Frequency Domain Packet Scheduling with MIMO for 3GPP LTE Downlink

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***Abstract*—In this paper, we formalize a general Frequency Domain Packet Scheduling (FDPS) problem for 3GPP LTE Downlink (DL). The DL FDPS problem incorporates the Single-User Multiple Input Multiple Output (SU-MIMO) technique, and can express various scheduling policies, including the *Proportional-Fair* metric, the *MaxWeight* scheduling, etc. For LTE DL SU-MIMO, the constraint of selecting only one MIMO mode (*transmit diversity* or *spatial multiplexing*) per user in each transmission time interval (TTI) increases the hardness of the FDPS problem. We prove the problem is MAX SNP-hard, which implies approximation algorithms with constant approximation ratios are the best we can expect. Subsequently, we propose an approximation algorithm of polynomial runtime. The solution is based on a greedy method for maximizing a non-decreasing *submodular function* over a *matroid*. The algorithm can solve the general DL FDPS problem with an approximation ratio of 4. We implement the proposed algorithm and compare its performance with other well-known schedulers.**

***Index Terms*—Long Term Evolution (LTE), downlink (DL), frequency domain packet scheduling (FDPS), optimization algorithm, approximation ratio, submodular function, matroid.**

I. Introduction

THE Third Generation Partnership Project (3GPP) Long Term Evolution (LTE) standardization is the next forward step in cellular network. LTE can achieve a high peak-data-rate that scales with scalable system bandwidths, system capacity and coverage improvements, spectrum efficiency, latency reduction and packet optimized radio access. The architecture of LTE network is depicted in Figure 1. The main functionality of LTE is divided into three domains: User Equipment (UE), Evolved UMTS Terrestrial Radio Access Network (EUTRAN) and System Architecture Evolution (SAE) core, also known as Evolved Packet Core (EPC). For the Downlink (DL), LTE provides transmission speed up to 100 Mbps over a 20 MHz channel. Because of the robustness against multi-path fading, higher spectral efficiency and bandwidth scalability, the Orthogonal Frequency Division Multiplexing Access (OFDMA) has been selected for the LTE DL [1].

In LTE, the system bandwidth is divided into separable chunks denoted as resource blocks (RBs) [2] (see Figure 1).

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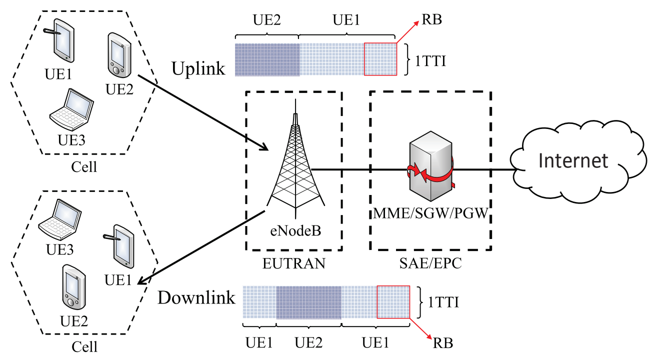

Fig. 1: The architecture of 3GPP LTE network.

An RB is considered as the minimum scheduling resolution in the time-frequency domain. The Frequency Domain Packet Scheduling (FDPS) allocates different RBs to individual users according to their current channel conditions, queue lengths and other information. The FDPS policy is conducted during each Transmission Time Interval (TTI, in LTE, 1TTI=1ms). The DL OFDMA is achieved by the FDPS assignment of different frequency portions of bandwidth, which simultaneously realizes frequency-domain multiplexing in concert with time-domain scheduling [1].

The Spatial Division Multiplexing (SDM) Multiple Input Multiple Output (MIMO) techniques form another essential part of LTE in order to accomplish the ambitious requirements for throughput and spectral efficiency. Resting with the spatial domain user selection over individual RBs, different MIMO schemes are incorporated in the 3GPP standard [3]. In the Single-User MIMO (SU-MIMO), the FDPS is restricted since at most one user can be scheduled over each RB. Moreover, all the RBs in one TTI (i.e. in one DL subframe) assigned to an individual user are transmitted either in *transmit diversity* mode or *spatial multiplexing* mode. The transmit diversity is an approach whereby information is spread across multiple transmit antennas to maximize the diversity advantage in fading channels. Spatial multiplexing, on the other hand, is an approach where the incoming data is divided into multiple substreams and each substream is transmitted on a different transmit antenna [4]. Both of the aforementioned MIMO modes can be utilized to obtain the most advantageous schedule. Although the Multi-User MIMO (MU-MIMO) provides greater spatial domain flexibility via allowing different users to be scheduled on different spatial streams over the same RB, we only focus on the SU-MIMO in this paper.

The LTE DL SU-MIMO FDPS problem has been addressed in some literatures. However, the selection of the scheduling policy for a specific LTE DL system depends on a case by case analysis and is not the focus of our work. In this paper, rather than a particular scheduling objective, we propose a universal solution for the LTE DL SU-MIMO FDPS problem. We define a general *profit function* which indicates the profit gained by allocating a set of RBs to an active user. The profit function is capable of expressing various scheduling objectives such as the *Proportional-Fair* metric in [1] that achieves good compromise between cell throughput and user fairness, the *MaxWeight* scheduling in [5] that combines utility maximization with queue stability, and so on. Based on the profit function, we formalize a general FDPS problem involving SU-MIMO enhancement for the LTE DL, which can cover many existing scheduling algorithms.

Furthermore, we address the hardness of the LTE DL SU-MIMO FDPS problem through an *L-Reduction* to a known problem. We prove the problem is MAX SNP-hard, which means that the problem is improbable to efficiently approximate within a certain ratio. Accordingly, the approximation algorithms with constant approximation ratios are the best solutions that we can expect. Afterwards, we provide an approximation algorithm for this problem in company with a provable constant approximation ratio of 4. We design the solution via a greedy method for maximizing a non-decreasing *submodular function* over a *matroid*. First, the necessary backgrounds of matroid and submodular function are briefly introduced. Second, we prove the *total profit function* as well as many other scheduling objectives are non-decreasing and submodular. Third, the 4-approximation algorithm is derived for the LTE DL SU-MIMO FDPS problem. We subsequently implement the proposed algorithm in Matlab and conduct comparative simulations to compare its performance with other well-known schedulers.

The remainder of the paper is organized as follows. Section II reports related works on the LTE DL FDPS problem. Section III describes the system model and formalizes the LTE DL SU-MIMO FDPS problem. Section IV proves the problem is MAX SNP-hard and shows that polynomial runtime approximation algorithms with guaranteed approximation ratios are necessary for practical LTE DL systems. In Section V, we propose the computable approximation algorithm in polynomial runtime and find its approximation ratio. We evaluate the performance and draw conclusions in Section VI and Section VII respectively. The necessary theoretic background of approximation algorithms and complexity utilized in this work is supplemented in the appendix.

