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Flexible Proportional-Rate Scheduling for OFDMA System

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***Abstract*—In this paper, we study the sum-rate maximization algorithms for downlink and uplink orthogonal frequency division multiple access (OFDMA) systems under proportional-rate constraint (PRC) and minimum-rate constraint. We develop a low-complexity weighted channel signal-to-noise ratio (w-SNR)-based ranking scheme for user selection on each subchannel in OFDMA combined with waterfilling (WF) power allocation. Both offline and online optimization algorithms are developed to optimize the SNR weight vector to maximize the sum rate while satisfying several constraints, such as PRC. The offline weight optimization technique relies on the analytical throughput results developed in this paper, and the online weight adaptation method tracks the user rates and meets the PRC using a sub-gradient search. Furthermore, we introduce a novel SNR operating region test to enhance the multiuser diversity gain and the sum rate. The proposed schemes have a low complexity which is linear to the numbers of users and subchannels. Simulation results verify the accuracy of the developed analytical rates and fairness formulas, and show that the proposed w-SNR schemes can achieve higher sum rates than several benchmark schemes which provide the PRC with either short-term or long-term fairness.**

***Index Terms*—OFDMA, proportional rate constraint (PRC), admission control, multiuser diversity, throughput maximization.**

I. Introduction

The orthogonal frequency division multiple access (OFDMA) is a promising candidate modulation and access scheme for the 4th generation (4G) communication systems [1]–[4]. The sum-rate maximization under the transmit power and quality-of-service (QoS) constraints has been a research focus for OFDMA systems. To avoid a certain subset of users monopolizing the resource allocation, the fairness provisioning has been regarded as an important QoS design technique. Many fairness metrics have been proposed in the literature, such as the proportional fairness (PF) [5]–[9], the max-min fairness [6], [7], and the channel access fairness [10], [11]. However, though these methods provide fairness in various measures, they cannot provide a desired target rate ratio between the users. For example, the proportional fairness tries to maximize the sum-logarithm of multiple users' rates, but it cannot control the rate allocation between the users precisely. The same problem exists for the max-min fairness, channel access fairness, and several other fairness schemes.

In practical systems, users may have different traffic priorities, arrival rates and deadlines. Thus, it is important to provide heterogenous service rates that match the different users' demands and channel conditions. Furthermore, when the required rates cannot be met exactly, it is desirable to decrease all users' rates by a specified ratio [12]. To meet this demand, the proportional rate constraint (PRC) has been proposed to match different users' rate requirements, and the rate or power allocation optimization methods under PRC have been considered in several works [3], [4], [12]–[15].

Besides the PRC, the minimum-rate constraint [16], [17] specifies that all admitted users must be provided with a minimum rate for their traffic class. Otherwise, the users who fail to meet the minimum rate constraint are not admitted. This is one aspect of admission control and will be considered jointly with the PRC for sum-rate maximization design.

In [3], [4], [13], downlink OFDMA sum-rate maximization schemes with short-term fairness (STF) PRC were proposed and their performances were evaluated via simulations. The results in [3], [4] included adaptive modulation on each subchannel, and the scheme in [13] considered a fixed rate modulation on each subchannel and used the generalized processor sharing (GPS) principle for subchannel allocation.

For uplink rate fairness scheduling, in [16] a low-complexity sum-rate maximization scheme was proposed subject to the minimum user rate constraint. In [7], the max-min and proportional fairness constraints were considered to maximize the uplink OFDMA utility functions. Also, a linear programming (LP) and several proportional fairness algorithms were designed in [18]. Yet, the PRC for OFDMA uplink scheduling has not been adequately addressed.

In these contributions, different short-term fairness constraints were considered. For example, it is required that the fairness was attained for every packet (or every channel realization) [3], [4], [13]. However, this constraint can sig-

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nificantly reduce the sum rate, when some users experience bad channel conditions. Relaxing the time scale of the constraint can improve the performance. For example, the results in [14], [19]–[21] showed that long-term fairness (LTF) constraint-based designs could provide better performance than the STF constraint schemes. Among them, a tradeoff between rate and allowed time-scale (or delay) has been studied in [21] for the proportional fair scheduling. In [8], [9], gradient scheduling (GS) algorithms were proposed under long-term fairness. The asymptotic optimality of the GS method in terms of sum utility was studied in [8], and in [9] a few GS algorithms were developed with sum rate maximization and PF scheduling under maximum and minimum user rate constraints. For delay tolerant applications, such as the best effort service defined for the WiMax system, the short-term constraint considered in [3], [4], [13] may be too stringent for some applications and the long-term constraint is adequate [8], [9], [14], [19]. By using the long-term constraint, the temporal diversity can be introduced to mitigate the loss caused by the fairness constraint.

It is known that the Lagrangian duality based convex optimization tool can provide optimal or near-optimal weighted sum rates for downlink [22], [23], uplink [7], [24], and ad hoc channels [17] in OFDMA systems. However, inclusion of the PRC and minimum-rate constraints to the objective functions will significantly increase the complexity and slow down the convergence speed for the duality methods. Therefore, we search for low-complexity and flexible methods instead, using subchannel multiuser diversity to meet the PRC and minimum-rate constraint for uplink and downlink OFDMA systems.

Our proposed schemes implement channel assignment based on *weighted* channel SNR (*w*-SNR) ranking on each subchannel, then use various transmit power allocation schemes, including the waterfilling methods and equal power allocation (EPA). To provide an enhanced multiuser diversity gain, we further introduce a novel operating region test method to the online SNR weighting scheme, which can achieve a substantially larger sum rate. We call this scheme the improved *w*-SNR scheme.

To find the optimal SNR weight vector, we design two techniques depending on the availability of channel distribution information (CDI): (1) an offline optimization method which requires to know the CDI and calculates the SNR weight vector to achieve the PRC and the minimum-rate constraint; and (2) an online adaptation method which tracks the time-average rate of each user and dynamically adjusts the SNR weights for OFDMA to meet the PRC. These methods have a low complexity which is linear to the numbers of users and subchannels. For comparison purposes, we also provide short-term PRC schemes with minimum-rate constraint, for both uplink and downlink OFDMA systems.

Major contributions of this paper include: (1) we provide low-complexity flexible PRC provisioning methods which achieve enhanced performance than several benchmark schemes; and (2) we derive accurate analytical rate and fairness formulas for the proposed w-SNR schemes with waterfilling and EPA, which provide insights into the effects of system and channel parameters.

Our PRC design method is flexible because (1) it can be combined with other QoS constraints (such as the minimum-rate constraint); (2) the weight optimization can be done either online or offline; and (3) it is feasible to extend our weight adaptation PRC method to other optimization techniques (such as duality based WSR maximization) to design new PRC schemes. Simulation results verify the accuracy of our analytical results, and show that the proposed *w*-SNR schemes can provide higher sum rates than the STF and LTF-based PRC benchmark techniques, and also provide a near-perfect modified Jain's fairness metric for the non-uniform PRC.

II. Signal Model

A. Downlink System

Consider a downlink OFDMA system where K active users have backlogged data at the base station (BS) and compete for N available subchannels. For simplicity, only single transmit antenna and receive antenna are considered, without using interference cancellation. At time t, the channel SNR of user k on subchannel n is given by (for $k = 1,\dots,K$) $\gamma^h_{k,n}(t) = |h_{k,n}(t)|^2/\mathcal{N}_{k,n}$, where $h_{k,n}(t)$ and $\mathcal{N}_{k,n}$ are the complex channel gain and the noise power of user k, respectively, both on subchannel n. For convenience to include the target bit error rate (BER) constraint, we define the equivalent SNR of user k on subchannel n as $\gamma_{k,n}(t) = \xi_k\gamma^h_{k,n}(t)$, where ξ_k is the SNR gap function of user k [25]. For example, to study the throughput of continuous-rate quadrature amplitude modulation (CR-QAM) under the BER constraint, we set $\xi_k = -1.5/\log(5P_{e,k})$, where $\log(x)$ is the natural logarithm, and $P_{e,k}$ is the target BER for user k [25]. Let $\mathcal{K}_{\text{Adm}}$ be the set of indices of all admitted users, and K_{Adm} be the number of non-zero elements in $\mathcal{K}_{\text{Adm}}$. $0 \le K_{\text{Adm}} < K$ holds due to the minimum rate constraint.

For downlink OFDMA, the rate maximization subject to (s.t.) the instantaneous sum power and long-term proportional-rate and minimum-rate constraints can be posed as

$$\max_{\{\mathcal{S}_k(t)\},\{P_{k,n}(t)\}} \sum_{k=1}^{K}\sum_{n\in\mathcal{S}_k(t)} R_{k,n}(t) \tag{1}$$

$$\text{subject to} \quad \sum_{k\in\mathcal{K}_{\text{Adm}}}\sum_{n\in\mathcal{S}_k(t)} P_{k,n}(t) \le P_T, \tag{2}$$

$$\bar{R}_1 : \cdots : \bar{R}_K = \alpha_1 : \cdots : \alpha_K,$$
$$\text{and} \quad \bar{R}_k > R_{k,\min} \ \ (\forall\, k \in \mathcal{K}_{\text{Adm}}) \tag{3}$$

where $\mathcal{S}_k(t)$ is the subchannel set assigned to user k, $\mathcal{S}_k(t)$ for $k \in \mathcal{K}_{\text{Adm}}$ are non-overlapping sets assuming exclusive

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channel assignment (ECA). This constraint is based on a popular design employed in the literature [2]–[4] for uplink and downlink OFDMA systems. $P_{k,n}(t)$ and P_T is the power for user k on subchannel n and the total transmit power, respectively. In (3), $\bar{R}_k = E_t[R_k(t)]$ where $E_t[\cdot]$ is the expectation with respect to (w.r.t.) t, $R_k(t) = \sum_{n\in\mathcal{S}_k(t)} R_{k,n}(t)$, and $R_{k,n}(t)$ is the throughput of user k on subchannel n. We have

$$R_{k,n}(t) = \tilde{B}\log(1 + P_{k,n}(t)\gamma_{k,n}(t)), \tag{4}$$

where $\tilde{B} = \frac{B_{\text{tot}}}{N}$ is the bandwidth per subchannel, and B_{tot} is the total available bandwidth. $\bar{R}_1 : \cdots : \bar{R}_K = \alpha_1 : \cdots : \alpha_K$ specifies the long-term PRC, and $[\alpha_1, \cdots, \alpha_K]$ is the target rate ratio vector. Furthermore, $\bar{R}_k > R_{k,\min}$ provides the minimum-rate constraint for link k once scheduled. This is termed as admission control in this paper. In our problem formulation, when not all K users can be admitted, the PRC is implemented for the admitted users only.

