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Performance analysis of decode-and-forward relaying schemes with adaptive quadrature amplitude modulation (QAM)

E.S. Altubaishi[1,2] *X. (Sherman) Shen*[1]

[1]*Electrical and Computer Engineering, University of Waterloo, Waterloo, Canada N2L 3G1*
[2]*Electrical Engineering, King Saud University, Riyadh 11421, Saudi Arabia*
E-mail: esaltuba@bbcr.uwaterloo.ca

Abstract: In this study, the authors propose two schemes to enhance the spectral efficiency of the decode-and-forward adaptive relaying with adaptive modulation. The first scheme, called the minimum error rate scheme, uses cooperative transmission all the time to achieve transmit diversity whereas the second scheme, called the maximum spectral efficiency scheme, tries to avoid cooperative transmission if the spectral efficiency cannot be improved. For each scheme, the authors introduce a policy that depends on the channel state information of each link. They provide performance analysis in terms of average spectral efficiency, average bit error rate and outage probability over independent non-identically Rayleigh fading channels. Our analysis is verified by simulation results. These results show that our proposed schemes lead to an improvement in the performance of cooperative wireless systems.

1 Introduction

In wireless communications, the characteristics of the channel are usually under continuous change because of multipath fading. As a result, the capacity of wireless link experiences high variability. Furthermore, the spectrum over wireless channel is limited, as well as the required radio spectrum is increased as a consequence of increasing demand of data services. Therefore more spectrally efficient transmission schemes are required. Cooperative diversity (CD) is one of the promising techniques that can be used to mitigate fading [1–9]. The two well-known schemes in CD networks are the decode-and-forward (DF) scheme and the amplify-and-forward (AF) scheme. If the relay node forwards the received signal all the time, it is called fixed relaying. CD schemes usually transmit using orthogonal channels in order to avoid interference between the source and relay links. However, this causes reduction in the spectral efficiency. One way to overcome this problem is to use adaptive cooperative schemes called incremental relaying [9, 10]. It basically tries to avoid cooperative transmission whenever the direct transmission can provide better performance. Fig. 1 shows an example of a wireless network that uses incremental relaying techniques to connect a user with a base station. Selection relaying is another adaptive cooperative scheme that controls the transmission of the relay node based on the predefined metric [9].

Recently, the cooperative schemes with variable-rate transmission have been appeared (e.g. [9–15]) because the variable-rate transmission has been proven to maximise the spectral efficiency in the conventional direct transmission systems [16–18]. In [14], AF fixed relaying (AFFR) was analysed under variable-rate transmission in Rayleigh fading environments. It is noticed the reduction of the spectral efficiency as the number of relay is increased. In [15], AF incremental relaying with adaptive modulation was investigated. The main idea is to use cooperative transmission if the direct link experiences an outage. In [19], DF scheme was investigated with adaptive modulation. It is assumed that the destination node may receive two different modulation modes, which increases the detection complexity especially when the multiple relays are used. This assumption is avoided in [20] where fixed modulation mode at the destination node was considered. However, the modulation mode selection was based only on the combined received signal-to-noise ratio (SNR) from relay–destination and source–destination links, which may not be supported by the source–relay link.

In this study, we propose two schemes for DF relaying with adaptive modulation. The first scheme presents a solution for enhancing the spectral efficiency of the DF wireless networks. In DF scheme, the relay node decodes, encodes and then retransmits the received signal. Thus, it is important to incorporate two factors: the effect of receiving errors from the relay node and the ability of the source–relay link to support the rate selected by the combined links at the destination node. Therefore we introduce a policy that has the capability to select suitable transmission parameters taking in account that the cooperative transmission is used all the time. This scheme is called minimum error rate scheme (MERS). The second scheme aims to further enhance the spectral efficiency by introducing another policy that avoids cooperative transmission whenever it is not useful. This scheme is called maximum spectral efficiency scheme

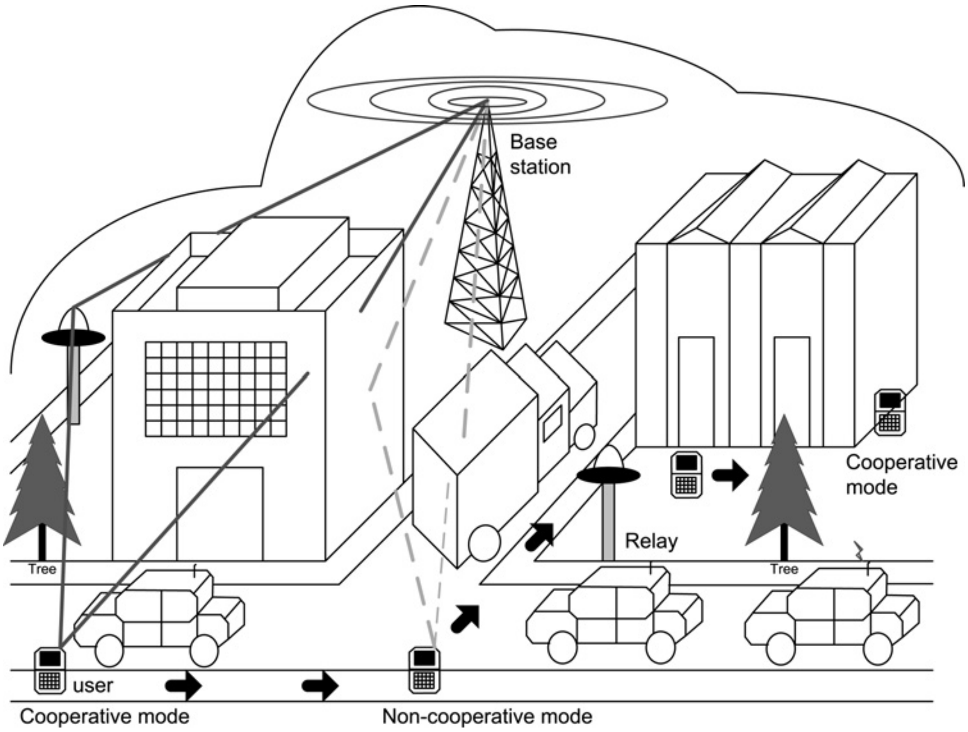

Fig. 1 *Example of adaptive cooperative wireless network*

(MSES). To evaluate these two schemes, we provide the performance analysis in terms of average spectral efficiency, average bit error rate (ABER) and outage probability over independent non-identically Rayleigh fading environments. We start the analysis by deriving the cumulative distribution function (CDF) and probability density function (PDF) of the output SNR of the two schemes. By using these results, the performance measures are derived and shown in closed-form.

The remainder of this study is organised as follows. Section 2 describes the system model and its parameters that includes channel model and adaptive modulation considered. Section 3 presents the MERS and the MSES. Performance analysis of the two proposed schemes for DF adaptive relaying with adaptive modulation is given in Section 4. Simulation results are presented in Section 5. Finally, Section 6 concludes the study.

2 System overview

2.1 System model

We consider a cooperative wireless system consists of a source node S, a destination node D, and a relay node R as shown in Fig. 2. Before we further describe the system, the following assumptions are made.

(*A1*) The links between nodes are modelled as independent but not identical quasi-static Rayleigh fading channels, that is, the channel gains do not change during the transmission of one frame.
(*A2*) The channel is assumed to be reciprocal [21], that is, the uplink and downlink gains are the same.
(*A3*) The transmit power from the source and relay are constant since adaptive modulation is used. Although varying the power in conjunction with varying the rate can be applied, a minimal gain is expected [16].

The relay node is equipped with error-detection techniques such as cyclic redundancy, and therefore the relay node is kept silent in the second phase of cooperative transmission if an error is detected [13, 22].