II. Related Works

B.Sadiq *et al.* [6] present a general scheduling framework for LTE DL. They extend existing work on single-user queue and channel-aware schedulers to multiuser ones for wideband systems. In [7], A.Pokhariyal *et al.* investigate the performance of FDPS under Fractional-Load (FL) in LTE DL. The system-level simulations indicate that FDPS under FL can provide an effective trade-off between cell throughput and coverage. R.Kwan *et al.* [8] show that the system performance improves with increasing correlation among OFDMA subcarriers, while only a limited amount of feedback information is needed to achieve fairly good performance. J.Huang *et al.* [9] consider the resource allocation for the downlink of a cellular OFDM system, with various practical considerations including integer tone allocations, different sub-channelization schemes, maximum SNR constraint per tone, and *self-noise* due to channel estimation errors and phase noise. A gradient-based scheduling scheme is exploited to solve the optimization problem in each time-slot. M.Assaad *et al.* [10] propose two novel schedulers with respectively joint and separate implementation of scheduling and the adaptive modulation and coding (AMC).

As a promising technique, the MIMO mechanism is introduced to enhance the FDPS performance, which is evaluated by N.Wei *et al.* in [11]. In particular, N.Wei *et al.* address the performance of SDM MIMO techniques together with FDPS for the LTE downlink [12]. Using the proportional fair criterion, the combination of FDPS and SDM is studied. The results show that the combination achieves increased gains up to 20% with precoding in the macro-cell scenario while 35% in the micro-cell scenario. Z.Lin et al. [13] analyze the average channel capacity and the SINR distribution for LTE DL MU-MIMO systems. They investigate the SU-MIMO and the MU-MIMO schemes and show that, the outage probability for system in SU-MIMO is larger than the one in MU-MIMO. In addition, the system performance of the two dual-codeword SU-MIMO schemes in LTE DL is evaluated by E.Virtej *et al.* [14]. S.Donthi *et al.* [15] develop closed-form expressions for the throughput achieved by the feedback schemes of LTE. Their analysis quantifies the joint effects of three critical components on the overall system throughput, including the scheduler, the multiple-antenna mode, and the feedback scheme.

The LTE DL SU-MIMO FDPS problem has been recently addressed in [1]. S.Lee *et al.* prove the NP hardness of the optimal MIMO mode selection (spatial multiplexing or transmit diversity) per user in each TTI. Specifically, they develop two approximation algorithms to maximize the Proportional-Fair (PF) criterion extended to frequency and spatial domains. The presented algorithms are based on full-channel feedback and partial-channel feedback respectively, which both achieve an approximation ratio of 2. In their follow-on work [16], the FDPS problem is fully extended to incorporate with MU-MIMO, which also aims at optimizing the PF criterion. H.Zhang *et al.* [17] consider all the aforementioned practical constraints imposed by LTE standards and address the scheduling problem in practical LTE systems under two traffic models: backlogged traffic model and finite queue model.

The assumption of an infinitely backlogged model in scheduling analysis is conventional in the above literatures such as [1], which imply there are always arrival packets to serve. However, this is not always the case in practical systems. In reality, packets are generated for each user according to a stochastic arrival process with respect to applications. M.Andrews et al. point out that the Proportional-Fair scheduling does not work so well when the queues are fed by an admissible arrival process [5]. In particular, it can result in the instability of queues [18]. Therefore, it is preferable to take system utility maximization and queueing stability into account simultaneously when we design scheduling policies. The main results and contributions of [5] consist of generalization of the *MaxWeight* scheduling algorithm already well-known in the single-carrier setting. The algorithm is accommodated to a variety of scheduling algorithms for multi-carrier and frame-based wireless data systems. The NP hardness of these problems are stated and algorithmic solutions with provable

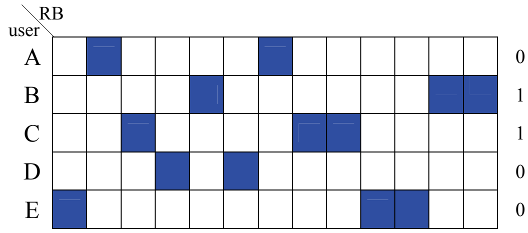

Fig. 2: A feasible SU-MIMO FDPS example for the LTE DL when m = 13 and n = 5. The blue colored (user, RB) pairs denote the RB-to-user assignment of a feasible schedule. Note that all the RBs assigned to each user belong to the same MIMO mode. 0 denotes the transmit diversity while 1 denotes the spatial multiplexing.

performance bounds are proposed. One of their algorithms has the approximation ratio of $1-\frac{1}{e}-\epsilon$ (for any $\epsilon>0$) while the other algorithms achieve at most a $\frac{1}{2}+\epsilon$ fraction of the optimal values. These conclusions are worthy of further investigation in different kinds of multi-carrier scenarios.

All of the existing literatures require case by case analysis and solution. In this paper, however, instead of a particular objective, we propose a general profit function to formalize the SU-MIMO LTE DL FDPS problem. The profit function can express various specific scheduling objectives, including the PF metric and the MaxWeight algorithms.

III. Problem Formalization

A. System Model

We consider the downlink of an LTE cellular network, where the system bandwidth is divided into m RBs. Besides, the network has a single base station and n active wireless users. We denote the set of all RBs by M ($M=\{1,2,\cdots,m\}$) and the set of all users by N ($N=\{1,2,\cdots,n\}$). With the LTE SU-MIMO support, we have two MIMO modes (transmit diversity and spatial multiplexing), which are represented by 0 and 1 respectively in the modes set L ($L=\{0,1\}$).

During each time slot, the base station allocates m RBs to n users. Specifically, each RB is assigned to at most one user, and all the RBs assigned to one user should belong to one MIMO mode. An example of a feasible SU-MIMO FDPS scheduling for the LTE DL is illustrated in Figure 2.

We denote by A the power set of M ($A=\mathcal{P}(M)$), i.e., the collection of all the subsets of M. $\forall a\in A$, we use the boolean variable x_i^a to indicate whether or not the set of RBs a is assigned to user i. User i gets the set $a\in A$ if and only if $x_i^a=1$.