The instantaneous power constraint considered in (3) can be replaced by the average power constraint, which is $E[\sum_{k\in\mathcal{K}_{\text{Adm}}}\sum_{n\in\mathcal{S}_k(t)} P_{k,n}(t)] \leq P_T$. Both instantaneous and average power constraints will be considered in this paper. The one-dimensional (1-D) waterfilling will be implemented over the subchannels, under the instantaneous power constraint; and the two-dimensional (2-D) waterfilling be implemented over both subchannels and time, under the time-average power constraint.

B. Uplink System

For uplink OFDMA, the rate maximization subject to the K mobile users' individual power and long-term fairness constraints can be posed as

$$\max_{\{\mathcal{S}_k(t)\},\{P_{k,n}(t)\}} \sum_{k=1}^{K}\sum_{n\in\mathcal{S}_k(t)} R_{k,n}(t) \tag{5}$$

$$\text{subject to} \quad \sum_{n\in\mathcal{S}_k(t)} P_{k,n}(t) \leq P_{T,k}, \tag{6}$$

$$\bar{R}_1 : \cdots : \bar{R}_K = \alpha_1 : \cdots : \alpha_K,$$
$$\text{and} \quad \bar{R}_k > R_{k,\min} \quad (\forall\, k \in \mathcal{K}_{\text{Adm}}) \tag{7}$$

where $P_{T,k}$ is the available transmit power for user k.

C. Weighted-SNR-based Multiuser Diversity Schemes

To provide the strict solutions for (1) to (7), the convex-optimization techniques, such as the Lagrangian duality approach [26], can be developed to provide an asymptotically optimal performance. However, this technique requires a large implementation complexity when multiple fairness constraints are involved and suffers a slow convergence. Instead, we will develop low-complexity multiuser diversity and temporal diversity techniques to provide approximate and near-optimal solutions of (1) – (7) and the new schemes may achieve much better performance than many available short-term constraint based PRC scheduling schemes. For both downlink and uplink OFDMA systems, we propose to use the weighted normalized SNR ranking to implement subchannel allocation. While they might exist other parameters other than the weighted normalized SNR ranking for subchannel allocation, we believe that the weighted normalized SNR is a natural low-complexity choice. The proposed optimization algorithms will provide guaranteed performance gain as long as the algorithms converge. As numerical results confirm, the proposed weighted normalized SNR-based algorithms converge fast and provide excellent overall performance. The time index t is suppressed when there is no confusion. The weighted normalized SNR of user k on subchannel n is defined as $z_{k,n} = w_k\gamma_{k,n}/\bar{\gamma}_k$, where $\bar{\gamma}_k = E[\gamma_{k,n}]$ is the average channel equivalent SNR (same for all the subchannels) of user k, and w_k is a weight factor of user k. Subcarrier n is assigned to user k^* if the following equation holds,

$$k^* = \text{argmax}_k z_{k,n}. \tag{8}$$

Let z_n^{SC} be the w-SNR selection combining output on subchannel n, and obviously $z_n^{\text{SC}} = \max\{z_{1,n}, \ldots, z_{K,n}\}$. The SC procedure is based on OFDMA ECA where only the user with the largest metric $z_{k,n}$ wins subchannel n. Also, let $z_{k,n}^{\text{SC}}$ be the realization of $z_{k,n}$ conditioned on that user k is selected using w-SNR ranking on subchannel n. Define $f_{z_{k,n}^{\text{SC}}}(x)$ as the probability density function (PDF) of $z_{k,n}^{\text{SC}}$. We have

$$\begin{aligned} f_{z_{k,n}^{\text{SC}}}(z) &= f_{z_k}(z)\prod_{k'\neq k,k'=1}^{K} F_{z_{k'}}(z) \\ &= \frac{1}{w_k}e^{-z/w_k}\prod_{k'=1,k'\neq k}^{K}(1 - e^{-z/w_{k'}}) \end{aligned} \tag{9}$$

where $f_{z_k}(z) = \frac{1}{w_k}\exp(-z/w_k)$ and $F_{z_k}(z) = 1 - \exp(-z/w_k)$ is the cumulative distribution function (CDF) of z_k, for all k. The pdf of z_n^{SC} is then given by $f_{z_n^{\text{SC}}}(x) = \sum_{k=1}^{K} f_{z_{k,n}^{\text{SC}}}(x)$.

For the stability purpose and without loss of generality, we assume $0 \leq w_k \leq w_{\max}$ for all k, where $w_{\max}$ is the maximum value for w_k. Increasing w_k will give user k a higher priority to compete for subchannels and transmission power, and obtain increased throughput R_k; and vice verse when w_k is decreased. We need to find the optimal $\{w_k\}_{k=1}^{K}$ so that both the long-term PRC $\bar{R}_1 : \cdots : \bar{R}_K = \alpha_1 : \cdots : \alpha_K$ and minimum-rate constraint $\bar{R}_k \geq R_{k,\min}$ (for all admitted users) hold.

For EPA in downlink channels we have $P_{k^*,n}(t) = P_T/N$, and in uplink channels $P_{k^*,n}(t) = P_{T,k^*}/N_{k^*}$, where $N_k = |\mathcal{S}_k|$ is the cardinality number (i.e., the number of non-empty elements) of subchannel set $\mathcal{S}_k$.

Assume the 1-D WF power allocation for downlink channels, we have [2], [11]

$$P_{k^*,n}(t) = (1/\lambda(t) - 1/\gamma_{k^*,n}(t))^+ \tag{10}$$

where $(x)^+ = \max(0,x)$, and $\lambda(t) = N_{\text{eff}}(t) \Big/ \left[P_T + \sum_{n=1}^{N_{\text{eff}}(t)} \frac{1}{\gamma_{k^*,n}(t)}\right]$. Here, $N_{\text{eff}}(t)$ is the number of all subchannels that have non-zero power allocation based on WF [11].

For uplink channels, for $n \in \mathcal{S}_k(t)$ using 1-D WF we obtain [27]

$$P_{k,n}(t) = (1/\lambda_k(t) - 1/\gamma_{k,n}(t))^+ \tag{11}$$

where $\lambda_k(t)$ is given by $\lambda_k(t) = N_{k,\text{eff}}(t) \Big/ \left[P_{T,k} + \sum_{n\in\mathcal{S}_{k,\text{eff}}(t)} \frac{1}{\gamma_{k,n}(t)}\right]$. Here, $N_{k,\text{eff}}(t) = |\mathcal{S}_{k,\text{eff}}(t)|$, and $\mathcal{S}_{k,\text{eff}}(t) \subseteq \mathcal{S}_k(t)$ is the subchannel set for user k after WF.

By using 2-D WF along both the subchannels and the time t, we need to find a WF level-related parameter λ (which is independent of t) using CDI. This method will be given in Section III.

Assume the optimal weight $\mathbf{w}_{\text{opt}}$ is obtained, the w-*SNR algorithm* is summarized below:

1) At each time t, for $n = 1, \ldots, N$, implement subchannel allocation using (8).
2) Use either EPA, 1-D WF, or 2-D WF to implement the power allocation along the assigned channels for downlink and uplink systems.

To find $\mathbf{w}_{\text{opt}}$, we design two different approaches: an offline optimization method which relies on the CDI knowledge and an online method which does not need it.

III. Offline Optimization to Find $\mathbf{w}_{\text{OPT}}$

This section provides performance analysis for the EPA and 2-D WF algorithms assuming a given SNR weight vector $\mathbf{w}$ for both downlink and uplink OFDMA systems. Furthermore, based on such analytical results, we provide an algorithm to find optimal $\mathbf{w}$ which can fulfill the target proportional rate ratio and minimum-rate constraints.

First, we need to find the rates of all users given an SNR weight vector $\mathbf{w} = [w_1, \ldots, w_K]^T$, denoted by $\bar{\mathbf{R}}(\mathbf{w}) = [\bar{R}_1(\mathbf{w}), \ldots, \bar{R}_K(\mathbf{w})]^T$. Next, we have to find the optimal vector $\mathbf{w}$ which achieves the target long-term PRC.

Assume a mapping function $\bar{\mathbf{R}}(\mathbf{w}) = G(\mathbf{w})$, where G is a multi-variate function between $\bar{\mathbf{R}}$ and $\mathbf{w}$. We can pose the target PRC as

$$\bar{\mathbf{R}}_{\text{PRC}}(\mathbf{w}) = \bar{R}_{\text{tot}}(\mathbf{w})\tilde{\boldsymbol{\alpha}} = G(\mathbf{w}) \tag{12}$$

where $\bar{R}_{\text{tot}}(\mathbf{w}) = \sum_{k\in\mathcal{K}_{\text{Adm}}} \bar{R}_k(\mathbf{w})$ is the sum rate, $\tilde{\boldsymbol{\alpha}} = [\tilde{\alpha}_1, \ldots, \tilde{\alpha}_K]^T$ is the normalized ratio vector, and $\tilde{\alpha}_k = \alpha_k / \sum_{n=1}^K \alpha_n$ such that $\sum_{n=1}^K \tilde{\alpha}_n = 1$ holds. We will design an iterative search technique to find $\mathbf{w}_{\text{opt}}$ based on (12).