In the DF scheme, the transmission starts with broadcasting a source information $x(t)$ to the relay and destination nodes in the first phase. The received signals at the relay and destination nodes are

$$y_{\mathrm{sr}}(t) = a_{\mathrm{sr}}x(t) + n_{\mathrm{sr}}(t), \quad t = 1, 2, \ldots, \frac{T}{2} \tag{1}$$

$$y_{\mathrm{sd}}(t) = a_{\mathrm{sd}}x(t) + n_{\mathrm{sd}}(t), \quad t = 1, 2, \ldots, \frac{T}{2} \tag{2}$$

respectively, where a_{sr} and a_{sd} are the fading coefficients between the source and relay nodes and the source and destination nodes, respectively, $n_{\mathrm{sr}}(t)$ and $n_{\mathrm{sd}}(t)$ are the additive white gaussian noise terms at the relay and destination nodes, respectively, with a variance of N_o for all links, and T is a time-slot duration. In the second phase, relay node detects, encodes and retransmits the received signal to the destination node. Then the received signal at the destination node can be written as

$$y_{\mathrm{rd}}(t) = a_{\mathrm{rd}}\hat{x}(t) + n_{\mathrm{rd}}(t), \quad t = \frac{T}{2} + 1, \ldots, T \tag{3}$$

where $\hat{x}(t)$ is the transmitted signal from the relay node to the destination node, which is the original source signal $x(t)$ if there is no error detected. With maximum ratio combining, the instantaneous DF effective received SNR becomes

$$\gamma_{\mathrm{df}} = \gamma_{\mathrm{sd}} + \gamma_{\mathrm{rd}} \tag{4}$$

where γ_{sd} and γ_{rd} are the instantaneous SNRs between the source and destination nodes, and the relay and destination

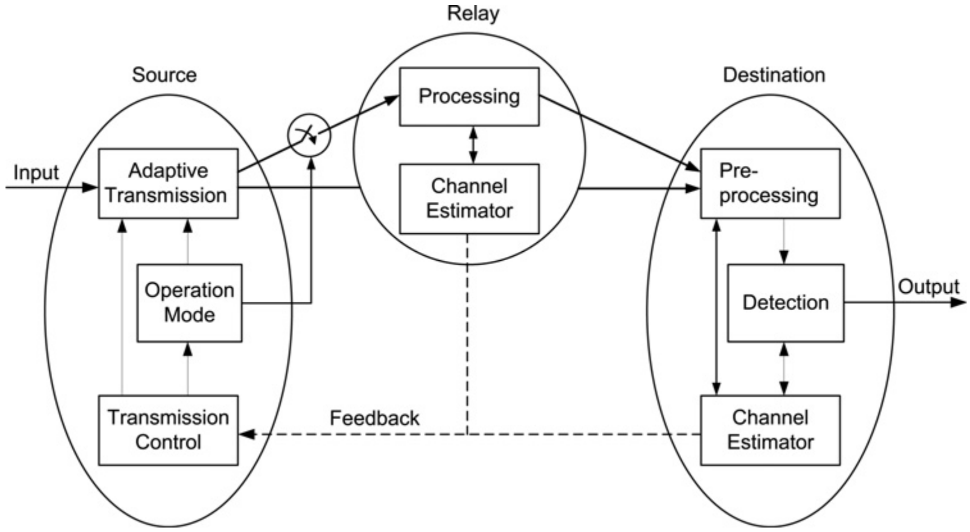

Fig. 2 *Incremental relaying scheme with adaptive transmission*

nodes, respectively. If cooperative transmission is not activated, the received signal at the destination node is

$$y_{sd}(t) = a_{sd}x(t) + n_{sd}(t), \quad t = 1, 2, \ldots, T \tag{5}$$

2.2 Channel model

For the Rayleigh fading channels, the CDF of the output SNR for each link from node i to node j is given by

$$F_{\gamma_{ij}}(x) = 1 - e^{-x/\bar{\gamma}_{ij}} \tag{6}$$

where $\bar{\gamma}_{ij} = E[\gamma_{ij}]$, and $E(.)$ is the expectation operator. The PDF is given by

$$f_{\gamma_{ij}}(x) = \frac{1}{\bar{\gamma}_{ij}} e^{-x/\bar{\gamma}_{ij}} \tag{7}$$

2.3 Adaptive modulation

For practical implementation of adaptive transmission, an adaptive M-QAM with constellation size of 2^n, $n = 2, 3, \ldots, N$, is considered, where N is the maximum spectral efficiency. The modulation mode selection depends on a predesigned target performance which can be represented by the target BER, BER_T. With this target performance, the range of the SNR is divided into regions $[\gamma_n, \gamma_{n+1})$, each region corresponds to a modulation mode. The transmission is declared to be under outage condition if SNR $< \gamma_2$, where $n = 0$. In M-QAM modulation with coherent detection and grey coding, the switching thresholds can be given as [23]

$$\gamma_n = \frac{2^n - 1}{3}\left(Q^{-1}\left(\frac{n\sqrt{2^n}\text{BER}_T}{2(\sqrt{2^n} - 1)}\right)\right)^2, \quad 2 \le n \le N \tag{8}$$

where $Q(.)$ is the Gaussian Q-function, and $\gamma_{N+1} = \infty$.

3 Mode of operation

One way of designing the system is to let the relay node estimate the relay links after receiving a ready-to-send signal from the source node, and a clear-to-send (CTS) signal from the destination node which includes the transmission parameters as designed in the conventional direct transmission. The relay node then sends the decision within a given time period using a predefined strategy. Otherwise, the source node uses the received parameters from CTS signal for the next frame transmission. The details of the predefined strategies are described in the following two schemes.

3.1 Minimum error rate scheme

The objective of the MERS is to achieve the same spectral efficiency of the AF scheme, but with better error rate. In the DF scheme that employs variable-rate transmission, it is not feasible to select the best transmission rate based only on the received effective SNR at the destination node, γ_{df}, because this transmission rate may not be supported by the source–relay link as well. Thus, we define the following strategy:

1. If $\gamma_{sr} < \gamma_{th}$, do not cooperate,
where γ_{sr} is the SNR of the source–relay link, and γ_{th} is the minimum switching threshold SNR in which the link can support the transmission.
2. If $\gamma_{sr} \ge \gamma_{th}$, cooperate.
Then, the output SNR of this scheme can be defined as

$$\gamma_{MERS} = \begin{cases} \gamma_{sd}, & \gamma_{sr} < \gamma_{th} \\ \gamma_{min}, & \gamma_{sr} \ge \gamma_{th} \end{cases} \tag{9}$$

where $\gamma_{min} = \min(\gamma_{sr}, \gamma_{df})$.

3.2 Maximum spectral efficiency scheme

The objective of the MSES is to enhance the spectral efficiency of the MERS, by avoiding cooperative transmission whenever it is not necessary (i.e. the direct transmission has higher transmission rate). Therefore we can define the following strategy.

1. If $\gamma_{sd} \ge \gamma_{\lfloor N/2 \rfloor + 1}$, do not cooperate,
where $\lfloor k \rfloor$ is the largest integer less than or equal k.
2. If $\gamma_{sd} < \gamma_{\lfloor N/2 \rfloor + 1}$, cooperate if $\gamma_{min} \ge \gamma_T$,
where γ_T is the switching threshold SNR used to guarantee that the cooperative transmission can maximise the spectral

efficiency, then the output SNR of this scheme, γ_{MSES}, can be defined as

$$\gamma_{\mathrm{MSES}} = \begin{cases} & \text{if } (\gamma_{\mathrm{sd}} \geq \gamma\lfloor N/2 \rfloor + 1), \quad \text{or} \\ \gamma_{\mathrm{sd}}, & \text{if } (\gamma_n \leq \gamma_{\mathrm{sd}} \leq \gamma_{n+1} \quad \text{and} \\ & \gamma_{\min} < \gamma_{\mathrm{T}}, \quad n = 2, \ldots, \left\lfloor \dfrac{N}{2} \right\rfloor\bigg) \\ \gamma_{\min}, & \text{otherwise} \end{cases} \tag{10}$$

For discrete variable-rate transmission, switching threshold γ_{T} can be set to be equal to γ_{2n}.

