+1}$. Let

$$
\Psi_{i}=\frac{\frac{f\left(n_{j}+1\right)}{n_{j}+1}}{\frac{f\left(n_{i}+1\right)}{n_{i}+1}} \Psi_{j}=\alpha \Psi_{j} .
$$

To prove

$$
\begin{aligned}
& \mathcal{E}_{1}-\mathcal{E}_{2} \\
= & \Psi_{i} f\left(n_{i}+1\right)+\Psi_{j} f\left(n_{j}\right)-\left(\Psi_{i} f\left(n_{i}\right)+\Psi_{j} f\left(n_{j}+1\right)\right) \\
= & \Psi_{j}\left[\alpha\left(f\left(n_{i}+1\right)-f\left(n_{i}\right)\right)+f\left(n_{j}\right)-f\left(n_{j}+1\right)\right]>0,
\end{aligned}
$$

is equivalent to prove

$$
\alpha=\frac{\frac{f\left(n_{j}+1\right)}{n_{j}+1}}{\frac{f\left(n_{i}+1\right)}{n_{i}+1}}<\frac{f\left(n_{j}\right)-f\left(n_{j}+1\right)}{f\left(n_{i}\right)-f\left(n_{i}+1\right)},
$$

since $f(n)-f(n+1)>0$.

$$
\begin{align*}
& \frac{\frac{f\left(n_{j}+1\right)}{n_{j}+1}}{\frac{f\left(n_{i}+1\right)}{n_{i}+1}}<\frac{f\left(n_{j}\right)-f\left(n_{j}+1\right)}{f\left(n_{i}\right)-f\left(n_{i}+1\right)} \\
\Leftrightarrow & \frac{\frac{f\left(n_{j}+1\right)}{n_{j}+1}}{f\left(n_{j}\right)-f\left(n_{j}+1\right)}<\frac{\frac{f\left(n_{i}+1\right)}{n_{i}+1}}{f\left(n_{i}\right)-f\left(n_{i}+1\right)} \\
\Leftrightarrow & g(n)=\frac{\frac{f(n+1)}{n+1}}{f(n)-f(n+1)} \text { increasing with } n \geq 1 \\
\Leftrightarrow & g^{\prime}(n)>0, \text { when } n \geq 1 . \tag{24}
\end{align*}
$$

We skip the tedious proof of (24) to simplify the exposition. Then, we have $\mathcal{E}_{1}>\mathcal{E}_{2}$.

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