A. Rate Proportions Based on Weight Factors

Performance of two power allocation schemes are analyzed below, 2-D WF and EPA. The analytical rate result for the 2-D WF scheme can be regarded as a tight approximation for the 1-D WF scheme.

1) Two-Dimensional Waterfilling Power Allocation: Besides the 1-D WF power allocation, we propose a 2-D WF algorithm over both subchannels and time. We will derive the analytical relation between $\mathbf{w}$ and $\mathbf{R}(\mathbf{w})$ for the 2-D WF, which is then used for the offline optimization.

Assume the downlink system first. Define $\gamma_{k,n}^{\text{sc}}$ as the equivalent channel SNR of user k on subchannel n conditioned on that user k is selected by the w-SNR ranking. This is equal to multiuser selection combining (SC) on each subchannel. Since $z_{k,n} = w_k\gamma_{k,n}/\bar{\gamma}_k$, we have $\gamma_{k,n} = z_{k,n}\bar{\gamma}_k/w_k$.

Define γ_n^{sc} as the w-SNR output SNR on subchannel n. We have $\gamma_n^{\text{sc}} = \gamma_{k^*,n}$ where $k^* = \text{argmax}_k z_{k,n}$. By using (1) and the Lagrangian approach, we obtain the following Karush-Kuhn-Tucker (KKT) conditions:

$$\begin{aligned} P_n^*(t) &= (1/\lambda(t) - 1/\gamma_n^{\text{sc}}(t))^+ \\ \text{subject to} \quad & \textstyle\sum_{n=1}^N E_t[P_n^*(t)] = P_T. \end{aligned} \tag{13}$$

This result leads to $\sum_{n=1}^N (1/\lambda(t) - 1/\gamma_n^{\text{sc}}(t))^+ = P_T(t)$. Taking the time average of this equality we have

$$E_t\left[\sum_{n=1}^N (1/\lambda(t) - 1/\gamma_n^{\text{sc}}(t))^+\right] = E_t[P_T(t)] = P_T. \tag{14}$$

With the assumption of stationary and ergodic channels, the time average of the left side of (14) is replaced by the ensemble average over the channel SNRs and we obtain

$$\begin{aligned} & N\int_{\lambda}^{\infty} (1/\lambda - 1/y) f_{\gamma_n^{sc}}(y)dy \qquad (15) \\ = & \sum_{k=1}^K N\int_{w_k\lambda/\bar{\gamma}_k}^{\infty} \left(\frac{1}{\lambda} - \frac{w_k}{z\bar{\gamma}_k}\right) f_{z_{k,n}^{\text{SC}}}(z)dz = P_T \end{aligned}$$

where $f_{z_{k,n}^{\text{SC}}}(z)$ is defined in (9). The solution of λ to (16) is a constant for different time slots. Generally speaking, solving (16) requires a numerical search. An efficient bisection search or sub-gradient based search can be used. The optimal 2-D WF algorithm is given below (***DL 2-D WF Algorithm***).

1) Find λ using (16).
2) Online implementation: At each time slot t, subchannel n is assigned to user k^* who satisfies that $k^* = \text{argmax}_k z_{k,n}(t)$, where $z_{k,n}(t) = w_k\gamma_{k,n}(t)/\bar{\gamma}_k$. The transmit power allocated to subchannel n is given by $P_{n,\text{WF-2D}}^*(t) = (1/\lambda - 1/\gamma_n^{\text{sc}}(t))^+$, where $\gamma_n^{\text{sc}}(t) = \gamma_{k^*,n}(t)$. ■

After λ is obtained, the time-average capacity of user k

can be calculated by

$$R_{k,\text{WF-2D}}(\mathbf{w}) = \tilde{B}\int_0^\infty \log(1+P^*_{n,\text{WF-2D}}\,y) f_{\gamma^{sc}_{k,n}}(y)dy = \tilde{B}\int_{w_k\lambda/\bar{\gamma}_k}^\infty \log\left(\frac{z\bar{\gamma}_k}{w_k\lambda}\right) f_{z^{\text{SC}}_{k,n}}(z)dz \quad (16)$$

A closed-form expression for (16) can be derived following an approach given in [11], [27], omitted here for brevity.

Next, we consider the uplink system. Define the Lagrangian

$$\mathcal{L}(\{\mathcal{S}_k(t)\},\{P_{k,n}(t)\}) \quad (17)$$
$$= \sum_{k=1}^K \sum_{n\in\mathcal{S}_k} \tilde{B}\log(1+P_{k,n}(t)\gamma_{k,n}(t)) - \lambda_k(t)\left(\sum_{n\in\mathcal{S}_k} P_{k,n}(t) - P_{T,k}(t)\right)$$

where $P_{T,k}(t)$ is the sum power at time t satisfying $E_t[P_{T,k}(t)] = P_{T,k}$. We obtain the following KKT conditions,

$$P^*_{k,n}(t) = \left(1/\lambda_k(t) - 1/\gamma^{\text{SC}}_{k,n}(t)\right)^+ \quad (18)$$
$$\text{subject to} \qquad \textstyle\sum_{n\in\mathcal{S}_k} E_t[P^*_{k,n}(t)] = P_{T,k}.$$

With the assumption of stationary and ergodic channels, we obtain

$$\overline{N}_k \int_{\lambda_k}^\infty (1/\lambda_k - 1/y) f_{\gamma^{sc}_{k,n}}(y)dy \quad (19)$$
$$= \overline{N}_k \int_{w_k\lambda_k/\bar{\gamma}_k}^\infty \left(\frac{1}{\lambda_k} - \frac{w_k}{z\bar{\gamma}_k}\right) f_{z^{\text{SC}}_{k,n}}(z)dz = P_{T,k}$$

The optimal 2-D power allocation algorithm is given below (**UL 2-D WF Algorithm**).

1) Find λ_k using (20) for all k.
2) Online channel and power allocation (similar to step (2) of the DL 2D-WF Algorithm): ■

After vector $\{\lambda_k\}$ is obtained, the time-average capacity of user k can be calculated by

$$R_{k,\text{WF-2D}}(\mathbf{w}) = \tilde{B}\int_0^\infty \log(1+P^*_{n,\text{WF-2D}}\,y) f_{\gamma^{sc}_{k,n}}(y)dy \quad (20)$$
$$= \tilde{B}\int_{w_k\lambda/\bar{\gamma}_k}^\infty \log\left(\frac{z\bar{\gamma}_k}{w_k\lambda_k}\right) f_{z^{\text{SC}}_{k,n}}(z)dz$$

2) Equal Power Allocation: For downlink OFDMA, the average rate for user k on subchannel n using EPA and a given weight vector $\mathbf{w}$ is given by

$$\bar{R}^{\text{SC}}_{k,n}(\mathbf{w}) = E[\tilde{B}\log(1+\gamma^{\text{SC}}_{k,n}P_T/N)] = \tilde{B}\int_0^\infty \log(1+z(\bar{\gamma}_k/w_k)P_T/N) f_{z^{\text{SC}}_{k,n}}(z)dz \quad (21)$$

The average rate of user k over all N subchannels is

$$\bar{R}_k(\mathbf{w}) = \sum_{n=1}^N \bar{R}^{\text{SC}}_{k,n}(\mathbf{w}) = N\bar{R}^{\text{SC}}_{k,n}(\mathbf{w}). \quad (22)$$

This result can be used to provide the rate fairness in the downlink offline optimization. A closed-form expression for $\bar{R}_k(\mathbf{w})$ can be derived following an approach given in [28], omitted here for brevity. A numerical trapezoidal summation technique can be used to evaluate (22) accurately.

For uplink OFDMA, the average rate of user k on subchannel n is given by $\bar{R}^{\text{SC}}_{k,n}(\mathbf{w}) = E[\log(1+\gamma^{\text{SC}}_{k,n}P_{T,k}/N_k(t))]$. However, it is difficult to evaluate this expression because $N_k(t)$ is a random variable.

To evaluate $N_k(t)$, we study channel access statistics for the proposed *w*-SNR schemes, including the average channel access probability (AAP) and average number of assigned subchannels $\overline{N}_k$ for each user k. This result can be used for evaluation of the individual user rate for the *w*-SNR scheme in uplink OFDMA. Define $\text{AAP}_{k,n}$ as the AAP that subchannel n is allocated to user k, and $\text{AAP}^{\text{w-SNR}}_{k,n} = \int_0^\infty f_{z^{\text{SC}}_{k,n}}(x)dx$, where $f_{z^{\text{SC}}_{k,n}}(x)$ is given by (9). A closed-form expression for $\text{AAP}^{\text{w-SNR}}_{k,n}$ is derived below. We define

$$\textstyle\prod_{k'=1,k'\neq k}^K \left(1-e^{-x/w_{k'}}\right) \quad (23)$$
$$= \sum_{\tau\in J_{K-1}\left(\{1/w_{k'}\}_{k=1,k'\neq k}^K\right)} \text{sign}(\tau)\exp(-x|\tau|)$$

where $J_{K-1}\left(\{1/w_{k'}\}_{k'\neq k}^K\right)$ are the expansion set [29] of $\{1/w_{k'}\}_{k'\neq k,k=1}^K$ and contains 2^{K-1} elements. Thus, we have

$$f_{z^{\text{SC}}_{k,n}}(x) = \sum_{\tau\in J_{K-1}} \frac{\text{sign}(\tau)}{w_k}\exp\left(-x\left[|\tau| + \frac{1}{w_k}\right]\right) \quad (24)$$

where J_{K-1} is a shorthand for $J_{K-1}\left(\{1/w_{k'}\}_{k'\neq k}^K\right)$. Using (24) we can readily show that

$$\text{AAP}^{\text{w-SNR}}_{k,n} = \sum_{\tau\in J_{K-1}} \frac{\text{sign}(\tau)}{(|\tau|w_k+1)} \quad (25)$$

The average number of subchannels assigned to user k is derived as

$$\overline{N}_k = \sum_{n=1}^N \text{AAP}^{\text{w-SNR}}_{k,n} = \sum_{n=1}^N \sum_{\tau\in J_{K-1}} \frac{\text{sign}(\tau)}{(|\tau|w_k+1)} \quad (26)$$

As a tight approximation, we replace $N_k(t)$ with its average value $\overline{N}_k$ given by (26). Then we obtain the rate of user k as

$$\bar{R}_k(\mathbf{w}) = N\bar{R}^{\text{SC}}_{k,n}(\mathbf{w}) \quad (27)$$
$$\simeq N\tilde{B}\int_0^\infty \log(1+z\bar{\gamma}_k/w_k\frac{P_{T,k}}{\overline{N}_k}) f_{z^{\text{SC}}_{k,n}}(z)dz$$

The approximate result in (27) is very tight for independent

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subchannels in uplink OFDMA. The assumption of independent subchannel is valid for the cases that a large number of resolvable paths exist, or that the subchannels are non-continuous and separated far away from each other in the broadband spectrum. On the other hand, for correlated uplink channels, the online weight adaptation scheme should be used.

