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Semi-supervised Laplacian regularized least squares algorithm for localization in wireless sensor networks

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ABSTRACT

In this paper, we propose a new approach for localization in wireless sensor networks based on semi-supervised Laplacian regularized least squares algorithm. We consider two kinds of localization data: signal strength and pair-wise distance between nodes. When nodes are close within their physical location space, their localization data vectors should be similar. We first propose a solution using the alignment criterion to learn an appropriate kernel function in terms of the similarities between anchors, and the kernel function is used to measure the similarity between pair-wise sensor nodes in the networks. We then propose a semi-supervised learning algorithm based upon manifold regularization to obtain the locations of the non-anchors. We evaluate our algorithm under various network topology, transmission range and signal noise, and analyze its performance. We also compare our approach with several existing approaches, and demonstrate the high efficiency of our proposed algorithm in terms of location estimation error.

1. Introduction

Recent technological advances in micro-electro-mechanical systems and wireless communication have led to the development of tiny, low-power, low-price sensor nodes for observation tasks in a wide range of environments [1]. There has been an increase number of wireless sensor networks (WSNs) applications for monitoring environmental information, e.g., healthcare monitoring [2], environmental monitoring [3] and target tracking [4], across an entire physical space, where the sensor network localization problem has received considerable attentions recently [5]. Typical networks of this type consist of a large number of densely deployed sensor nodes which can gather local data and communicate with other nodes. The sensor data from these nodes are relevant only when the location they refer to is known. Therefore, knowledge of the node positions becomes imperative.

On the other hand, sensor nodes could be equipped with a global positioning system (GPS) to provide them with their absolute position, and this is currently a costly solution or impossible solution to some indoor cases. Therefore, it is often the case with a general assumption that the positions of some nodes (called *anchor*), are known exactly, so that it is possible to find the absolute positions of the remaining nodes (called *non-anchor*) in the WSNs. The main task of WSNs localization algorithm is to determine the positions of sensor nodes in a network given incomplete and noisy pairwise time-of-arrival (TOA), time-difference-of-arrival (TDOA), received signal strength (RSS) and/or angle-of-arrival (AOA) measurements [6–11], which are acquired by the sensor nodes during communications with their neighbors.

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The well-known techniques used for localizing sensor nodes to handle these noisy measurements are based upon multidimensional scaling (MDS) [12] or semidefinite programming (SDP) [13]. The MDS algorithm uses connectivity information or inner-distance to derive the location of each node by applying an orthogonal basis transformation. However, if the data are linked to coordinate by an unknown nonlinear function (*e.g.*, signal strength in noisy environment), MDS fails to accurately estimate the node locations. In the SDP approach, measurements between nodes are expressed as geometric convex constraints. The location estimation problem can be solved by SDP. Although the SDP approach can achieve high localization accuracy, solving SDP is time consuming, which is not suitable for large scale deployment. Recently, several algorithms making use of kernel-based machine learning have been proposed for the localization problem based on inaccurate measurement in WSNs [14,15]. They employ kernel function to measure the similarities between sensor nodes, and then train an estimator based upon the locations and measured similarities of the anchors. The locations of non-anchors are obtained from the measurements between the anchors and the non-anchors by employing the trained estimator. Wang et al. [16] formulate the localization problem as a graph embedding problem, and then use the kernel locality preserving projection technique to estimate the relative locations of all sensor nodes. Nguyen et al. [14] divide the interested region into overlapping subregions, and then use support vector machine (SVM) to select a set of subregions where the sensor nodes are possibly located. The SVM model is trained based upon the similarity between anchors. Finally, they calculate the centers of the selected subregions as the estimated location. Kuh et al. [17] and Brunato et al. [18] also formulate the localization problem as a kernel-based regression problem, and use both least squares kernel regression (LSKR) and support vector regression (SVR) for training location estimators. Essoloh et al. [19] consider an alignment criterion to learn the optimal parameter of a given kernel function, and then use incremental kernel principal component analysis to build the nonlinear manifold model linking anchors. The locations of non-anchors are estimated by the pre-image projections of the trained model. Because of the nonlinear property of the kernel function, most kernel-based localization algorithms can capture the nonlinearity of the measured data [14,19].