4 Performance analysis

4.1 Average spectral efficiency

4.1.1 Minimum error rate scheme: The average spectral efficiency of the MERS can be expressed as

$$\eta^{\mathrm{MERS}} = \Pr(\gamma_{\mathrm{sr}} < \gamma_{\mathrm{th}}) \sum_{n=2}^{N} n a(n) + (1 - \Pr(\gamma_{\mathrm{sr}} < \gamma_{\mathrm{th}})) \sum_{m=2}^{N} \frac{m}{2} b(m) \tag{11}$$

where m is the spectral efficiency of the cooperative transmission and divided by two because of the half duplex constraint. $a(n)$ and $b(m)$ are the probability that γ_{sd} and $\gamma_{\min}$ fall in region n and m, respectively. Then $a(n)$ can be represented as

$$a(n) = \int_{\gamma_n}^{\gamma_{n+1}} f_{\gamma_{\mathrm{sd}}}(x)\,\mathrm{d}x = F_{\gamma_{\mathrm{sd}}}(\gamma_{n+1}) - F_{\gamma_{\mathrm{sd}}}(\gamma_n) \tag{12}$$

where $f_{\gamma_{\mathrm{sd}}}(x)$ is the PDF of γ_{sd} and $F_{\gamma_{\mathrm{sd}}}(x)$ is its CDF. Similarly, $b(m)$ can be represented as

$$\begin{aligned} b(m) &= \int_{\gamma_m}^{\gamma_{m+1}} f_{\gamma_{\min}}(z/\gamma_{\mathrm{sr}} > \gamma_{\mathrm{th}})\,\mathrm{d}z \\ &= F_{\gamma_{\min}}(\gamma_{m+1}/\gamma_{\mathrm{sr}} > \gamma_{\mathrm{th}}) - F_{\gamma_{\min}}(\gamma_m/\gamma_{\mathrm{sr}} > \gamma_{\mathrm{th}}) \end{aligned} \tag{13}$$

where $f_{\gamma_{\min}}(z/\gamma_{\mathrm{sr}} > \gamma_{\mathrm{th}})$ is the conditional PDF of $\gamma_{\min}$ given that γ_{sr} is greater than the minimum switching threshold, γ_{th}, and $F_{\gamma_{\min}}(z/\gamma_{\mathrm{sr}} > \gamma_{\mathrm{th}})$ is its conditional CDF. From Appendix 1, the conditional CDF of $\gamma_{\min}$ and its corresponding conditional PDF can be given by

$$F_{\gamma_{\min}}(z/\gamma_{\mathrm{sr}} > \gamma_{\mathrm{th}}) = \begin{cases} 1 - \dfrac{\mathrm{e}^{-z/\bar{\gamma}_{\mathrm{sr}}}}{\mathrm{e}^{-\gamma_{\mathrm{th}}/\bar{\gamma}_{\mathrm{sr}}}}\left[\mathrm{e}^{-z/\bar{\gamma}_{\mathrm{sd}}} + \dfrac{\bar{\gamma}_{\mathrm{rd}}}{\bar{\gamma}_{\mathrm{rd}} - \bar{\gamma}_{\mathrm{sd}}}(\mathrm{e}^{-z/\bar{\gamma}_{\mathrm{rd}}} - \mathrm{e}^{-z/\bar{\gamma}_{\mathrm{sd}}})\right], & z > \gamma_{\mathrm{th}} \\ 1 - \mathrm{e}^{-z/\bar{\gamma}_{\mathrm{sd}}} - \dfrac{\bar{\gamma}_{\mathrm{rd}}}{\bar{\gamma}_{\mathrm{rd}} - \bar{\gamma}_{\mathrm{sd}}}(\mathrm{e}^{-z/\bar{\gamma}_{\mathrm{rd}}} - \mathrm{e}^{-z/\bar{\gamma}_{\mathrm{sd}}}), & z \leq \gamma_{\mathrm{th}} \end{cases} \tag{14}$$

and

$$f_{\gamma_{\min}}(z/\gamma_{\mathrm{sr}} > \gamma_{\mathrm{th}}) = \begin{cases} \dfrac{1}{\mathrm{e}^{-\gamma_{\mathrm{th}}/\bar{\gamma}_{\mathrm{sr}}}}\left[\dfrac{(\bar{\gamma}_{\mathrm{sr}} + \bar{\gamma}_{\mathrm{rd}})}{\bar{\gamma}_{\mathrm{sr}}(\bar{\gamma}_{\mathrm{rd}} - \bar{\gamma}_{\mathrm{sd}})}\mathrm{e}^{-z((1/\bar{\gamma}_{\mathrm{sr}})+(1/\bar{\gamma}_{\mathrm{rd}}))} \right. \\ \left. \quad - \dfrac{(\bar{\gamma}_{\mathrm{sr}} + \bar{\gamma}_{\mathrm{sd}})}{\bar{\gamma}_{\mathrm{sr}}(\bar{\gamma}_{\mathrm{rd}} - \bar{\gamma}_{\mathrm{sd}})}\mathrm{e}^{-z((1/\bar{\gamma}_{\mathrm{sr}})+(1/\bar{\gamma}_{\mathrm{sd}}))}\right], & z > \gamma_{\mathrm{th}} \\ \dfrac{1}{\bar{\gamma}_{\mathrm{rd}} - \bar{\gamma}_{\mathrm{sd}}}(\mathrm{e}^{-z/\bar{\gamma}_{\mathrm{rd}}} - \mathrm{e}^{-\gamma_{\mathrm{th}}/\bar{\gamma}_{\mathrm{sd}}}), & z \leq \gamma_{\mathrm{th}} \end{cases} \tag{15}$$

respectively. Substituting (14) into (13) and then substituting (12) into (11), the average spectral efficiency of the MERS can be given by

$$\begin{aligned} \eta^{\mathrm{MERS}} &= (1 - \mathrm{e}^{-\gamma_{\mathrm{th}}/\bar{\gamma}_{\mathrm{sr}}}) \sum_{n=2}^{N} n\psi(n, 1/\bar{\gamma}_{\mathrm{sd}}) \\ &\quad + \frac{1}{\bar{\gamma}_{\mathrm{sd}} - \bar{\gamma}_{\mathrm{rd}}} \sum_{m=2}^{N} \frac{m}{2}\left[\bar{\gamma}_{\mathrm{sd}}\psi\left(m, \frac{1}{\bar{\gamma}_{\mathrm{sr}}} + \frac{1}{\bar{\gamma}_{\mathrm{sd}}}\right)\right. \\ &\quad \left. - \bar{\gamma}_{\mathrm{rd}}\psi\left(m, \frac{1}{\bar{\gamma}_{\mathrm{sr}}} + \frac{1}{\bar{\gamma}_{\mathrm{rd}}}\right)\right] \end{aligned} \tag{16}$$

where $\psi(\alpha, \beta) = \mathrm{e}^{-\gamma_\alpha \beta} - \mathrm{e}^{-\gamma_{\alpha+1}\beta}$.

4.1.2 Maximum spectral efficiency scheme: The average spectral efficiency of the MSES can be expressed as

$$\eta^{\mathrm{MSES}} = \sum_{n=\lfloor N/2 \rfloor + 1}^{N} n a(n) + \sum_{n=2}^{\lfloor N/2 \rfloor} n c(n) + \sum_{\substack{n=0 \\ n \neq 1}}^{\lfloor N/2 \rfloor} \sum_{m=I^n}^{N} \frac{m}{2} d(n, m) \tag{17}$$

where $c(n)$ is the probability that $\gamma_{\mathrm{sd}} < \gamma_{\lfloor N/2 \rfloor + 1}$ and $\gamma_{\min} < \gamma_{\mathrm{T}}$, $d(n, m)$ is the probability that $\gamma_{\mathrm{sd}} < \gamma_{\lfloor N/2 \rfloor + 1}$ and $\gamma_{\min} \geq \gamma_{\mathrm{T}}$, and $I^n = 2$ if $n = 0$ and $I^n = 2n$ if $n \neq 0$. Then $c(n)$ can be represented as

$$c(n) = F_{\gamma_{\min}, \gamma_{\mathrm{sd}}}(\gamma_{\mathrm{T}}, \gamma_{n+1}) - F_{\gamma_{\min}, \gamma_{\mathrm{sd}}}(\gamma_{\mathrm{T}}, \gamma_n) \tag{18}$$

where the joint CDF of $\gamma_{\min}$ and γ_{sd}, $F_{\gamma_{\min}, \gamma_{\mathrm{sd}}}(z, x)$, and its corresponding joint PDF are derived in Appendix 2, and given by (see (19))

$$F_{\gamma_{\min}, \gamma_{\mathrm{sd}}}(z, x) = \begin{cases} 1 - \mathrm{e}^{-x/\bar{\gamma}_{\mathrm{sd}}} - \dfrac{\bar{\gamma}_{\mathrm{rd}}}{\bar{\gamma}_{\mathrm{rd}} - \bar{\gamma}_{\mathrm{sd}}}\mathrm{e}^{-z((1/\bar{\gamma}_{\mathrm{sr}})+(1/\bar{\gamma}_{\mathrm{rd}}))}(1 - \mathrm{e}^{-x((1/\bar{\gamma}_{\mathrm{sd}})+(1/\bar{\gamma}_{\mathrm{rd}}))}), & x \leq z \\ 1 - \mathrm{e}^{-x/\bar{\gamma}_{\mathrm{sd}}} + \mathrm{e}^{-z/\bar{\gamma}_{\mathrm{sr}}}\mathrm{e}^{-x/\bar{\gamma}_{\mathrm{sd}}} + \dfrac{1}{\bar{\gamma}_{\mathrm{rd}} - \bar{\gamma}_{\mathrm{sd}}} & x > z \\ \quad \times[\bar{\gamma}_{\mathrm{sd}}\mathrm{e}^{-z((1/\bar{\gamma}_{\mathrm{sr}})+(1/\bar{\gamma}_{\mathrm{sd}}))} - \bar{\gamma}_{\mathrm{rd}}\mathrm{e}^{-z((1/\bar{\gamma}_{\mathrm{sr}})+(1/\bar{\gamma}_{\mathrm{rd}}))}], & \end{cases} \tag{19}$$