After the analytical relation between $\mathbf{w}$ and $\bar{\mathbf{R}}(\mathbf{w})$ for both EPA and WF is obtained, we still need to find the $\mathbf{w}_{\text{opt}}$ which provides the target PRC, and this step is designed next.

B. Offline Optimization Approach to Find Weight Factors

To maximize the sum rate under both PRC and minimum-rate constraint, we propose a joint optimization algorithm which has two steps: first, find $\mathbf{w}$ to realize PRC for a given set of users; second, implement admission control by removing users who cannot meet the minimum-rate constraint. These two steps should be implemented iteratively. The target is to find $\mathbf{w}_{\text{opt}}$ such that both the PRC and $\bar{R}_k > R_{k,\min}$ $(\forall\ k)$ hold.

Our method to achieve the PRC is proposed below. The function $G(\mathbf{w})$ can be analytically calculated using the results in Subsection III-A for EPA and 2-D WF, respectively. However, the analytical solution for optimal $\mathbf{w}$ is difficult. Therefore, we design an iterative search to meet the PRC for a given set of users. Define $\mathbf{r} = [r_1, r_2, \ldots, r_K]^T = [\bar{R}_1/\tilde{\alpha}_1, \bar{R}_2/\tilde{\alpha}_2, \ \ldots, \bar{R}_K/\tilde{\alpha}_K]^T/\bar{R}_{\text{tot}}$ as the normalized rate ratio vector. When $\mathbf{r} = \mathbf{1}_{K\times 1}$, where $\mathbf{1}_{K\times 1}$ is a $K \times 1$ all-one vector, the target PRC is perfectly attained.

Let the value of $\mathbf{r}$ at stage m $(m = 1, 2, \ldots)$ be given by $\mathbf{r}(m) = [\tilde{r}_1(m), \ldots, \tilde{r}_K(m)]^T$. Also, define $r_{\max}(m) = \max_k\{r_1(m), \ldots, r_K(m)\}$, $r_{\min}(m) = \min_k\{r_1(m), \ \ldots, r_K(m)\}$, and $e(m) = r_{\max}(m) - r_{\min}(m)$ as the relative error. When $|e(m)|$ is less than a pre-specified threshold ϵ_e $(0 < \epsilon_e \ll 1)$, we assume the required PRC is achieved. Certainly, the error metric $e(m)$ can be defined in many different ways, such as $e(m) = \sum_{k=1}^{K} |r_k(m) - 1|^p$, where p is a real positive number. The algorithm for the proposed iterative search is given below (***Offline PRC Optimization Algorithm***):

1) Let $m = 1$. For a given initial vector $\mathbf{w}(1)$ (say $\mathbf{w}(1) = \mathbf{1}_{K\times 1}$), find the rate vector $\tilde{\mathbf{R}}(1)$ and the error metric $e(1)$.
2) While $|e(m)| > \epsilon_e$, repeat Steps 2.1) – 2.3); otherwise go to Step 3).
   2.1) Find user $k_{\min}$ whose current PRC is the lowest (i.e., least satisfied) among all the K users, and user $k_{\max}$ whose current PRC is the highest, i.e., $k_{\min} = \text{argmin}_k\{r_k(m)\}$, and $k_{\max} = \text{argmax}_k\{r_k(m)\}$.
   2.2) Let $w_{k_{\min}}(m+1) = w_{k_{\min}}(m) + \beta(m)|e(m)|$, and $w_{k_{\max}}(m+1) = w_{k_{\max}}(m) - \beta(m)|e(m)|$, where $\beta(m)$ $(0 < \beta(m) < 1)$ is a scalar whose value should decrease as stage m increases. The remaining elements of $\mathbf{w}(m+1)$ are given directly by those in $\mathbf{w}(m)$.
   2.3) Based on $\mathbf{w}(m+1)$, re-calculate rate vector which is given by $\bar{\mathbf{R}}(\mathbf{w}(m+1))$. Find the normalized ratio vector $\mathbf{r}(m+1)$. Increase m by one.
3) The convergence is achieved. The vector $\mathbf{w}(m)$ gives the solution $\mathbf{w}_{\text{opt}}$, which is then used in the proposed w-SNR ranking scheme to achieve the long-term PRC. ■

The proposed algorithm is very numerically stable as verified by simulations for different K's and N's. Based on obtained $\mathbf{w}_{\text{opt}}$, we can calculate the optimal average rate vector $\bar{\mathbf{R}}_{\text{PRC}}$. Finally, since the analytical rate expression for the 2-D WF is a tight approximation for the 1-D WF, the $\mathbf{w}_{\text{opt}}$ calculated based on the 2-D WF can be used in the 1-D WF algorithm to approximately achieve its PRC target.

After the target PRC is achieved for a given set of users, the admission control should be performed. For simplicity, we assume that all users have the same minimum-rate constraint $R_{\min}$. The Algorithm to achieve both PRC and minimum-rate constraints is given below (***Joint PRC and Admission Control Algorithm***):

Algorithm input: The set of users to be scheduled, their SNRs, PRC vector $\boldsymbol{\alpha}$, and the minimum-rate $R_{\min}$. Algorithm output: Optimal weights for the set of admitted users, and their subchannel and power allocation.

(1) Initialize $\mathcal{K}_{\text{Adm}}$ to be the set of indices of all users with backlogged data.
(2) For all users in $\mathcal{K}_{\text{Adm}}$, implement the *Offline PRC Optimization Algorithm.*
(3) Rank the rates of the users in descending order and check the minimum-rate constraint. Let K_f be the number of users whose rates are below $R_{\min}$ (or called failed users). If $K_f = 0$, go to step (4); otherwise, find the $\lceil K_f/2 \rceil$ failed users who have the lowest rates, remove their user indexes from $\mathcal{K}_{\text{Adm}}$, and go to step (2).
(4) The algorithm converges and outputs the optimal weight vector $\mathbf{w}_{\text{opt}}$ and admitted user set $\mathcal{K}_{\text{Adm}}$.

Some comments are in order: First, if we set $\boldsymbol{\alpha} = \mathbf{1}$, this algorithm will achieve the equal rate proportions and the Jain's fairness metric was defined for this case [30]. Second, if we drop either the PRC or the minimum-rate constraint, this algorithm can be simplified to either a PRC optimization algorithm or an admission control algorithm. Third, this method removes $\lceil K_f/2 \rceil$ out of K_f failed users in each iteration, and converges in at most $\lceil \log_2 |\mathcal{K}_{\text{Adm}}| \rceil$ iterations. This achieves a must faster convergence than the one-user-removal method which removes only one worst user in each iteration (which may take up to $|\mathcal{K}_{\text{Adm}}|$ iterations), and yet provides virtually identical sum-rate performance. Additionally, this method performs significantly better than the method which removes all failed users in each iteration, especially in the low SNR region.

IV. Online Optimization for Uplink and Downlink Channels

The proposed offline optimization method needs to know the CDI at the scheduler. To avoid such a need, we propose an adaptive approach to update $\mathbf{w}$ online to maximize the sum rate under the PRC and minimum-rate constraint.

A. Rate Tracking and Weight-Adaptation Scheme

Assuming an exponentially-decayed observation window, the average rate for user k at time t, $\bar{R}_k(t)$, is defined as

$$\bar{R}_k(t) = (1-\beta_k)\bar{R}_k(t-T) + \beta_k R_k(t) \tag{28}$$

where $R_k(t)$ is the instantaneous rate at time t, and β_k is a factor of user k related to observation window size t_c and $\beta_k = 1/t_c$ holds.

The target rate of user k under both PRC and minimum-rate constraints is given by

$$\bar{R}_{k,\text{tar}}(t) = \max(\tilde{\alpha}_k \bar{R}_{\text{tot}}(t), R_{\min}) \tag{29}$$

where $\bar{R}_{\text{tot}}(t) = \sum_{k\in\mathcal{K}_{\text{Adm}}} \bar{R}_k(t)$ is the tracked average sum rate for all admitted users. Note that $\{\tilde{R}_k(t)\}_{k\in\mathcal{K}_{\text{Adm}}}$ is a function of current and past SNR weight vectors. We dynamically adjust $\mathbf{w}(t)$ so that the time-average rate vector $\{\bar{R}_k(t)\}_{k=1}^K$ follows the rate ratio vector $\boldsymbol{\alpha}$ and meets the minimum-rate constraint. In order for $\bar{R}_k(t)$ to closely track $\bar{R}_{k,\text{tar}}(t)$, we propose to update $\mathbf{w}(t)$ based on a sub-gradient tracking. Define the mean square error (MSE) of rate tracking for user k as $E_k(t) = |e_k(t)|^2$, where $e_k(t)$ is the error metric for user k and is given by $e_k(t) = [\bar{R}_k(t) - \bar{R}_{k,\text{tar}}(t)]/\bar{R}_{k,\text{tar}}(t)$.