In this paper, we propose a viable kernel-based algorithm to solve the localization problem based on noisy measurements. To reduce the calibration effort in the localization process, the number of anchors is always small, even if less than 10% in some real sensor network applications. Such small sample size makes some kernel-based localization approaches fail to accurately achieve the locations of non-anchors, *e.g.*, the SVM-based method proposed by Nguyen et al. [14], LSKR-based method proposed in [17]. On the other hand, how to choose an optimal kernel function is also a challenging problem in most kernel-based localization algorithms. We consider two kinds of data model for localization: received signal strength and measured pair-wise distance between nodes. In general, when two sensor nodes are close in their physical locations, their measured location features should be similar, i.e., the high dimensional localization data lies (roughly) on a low-dimensional manifold. Our algorithm has two stages. In the first stage, we train a suitable kernel function based upon kernel alignment criterion [20]. In the second stage, we construct a mapping between the localization data space and the physical location space under the semi-supervised learning framework, and then determine the locations of the sensor nodes based upon the mapping. We will show the first stage is a kernel learning problem. The optimal kernel function exists in a kernel conical hull which is spanned by a set of basis kernel functions, and their corresponding Gram matrices are determined in terms of the measurements between anchors. By considering the kernel alignment criterion, this problem is reduced to finding a set of coefficients, and can be formulated as a quadratically constrained quadratic programming (QCQP) problem, which can be efficiently solved by standard optimization tools. The second stage is a semi-supervised kernel regression problem [21]. We use the optimal kernel function, which is a weighted combination of basis kernel functions, to measure the similarity between all of the sensor nodes, and then employ the semi-supervised Laplacian regularized least squares (S^2LapRLS) algorithm to build the relationship between the signal space and the physical location space, or the measured pair-wise distance space and the physical location space. The location of non-anchors can be estimated by this relationship. Compared with the related kernel-based localization algorithms [14,17,18] or the traditional range-based localization algorithms [12,22], our algorithm is under the semi-supervised framework and takes into consideration the manifold property of the localization data. In other words, we use the non-anchor information to train the model for improving the localization accuracy.

The remainder of the paper is organized as follows. Section 2 introduces the semi-supervised Laplacian regularized least squares regression algorithm. Section 3 presents the main contribution of this paper, which consists of the optimal kernel selection procedure and the sensor node location estimation algorithm. Extensive simulation results are given in Section 4. Finally, Section 5 concludes the paper.

2. Semi-supervised laplacian regularized least squares regression

Consider a 1-dimensional regression problem which is to learn a function $f : \mathcal{X} \to \mathcal{Z}$, where $\mathcal{X}$ denotes the input or instance space, which is a subspace of $\mathbb{R}^p$, and $\mathcal{Z} \subseteq \mathbb{R}$ denotes the output space. An input–output pair $(\mathbf{v}, z)$ is called an example, if $\mathbf{v} \in \mathcal{X}$ and $z \in \mathcal{Z}$. We assume that the examples are drawn randomly and independently from an unknown, but fixed, underlying distribution over $\mathcal{X} \times \mathcal{Z}$. Let $\kappa : \mathcal{X} \times \mathcal{X} \to \mathbb{R}$ be a Mercer kernel that satisfies the finite positive property, *i.e.*, there is an associated reproducing kernel Hilbert space (RKHS) $\mathcal{H}_\kappa$ with the corresponding norm $\|\cdot\|_\kappa$ [23]. Suppose there is a set of m labeled examples $\{\mathbf{v}_i, z_i\}_{i=1}^m$ and $n-m$ unlabeled examples $\{\mathbf{v}_i\}_{i=m+1}^n$.

The learning problem corresponds to solve the following optimization problem:

$$\min_f \quad \frac{1}{m}\sum_{i=1}^{m}\|f(\mathbf{v}_i)-z_i\|^2+v\|f\|_\kappa^2+\frac{\gamma}{n^2}\hat{f}^T L\hat{f}, \tag{1}$$

where $\hat{f}=[f(\mathbf{v}_1),\ldots,f(\mathbf{v}_n)]^T$ and L is the graph Laplacian matrix [24], v and γ are the trade-off parameters, v is corresponding to the regularization term [23], and γ is corresponding to the smoothness term [21]. This is the so-called semi-supervised Laplacian regularized least squares regression algorithm. The first term is the fitting over the labeled examples. The second term is used to avoid poor generalizabilities due to limited labeled data. The third term is called the manifold regularization term, which exploits the geometric structure of the marginal distribution of the data. The basic underlying assumption is that if two points are close in the intrinsic geometry of the marginal distribution, their regression function values are similar.