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and (see (20))

respectively. $d(n,m)$ is the probability of activation the cooperative transmission which can be represented as

$$d(n, m) = F_{\gamma_{\min},\gamma_{sd}}(\gamma_{m+1}, \gamma_{n+1}) - F_{\gamma_{\min},\gamma_{sd}}(\gamma_{m+1}, \gamma_n) - F_{\gamma_{\min},\gamma_{sd}}(\gamma_m, \gamma_{n+1}) + F_{\gamma_{\min},\gamma_{sd}}(\gamma_m, \gamma_n) \quad (21)$$

Substituting (19) into (18) and (21), then substituting (12) into (17), the average spectral efficiency of the MSES can be given as

$$\eta^{\mathrm{MSES}} = \sum_{n=2}^{N} n\psi(n, 1/\bar{\gamma}_{sd}) + \frac{\bar{\gamma}_{rd}}{\bar{\gamma}_{sd} - \bar{\gamma}_{rd}} e^{-\gamma_T((1/\bar{\gamma}_{sr})+(1/\bar{\gamma}_{rd}))} \times \sum_{n=2}^{\lfloor N/2 \rfloor} n\,\psi\left(n, \frac{1}{\bar{\gamma}_{sd}} - \frac{1}{\bar{\gamma}_{rd}}\right) + \frac{\bar{\gamma}_{rd}}{\bar{\gamma}_{rd} - \bar{\gamma}_{sd}} \times \sum_{\substack{n=0 \\ n\neq 1}}^{\lfloor \frac{N}{2} \rfloor} \sum_{m=I^n}^{N} \frac{m}{2}\psi\left(n, \frac{1}{\bar{\gamma}_{sd}} - \frac{1}{\bar{\gamma}_{rd}}\right)\psi\left(m, \frac{1}{\bar{\gamma}_{sr}} + \frac{1}{\bar{\gamma}_{rd}}\right) \quad (22)$$

4.2 Average bit error rate

4.2.1 Minimum error rate scheme: In general, the ABER can be defined as the average number of bits in error divided by the total average number of transmitted bits. Therefore the ABER of the MERS can be given as

$$\mathrm{ABER}^{\mathrm{MERS}} = \frac{1}{\eta^{\mathrm{MERS}}}\left[\Pr(\gamma_{sr} < \gamma_{th}) \sum_{n=2}^{N} n\mathrm{ABER}_{a(n)} + \sum_{m=2}^{N} \frac{m}{2} \mathrm{ABER}_{b(m)}\right] \quad (23)$$

where $\mathrm{ABER}_{a(n)}$ and $\mathrm{ABER}_{b(m)}$ are the ABER of the direct and cooperative transmissions, respectively. Then $\mathrm{ABER}_{a(n)}$ can be given as

$$\mathrm{ABER}_{a(n)} = \int_{\gamma_n}^{\gamma_{n+1}} \mathrm{BER}(n, x) f_{\gamma_{sd}}(x)\,dx \quad (24)$$

where BER(.,.) is the BER of the M-QAM which can be approximated as [23]

$$\mathrm{BER}(n, \gamma) = \frac{2(\sqrt{2^n} - 1)}{n\sqrt{2^n}} Q\left(\sqrt{\frac{3\gamma}{2^n - 1}}\right), \quad n \geq 2 \quad (25)$$

To solve the ABER, it is necessary to define a common finite integral as

$$\Phi(\gamma_{s1}, \gamma_{s2}, \bar{\gamma}, B) = \int_{\gamma_{s1}}^{\gamma_{s2}} Q\left(\sqrt{Bx}\right) \frac{e^{-(x/\gamma)}}{\bar{\gamma}}\,dx \quad (26)$$

Equation (26) can be solved using integration by parts as shown in Appendix 3, which equals to

$$\Phi(\gamma_{s1}, \gamma_{s2}, \bar{\gamma}, B) = Q\left(\sqrt{B\gamma_{s1}}\right)e^{-\gamma_{s1}/\bar{\gamma}} - \sqrt{\frac{B}{(2/\bar{\gamma}) + B}} Q\left(\sqrt{2\gamma_{s1}\left(\frac{1}{\bar{\gamma}} + \frac{B}{2}\right)}\right) - Q\left(\sqrt{B\gamma_{s2}}\right)e^{-\gamma_{s2}/\bar{\gamma}} + \sqrt{\frac{B}{(2/\bar{\gamma}) + B}} Q\left(\sqrt{2\gamma_{s2}\left(\frac{1}{\bar{\gamma}} + \frac{B}{2}\right)}\right) \quad (27)$$

then, the ABER of the direct transmission can be written as

$$\mathrm{ABER}_{a(n)} = \mathrm{A}_n \Phi(\gamma_n, \gamma_{n+1}, \bar{\gamma}_{sd}, B_n) \quad (28)$$

where $A_n = 2(\sqrt{2}^n - 1)/(n\sqrt{2}^n)$ and $B_n = 3/(2^n - 1)$.

In the DF scheme, to have error-free transmission, there must be a successful transmission from source–relay link, then a successful transmission after combining the two links from the source and relay nodes. Hence, the ABER at the destination node can be given as [13]

$$\mathrm{ABER}_{b(m)} = \mathrm{ABER}^{\mathrm{sr}}_{b(m)} \cdot \mathrm{ABER}^{\mathrm{sd}}_{b(m)} + (1 - \mathrm{ABER}^{\mathrm{sr}}_{b(m)}) \cdot \mathrm{ABER}^{df}_{b(m)} \quad (29)$$

The ABER at the relay node, $\mathrm{ABER}^{\mathrm{sr}}_{b(m)}$, should be calculated based on whether γ_{sr} is the minimum or not, therefore $\mathrm{ABER}^{\mathrm{sr}}_{b(m)}$ can be determined as

$$\mathrm{ABER}^{\mathrm{sr}}_{b(m)} = \int_{\gamma_m}^{\gamma_{m+1}} \mathrm{BER}(m, z)(1 - F_{\gamma_{df}}(z)) f_{\gamma_{sr}}(z)\,dz + \int_{\gamma_m}^{\gamma_{m+1}} \int_{z}^{\infty} \mathrm{BER}(m, y) f_{\gamma_{sr}}(y) f_{\gamma_{df}}(z)\,dy\,dz \quad (30)$$

Similarly

$$\mathrm{ABER}^{\mathrm{df}}_{b(m)} = \int_{\gamma_m}^{\gamma_{m+1}} \mathrm{BER}(m, z)(1 - F_{\gamma_{sr}}(z)) f_{\gamma_{df}}(z)\,dz + \int_{\gamma_m}^{\gamma_{m+1}} \int_{z}^{\infty} \mathrm{BER}(m, y) f_{\gamma_{df}}(y) f_{\gamma_{sr}}(z)\,dy\,dz \quad (31)$$

$$f_{\gamma_{\min},\gamma_{sd}}(z, x) = \begin{cases} \dfrac{\bar{\gamma}_{sr} + \bar{\gamma}_{rd}}{\bar{\gamma}_{sr}(\bar{\gamma}_{rd} - \bar{\gamma}_{sd})} e^{-z((1/\bar{\gamma}_{sr})+(1/\bar{\gamma}_{rd}))}(1 - e^{-x((1/\bar{\gamma}_{sd})-(1/\bar{\gamma}_{rd}))}), & x \leq z \\ \dfrac{1}{\bar{\gamma}_{sr}(\bar{\gamma}_{rd} - \bar{\gamma}_{sd})}[(\bar{\gamma}_{sr} + \bar{\gamma}_{rd})e^{-z((1/\bar{\gamma}_{sr})+(1/\bar{\gamma}_{rd}))} - (\bar{\gamma}_{sr} + \bar{\gamma}_{sd})e^{-z((1/\bar{\gamma}_{sr})+(1/\bar{\gamma}_{sd}))}] - \dfrac{1}{\bar{\gamma}_{sr}} e^{-z/\bar{\gamma}_{sr}} e^{-x/\bar{\gamma}_{sd}}, & x > z \end{cases} \quad (20)$$

and

$$\text{ABER}_{b(m)}^{\text{sd}} = \int_0^\infty \text{BER}(m, x) f_{\gamma_{\text{sd}}}(x)\,\text{d}x \tag{32}$$