Based on the sub-gradient search the weight vector is updated by $w_k(t+T_f) = \min\{\left(w_k(t) - \beta_w \frac{\partial E_k(t)}{\partial w_k(t)}\right)^+, w_{\max}\}$, where T_f is the inter-frame interval, which is equal to the scheduling interval. β_w is a step size used to control the stability and convergence speed, and $\frac{\partial E_k(t)}{\partial w_k(t)} = 2e_k^*(t)\frac{\partial e_k(t)}{\partial w_k(t)}$. However, $\frac{\partial e_k(t)}{\partial w_k(t)}$ is difficult to evaluate. We use an approximation that

$$w_k(t+T_f) = \left(w_k(t) - \tilde{\beta}_w e_k(t)\right)^+, \tag{30}$$
$$\text{subject to} \quad w_{k,\min} \le w_k(t+T_f) \le w_{k,\max}$$

where $\tilde{\beta}_w$ is a modified step size factor, and $w_{k,\min}$ and $w_{k,\max}$ are the specified lower and upper bounds of w_k to maintain numerical stability.

The online scheduling algorithm is given below, which realizes PRC and admission control for downlink and uplink OFDMA systems (***Weighted-SNR rate-tracking (w-SNR RT) Algorithm***):

(i) Initialize. Let $\mathcal{K}_{\text{Adm}}$ be the set of indices of all users with backlogged data.
(ii) At time t, all users in $\mathcal{K}_{\text{Adm}}$ compete for the N subchannels using the *w*-SNR algorithm based on $\mathbf{w}(t)$ and SNRs $\{\gamma_{k,n}\}_{k\in\mathcal{K}_{\text{Adm}}, n=1,\ldots,N}$.
(iii) The scheduler implements power allocation (EPA or 1-D WF), and measures the rate $R_k(t)$ and tracked rate $\bar{R}_k(t)$ for all k.
(iv) Rank the user rates in descending order. Among the users whose rates are below $R_{\min}$, find the user who has the lowest rate and remove it from $\mathcal{K}_{\text{Adm}}$.
(v) The BS updates the weight factor $w_k(t+T_f)$ using (28), (29) and (30) for each user k. Increase t to $t+T_f$, and go to step (ii).

Here, we remove no more than one failed user in one slot, so that other low-rate users can adapt their weights and enhance their rates in the next time slots.

For Step (ii) in the online algorithm above, we introduce a novel SNR operating region-based *w*-SNR scheme next, which provides enhanced sum-rate than the original *w*-SNR ranking scheme. In the new scheme, for $|\mathcal{K}_{\text{Adm}}|$ users to compete for the N subchannels using $k^* = \text{argmax}_k z_{k,n}$, we redefine the decision variable $z_{k,n}$ as

$$z_{k,n} = \begin{cases} w_k\gamma_{k,n}/\bar{\gamma}_k & (\gamma_{k,n}/\bar{\gamma}_k > 1 \text{ and } w_k > 1) \\ & \text{ or } (\gamma_{k,n}/\bar{\gamma}_k < 1 \text{ and } w_k < 1) \\ \gamma_{k,n}/\bar{\gamma}_k & \text{elsewhere.} \end{cases} \tag{31}$$

The SNR operation region test introduced above can significantly enhance the multiuser diversity gain for the *w*-SNR scheme. When $\gamma_{k,n}/\bar{\gamma}_k > 1$ user k is operating at favorable SNR region; and when $w_k > 1$ user k is gaining advantage in channel competition compared to the other users. The algorithm above tries to increase the chance that user k gains subchannels when it is in favorable SNR region, and decrease such chance vice versa. For example, when $\gamma_{k,n}/\bar{\gamma}_k < 1$ and $w_k > 1$, user k is in an unfavorable channel condition, using $z_{k,n} = \gamma_{k,n}/\bar{\gamma}_k$ instead of $z_{k,n} = w_k\gamma_{k,n}/\bar{\gamma}_k$ reduces the sum rate loss by reducing the probability that user k gains a large number of subchannels. This method can significantly increase the sum rate, especially in uplink OFDMA systems, and we call it *improved RT* w*-SNR scheme*.

B. Comparison With Other Algorithms

For comparison purposes, we provide both STF and LTF-based PRC benchmark schemes. First, we extend a downlink STF PRC scheme [3] to uplink, and also introduce the minimum-rate constraint to make a fair comparison with the proposed algorithms.

The ***Uplink STF-Based PRC and Admission Control Algorithm*** is given below.

(i) Initialize. Let set $\mathcal{S}_k$ be empty for all k and $\tilde{\mathcal{S}} = \{1, 2, \ldots, N\}$ be the set of available subchannels. Let $\mathcal{K}_{\text{Adm}}$ be the set of backlogged users.
(ii) At time slot $t = nT_f$. If $N < K$, skip step (ii); otherwise for $k = 1, \ldots, K$, do the following:
User k gets subchannel n^* if $n^* = \text{argmax}_n \gamma_{k,n}$, where $n \in \tilde{\mathcal{S}}$.
Add element n^* to set $\mathcal{S}_{k^*}$ and remove it from $\tilde{\mathcal{S}}$.

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(iii) For the remaining subchannels in $\tilde{\mathcal{S}}$, do the following to achieve the short-term PRC,
   (1) Compute normalized rate ratio $r_k = \frac{\bar{R}_k(t)}{\bar{R}_{\text{tot}}(t)\tilde{\alpha}_k}$. Find the minimum element in $\{r_k\}_{k=1}^K$, denoted as $r_{k_{\min}}$. User $k^* = k_{\min}$ is the least satisfied in proportional rate.
   (2) User k^* is assigned subchannel n^*, where $n^* = \text{argmax}_n \gamma_{k^*,n}$, and $n \in \tilde{\mathcal{S}}$.
   Add element n^* to set $\mathcal{S}_k$ and remove it from $\tilde{\mathcal{S}}$. Update $R_{k^*}(t)$ based on EPA or WF along subchannels in $\mathcal{S}_{k^*}$.
   (3) If $\tilde{\mathcal{S}}$ is non-empty, go to Step (iii.1); Otherwise, proceed to Step (iv).
(iv) Implement the admission control. Rank the rates of the users in $\mathcal{K}_{\text{Adm}}$ in descending order. Let K_f be the number of users whose rates are below $R_{\min}$ (or called failed users). If $K_f = 0$, go to Step (v); otherwise, find the worst user who has the lowest rate, remove this user from $\mathcal{K}_{\text{Adm}}$, and go to Step (ii).
(v) The algorithm converges and outputs the admitted user set $\mathcal{K}_{\text{Adm}}$ and power and rate allocation results.

The ***downlink STF-Based PRC and Admission Control Algorithm*** can be obtained from the algorithm above by changing the power allocation method in Step (iii.2).

This method has a complexity of $O(NK)$ for the PRC scheduling, but to meet the minimum-rate constraint, the largest complexity is about $O(NK^2/2)$. In comparison, the proposed online algorithm has approximately the same complexity of $O(NK)$ with or without the minimum rate constraint. The rate maximization in the STF algorithm is attempted by frequency diversity-based subchannel allocation and WF power allocation. However, this approach does not use multiuser diversity or temporal diversity for channel competition, which are exploited by the proposed w-SNR schemes.

Next, we design a LTF PRC scheme for comparison. An interesting method using LTF scheduling was considered in [8], [9]. Notice that the GS methods in [9] assumed a wideband code division multiple access (CDMA) signal model with PF (or maximum rate) scheduling and bounded rate constraints. For comparison purposes, based on GS with token control (GST) idea given in [9], we design a LTF-based PRC algorithm for OFDMA channels. Since the methods in [9] deal with CDMA systems such as CDMA2000 3G1xEV-DO downlink scheduling and we are dealing with OFDM channels in this paper, our LTF-based PRC algorithm for OFDMA is not a straightforward extension of the algorithms in [9]. In a CDMA system there are spreading codes and power to allocate, while in OFDMA there are subcarriers and power to allocate. The proposed LTF-based PRC algorithm exploits the design philosophy in [9] and uses the token counter mechanism to allocate rates in OFDMA. The details are given below. Let $T_k(t)$ be the token of user k at time t. When user k's allocated rate exceeds its normalized rate $\bar{R}_{\text{tot}}(t)\alpha_k$, $T_k(t)$ is decreased; otherwise, $T_k(t)$ is increased. The gradient search update is implemented for each scheduling slot.

The ***Uplink LTF-Based PRC GST Algorithm*** is given below.

(i) Initialize. Let $\bar{R}_k(t) = (1-\beta)\bar{R}_k(t-T)$ be the tentative average rate for time t, where $0 < \beta < 1$.
(ii) For subchannel index $n = 1, \ldots, N$, do the following until all subchannels are allocated,
   (1) Given subchannel n, compute metrics $I(k,n) = e^{T_k(t)} R_{k,n}$ for $k = 1, \ldots, K$, where $R_{k,n}$ is a tentative rate of user k if subchannel n is assigned to him. Subchannel n is assigned to user k^* if $k^* = \text{argmax}_k I(k,n)$.
   (2) After subchannel n is assigned, do EPA or WF for allocated subchannels for user k^*.
   (3) Update $\bar{R}_k(t)$ using $\bar{R}_k(t) = (1-\beta)\bar{R}_k(t-T) + \beta R_k(t)$. Find the sum tracked rate $\bar{R}_{\text{tot}}(t)$, and update $T_k(t)$ for all K users by
$$T_k(t+T_f) = \Big(T_k(t) - \tilde{\beta}_{GS} e_k(t)\Big)^+, \qquad (32)$$
$$\text{subject to} \quad T_{k,\min} \le T_k(t+T_f) \le T_{k,\max}$$
   where $\tilde{\beta}_{GS}$ is a small step size factor, $e_k(t)$ is an error for tracking the target rate similar to that used in (30), and $T_{k,\min}$ and $T_{k,\max}$ are the specified lower and upper bounds of T_k to maintain numerical stability.
(iii) Implement the admission control by checking the rates of users with a threshold.
(iv) Increase t by T and process the next time slot.