The extended Representor theorem [25] states that the optimal f^* exists in $\mathcal{H}_\kappa$ and can be written as

$$f^*(\mathbf{v})=\sum_{i=1}^{n}\alpha_i\kappa(\mathbf{v}_i,\mathbf{v}). \tag{2}$$

Then the problem (1) can be reduced to find an optimal solution over the finite dimensional space of the coefficients α_i, and can be rewritten as follows

$$\min_\alpha \quad \frac{1}{m}\sum_{i=1}^{m}\left\|\sum_{j=1}^{n}\alpha_j\kappa(\mathbf{v}_i,\mathbf{v}_j)-z_i\right\|^2+v\alpha^T K\alpha+\frac{\gamma}{n^2}\alpha^T KLK\alpha. \tag{3}$$

By setting the partial derivative of (3) to zero, we obtain the optimal solution $\alpha^*=\left[\alpha_1^*,\ldots,\alpha_n^*\right]^T$.

$$\alpha^*=(JK+CI+qLK)^{-1}J\mathbf{z}, \tag{4}$$

where K is the Gram matrix over all n examples with $K_{ij}=\kappa(\mathbf{v}_i,\mathbf{v}_j)$, J is a $n\times n$ diagonal matrix given by $J=\mathrm{diag}(1,\ldots,1,0,\ldots,0)$ with the first m diagonal entries equal to 1 and the rest being 0, I is a $n\times n$ identity matrix, $C=mv$, $\tau=\gamma m/n^2$, and $\mathbf{z}$ is a n-dimensional label vector given by $\mathbf{z}=[z_1,\ldots,z_m,0,\ldots,0]^T$.

3. Sensor node location estimation

3.1. Problem statement

Consider a p-dimensional localization problem ($p=2$ for planar localization). Suppose there are n sensor nodes $\{X_i\}_{i=1}^n$ placed in a geographical region $\mathcal{C}\subseteq\mathbb{R}^p$. Let $\mathbf{x}_i\in\mathbb{R}^p$ denote the location of the node X_i. Without loss of generability, let the first m ($m\ll n$) sensor nodes be anchors, whose locations are known. We assume that each sensor node is capable of transmitting localization data to each of its neighbors, up some communication range, and there are two kinds of locations data, i.e., signal strength and measured pair-wise distance.

- *Signal strength*. We use s_{ij} to denote the signal strength that node X_i receives from node X_j. We set $s_{ii}=1$ for all $i=1,\ldots,n$. If X_i is out of the communication range of X_j, the signal strength is missing, we simply set $s_{ij}=0$ in this case.
- *Measured pair-wise distance*. For every pair of nodes X_i and X_j, we use d_{ij} denote their measured Euclidean distance,

$$d_{ij}=\left(\sum_{k=1}^{p}(\mathbf{x}_{ik}-\mathbf{x}_{j_k})^2\right)^{\frac{1}{2}}. \tag{5}$$

If node X_i is out of the communication range of node X_j, d_{ij} cannot be obtained directly, and therefore, we denote $d_{ij}=\infty$.

Our objective is to determine the locations $\{\mathbf{x}_i\}_{i=m+1}^n$ of the $n-m$ non-anchors based upon the locations of anchors $\{\mathbf{x}_i\}_{i=1}^m$ and the localization data (signal strength $\{s_{ij}\}_{i,j=1}^n$ or pair-wise distance $\{d_{ij}\}_{i,j=1}^n$). Note that localizations using pair-wise distances are known as range-based methods, while those using signal strengths are range-free methods.

3.2. Localization algorithm

We try to build a mapping between the localization data space and the physical location space under the semi-supervised learning setting. Intuitively, there are two main characteristics about the localization data.

- Consider sensor nodes X_i and X_j. If they are close in the physical location space, their localization data vectors should be similar. Suppose $\mathbf{s}_i=[s_{i1},\ldots,s_{in}]$ and $\mathbf{s}_j=[s_{j1},\ldots,s_{jn}]$ are the signal strength vectors that X_i and X_j received from all other nodes. If $\|\mathbf{x}_i-\mathbf{x}_j\|$ is small, then $\|\mathbf{s}_i-\mathbf{s}_j\|$ should be small. Similarly, this should also hold for the measured pair-wise distances.
- If sensor nodes X_i and X_j are spatially close, the signal strength between them is likely to be high, and the measured distance between them is likely to be small.

These characteristics are related to the assumption of manifold techniques: When the locations of some sensor nodes are close, their localization data are similar, i.e., the high dimensional localization data lie on a low-dimensional manifold determined by the physical location space. In other words, when the locations of some sensor nodes are known, we can ground the unknown locations by exploiting the geometry of the distribution of localization data, assuming their conditional distributions are similar.