Equations (30)–(32) are solved and simplified as

$$\begin{aligned}\text{ABER}_{b(m)}^{\text{sr}} = \frac{A_m}{\bar{\gamma}_{\text{sd}} - \bar{\gamma}_{\text{rd}}} &[\bar{\gamma}_{\text{sd}}\Phi(\gamma_m,\, \gamma_{m+1},\, \bar{\gamma}_s,\, B_m) \\ &- \bar{\gamma}_{\text{sd}}\sqrt{\frac{B_m}{(2/\bar{\gamma}_{\text{sr}}) + B_m}}\Phi(\gamma_m,\, \gamma_{m+1},\, \bar{\gamma}_{\text{sd}},\, C_m) \\ &+ \bar{\gamma}_{\text{rd}}\sqrt{\frac{B_m}{(2/\bar{\gamma}_{\text{sr}}) + B_m}}\Phi(\gamma_m,\, \gamma_{m+1},\, \bar{\gamma}_{\text{rd}},\, C_m) \\ &- \bar{\gamma}_{\text{rd}}\Phi(\gamma_m,\, \gamma_{m+1},\, \bar{\gamma}_r,\, B_m)]\end{aligned} \tag{33}$$

$$\begin{aligned}\text{ABER}_{b(m)}^{\text{df}} = \frac{A_m}{\bar{\gamma}_{\text{sd}} - \bar{\gamma}_{\text{rd}}} &[\bar{\gamma}_{\text{sd}}\Phi(\gamma_m,\, \gamma_{m+1},\, \bar{\gamma}_s,\, B_m) \\ &- \bar{\gamma}_{\text{sd}}\sqrt{\frac{B_m}{(2/\bar{\gamma}_{\text{sd}}) + B_m}}\Phi(\gamma_m,\, \gamma_{m+1},\, \bar{\gamma}_{\text{sr}},\, D_m). \\ &+ \bar{\gamma}_{\text{rd}}\sqrt{\frac{B_m}{(2/\bar{\gamma}_{\text{rd}}) + B_m}}\Phi(\gamma_m,\, \gamma_{m+1},\, \bar{\gamma}_{\text{sr}},\, E_m) \\ &- \bar{\gamma}_{\text{rd}}\Phi(\gamma_m,\, \gamma_{m+1},\, \bar{\gamma}_r,\, B_m)]\end{aligned} \tag{34}$$

and

$$\text{ABER}_{b(m)}^{\text{sd}} = A_m\left(1 - \sqrt{\frac{B_m}{(2/\bar{\gamma}_{\text{sd}}) + B_m}}\right) \tag{35}$$

respectively, where

$$\gamma_s = \bar{\gamma}_{\text{sd}}\bar{\gamma}_{\text{sr}}/(\bar{\gamma}_{\text{sd}} + \bar{\gamma}_{\text{sr}})$$

$$\bar{\gamma}_{\text{r}} = \bar{\gamma}_{\text{sr}}\bar{\gamma}_{\text{rd}}/(\bar{\gamma}_{\text{sr}} + \bar{\gamma}_{\text{rd}})$$

$$C_m = 2(1/\bar{\gamma}_{\text{sr}} + B_m/2)$$

$$D_m = 2(1/\bar{\gamma}_{\text{sd}} + B_m/2)$$

$$E_m = 2(1/\bar{\gamma}_{\text{rd}} + B_m/2)$$

By substituting (33)–(35) into (29) and then substituting (28) into (23), a closed-form expression for the ABER of the MERS can be obtained.

4.2.2 Maximum spectral efficiency scheme: The ABER of the MSES can be given as

$$\begin{aligned}\text{ABER}^{\text{MSES}} = \frac{1}{\eta^{\text{MSES}}}&\left[\sum_{n=\lfloor N/2\rfloor+1}^{N} n\,\text{ABER}_{a(n)} + \sum_{n=2}^{\lfloor N/2\rfloor} n\,\text{ABER}_{c(n)}\right. \\ &\left. + \sum_{\substack{n=0\\n\neq 1}}^{\lfloor N/2\rfloor}\sum_{m=I^n}^{N}\frac{m}{2}\text{ABER}_{d(n,m)}\right]\end{aligned} \tag{36}$$

where $\text{ABER}_{c(n)}$ is the ABER when $\gamma_{\min} < \gamma_{\text{T}}$ (i.e. cooperative transmission is not activated), which can be determined as

$$\begin{aligned}\text{ABER}_{c(n)} &= \int_{\gamma_n}^{\gamma_{n+1}} \text{BER}(n, x) f_{\gamma_{\min},\gamma_{\text{sd}}}(\gamma_T, x)\,\text{d}x \\ &= A_n\Phi(\gamma_n,\, \gamma_{n+1},\, \bar{\gamma}_{\text{sd}}, B_n) - \frac{A_n\bar{\gamma}_{\text{rd}}}{\bar{\gamma}_{\text{rd}} - \bar{\gamma}_{\text{sd}}} \\ &\quad\times \text{e}^{-\gamma_T((1/\bar{\gamma}_{\text{sr}})+(1/\bar{\gamma}_{\text{rd}}))}\Phi(\gamma_n,\, \gamma_{n+1},\, \bar{\gamma}_{\text{df}},\, B_n)\end{aligned} \tag{37}$$

where $\bar{\gamma}_{\text{df}} = \bar{\gamma}_{\text{sd}}\bar{\gamma}_{\text{rd}}/(\bar{\gamma}_{\text{rd}} - \bar{\gamma}_{\text{sd}})$. $\text{ABER}_{d(n,m)}$ is the ABER when cooperative transmission is activated, which can be defined similar to (29) as

$$\begin{aligned}\text{ABER}_{d(n,m)} &= \text{ABER}_{d(n,m)}^{\text{sr}} \cdot \text{ABER}_{d(n,m)}^{\text{sd}} \\ &\quad + (1 - \text{ABER}_{d(n,m)}^{\text{sr}}) \cdot \text{ABER}_{d(n,m)}^{\text{df}}\end{aligned} \tag{38}$$

where

$$\begin{aligned}\text{ABER}_{d(n,m)}^{\text{sr}} &= \int_{\gamma_m}^{\gamma_{m+1}}\int_{\gamma_n}^{\gamma_{n+1}} \text{BER}(m, z)(1 - F_{\gamma_{\text{rd}}}(z - x)) \\ &\quad\times f_{\gamma_{\text{sd}}}(x) f_{\gamma_{\text{sr}}}(z)\,\text{d}x\,\text{d}z \\ &\quad+ \int_{\gamma_m}^{\gamma_{m+1}}\int_{\gamma_n}^{\gamma_{n+1}}\int_z^\infty \text{BER}(n, u) f_{\gamma_{\text{sr}}}(u) f_{\gamma_{\text{sd}}}(x) \\ &\quad\times f_{\gamma_{\text{rd}}}(z - x)\,\text{d}u\,\text{d}x\,\text{d}z \\ &= \frac{A_m\bar{\gamma}_{\text{df}}}{\bar{\gamma}_{\text{sd}}}\psi(n, 1/\bar{\gamma}_{\text{df}})\left[\Phi(\gamma_m,\, \gamma_{m+1},\, \bar{\gamma}_{\text{r}}, B_m)\right. \\ &\quad\left.- \sqrt{\frac{B_m}{(2/\bar{\gamma}_{\text{sr}}) + B_m}}\Phi(\gamma_m,\, \gamma_{m+1},\, \bar{\gamma}_{\text{rd}}, C_m)\right]\end{aligned} \tag{39}$$

and

$$\begin{aligned}\text{ABER}_{d(n,m)}^{\text{df}} &= \int_{\gamma_m}^{\gamma_{m+1}}\int_{\gamma_n}^{\gamma_{n+1}} \text{BER}(m, z)(1 - F_{\gamma_{\text{sr}}}(z)) \\ &\quad\times f_{\gamma_{\text{rd}}}(z - x) f_{\gamma_{\text{sd}}}(x)\,\text{d}x\,\text{d}z \\ &\quad+ \int_{\gamma_m}^{\gamma_{m+1}}\int_z^\infty\int_{\gamma_n}^{\gamma_{n+1}} \text{BER}(n, u) f_{\gamma_{\text{rd}}}(u - x) \\ &\quad\times f_{\gamma_{\text{sd}}}(x) f_{\gamma_{\text{sr}}}(z)\,\text{d}x\,\text{d}u\,\text{d}z \\ &= \frac{A_m\bar{\gamma}_{\text{df}}}{\bar{\gamma}_{\text{sd}}}\psi(n, 1/\bar{\gamma}_{\text{df}})\left[\Phi(\gamma_m,\, \gamma_{m+1},\, \bar{\gamma}_{\text{r}}, B_m)\right. \\ &\quad\left.- \sqrt{\frac{B_m}{(2/\bar{\gamma}_{\text{rd}}) + B_m}}\Phi(\gamma_m,\, \gamma_{m+1},\, \bar{\gamma}_{\text{sr}}, E_m)\right]\end{aligned} \tag{40}$$