We define the profit function $p(a,i,j)$ as follows[1],

$$p: A\times N\times L\rightarrow \mathbb{R}^{\geq 0} \tag{1}$$

where $p(a,i,j)$ indicates the profit gained by assigning $a\in A$ to user $i\in N$ with MIMO mode $j\in L$ in one feasible schedule (in one TTI). Moreover, we append the following assumptions to the profit function to ensure it is a *non-decreasing submodular* function. $\forall a\in A, i\in N, j\in L, a_1\subseteq a_2\subseteq M, b\in M$, we assume that,

$$p(a_1,i,j)\leq p(a_2,i,j) \tag{2}$$

$$p(\emptyset,i,j)=0 \tag{3}$$

[1]$\mathbb{R}^{\geq 0}$ denotes the non-negative real number set, i.e., $\forall a\in A, i\in N, j\in L, p(a,i,j)\geq 0$.

$$p(a_1\cup\{b\},i,j)-p(a_1,i,j)\geq p(a_2\cup\{b\},i,j)-p(a_2,i,j) \tag{4}$$

(2) indicates that, the more RBs are allocated to the user, the more profit he obtains, whereas allocating no RB delivers 0 profit to the user (in (3)). Via acquiring the same new RB, the profit increase will be larger in case that the fewer RBs are already been assigned (in (4)). Under these reasonable assumptions, the profit function can be considered as non-decreasing and submodular. In Section V, we develop an algorithm to maximize the *non-decreasing submodular* profit function over a *matroid*, which approximately solves the SU-MIMO FDPS problem in the LTE DL.

The most attractive point of profit function lies in its *generality*. We can use $p(a,i,j)$ to model various specific scheduling policies. In the following, we take the Proportional-Fair (PF) scheduling objective $\sum_{c\in a}\lambda_{i,j}^c$ studied in [1] as an example to demonstrate its generality.

Lemma 1: The profit function $p(a,i,j)$ can model the PF scheduling objective $\sum_{c\in a}\lambda_{i,j}^c$.

Proof: We define,

$$p(a,i,j)=\sum_{c\in a}\lambda_{i,j}^c \tag{5}$$

where $\lambda_{i,j}^c=\hat{r}_{i,j}^c/R_i$ is the PF metric value that user i achieves on RB c in MIMO mode j.

Since $\hat{r}_{i,j}^c\geq 0$ is the *current data rate*[2] and $R_i>0$ is the *average service rate*[3], then $\forall i,j,c,\lambda_{i,j}^c\geq 0$ and $\sum_{c\in a}\lambda_{i,j}^c\geq 0$.

Evidently, the more RBs are allocated, the $\sum_{c\in a}\lambda_{i,j}^c$ is larger. Thus $\forall a_1\subseteq a_2\subseteq M$, $\sum_{c\in a_1}\lambda_{i,j}^c\leq\sum_{c\in a_2}\lambda_{i,j}^c$.

If no RB is allocated, $\hat{r}_{i,j}^c=0$ and then $\lambda_{i,j}^c=0$. When the fewer RBs are already assigned, the current average service rate R_i is smaller and the PF metric is larger. Therefore, the increase between two PF objectives will be larger when the fewer RBs are already assigned. Namely, $\forall a_1\subseteq a_2\subseteq M, b\in M$, where a_1 and a_2 denote the assigned RBs and b denotes a new RB, we have,

$$\sum_{c\in a_1\cup\{b\}}\lambda_{i,j}^c-\sum_{c\in a_1}\lambda_{i,j}^c\geq\sum_{c\in a_2\cup\{b\}}\lambda_{i,j}^c-\sum_{c\in a_2}\lambda_{i,j}^c$$

So far, the PF objective $\sum_{c\in a}\lambda_{i,j}^c$ is proved to satisfy the assumptions (2)-(4) in turn, and hence we proof the lemma. ∎

The profit function is more general than the PF metric since the PF only represents the additive objective function. The profit function, however, involves both additive and non-additive situations. Furthermore, we can express the three objective functions in [5] as,

$$\begin{aligned} p(a,i,j)&=Q_i^s\sum_{c\in a}\hat{r}_{i,j}^c\\ p(a,i,j)&=Q_i^s\min\{Q_i^s,\sum_{c\in a}\hat{r}_{i,j}^c\}\\ p(a,i,j)&=(Q_i^s)^2-(\max\{0,Q_i^s-\sum_{c\in a}\hat{r}_{i,j}^c\})^2\end{aligned} \tag{6}$$

[2]The current data rate of user i is achieved on RB c in MIMO mode j within the present TTI.

[3]The average service rate is the data rate that a user has already been served in the past few TTIs, which is commonly achieved through an exponentially weighted moving average method.

where Q_i^s is the queue length for user i at the beginning of TTI, and $\hat{r}_{i,j}^c$ is the data rate for user i, RB c in MIMO mode j in this TTI. These three objective functions combine throughput maximization and queue stability together. We can similarly derive the same conclusions in Lemma 1 for the objectives in (6). So on the whole, the profit function might vary with different scheduling policies and still maintain its generality even though (2)-(4) appended.

B. LTE DL SU-MIMO FDPS

We consider a general SU-MIMO FDPS problem for the LTE DL system with m RBs and n users. In each TTI, for each set of RBs a ($a \in A$ and should be allocated in only one mode $j \in L$), we have a profit $p(a,i,j)$ for each user i. Our goal is to figure out a feasible FDPS solution in each TTI. More specifically, we intend to find the most advantageous way to assign a set a ($a \in A$) to user i in mode j so that the total profit is maximized. Thus the LTE DL SU-MIMO FDPS problem is formalized as the following combinatorial optimization problem.

$$\max_{j \in L} \sum_{(a,i) \in A \times N} p(a,i,j) \cdot x_i^a$$

$$\begin{aligned} &\text{subject to:} \\ &\text{for each RB } c \in M: && \sum_{i \in N, a: c \in a} x_i^a \le 1 \\ &\text{for each user } i \in N: && \sum_{a \in A} x_i^a \le 1 \\ &\forall i \in N, a \in A: && x_i^a \in \{0,1\} \end{aligned} \tag{7}$$

As a matter of fact, the objective function already contains the constraint that each user is scheduled in only one MIMO mode. Besides, the first constraint in (7) shows that every RB is assigned to at most one user, and the second constraint ensures each user gets no more than one set of RBs. Evidently, problem (7) is a binary integer programming and it is not hard to figure out the PF-FDPS problem studied in [1] is a special case of (7). The SU-MIMO FDPS algorithm aims at finding a subset of $A \times N \times L$ which maximizes the total profit in each TTI. Next, we analyze the hardness of (7) and design an approximation algorithm as its solution.

IV. Hardness Results

A. Hardness of (7)

It is straightforward to claim the LTE DL SU-MIMO FDPS problem (7) is NP-hard. S.Lee *et al.* [1] have shown that the LTE DL SU-MIMO PF-FDPS problem, a special case of LTE DL FDPS, is NP-hard. Obviously, if we reduce the LTE DL PF-FDPS problem to the problem (7) by setting $p(a,i) = \sum_{c \in a} \lambda_i^c$, the NP hardness of (7) is acquired as well.