The ***downlink LTF-Based PRC GST Algorithm*** can be obtained from the algorithm above by changing the power allocation method in Step (ii.2). Some comments are in order. As discussed earlier, the proposed GST method also utilized some design philosophy in [9], but is not a straightforward extension of the algorithms given in [9], and there are noticeable differences. Beside the differences in the system models, the token design was used to control the rate bound in [9], but is exploited to provide PRC here. The developed benchmark GST scheme provides temporal diversity, multi-carrier frequency diversity, and multiuser diversity.

The Jain's fairness metric [30] was designed for the equal-rate fairness, and its fairness metric for time t is $\text{FA}(t) = \left[\sum_{k=1}^K R_k(t)\right]^2 \Big/ \left[K \sum_{k=1}^K R_k^2(t)\right]$. In this paper, to consider the effect of unequal proportional rates and the minimum rate, we define a modified PRC fairness metric as
$$\text{FA}(t) = \left[\sum_{k \in \mathcal{K}_{\text{Adm}}} \bar{R}_k(t)/\tilde{\alpha}_k\right]^2 \Bigg/ \left[|\mathcal{K}_{\text{Adm}}| \sum_{k \in \mathcal{K}_{\text{Adm}}} \bar{R}_k^2(t)/\tilde{\alpha}_k^2\right] \quad (33)$$
Note that the fairness metric defined here considers the tracked rates for only the admitted users. The average fair-

ness metric is given by $\overline{\text{FA}} = E[\text{FA}(t)]$. It is known that $\overline{\text{FA}} \in (1/K, 1)$, where $\overline{\text{FA}} = 1/K$ corresponds to the most unfair case and $\overline{\text{FA}} = 1$ provides the best fairness.

V. Numerical Results

In this section, we study the effects of various parameters and constraints on the performance of proposed *w*-SNR schemes in downlink and uplink OFDMA systems. For comparison purposes, simulation results of the absolute channel SNR (*a*-SNR) based subchannel allocation scheme [2], [28], the developed STF PRC, and the LTF GST PRC algorithms are also presented. We assume that the total channel bandwidth is $B_{\text{tot}} = 1.25$ MHz for all numerical examples. For convenience, we define $\tilde{R}_{\min} = R_{\min}/\tilde{B}$ as the minimum spectral efficiency per subchannel, which is used for admission control. For the unequal channel SNR case, we assume the average channel SNRs decrease successively by 1 dB from the strongest user to the weakest one, i.e., $\bar{\gamma}^h_{k+1} = \bar{\gamma}^h_k 10^{-1/10}$, for $k = 1, \ldots, K-1$, and the average channel SNR per user is set to $\bar{\gamma}^h = \frac{1}{K}\sum_{k=1}^{K} \bar{\gamma}^h_k = 1$. For the exponentially decayed power delay profile (PDP), the path power decreases by 1.5 dB successively from the first path to the last one. After discrete Fourier transform (DFT), the multipath response is converted to the frequency subchannel gains. The case of unequal target BERs is considered, where the first $K/2$ users have the target BER of 10^{-3} and the remaining $K/2$ users have the target BER of 10^{-5}. We set $w_{\max} = K/2$ for all numerical examples.

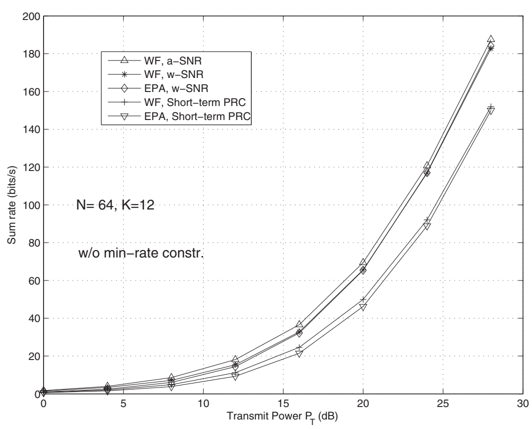

Fig. 1. Simulated sum rates vs. transmit power P_T for the proposed *w*-SNR and others schemes in downlink OFDMA, with $K = 12$, $N = 64$, unequal target BERs, equal channel ASNRs, $\boldsymbol{\alpha} = [12 : 11 : \cdots : 2 : 1]$, without the minimum link-rate constraint. $L = 8$ paths with an exponentially decayed PDP.

In Fig. 1 we present the simulated sum rate vs. BS transmit power P_T in downlink OFDMA for the proposed offline optimized *w*-SNR scheme, with $N = 32$ subchannels, $K = 12$ users, and $L = 8$ paths with an exponentially decayed PDP. The sum rates of the *a*-SNR scheme [2], [28] and the downlink short-term PRC schemes with WF [3] are shown for comparison. The result shows that the proposed *w*-SNR scheme with EPA or 1-D WF provides a substantially higher rate than the short-term PRC scheme [3], and at $P_T = 28$ dB, the sum rate improvement is about 20%. On the other hand, the performance loss of the *w*-SNR scheme compared to the *a*-SNR scheme (without PRC) is very small in this example, though it generally depends on the channel and system parameters.

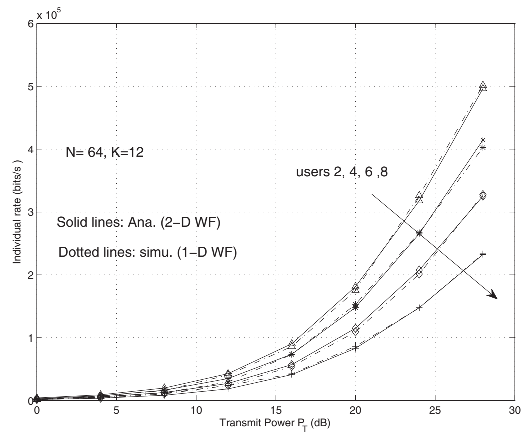

Fig. 2. Analytical and simulated individual rate vs. P_T in downlink OFDMA of users 2,4,6,8 with 1-D and 2-D WF power allocation of the optimal *w*-SNR scheme, with $K = 12$, $N = 64$, unequal target BERs, equal channel ASNRs, $\boldsymbol{\alpha} = [12 : 11 : \cdots : 2 : 1]$, without the minimum link-rate constraint. $L = 8$ paths with an exponentially decayed PDP.

To verify the accuracy of our rate analysis for the *w*-SNR scheme, we present the analytical and simulated individual rates in Fig. 2, where the analytical rate is based on 2-D WF and the simulated rate is based on 1-D WF. The rate performance difference between the 2-D WF and 1-D WF schemes is shown to be negligible. The results show excellent matching between analytical and simulated rates, and also verify that the proposed *w*-SNR schemes can achieve the target PRC for different users.

Besides the PRC, the access fairness of the resource allocation schemes has been studied in various works, e.g., for TDMA [10], [31], CDMA [32], [33] and multi-carrier-based networks [11], [28]. Following the definition of the access fairness metric for the single-channel system [10], [34], we define the degree of access fairness (DoF) for the OFDMA system as

$$\text{DoF}_{\text{all}} = \frac{1}{N}\sum_{k=1}^{K}\sum_{n=1}^{N} \text{AAP}_{k,n} \frac{\log(\text{AAP}_{k,n})}{\log(1/K)} \tag{34}$$

where $\text{AAP}_{k,n}$ is the average access probability for user k on subchannel n, and its expression for the *w*-SNR schemes is given in (25). We can show that $0 < \text{DoF}_{\text{all}} \leq 1$. A larger DoF value corresponds to a better access fairness. In Fig. 3 we show the DoF of the *w*-SNR (with EPA and 1-D WF) and the *a*-SNR (with EPA and 1-D WF) schemes for downlink

OFDMA with $N = 32$ and $K = 12$. The result shows that EPA schemes can generally provide better fairness than the WF scheme, and the *w*-SNR EPA scheme provides the best access fairness. For the case of a small P_T, the *w*-SNR WF scheme has to meet both the PRC and the waterfilling threshold, and many subchannels are left unused, and thus it has a low DoF. When P_T increases, its DoF performance improves and becomes better than that of the *a*-SNR scheme.

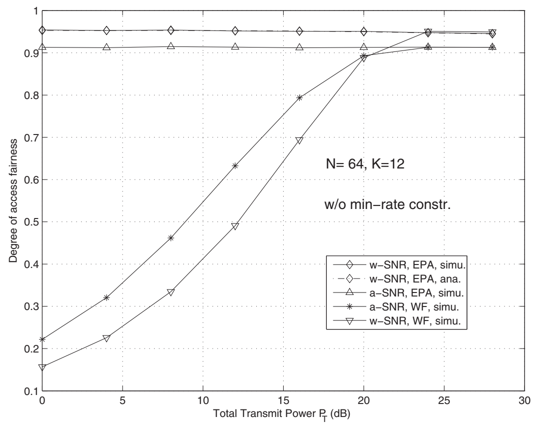

Fig. 3. Degree of access fairness vs. P_T in downlink OFDMA of the *w*-SNR and the *a*-SNR schemes, with $K = 12$, $N = 64$, unequal target BERs, equal channel ASNRs, $\boldsymbol{\alpha} = [12 : 11 : \cdots : 2 : 1]$, without the minimum link-rate constraint. $L = 8$ paths with an exponentially decayed PDP.