IET Commun., 2012, Vol. 6, Iss. 6, pp. 649–658
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Finally

$$\begin{aligned}\mathrm{ABER}^{\mathrm{sd}}_{d(n,m)} &= \int_{\gamma_m}^{\gamma_{m+1}} \int_{\gamma_n}^{\gamma_{n+1}} \mathrm{BER}(m,x)(1-F_{\gamma_{\mathrm{rd}}}(z-x)) \\ &\quad \times f_{\gamma_{\mathrm{sd}}}(x) f_{\gamma_{\mathrm{sr}}}(z)\,\mathrm{d}x\,\mathrm{d}z \\ &\quad + \int_{\gamma_m}^{\gamma_{m+1}} \int_{\gamma_n}^{\gamma_{n+1}} \mathrm{BER}(m,x) f_{\gamma_{\mathrm{rd}}}(z-x) \\ &\quad \times f_{\gamma_{\mathrm{sd}}}(x)(1-F_{\gamma_{\mathrm{sr}}}(z))\,\mathrm{d}x\,\mathrm{d}z \\ &= \frac{A_m \bar{\gamma}_{\mathrm{df}}}{\bar{\gamma}_{\mathrm{sd}}} \psi(m, 1/\bar{\gamma}_{\mathrm{r}})[\Phi(\gamma_m, \gamma_{m+1}, \bar{\gamma}_{\mathrm{df}}, B_m)] \qquad (41)\end{aligned}$$

By substituting (39)–(41) into (38) and then substituting (37) and (28) into (36), a closed-form expression for the ABER of the MSES can be obtained.

4.3 Outage probability

4.3.1 Minimum error rate scheme: In general, the outage probability under adaptive modulation transmission can be defined as the probability when the output SNR is below the minimum switching threshold. In the MERS, the outage probability depends on the output SNR from source–relay link, γ_{sr} and the combined output SNR from source–destination and relay–destination links, γ_{df}. Hence, the outage probability of the MERS can be obtained as

$$\begin{aligned}P^{\mathrm{MERS}}_{\mathrm{out}} &= \Pr[\gamma_{\mathrm{sd}} < \gamma_{\mathrm{th}} / \gamma_{\mathrm{sr}} < \gamma_{\mathrm{th}}]\Pr(\gamma_{\mathrm{sr}} < \gamma_{\mathrm{th}}) \\ &\quad + \Pr[\gamma_{\min} < \gamma_{\mathrm{th}} / \gamma_{\mathrm{sr}} \geq \gamma_{\mathrm{th}}]\Pr[\gamma_{\mathrm{sr}} \geq \gamma_{\mathrm{th}}] \\ &= \Pr[\gamma_{\mathrm{sd}} < \gamma_{\mathrm{th}}]\Pr[\gamma_{\mathrm{sr}} < \gamma_{\mathrm{th}}] \\ &\quad + \Pr[\gamma_{\mathrm{df}} < \gamma_{\mathrm{th}}](1 - \Pr[\gamma_{\mathrm{sr}} < \gamma_{\mathrm{th}}]) \qquad (42)\end{aligned}$$

By substituting (46) into (42), a closed-form expression for the outage probability of the MERS can be written as

$$\begin{aligned}P^{\mathrm{MERS}}_{\mathrm{out}} &= 1 - \mathrm{e}^{-\gamma_{\mathrm{th}}/\bar{\gamma}_{\mathrm{sd}}} - \frac{\bar{\gamma}_{\mathrm{rd}}}{\bar{\gamma}_{\mathrm{sd}} - \bar{\gamma}_{\mathrm{rd}}}(\mathrm{e}^{-\gamma_{\mathrm{th}}(((1/\bar{\gamma}_{\mathrm{sd}})+(1/\bar{\gamma}_{\mathrm{sr}}))} \\ &\quad - \mathrm{e}^{-\gamma_{\mathrm{th}}((1/\bar{\gamma}_{\mathrm{sr}})+(1/\bar{\gamma}_{\mathrm{rd}}))}) \qquad (43)\end{aligned}$$

4.3.2 Maximum spectral efficiency scheme: The outage probability of the MSES can be defined as the probability when both γ_{sd} and $\gamma_{\min}$ are below the minimum switching threshold, γ_{th}. Then, the outage probability can be obtained as

$$\begin{aligned}P^{\mathrm{MSES}}_{\mathrm{out}} &= \Pr[\gamma_{\min} < \gamma_{\mathrm{th}}, \gamma_{\mathrm{sd}} < \gamma_{\mathrm{th}}] = F_{\gamma_{\min},\gamma_{\mathrm{sd}}}(\gamma_{\mathrm{th}}, \gamma_{\mathrm{th}}) \\ &= P^{\mathrm{MERS}}_{\mathrm{out}} \qquad (44)\end{aligned}$$

It is easy to show that the outage probability of the MERS and MSES are the same by substituting γ_{th} into (49) and doing some manipulation.

5 Numerical results

This section presents the analytical results of the average spectral efficiency, the ABER, and the outage probability of the MERS and MSES. These results are verified by the simulation ones. To study the effect of the channel condition of the relay path, we consider two cases. In the first case, the relay node is close to the destination node where we have low SNR in the source–relay link. The second case assumes that both source–relay link and relay–destination link have strong SNR. We set the maximum spectral efficiency, N, to be equal to 8, and the target BER, $\mathrm{BER}_{\mathrm{T}}$, to be equal to 10^{-3}. We compare the performance of the two proposed schemes, MERS and MSES, with that of the AFFR [14], the outage-based cooperative scheme (OBCS) [15] and the direct transmission [17].

Fig. 3 shows the average spectral efficiency of the proposed MERS and the AFFR. In the first case, there is a reduction in the average spectral efficiency of the MERS as the source–relay link becomes a bottle neck in the high SNR. Although in the second case, both the MERS and the AFFR have almost the same performance. Furthermore, in both cases, the average spectral efficiency of the proposed MERS is better compared with that of the AFFR in relatively low average SNR (i.e. average SNR <15 dB).

Fig. 4 shows the ABER of the proposed MERS and that of the AFFR. In both cases, MERS can significantly improve the ABER. The improvement is due to the possibility of reducing the modulation level even though the combined links at the destination node can support higher modulation level. Therefore with high effective gain of the relay path, MERS provides better performance as the average spectral efficiency is similar to that of the AFFR.

Fig. 5 shows the average spectral efficiency of the two proposed schemes, MERS and MSES, the AFFR, the OBCS and the direct transmission. We can see the proposed MSES can improve the spectral efficiency all the time. This is due to the utilisation of the degrees of freedom of the channels and avoiding cooperative transmission whenever it is not beneficial. For instant, when the spectral efficiency of the direct transmission intersects with the spectral efficiency of both the MERS and the AFFR, almost 30% gain can be achieved. In addition, if we compare the MSES with the direct transmission, the gain achieved by the MERS is found notably in the low and moderate SNR (i.e. SNR <20 dB). This is because in high SNR the cooperative transmission is rarely activated. In addition, when we compare the proposed MSES with OBCS, there is a small improvement in the spectral efficiency of the MSES