In [5], M.Andrews *et al.* formalize three optimization objectives and design five corresponding algorithms in multi-carrier wireless data systems. In such systems, the queue size of user i at the beginning of time slot t is denoted by $Q_i^s(t)$, while the data rate for user i on RB c is denoted by $r(i,c,t)$. We consider the second objective in [5], which aims at keeping the stability properties of *MaxWeight* algorithm and avoiding unnecessary service to a user, i.e.,

$$max \quad \sum_i Q_i^s(t) min\{Q_i^s(t), \mu(i,t)\} \tag{8}$$

where $\mu(i,t) = \sum_c r(i,c,t)x(i,c,t)$ represents the amount of service that user i receives at time slot t.

The hardness of (8) is proved through a reduction from a known problem in [19] similar to the reduction technique utilized in [20]. More formally, the hardness results of (8) in [5] is quoted below.

Lemma 2: For some $\delta > 0$, there is no $(1-\delta)$-approximation algorithm for objective (8) unless P=NP.

Since Lemma 2 actually shows there is no PTAS for (8), it indicates the MAX-SNP hardness of (8) in another way. Consequently, we can derive the MAX-SNP hardness of problem (7) via a similar reduction from the known problem (8). We demonstrate that hardness result in Theorem 1.

Theorem 1: The LTE DL SU-MIMO FDPS problem (7) is MAX SNP-hard, and accordingly it does not have a PTAS assuming $P \neq NP$.

Proof: Based on Lemma 5, to prove that a problem $\mathcal{B}$ is MAX SNP-hard, it suffices to present an L-Reduction from a known MAX SNP-hard problem $\mathcal{A}$ to $\mathcal{B}$. Accordingly, we prove this theorem by an L-Reduction from problem (8) to the LTE DL SU-MIMO FDPS problem (7).

To simplify notations, all the dependence on TTI t of (8) is ignored hereafter. The n queues corresponding to n users at the beginning of each TTI are hence denoted by $Q_1^s, Q_2^s, \cdots, Q_n^s$. The $r(i,c)$ indicates the data rate for user i on RB c. Now we start to construct the mapping relationship between instances via function R. Assume that X is an instance of problem (8). An instance of problem (7) is expressed by $R(X)$, which describes an LTE DL system with n users and m RBs using two feasible MIMO modes. $N = \{1, 2, \cdots n\}$ is the set of active users. A is the power set of all RBs such that $\forall a \in A$, a is the subset of RBs allocated to one user. Chosen a MIMO mode (take the transmit diversity for example), the profit function $p: A \times N \rightarrow \mathbb{R}^{\geq 0}$ is anew defined as follows.

$$p(a,i) = \begin{cases} Q_i^s min\{Q_i^s, \sum_{a:c \in a} r(i,c)\} & \forall c \in a \text{ in mode } 0, \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

As for problem (8), the profit acquired by user i with queue length Q_i^s is $p(a,i)$, as long as $\forall c \in a$ are scheduled in MIMO mode 0 (0 stands for transmit diversity). Otherwise, the profit is set to 0 when the user is scheduled in mode 1 (1 stands for namely spatial multiplexing). Similarly, provided that mode 1 is chosen, the profit $p(a,i)$ can be also defined. So far, the instance mapping from (8) to (7) is complete.

Afterwards, we establish the mapping relationship between solutions. Assume that s is a feasible solution of $R(X)$. Evidently, s indicates the set of RBs allocated to each user consisting of the boolean variable x_i^a. According to the renewed definition of the profit function in (9), the feasible solution of $R(X)$ can be decomposed directly to feasible solution $S(s)$ for X with equal VALs. As a result, the mapping function $S(s)$ is defined through x_i^c, which denotes whether one RB c of the set a is scheduled or not for problem (8).

$$x_i^c = \begin{cases} 1 & \forall c \in a \text{ when } x_i^a = 1, \\ 0 & \text{otherwise.} \end{cases} \tag{10}$$

Next we need to check if (25) and (26) hold for mapping R and S respectively. To validate (25), we assume s_0 is the optimal solution of $R(X)$. Apparently,

$$VAL(s) = OPT(R(X)) \tag{11}$$

Since $S(s_0)$ is a decomposed feasible solution of X, and all the profits are set to 0 if scheduling is made with mode 1, we have

$$VAL(s) = VAL(S(s_0)) \leq OPT(X) \tag{12}$$

Therefore, we obtain $OPT(R(X)) \leq OPT(X)$. So the inequality (25) holds when $\alpha = 1$.

Combining (11) and (12), we have $OPT(X) - VAL(S(s)) \leq OPT(R(X)) - VAL(s)$. So (26) holds when $\beta = 1$.

In other words, (R, S) is an L-Reduction from (8) to (7). Since problem (8) is MAX SNP-hard, the LTE DL SU-MIMO FDPS problem (7) is also MAX SNP-hard and it does not have a PTAS unless $P = NP$. ■

Theorem 1 addresses the nonexistence of PTAS, which implies that for some constant $\delta > 0$, there are no polynomial time $(1+\delta)$-approximation algorithms for the LTE DL SU-MIMO FDPS problem (7) unless $P = NP$. In other words, we could at most hope for approximation algorithms with constant approximation ratios[4].

B. Search Space of (7)

In this section, we calculate the number of feasible schedules and estimate the running time of the *exhaustive search*. Suppose the LTE DL system has m RBs and n active users. According to (5), we only need to cover the situation that all the RBs are used up. Assume that in a feasible schedule without MIMO enhancement, at most k out of n users are supplied with m RBs ($k \leq min(n, m)$). $f_1, f_2, \cdots, f_k$ denote the number of scheduling policies when all the m RBs are assigned to $1, 2, \cdots, k$ users, respectively, which yields (13),

$$\binom{k}{k} \cdot f_k + \binom{k}{k-1} \cdot f_{k-1} + \cdots \binom{k}{1} f_1 = k^m \tag{13}$$

The right part of (13) indicates that, since each RB has k choices, the total options are k^m. It is straightforward to obtain $f_1 = 1$ (all the m RBs are distributed to one user). Similarly, we can obtain $f_2 = 2^m - 2, f_3 = 3^m - 3 \cdot 2^m + 3, \cdots$, and iteratively derive $f_{min}(n, m)$. Specifically, $f_k = 0$ whenever $k > min(n, m)$.

Taking the MIMO enhancement into account, the search space F of the LTE DL SU-MIMO FDPS problem (7) is computed eventually,

$$\begin{aligned} F = \ & \binom{n}{1} \cdot f_1 \cdot 2 + \binom{n}{2} \cdot f_2 \cdot 2^2 + \cdots \\ & + \binom{n}{\min(n,m)} \cdot f_{\min(n,m)} \cdot 2^{\min(n,m)} \end{aligned}$$

where $F > min(n, m)^m$ obviously.