Since the *w*-SNR EPA scheme achieves a very similar sum rate as the *w*-SNR WF scheme, and possesses a low-complexity, we will assume EPA for the *w*-SNR schemes from here on. In Fig. 4 we present analytical and simulated sum rates vs. P_T for the offline optimal *w*-SNR scheme and the short-term PRC scheme in downlink OFDMA, with $K = 12$, $N = 64$, unequal target BERs, unequal channel ASNRs, $\boldsymbol{\alpha} = [12 : 11 : \cdots : 2 : 1]$, with and without the minimum link-rate constraint ($\tilde{R}_{\min} = 2$ or 0), and $L = 8$ paths with an exponentially decayed PDP.

Assuming the same parameters, the individual rates for some selected users are shown in Fig. 5. Figs. 4 and 5 show excellent matching between analytical and simulation results. Fig. 4 further shows that the sum rates with the minimum-rate constraint is improved than the case without it. This is because the minimum-rate constraint cuts off some users with weaker ASNRs, and thus improves the sum rate. The results also confirm the sum rate advantage of the *w*-SNR scheme than the STF scheme, which is about 25% at $P_T = 28$ dB.

The rate tracking performance of the proposed *w*-SNR schemes is studied next. We assume the normalized inter-frame Doppler bandwidth is $B_f T_f = 0.04$ for both downlink and uplink channels, where B_f is the Doppler bandwidth. The sum rates of the *w*-SNR schemes, and the short-term PRC and long-term GST PRC schemes, all with admission control are presented in Fig. 6, with $K = 8$, $N = 32$ and a

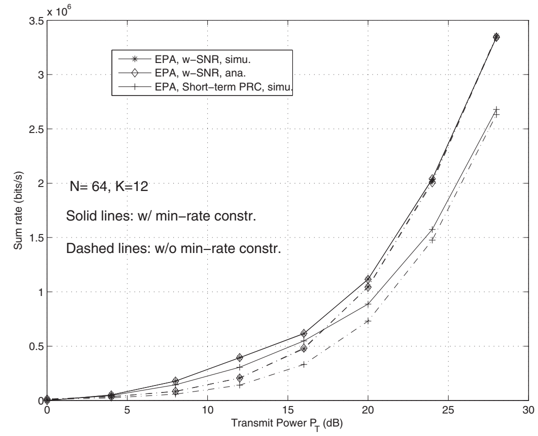

Fig. 4. Analytical and simulated sum rates vs. transmit power P_T for the optimal *w*-SNR scheme and the short-term fairness scheme in downlink OFDMA, with $K = 12$, $N = 64$, unequal target BERs, unequal channel ASNRs, $\boldsymbol{\alpha} = [12 : 11 : \cdots : 2 : 1]$, with and without the minimum link-rate constraint ($\tilde{R}_{\min} = 2$ bits/s/Hz or 0), and $L = 8$ paths with an exponentially decayed PDP.

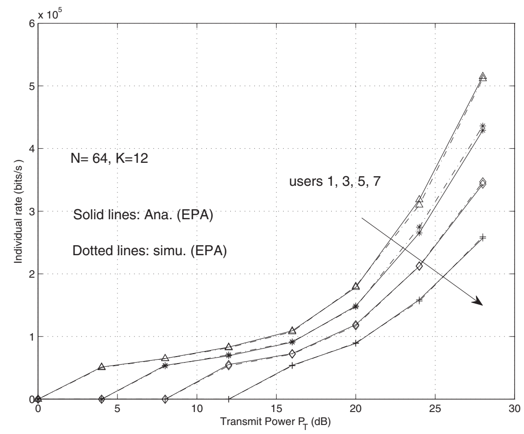

Fig. 5. Analytical and simulated individual rates vs. transmit power P_T for the optimal *w*-SNR scheme with EPA in downlink OFDMA, with $K = 12$, $N = 64$, unequal target BERs, unequal channel ASNRs, $\boldsymbol{\alpha} = [12 : 11 : \cdots : 1]$, with the minimum link-rate constraint ($\tilde{R}_{\min} = 2$ bits/s/Hz), and $L = 8$ paths with an exponentially decayed PDP.

non-uniform $\boldsymbol{\alpha}$. We set $\beta = 0.98$, $\tilde{\beta}_{GS} = 0.4$, $T_{k,\min} = -1$ and $T_{k,\max} = 1$ in the LTF GST algorithm. Fig. 6 shows that the offline *w*-SNR (with fixed $\mathbf{w}$ based on the offline statistical optimization) scheme provides the highest sum rate. The rate-tracking *w*-SNR scheme can provide about 12% to 20% sum rate improvement than the STF scheme when $P_T \geq 20$ dB.

Assuming the same parameters as in Fig. 6, the individual tracked rates for the offline optimized *w*-SNR scheme and the rate-tracking *w*-SNR scheme are given in Figs. 7 and 8, respectively. The results show that the RT *w*-SNR scheme can converge faster and track the target rates more closely than the offline *w*-SNR scheme.

The performance of the *w*-SNR schemes in uplink systems

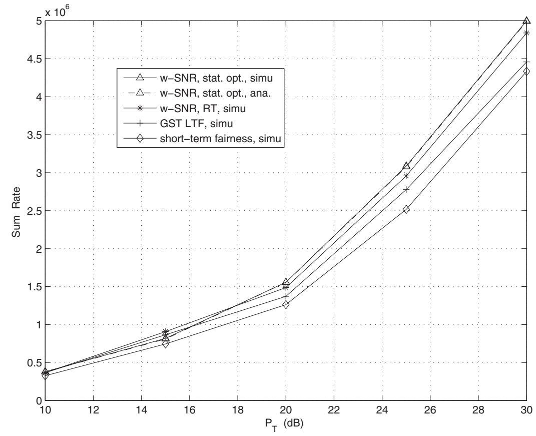

Fig. 6. Analytical and simulated sum rates vs. transmit power P_T for the w-SNR, the STF, and LTF GST schemes in downlink OFDMA, with $K = 8$, $N = 32$, unequal target BERs, unequal channel ASNRs, $\boldsymbol{\alpha} = [2 : 2 : 2 : 2 : 1 : 1 : 1 : 1]$, $\tilde{R}_{\min} = 2$ bits/s/Hz, and $L = 8$ paths with an exponentially decayed PDP.

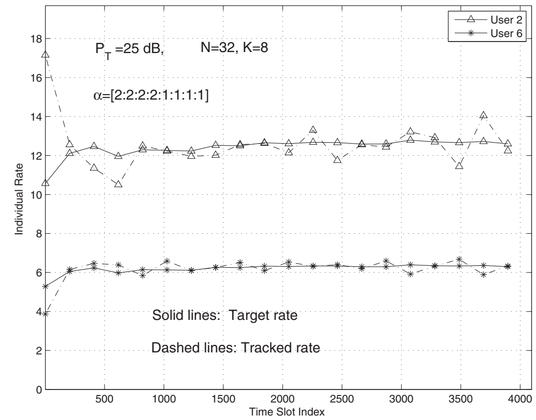

Fig. 8. Individual tracked and targets rates vs. time for the adaptive w-SNR scheme in downlink OFDMA, with $K = 8$, $N = 32$, $P_T = 25$ dB, unequal target BERs, unequal channel ASNRs, $\boldsymbol{\alpha} = [2 : 2 : 2 : 2 : 1 : 1 : 1 : 1]$, $\tilde{R}_{\min} = 2$ bits/s/Hz, and $L = 8$ paths with an exponentially decayed PDP.

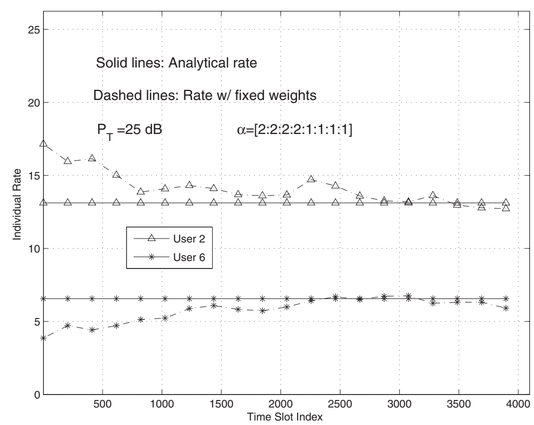

Fig. 7. Individual tracked and analytical rates vs. time for the optimal w-SNR scheme in downlink OFDMA, with $K = 8$, $N = 32$, $P_T = 25$ dB, unequal target BERs, unequal channel ASNRs, $\boldsymbol{\alpha} = [2 : 2 : 2 : 2 : 1 : 1 : 1 : 1]$, $\tilde{R}_{\min} = 2$ bits/s/Hz, and $L = 8$ paths with an exponentially decayed PDP.

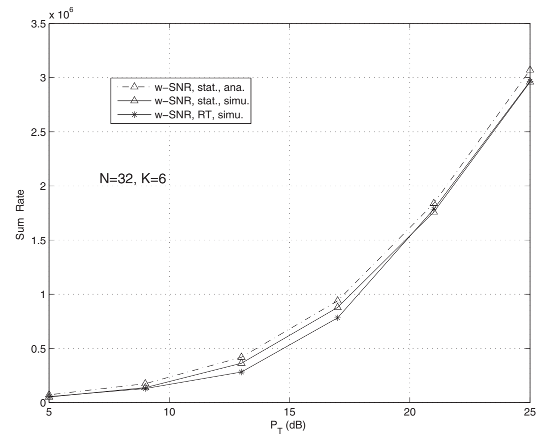

Fig. 9. Sum rates vs. transmit power P_T for the w-SNR schemes with statistical $\mathbf{w}$ and adaptive $\mathbf{w}$, respectively, in uplink OFDMA, with $K = 6$, $N = 32$ independent subchannels, unequal target BERs, equal channel ASNRs, $\boldsymbol{\alpha} = [2 : 2 : 2 : 1 : 1 : 1]$, $\tilde{R}_{\min} = 0$, and $P_{T,k} = P_T/K$ for all k.

is studied next. Fig. 9 shows that analytical and simulated sum rates vs. P_T for the offline optimized w-SNR scheme and the conventional w-SNR RT scheme, with $K = 6$ users and $N = 32$ independent subchannels. The available transmit power of all users are assumed to be equal to $P_{T,k} = P_T/K$ for all k. Fig. 9 shows that the analytical rate result provides a tight approximation of the simulation result for the offline optimal w-SNR scheme. Generally, when the subchannels are highly correlated, our analytical sum rate result provides only a performance upper bound. The result in Fig. 9 also shows that the conventional w-SNR RT scheme performs closely to the offline optimized w-SNR scheme.