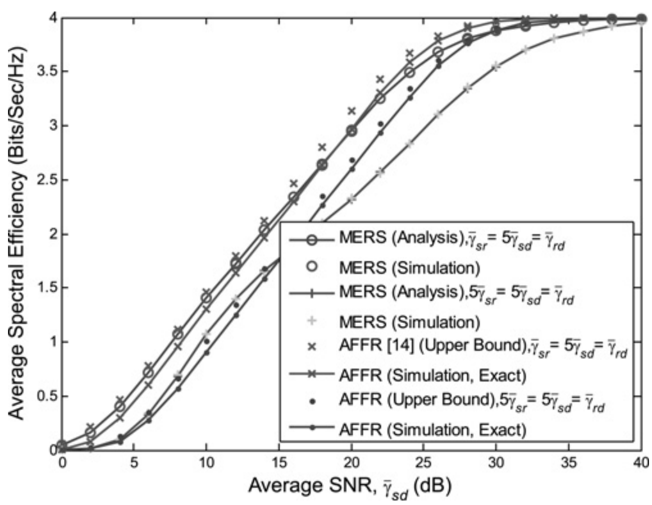

Fig. 3 *Average spectral efficiency of the MERS*

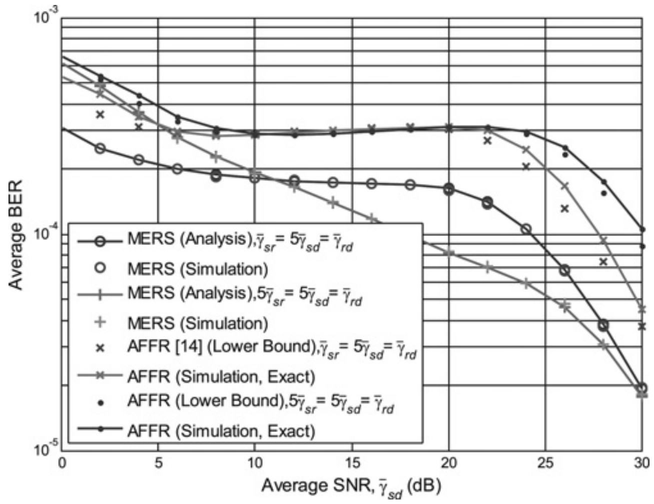

Fig. 4 *ABER of the MERS*

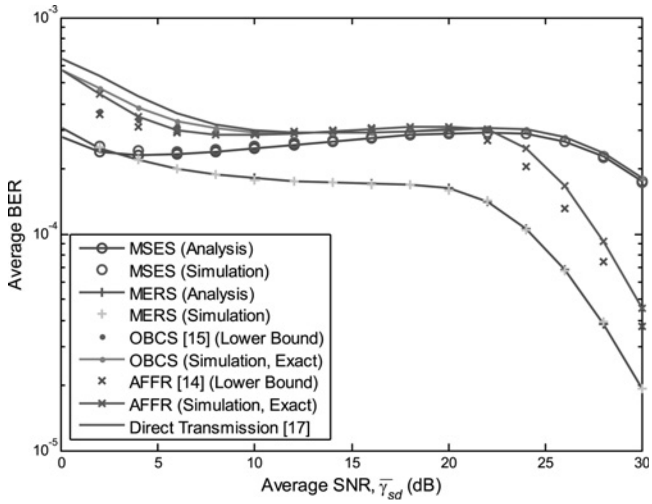

Fig. 6 *ABER of the MERS and MSES,* $\bar{\gamma}_{sr} = 5\bar{\gamma}_{sd} = \bar{\gamma}_{rd}$

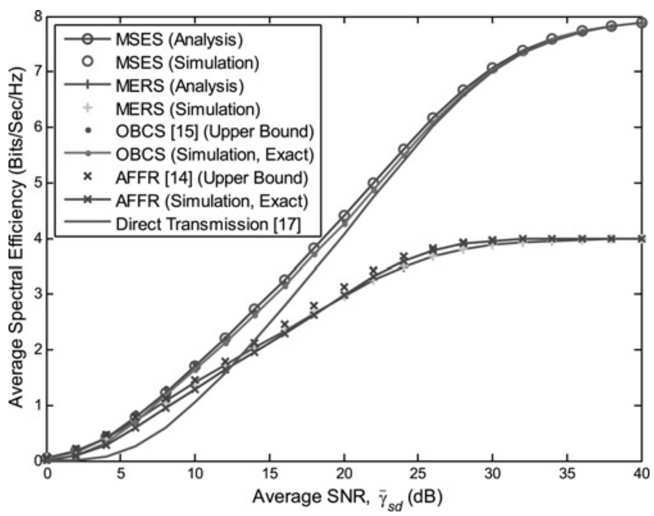

Fig. 5 *Average spectral efficiency of the MERS and MSES,* $\bar{\gamma}_{sr} = 5\bar{\gamma}_{sd} = \bar{\gamma}_{rd}$

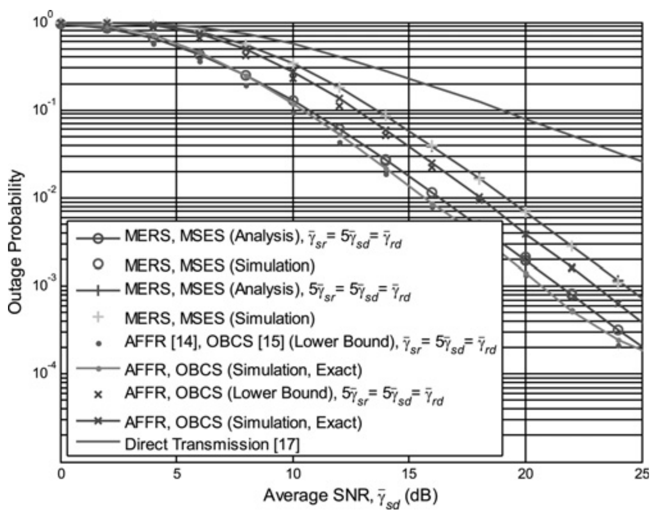

Fig. 7 *Outage probability of the MERS and the MSES*

because the OBCS avoids also cooperative transmission but only when the direct transmission experiences an outage.

Fig. 6 shows the ABER of all above schemes. It can be seen that the proposed MERS outperforms all the other schemes. Also in low SNR, even though the MSES has the best average spectral efficiency, the ABER is still below that of the AFFR, the OBCS and the direct transmission. This observation along with the previous one in Fig. 5 indicates the benefit of using the proposed MSES over the OBCS.

Fig. 7 shows the last performance measure which is the outage probability. The direct observation is the improvement of the outage probability of all cooperative schemes when compared with that of the direct transmission because of the diversity gain. Furthermore, the slope of the outage curves of the cooperative schemes is the same, because they have equal diversity order. Furthermore, the outage probability of the MERS and the MSES has a slight reduction because of the effect of the channel condition in the source–relay link. This reduction is decreased as the channel condition improves. Finally, there is a good match between analytical and simulation results.

6 Conclusion

In this study, we have proposed two adaptive DF schemes: the MERS and the MSES. We have derived the average spectral efficiency, the ABER and the outage probability. Both analytical and simulation show that our schemes can achieve better performance. More specifically, the MERS improves the ABER significantly and provides acceptable average spectral efficiency when the channel gain of the relay path is good. In contrast, the MSES provides the best average spectral efficiency while maintaining the required BER performance, so it is characterised by efficient utilisation of the resources. Finally, the MERS suffers reduction in the spectral efficiency when the channel gain from source–relay link is low; therefore it is important to select the best relay that can be used during cooperative transmission.

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9 Appendix 1: conditional statistics of $\gamma_{\min}$

The conditional CDF of $\gamma_{\min}$ given that γ_{sr} is greater than the minimum switching threshold, γ_{th}, can be obtained as

$$\begin{aligned} F_{\gamma_{\min}}(z/\gamma_{sr} > \gamma_{th}) &= \Pr[\min(\gamma_{sr}, \gamma_{df}) \le z/\gamma_{sr} > \gamma_{th}] \\ &= 1 - \Pr[(\gamma_{df} > z, \gamma_{sr} > z)/\gamma_{sr} > \gamma_{th}] \\ &= \begin{cases} 1 - \left[\dfrac{1 - F_{\gamma_{sr}}(z)}{1 - F_{\gamma_{sr}}(\gamma_{th})}(1 - F_{\gamma_{df}}(z))\right], & \text{if } z > \gamma_{th} \\ F_{\gamma_{df}}(z), & \text{if } z \le \gamma_{th} \end{cases} \end{aligned} \tag{45}$$

where $F_{\gamma_{df}}(z)$ is the CDF of the combined output SNR at the destination node, γ_{df}, which can be determined as

$$\begin{aligned} F_{\gamma_{df}}(z) &= \int_0^z F_{\gamma_{rd}}(z-x) f_{\gamma_{sd}}(x)\, dx = 1 - e^{-z/\bar{\gamma}_{sd}} \\ &\quad - \frac{\bar{\gamma}_{rd}}{\bar{\gamma}_{rd} - \bar{\gamma}_{sd}}(e^{-z/\bar{\gamma}_{rd}} - e^{-z/\bar{\gamma}_{sd}}) \end{aligned} \tag{46}$$