In practical systems, the 3GPP LTE Release 8 [21] specifies the set of allowed values for m. Here, the set is given as $m \in \{25, 50, 75, 100\}$. Meanwhile, it is common that dozens of active users coexist in a cell. Provided that $n = 10, m = 25$, then we have,

$$F > \min(n, m)^m = 10^{25}$$

If it takes 1×10^{-9}s to check one feasible schedule scheme. The giant running time of an exhaustive search ($> 1 \times 10^{16}$s$\gg$TTI=1×10^{-3}s) is entirely unacceptable.

V. A Greedy Approximation Algorithm

In this section, we design a greedy approximation algorithm for the LTE DL SU-MIMO FDPS problem (7), which is based on maximizing a non-decreasing *submodular function* over a *matroid*. First, the necessary backgrounds of matroid and submodular function are briefly introduced. Second, we compute the total profit which is the objective of (7), and prove that objective function as well as many other scheduling metrics are non-decreasing submodular functions. Third, we prove the algorithm approximately solves problem (7) with ratio 4, which invokes a sub-algorithm to maximize the submodular objective function.

A. Matroid and Submodular Function

A rich variety of combinatorial optimization problems can be modeled as the maximization of submodular functions over a matroid [22]. We begin with the definition of *matroid* [5] [23].

Definition 1: Let S be a finite ground set and $\mathcal{L}$ be a nonempty collection of subsets of S. The ordered pair $(S, \mathcal{L})$ is called a *matroid* if:

1) $\emptyset \in \mathcal{L}$.
2) If $A \in \mathcal{L}$ and $B \subseteq A$, then $B \in \mathcal{L}$.
3) If $A \in \mathcal{L}$, $B \in \mathcal{L}$, and $|A| > |B|$, then there exists an element $x \in A \setminus B$ such that $B \cup \{x\} \in \mathcal{L}$.

The members of $\mathcal{L}$ are called the *independent* subset [22] of S. Furthermore, the conditions 2) and 3) indicate the inheritance of $\mathcal{L}$ and the exchangeability of M [23], respectively.

A special case of matroid is the *partition matroid* [5]. The matroid $(S, \mathcal{L})$ is called a partition matroid if there is partition of S into components $\{\Gamma_1, \Gamma_2, \dots\}$ [5] such that,

$$A \in \mathcal{L} \Longleftrightarrow \forall k, |A \cap \Gamma_k| \leq 1$$

Subsequently, we present the definition of *submodular set function* [22] [5].

Definition 2: Given a matroid $M = (S, \mathcal{L})$, let f$(\cdot)$ be a function on sets in $\mathcal{L}$. We say f$(\cdot)$ is *submodular* if $\forall a \in S, A \in \mathcal{L}, B \in \mathcal{L}$, $B \subseteq A$ and $(A \cup \{a\}) \in \mathcal{L}$,

$$f(A \cup \{a\}) - f(A) \leq f(B \cup \{a\}) - f(B) \tag{14}$$

where the *incremental value* of adding element a to the set A is often used,

$$\rho_a(A) = f(A \cup \{a\}) - f(A)$$

In addition, we say the submodular function f$(\cdot)$ is *non-decreasing* if,

$$f(\emptyset) = 0 \qquad \text{and} \qquad \rho_a(A) \geq 0.$$

[4]For PTAS, we can find out a $(1+\epsilon)$-approximation algorithm computable in polynomial time for any $\epsilon > 0$.

[5]$\{\Gamma_1, \Gamma_2, \dots\}$ is a partition of S if $\cup_i \Gamma_i = S$ and $\Gamma_i \cap \Gamma_j = \emptyset$, $\forall i \neq j$.

Besides, [22] addresses that the following three statements are equivalent to define a non-decreasing submodular set function f(·). $\forall a \in S, A \in \mathcal{L}, B \in \mathcal{L}, B \subseteq A$,

$$\begin{cases} f(A) + f(B) \geq f(A \cup B) + f(A \cap B) \\ \qquad\qquad f(A) \geq f(B) \end{cases} \tag{15}$$

$$\rho_a(B) \geq \rho_a(A) \geq 0 \tag{16}$$

$$f(A) \leq f(B) + \sum_{x \in A \setminus B} \rho_x(B) \tag{17}$$

Now that the *matroid* and the *submodular function* are well defined, we turn to the optimization problem. A generalized optimization problem of maximizing the submodular function over a matroid is established in [24], i.e.,

$$\begin{aligned} max_{A \subseteq S}\{f(A) : A \in \mathcal{L}, M = (S, \mathcal{L}) \text{ is a matroid,} \\ f(A) \text{ is submodular and non-decreasing}\} \end{aligned} \tag{18}$$

In [24], a greedy solution is also proposed for the problem (18), which is described in Lemma 3.

Lemma 3: The Greedy-Sub algorithm is a 2-approximation algorithm to maximize the non-decreasing submodular function $f(\cdot)$ over a matroid $(S, \mathcal{L})$. Moreover, the approximation ratio 2 is tight.

Algorithm 1 GREEDY-SUB

input S, $\mathcal{L}$, $f(\cdot)$
$A \leftarrow \emptyset$
while $\exists\ a \in S$ such that $A \cup \{a\} \in \mathcal{L}$ **do**
  $a \leftarrow \underset{a \in S, A \cup \{a\} \in \mathcal{L}}{\arg\max} \ (f(A \cup \{a\}) - f(A))$
  $A \leftarrow A \cup \{a\}$
end while
return A and $f(A)$

B. Non-decreasing Submodular Objective Function

To utilize Lemma 3, we illuminate the objective function of problem (7) as well as many other scheduling objectives are non-decreasing and submodular functions.

Recall that $M = \{1, 2, \cdots, m\}$ is the set of all RBs while $N = \{1, 2, \cdots, n\}$ is the set of all users. Let $S = M \times N$ be the ground set. Without MIMO mechanism, a valid schedule can be denoted by $\{(a_1, 1), (a_2, 2), \cdots, (a_n, n)\}$, where $a_i \in \mathcal{P}(M)$, $\forall i, j, a_i \cap a_j = \emptyset$ and (a_i, i) indicates the set of RBs a_i is assigned to user i ($i \in N$). If we refine a_i by $a_i = \{b_{i1}, b_{i2}, \cdots, b_{ik}, \cdots\}$, where b_{ik} indicates the kth element in the set a_i allocated to user i. Then the valid schedule can be rewritten,

$$\begin{aligned} \{&(a_{11}, 1), (a_{12}, 1), \ldots, (a_{21}, 2), (a_{22}, 2), \ldots, \\ &(a_{n1}, n), (a_{n2}, n), \ldots\} \end{aligned} \tag{19}$$

Let $\mathcal{L}$ be the collection of all valid schedules for the LTE DL without MIMO enhancement. The representations of elements in $\mathcal{L}$ comply with (19). Evidently, the ordered pair $(S, \mathcal{L})$ is a matroid according to Definition 1. Then $\exists\ l \in \mathcal{L}$ such that,

$$\begin{aligned} l = \{&(a_{11}, 1), (a_{12}, 1), \ldots, (a_{21}, 2), (a_{22}, 2), \ldots, \\ &(a_{n1}, n), (a_{n2}, n), \ldots\} \\ = \{&(a_1, 1), (a_2, 2), \ldots (a_n, n)\}, \end{aligned}$$

and functions p_0, p_1 can be defined on $\mathcal{L}$,

$$\begin{aligned} p_0(l) &= \sum_{i=1}^{n} p(a_i, i, 0) \\ p_1(l) &= \sum_{i=1}^{n} p(a_i, i, 1) \end{aligned} \tag{20}$$

where $p(a_i, i, 0)$ is the profit attained when a_i is scheduled to user i in MIMO mode 0 (transmit diversity). Likewise, the $p(a_i, i, 1)$ is the profit in MIMO mode 1 (spatial multiplexing).