In Fig. 10, we present the sum rates vs. P_T for $K = 8$, $N = 32$, with a uniform PRC $\boldsymbol{\alpha} = \mathbf{1}$ and admission control ($\tilde{R}_{\min} = 1$ bits/s/Hz), and $L = 8$ paths with an exponential PDP. The subchannels are highly correlated since $L < N$. The improved and conventional RT w-SNR schemes use $z_{k,n}$ in (31) and $z_{k,n} = w_k \gamma_{k,n}/\bar{\gamma}_k$, respectively. The result shows that the improved w-SNR scheme provides a substantially higher rate than the conventional scheme. Furthermore, it can outperform the STF scheme by about 10% at $P_T = 26$ dB.

In Fig. 11 we show a comparison between the adaptive w-SNR, the short-term fairness, and the LTF GST schemes in uplink OFDMA channels, with $K = 4$, $N = 32$, unequal target BERs and unequal channel ASNRs. The result shows that the w-SNR performs the best, and the GST method is better than the short-term fairness method for small to median P_T.

Fig. 12 presents the individual tracked rate for the improved RT scheme with $K = 8$, $N = 32$ and non-uniform PRC $\boldsymbol{\alpha}$. We assume that the interested long-term time scale

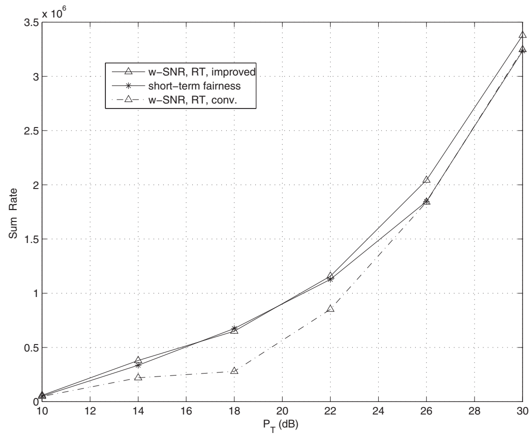

Fig. 10. Sum rates vs. transmit power P_T for the adaptive w-SNR and short-term fairness schemes in uplink OFDMA, with $K = 8$, $N = 32$, unequal target BERs, unequal channel ASNRs, $\boldsymbol{\alpha} = [1 : 1 : 1 : 1 : 1 : 1 : 1 : 1]$, $\tilde{R}_{\min} = 1$ bits/s/Hz, $P_{T,k} = P_T/K$ for all k, and $L = 8$ paths with an exponentially decayed PDP.

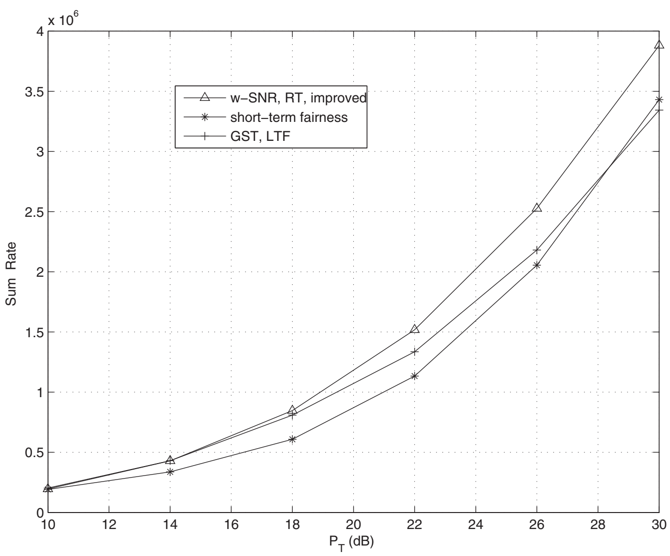

Fig. 11. Sum rates vs. transmit power P_T for the adaptive w-SNR, short-term fairness, and GST schemes in uplink OFDMA, with $K = 4$, $N = 32$, unequal target BERs, unequal channel ASNRs, $\boldsymbol{\alpha} = [1 : 1 : 1 : 1]$, $\tilde{R}_{\min} = 0$ bits/s/Hz, $P_{T,k} = P_T/K$ for all k, and $L = 8$ paths with an exponentially decayed PDP.

size is 1000 OFDM symbols' duration. We use $\beta_k = 0.01$ to find the initially tracked rate, and based on it to update $\mathbf{w}$. Then we use $\beta_k = 0.001$ to present the tracked rates. The SNR weights are updated using $\tilde{\beta}_w = 0.1$. The target rates changes with time due to the time-varying nature of the channel and the users' sum rate.

Finally, in Fig. 13 we present the modified Jain's PRC fairness for the RT w-SNR scheme under admission control and PRC in downlink and uplink OFDMA channels. The near-perfect fairness (larger than 0.95) is achieved for all cases. The fairness metric is defined based on the users who

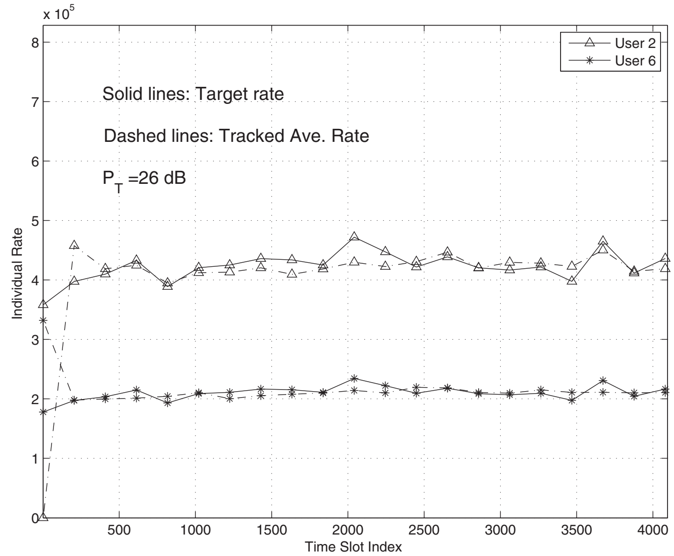

Fig. 12. Individual tracked and targets rates vs. time for the adaptive w-SNR scheme in uplink OFDMA, with $K = 8$, $N = 32$, $P_T = 26$ dB, unequal target BERs, unequal channel ASNRs, $\boldsymbol{\alpha} = [2 : 2 : 2 : 2 : 1 : 1 : 1 : 1]$, $\tilde{R}_{\min} = 1$ bits/s/Hz, $P_{T,k} = P_T/K$ for all k, and $L = 8$ paths with an exponentially decayed PDP.

are admitted. For the uplink case, when P_T increases from 10 dB to 18 dB, more users are admitted but the PRC between the users could be less accurately tracked, and the modified Jain's metric dropped from 1 to 0.96 for the uniform PRC case. When P_T increases further, the Jain's metric rises back to close to unity.

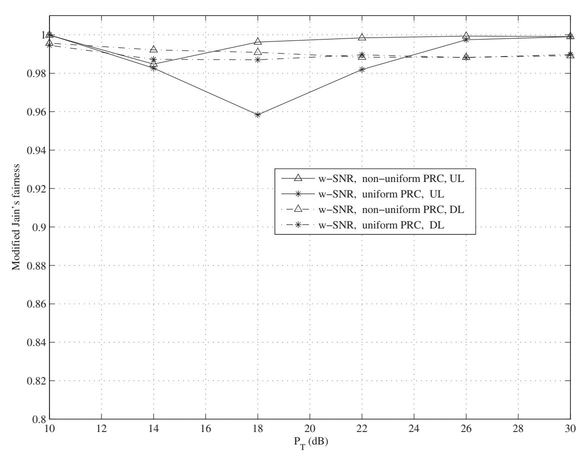

Fig. 13. Modified Jain's fairness metric vs. transmit power P_T for the adaptive w-SNR schemes in uplink and downlink OFDMA systems, with $K = 8$, $N = 32$, unequal target BERs, unequal channel ASNRs, with uniform and non-uniform target rate proportions, and $L = 8$ paths with an exponentially decayed PDP.

VI. Conclusions

In this paper, we have proposed the weighted-SNR-ranking-based multiuser diversity schemes to maximize the sum rate under long-term proportional-rate and minimum-rate constraints, using EPA, 1-D WF and 2-D WF power allocation, for both downlink and uplink OFDMA systems. Both the offline SNR-weight optimization and the online rate-tracking weight adaptation techniques have been developed with low complexity of implementation, and closed-form analytical formulas and access fairness metrics have been derived. Simulation results have verified the accuracy of our analytical results, and showed that the proposed schemes generate substantially higher sum rates than the short-term PRC scheme, and perform more favorably than a benchmark long-term PRC scheme. These results provide new insights on how to implement the PRC and other constraints for OFDMA scheduling, and may be absorbed into other non-SNR-diversity based scheduling techniques.

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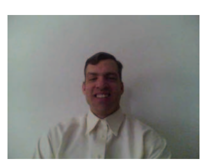

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