Substituting (46) into (45), we have $F_{\gamma_{\min}}(z/\gamma_{sr} > \gamma_{th})$ given in (14). By differentiating (46) with respect to z, the PDF of γ_{df} can be written as

$$\begin{aligned} f_{\gamma_{df}}(z) &= \int_0^z f_{\gamma_{rd}}(z-x) f_{\gamma_{sd}}(x)\, dx \\ &= \frac{1}{\bar{\gamma}_{rd} - \bar{\gamma}_{sd}}(e^{-z/\bar{\gamma}_{rd}} - e^{-z/\bar{\gamma}_{sd}}) \end{aligned} \tag{47}$$

The conditional PDF of $\gamma_{\min}$ can be obtained by differentiating (45) with respect to z, yielding

$$\begin{aligned} &f_{\gamma_{\min}}(z/\gamma_{sr} > \gamma_{th}) \\ &= \begin{cases} \dfrac{1}{1 - F_{\gamma_{sr}}(\gamma_{th})}[f_{\gamma_{sr}}(z)(1 - F_{\gamma_{df}}(z)) \\ \quad + f_{\gamma_{df}}(z)(1 - F_{\gamma_{sr}}(z))], & z > \gamma_{th} \\ f_{\gamma_{df}}(z), & z \le \gamma_{th} \end{cases} \end{aligned} \tag{48}$$

Similarly, substituting (47) into (48), we have $f_{\gamma_{\min}}(z/\gamma_{sr} > \gamma_{th})$ given in (15).

10 Appendix 2: joint statistics of $\gamma_{\min}$ and γ_{sd}

The joint CDF of $\gamma_{\min}$ and γ_{sd} can be obtained as

$$\begin{aligned} F_{\gamma_{\min},\gamma_{sd}}(z,x) &= \Pr[\min(\gamma_{sr}, \gamma_{df}) \le z/\gamma_{sd} \le x]\Pr[\gamma_{sd} \le x] \\ &= (1 - \Pr[(\gamma_{sr} > z]\Pr[\gamma_{df} > z/\gamma_{sd} \le x])\Pr[\gamma_{sd} \le x] \\ &= (1 - F_{\gamma_{sr}}(z))F_{\gamma_{df},\gamma_{sd}}(z,x) + F_{\gamma_{sr}}(z)F_{\gamma_{sd}}(x) \end{aligned} \tag{49}$$

where $F_{\gamma_{df},\gamma_{sd}}(z,x)$ is the joint CDF of γ_{df} and γ_{sd} which can be determined as

$$F_{\gamma_{df},\gamma_{sd}}(z,x) = \begin{cases} \int_0^x F_{\gamma_{rd}}(z-t) f_{\gamma_{sd}}(t)\,dt, & x \le z, \\ \int_0^z \int_0^u f_{\gamma_{rd}}(u-t) f_{\gamma_{sd}}(t)\,dt\,du, & x > z \end{cases}$$
$$= \begin{cases} 1 - e^{-x/\bar{\gamma}_{sd}} - \dfrac{\bar{\gamma}_{rd}}{\bar{\gamma}_{rd} - \bar{\gamma}_{sd}} e^{-z/\bar{\gamma}_{rd}} & x \le z \\ \quad \times (1 - e^{-x((1/\bar{\gamma}_{sd})-(1/\bar{\gamma}_{rd}))}), & \\ \dfrac{1}{\bar{\gamma}_{rd} - \bar{\gamma}_{sd}} [\bar{\gamma}_{rd}(1 - e^{-z/\bar{\gamma}_{rd}}) & x > z \\ \quad - \bar{\gamma}_{sd}(1 - e^{-z/\bar{\gamma}_{sd}})], & \end{cases} \tag{50}$$

By differentiating (50) with respect to z, the joint PDF of γ_{df} and γ_{sd} can be given as

$$f_{\gamma_{df},\gamma_{sd}}(z,x) = \int_0^{\min[x,z]} f_{\gamma_{rd}}(z-t) f_{\gamma_{sd}}(t)\,dt$$
$$= \frac{e^{-z/\bar{\gamma}_{rd}}}{\bar{\gamma}_{rd} - \bar{\gamma}_{sd}} (1 - e^{-\min[x,z]((1/\bar{\gamma}_{sd})-(1/\bar{\gamma}_{rd}))}) \tag{51}$$

Substituting (50) into (49), $F_{\gamma_{\min},\gamma_{sd}}(z,x)$ is given in (19). Similarly, differentiating $F_{\gamma_{\min},\gamma_{sd}}(z,x)$ with respect to z, the joint PDF of $\gamma_{\min}$ and γ_{sd} can be written as

$$f_{\gamma_{\min},\gamma_{sd}}(z,x) = \begin{cases} (1 - F_{\gamma_{sr}}(z)) \int_0^x f_{\gamma_{rd}}(z-t) f_{\gamma_{sd}}(t)\,dt & x \le z \\ \quad + f_{\gamma_{sr}}(z) \int_0^x (1 - F_{\gamma_{rd}}(z-t)) f_{\gamma_{sd}}(t)\,dt, & \\ (1 - F_{\gamma_{sr}}(z)) \int_0^z f_{\gamma_{rd}}(z-x) f_{\gamma_{sd}}(x)\,dx & \\ \quad + f_{\gamma_{sr}}(z)[F_{\gamma_{sd}}(x) - \int_0^z \int_0^u f_{\gamma_{rd}}(u-t) & x > z \\ \quad \times f_{\gamma_{sr}}(t)\,dt\,du], & \end{cases} \tag{52}$$

Solving (52) yields to closed-form of the joint PDF of $\gamma_{\min}$ and γ_{sd}, as given in (20).

11 Appendix 3: solving the finite integral in (26)

$$\Phi(\gamma_{s_1}, \gamma_{s_2}, \bar{\gamma}, B) = \int_{\gamma_{s1}}^{\gamma_{s2}} Q(\sqrt{Bx}) \frac{e^{-x/\bar{\gamma}}}{\bar{\gamma}}\,dx \tag{53}$$

The integration in (53) can be solved using integration by parts, which can be defined as

$$\int u\,dv = uv - \int v\,du \tag{54}$$

where $u = Q(\sqrt{Bx})$, $du = -\left(\sqrt{B}/2\sqrt{2\pi x}\right) e^{-xB/2}\,dx$, $dv = (e^{-x/\bar{\gamma}}/\bar{\gamma})\,dx$, and $v = -e^{-x/\bar{\gamma}}$, then

$$\Phi(\gamma_{s_1}, \gamma_{s_2}, \bar{\gamma}, B) = \left[-e^{-x/\bar{\gamma}} Q(\sqrt{Bx})\right]_{\gamma_{s1}}^{\gamma_{s2}}$$
$$- \int_{\gamma_{s_1}}^{\gamma_{s_2}} \frac{\sqrt{B}}{2\sqrt{2\pi x}} e^{-(B\bar{\gamma}+2/2\bar{\gamma})x}\,dx \tag{55}$$

The second part of (55) can be written as

$$q = \int_{\gamma_{s_1}}^{\gamma_{s_2}} \frac{\sqrt{B}}{2\sqrt{2\pi x}} e^{-xB/(2B\bar{\gamma}/B\bar{\gamma}+2)}\,dx \tag{56}$$

Let $w^2 = Bx/((B\bar{\gamma})/(B\bar{\gamma}+2))$, and $dx = 2\sqrt{(x/B)}\sqrt{(B\bar{\gamma}/B\bar{\gamma}+2)}\,dw$, then (56) can be expressed as

$$q = \sqrt{\frac{B\bar{\gamma}}{B\bar{\gamma}+2}} \frac{1}{\sqrt{2\pi}} \int e^{-w^2/2}\,dw \tag{57}$$

this yields

$$q = \sqrt{\frac{B\bar{\gamma}}{B\bar{\gamma}+2}} \left[Q\left(\sqrt{B\gamma_{s_1} + \frac{2\gamma_{s_1}}{\bar{\gamma}}}\right) - Q\left(\sqrt{B\gamma_{s_2} + \frac{2\gamma_{s_2}}{\bar{\gamma}}}\right) \right] \tag{58}$$

and finally, substituting (58) into (55), we have the solution of the finite integral in (26), which is given in (27).

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