Obviously, $p_0(l)$ and $p_1(l)$ indicate the total profits gained via some valid schedule l in different MIMO modes respectively (all the users belong to mode 0 or all belong to mode 1). We demonstrate they are non-decreasing and submodular in the following lemma.

Lemma 4: p_0 and p_1 defined in (20) are non-decreasing submodular functions over $\mathcal{L}$.

Proof: We only need to prove the lemma for p_0 since p_1 is defined similarly.

Suppose $l_1 \in \mathcal{L}$ and $l_2 \in \mathcal{L}$ are two valid schedule and $l_1 \subseteq l_2$. That is to say, there is at least one user u is provided with set of RBs such that $(a_u^1, u) \in l_1$, $(a_u^2, u) \in l_2$ and $a_u^1 \subseteq a_u^2$, whereas all the other users could be the same. Based on (4) for a single user, $\forall (a, v) \in S$, we have,

$$\begin{aligned} &p_0(l_1 \cup \{a, v\}) - p_0(l_2 \cup \{(a, v)\}) \\ &= p(a_1^1, 1, 0) + p(a_2^1, 2, 0) \cdots + p(a_v^1 \cup \{a\}, v, 0) \cdots \\ &\quad - \ p(a_1^2, 1, 0) - p(a_2^2, 2, 0) \cdots - p(a_v^2 \cup \{a\}, v, 0) \cdots \\ &\geq p(a_1^1, 1, 0) + p(a_2^1, 2, 0) \cdots + p(a_v^1, v, 0) \cdots \\ &\quad - \ p(a_1^2, 1, 0) - p(a_2^2, 2, 0) \cdots - p(a_v^2, v, 0) \cdots \\ &= p_0(l_1) - p_0(l_2) \end{aligned}$$

Hence the inequality can be rewritten as,

$$p_0(l_1 \cup \{a, v\}) - p_0(l_1) \geq p_0(l_2 \cup \{(a, v)\}) - p_0(l_2) \tag{21}$$

which satisfies the submodular requirement. Meanwhile, based on the non-decreasing proposition (2) for a single user, it is straightforward to deduce that,

$$\begin{aligned} &p_0(l_2) \\ &= p(a_1^2, 1, 0) + p(a_2^2, 2, 0) \cdots + p(a_v^2, v, 0) \cdots \\ &\leq p(a_1^2, 1, 0) + p(a_2^2, 2, 0) \cdots + p(a_v^2 \cup \{a\}, v, 0) \cdots \\ &= p_0(l_2 \cup \{(a, v)\}) \end{aligned} \tag{22}$$

which implies the non-decreasing property. Besides,

$$p_0(\emptyset) = \sum_{i=1}^{n} p(\emptyset, i, 0) = 0 \tag{23}$$

Combine (21), (22) and (23), we prove the lemma. ■

Based on the discussions in Lemma 1, we can also consider the other objective functions like the *Proportional-Fair* metric in [1] and *MaxWeight* scheduling in [5] are non-decreasing submodular functions. This accounts for the generalized representativeness of the profit function as well.

Thereby, Algorithm 1 is applicable to the LTE DL FDPS problem without MIMO. When we try to optimize (7) without MIMO enhancement, we are trying to find an assignment (which corresponds to an element of a partition matroid) that maximizes a submodular function.

C. Integrated Solution for (7)

Now we take the SU-MIMO mechanism into account to design the integrated algorithm for problem (7). The algorithm takes the inputs consisting of the user collection N, the RB collection M and the profit function $p(a,i,j)$. In fact, the algorithm invokes Algorithm 1 in succession to output A_0 and A_1, which are feasible schedules of problem (7). The total profit values of the schedules are $p_0(A_0)$ and $p_1(A_1)$. The algorithm chooses the larger value and its corresponding schedule as the output.

Algorithm 2 GREEDY-DOWNLINK (G-D for short)

input N, M, p
$S \leftarrow M \times N$, $\mathcal{L} \leftarrow \mathcal{P}(S)$, $p_0 \leftarrow \sum_{i\in N} p(a_i,i,0)$, $p_1 \leftarrow \sum_{i\in N} p(a_i,i,1)$
$[A_0,\ p_0(A_0)] \leftarrow$ GREEDY-SUB$(S,\mathcal{L},p_0)$ // invoke the Algorithm 1
$[A_1,\ p_1(A_1)] \leftarrow$ GREEDY-SUB$(S,\mathcal{L},p_1)$
$j \leftarrow \underset{i\in\{0,1\}}{\arg\max}(p_i(A_i))$
return A_j, $p_j(A_j)$, j

Subsequently, we demonstrate Algorithm 2 approximately solves problem (7) with constant ratio.

Theorem 2: Algorithm 2 is a 4-approximation algorithm for the LTE DL SU-MIMO FDPS problem (7).

Proof: We first denote the optimal schedule for problem (7) as follows,

$$\{(a_1^*,1,j_1^*),(a_2^*,2,j_2^*),\ldots,(a_n^*,n,j_n^*)\}$$

where (a_k^*,k,j_k^*) implies the set of RBs a_k^* is assigned to user k in MIMO mode j_k^*.

Let $p^* = \sum_{i=1}^n p(a_i^*,i,j_i^*)$ be the optimal total profit gained. Consider the dual scheme (all schedules are in the opposite MIMO mode) of the optimal schedule,

$$\{(a_1^*,1,1-j_1^*),(a_2^*,2,1-j_2^*),\ldots,(a_n^*,n,1-j_n^*)\}$$

and obtain the total profit $\sum_{i=1}^n p(a_i^*,i,1-j_i^*) \geq 0$. Therefore we have,

$$\begin{aligned} p^* &\leq \sum_{i=1}^n p(a_i^*,i,j_i^*) + p(a_i^*,i,1-j_i^*) \\ &= \sum_{i=1}^n p(a_i^*,i,0) + p(a_i^*,i,1) \end{aligned}$$

Recall that in Lemma 3, Algorithm 1 is a 2-approximation algorithm for maximizing a non-decreasing submodular function over a matroid. Accordingly, we have,

$$\begin{aligned} p^* &\leq \sum_{i=1}^n p(a_i^*,i,0) + p(a_i^*,i,1) \\ &\leq 2p_0(A_0) + 2p_1(A_1) \\ &\leq 4\max(p_0(A_0),p_1(A_1)) \end{aligned}$$

where $\max(p_0(A_0),p_1(A_1))$ is the return profit value of Algorithm 2. Thus we prove the theorem. ∎

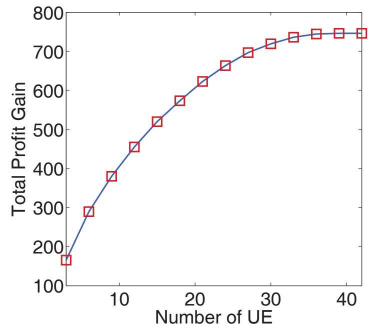

Fig. 3: The numerical example of the Greedy-Downlink algorithm.

TABLE I: Simulation Parameters

Parameter	Value
Frequency	2.0 GHz
Bandwidth	20 MHz
MIMO Antennas	2×2
SU-MIMO Mode	Transmit Diversity/Spatial Multiplexing
Simulation Length	10000 TTIs
Number of Users	10/20/30/40/50
User Speed	5/50 KM/h
eNodeB Transmit Power	49 dBm
Uplink Feedback Channel Delay	0 TTI

D. Time Complexity of the Algorithm

The time complexity of the Algorithm 2 depends on that of Algorithm 1, which operates one after another.

The Algorithm 1 assigns one RB to some user in each loop, as long as there is still RB available. In the kth loop, the algorithm need to check another $n \times (m-(k-1))$ ordered pairs (a,i) $(a \in \mathcal{P}(M), i \in N)$, since $k-1$ out of m RBs have been already scheduled. So the total running time of Algorithm 1 can be calculated as,

$$\sum_{j=1}^{m} n(m-j+1) = O(nm^2)$$

In consequence, the time complexity of the Algorithm 2 denoted by T_{G-D} is,

$$T_{G\text{-}D} = O(nm^2). \tag{24}$$

The time complexity of Algorithm 1 in [1] that Lee et al. have proposed is calculated as $O(nm)$. It is indicated that for both algorithms, along with the increasing user number n, the time complexities of them increase linearly given a fixed number of resource blocks m. Since in most cases the number of resource block in a TTI is relatively small, e.g., $m = 25$, the time complexity of Algorithm 2 is still acceptable compared to that in [1].

VI. Performance Evaluation

The Algorithm 2 is implemented and evaluated in the MATLAB simulator. We first give numerical example to illustrate the total profit scheduled by Algorithm 2. Then, we perform simulations for Algorithm 2 and other well-known schedulers.

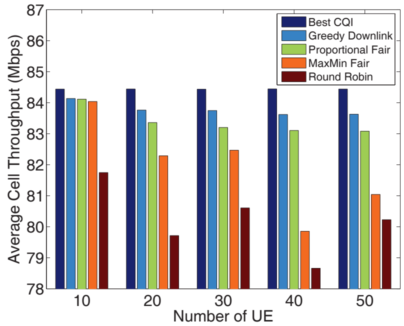

Fig. 4: The average cell throughput of all the downloading UEs. (User speed is 5 km/h.)

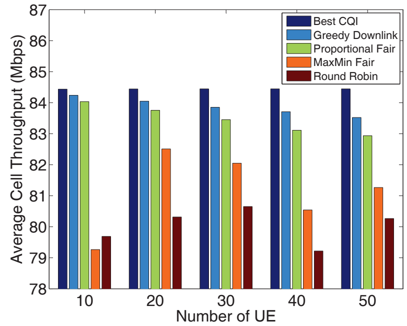

Fig. 5: The average cell throughput of all the downloading UEs. (User speed is 50 km/h.)

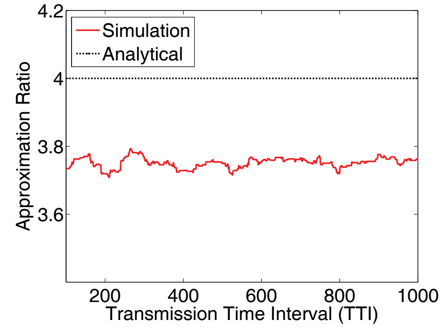

Fig. 6: The approximation ratio between the Optimal and the the Algorithm 2.

A. Numerical Example

The numerical example is acquired in one TTI. Namely, we calculate the numerical values in one schedule. Assume there are N active users that communicate with the eNodeB through 80 RBs in one TTI. The profit value of each RB is randomly generated between 1 and 10, thus the upper bound of the total profit is 800. We run the Algorithm 2 and compute the total profit in different cases of N (3, 6, 9, 12 and so on). As illustrated in Figure 3, the total profit is approximating the upper bound when N becomes larger, especially when it is larger than 36. The numerical example shows the system can approximate the optimal profit when it is scheduled by Algorithm 2.

B. Comparative Simulations

Here, we utilize the LTE DL system-level simulator [25], which is an open platform for academic research. On that platform, we deploy the Algorithm 2 and perform simulations for the Greedy-Downlink (Algorithm 2) scheduler and other well-known schedulers, including Proportional-Fair (PF), Max-Min fairness, Best CQI and Round Robin (RR). Some simulation parameters are summarized in Table I. To implement Algorithm 2, the profit function is specified as the current data rate, namely, $p(a,i,j) = \sum_{c\in a} \hat{r}^c_{i,j}$, where $\hat{r}^c_{i,j}$ is the current data rate of user i on RB c in MIMO mode j within the present TTI. Moreover, $p(a,i,j)$ can also be specified as other scheduling objectives in the implementation. For example, when throughput and fairness are simultaneously considered, we can set $p(a,i,j) = \sum_{c\in a} \lambda^c_{i,j}$ like (5), where $\lambda^c_{i,j}$ is the proportional fair metric value that user i achieves on RB c in MIMO mode j. In other words, the profit function is verified to represent various scheduling schemes.

We carry out five similar simulations and calculate the average *cell throughput*[6] for each scheduler. The results are provided with the average values of a 10000 TTIs duration for each number of UEs. Two kinds of user speed are set, i.e., the walking mode (5 km/h) and the vehicular mode (50 km/h), respectively. As illustrated in Figures 4 and 5, the RR and Best CQI (always schedules user at the best channel) algorithms are the lower and upper bounds of all schedulers in terms of system performance (represented by cell throughput in this case), respectively. The performance of Greedy-Downlink Algorithm 2 is between the PF and the upper bound, which indicates its good performance. When the user speed is set to 50 km/h, due to influence of serious channel fading, the average cell throughput of all schedulers are degraded (Figure 5). However, in this case, the Algorithm 2 still achieves better performance than the PF scheduler.

[6]The cell throughput is the total throughput of all the downloading UEs that an eNodeB covers.

C. Approximation Ratio

Furthermore, we present the approximation ratio between the Optimal and the the Algorithm 2, in terms of simulated and analytical values respectively. In each TTI, the optimal objective value of the problem (7) is calculated through an exhausted search. Then the optimal value is divided by the simulation value of problem (7) when Algorithm 2 is performed, and thereby we obtain the simulated approximation ratio. The simulated result shown in Figure 6 (solid line) with the theoretical approximation ratio of 4 (dash line) indicates that the worst case of 4 can be a fairly tight performance bound for the proposed Algorithm 2.

VII. Conclusion

In this paper, we have studied a general FDPS problem for the LTE DL. As one of the promising techniques, the MIMO is incorporated in the LTE DL to enhance the system performance. We have focused on the SU-MIMO, which involves two constrains for the FDPS. Namely, each RB can be assigned to at most one user while all the RBs assigned to one user should belong to the same MIMO mode, either *transmit diversity* or *spatial multiplexing*.

Instead of a specific scheduling objective, we have proposed a general *profit* function to indicate the profit gained by a valid schedule. The most attractive point of profit function lies in its generality. Utilizing profit as the objective, the LTE DL SU-MIMO FDPS problem can be formalized as a combinatorial optimization problem. This formalization can be generalized to many scheduling policies such as *Proportional-Fair* and *MaxWeight*. We have proved the formalized problem is MAX SNP-hard through an *L-Reduction*, which implies the problem is so difficult to be approximated that there is no PTAS for it. Thus we have proposed an approximation algorithm with constant ratio of 4 to solve the LTE DL SU-MIMO FDPS problem. The proposed algorithm is based on a greedy method that maximizes a *submodular function* over a *matroid*. The simulation results demonstrate the profit model can express other well-known schedulers and the proposed algorithm can achieve fairly good performance.

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Appendix A
Approximation Algorithm and Approximation Ratio

Assume that X is an instance of a maximization problem[7]. We denote the size of the input by $|X|$ and its optimal value by $OPT(X)$. Suppose ALG is a feasible solution for the maximization problem. For instance X, we denote the cost of ALG by $ALG(X)$. We say that ALG has an approximation ratio of $r(|X|)$ [23] if, for any instance X, $OPT(X)$ is within a factor of $r(|X|)$ of $ALG(X)$:

$$OPT(X) \leq r(|X|) \cdot ALG(X)$$

We also call ALG a $r(|X|)$-approximation algorithm. When the approximation ratio is independent of the input size $|X|$, we will use the terms approximation ratio of r and r-approximation algorithm, indicating no dependence on $|X|$.

Note that $r(|X|) \geq 1$ (or $r \geq 1$), and a smaller value of $r(|X|)$ (or r) indicates that the approximation algorithm has a better performance in a worst-case sense. In particular, when $r = 1$, the approximation algorithm ALG is essentially the optimal solution for any instance X.

Appendix B
Polynomial-Time Approximation Scheme

A polynomial-time approximation scheme (PTAS) [23] for a maximization problem is an approximation algorithm that takes as an input not only an instance of the problem, but also a value $\epsilon > 0$ such that for any fixed ϵ, the scheme is a $(1+\epsilon)$-approximation algorithm which is computable in polynomial time in the size of the input instance.

In a technical sense, a PTAS is the best that one can hope for an NP-hard optimization problem, assuming $P \neq NP$.

Appendix C
L-Reduction

Suppose that $\mathcal{A}$ and $\mathcal{B}$ are maximization problems,. An L-Reduction [26] from $\mathcal{A}$ to $\mathcal{B}$ is a pair of functions R and S, both computable in polynomial time, with the following two additional properties:

First, if X is an instance of $\mathcal{A}$ with optimum $OPT(X)$, then $R(X)$ is an instance of $\mathcal{B}$ with optimum $OPT(R(X))$ that satisfies,

$$OPT(R(X)) \leq \alpha \cdot OPT(X) \tag{25}$$

where α is a positive constant.

[7]We only focus on maximization problems in this paper since the LTE DL SU-MIMO FDPS is a maximization problem.

Second, if s is any feasible solution of $R(X)$, then $S(s)$ is a feasible solution of X such that,

$$OPT(X) - VAL(S(s)) \leq \beta \cdot (OPT(R(X)) - VAL(s)) \tag{26}$$

where β is another positive constant particular to the reduction and VAL denotes the value of the feasible solution in both instances. (26) guarantees that S returns a feasible solution of X which is not much more suboptimal than the given solution of $R(X)$. In particular, if s is the optimal solution of $R(X)$, then $S(s)$ is the optimal solution of X.

The L-Reductions have the *composition* property [26], illuminated in Lemma 5.

Lemma 5: If (R, S) is an L-Reduction from problem $\mathcal{A}$ to problem $\mathcal{B}$, and (R', S') is an L-Reduction from problem $\mathcal{B}$ to problem $\mathcal{C}$, then their composition $(R \cdot R', S' \cdot S)$ is an L-Reduction from $\mathcal{A}$ to $\mathcal{C}$.

Appendix D
MAX SNP-Hardness

In computational complexity theory, SNP (from Strict NP) is a complexity class containing a limited subset of NP based on its logical characterization in terms of graph-theoretical properties. The class MAX SNP is a subset of optimization problems derived from SNP [26]. A problem is said to be MAX SNP-hard if all MAX SNP problems can be L-reduced to this problem. Note that the problem itself may not necessarily be MAX SNP.

MAX SNP-hard problems are hard to approximate. It is shown in [26] that,

Lemma 6: Any MAX SNP-hard problem does not have a PTAS unless $P = NP$.

Suppose $\mathcal{A}$ is a known MAX SNP problem, thus all the MAX SNP problems can be L-reduced to $\mathcal{A}$. Once $\mathcal{B}$ can be L-reduced from $\mathcal{A}$, according to the composition property in Lemma 5, all the MAX SNP problems can also be L-reduced to $\mathcal{B}$, which indicates $\mathcal{B}$ is MAX SNP-Hard. In other words, to prove that a problem $\mathcal{B}$ is MAX SNP-hard, it suffices to present an L-reduction from a known MAX SNP-hard problem $\mathcal{A}$ to $\mathcal{B}$.

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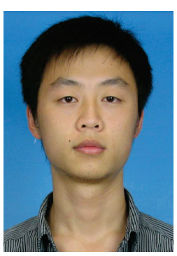

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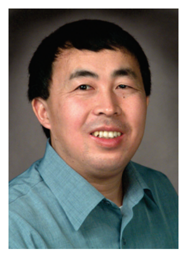

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