More specifically, we can estimate the locations of non-anchors by using S^{2}LapRLS algorithm. We first define the Gram matrix $K=(\kappa(\mathbf{x}_i,\mathbf{x}_j))_{i,j=1}^n$. Following the work proposed in [14], we envision a hierarchy of kernels based upon the localization data matrix. Take the radial basis function (RBF) kernel function for example, for a parameter w_g,

- if the localization data is the signal strength, then

$$\kappa(\mathbf{x}_i,\mathbf{x}_j)=\exp\left(-\frac{\|\Phi(\mathbf{x}_i)-\Phi(\mathbf{x}_j)\|^2}{2w_g^2}\right)=\exp\left(-\frac{\sum_{l=1}^{N}(\mathbf{s}_{il}-\mathbf{s}_{jl})^2}{2w_g^2}\right), \tag{6}$$

where $\|\cdot\|$ is the Euclidean norm, and $\Phi(\cdot)$ is the image of the input vector in H_κ;

- if the localization data is the measured pair-wise distance, then

$$\kappa(\mathbf{x}_i,\mathbf{x}_j)=\exp\left(-\frac{\|\Phi(\mathbf{x}_i)-\Phi(\mathbf{x}_j)\|^2}{2w_g^2}\right)=\exp\left(-\frac{G^2(\mathbf{x}_i,\mathbf{x}_j)}{2w_g^2}\right), \tag{7}$$

where $G(\mathbf{x}_i,\mathbf{x}_j)$ is the geodesic distance [26], which contains the shortest path distance between nodes X_i and X_j. The geodesic distance matrix G can be obtained by two steps. First, we set $G(\mathbf{x}_i,\mathbf{x}_j)=d_{ij}$ if node X_i is within the communication range of node X_j or X_j is within the communication range of X_i; $G(\mathbf{x}_i,\mathbf{x}_j)=\infty$ otherwise. Second, for each value of $l=1,\ldots,n$, replace all entries $G(\mathbf{x}_i,\mathbf{x}_j)$ in turn by $\min\{G(\mathbf{x}_i,\mathbf{x}_j),G(\mathbf{x}_i,\mathbf{x}_l)+G(\mathbf{x}_l,\mathbf{x}_j)\}$. The final matrix $[G_{ij}]_{n\times n}$ is the geodesic distance matrix.

Next, we the calculate the graph Laplcian matrix L. Let X_i be connected to its k nearest neighbors, where the distance is measured by the Euclidean norm of their localization data. Suppose W is the weight matrix with W_{ij} being the weight of the edge between X_i and X_j. If X_i is connected to X_j, we choose heat kernel to calculate W_{ij} [27], otherwise we simply set $W_{ij}=0$. Then

$$L=D-W, \tag{8}$$

where D is a degree matrix with $D_{ii}=\sum_{j=1}^{n}W_{ij}$ and reflects the weighted degree of each sensor node.

Finally, for our localization problem, the objective function will be

$$\min_{\alpha_k}\ \frac{1}{m}\sum_{i=1}^{m}\left\|\sum_{j=1}^{n}\alpha_j^{(k)}\kappa(\mathbf{x}_i,\mathbf{x}_j)-\mathbf{x}_i^{(k)}\right\|^2+v\alpha_k^TK\alpha_k+\frac{\gamma}{n^2}\alpha_k^TKLK\alpha_k, \tag{9}$$

where $\alpha_j^{(k)}$ is the jth element of α_k, and $\mathbf{x}_i^{(k)}$ is the ith element of $\mathbf{x}_k$, for all $k=1,\ldots,p$. By solving the above problem p times, we obtain the p coefficient vectors. The coordinates of non-anchors can be estimated by these vectors. Furthermore, the problem can be rewritten in matrix form.

$$A^*=\underset{A\in\mathbb{R}^{n\times p}}{\operatorname{argmin}}\,(KA-X_A)^TJ(KA-X_A)+CA^TKA+qA^TKLKA, \tag{10}$$

where X_A corresponds to the coordinates of the anchors given by $X_A=[\mathbf{x}_1,\ldots,\mathbf{x}_m,0,\ldots,0]^T$.

Setting the derivative of Eq. (10) to zero yields

$$J(KA-X_A)+CA+qLKA=0,$$

and the optimal solution is given by

$$A^*=(JK+CI+qLK)^{-1}JX_A. \tag{11}$$

The location of the non-anchor X_j can be estimated by

$$\tilde{\mathbf{x}}_j^{(k)}=\sum_{i=1}^{n}A_{ik}^*\kappa(\mathbf{x}_j,\mathbf{x}_i),\quad j=m+1,\ldots,n \tag{12}$$

where $\tilde{\mathbf{x}}_j^{(k)}$ is the kth column of vector $\tilde{\mathbf{x}}_j$.

3.3. Optimal kernel selection via anchor similarities

In order to have a valid RKHS, an appropriate kernel function should be selected. Most of the related methods choose the RBF kernel function, because the RBF kernel has the ability of universally approximating any distribution [28]. However, the localization data in a complicated environment usually attenuates in a way that is highly nonlinear and uncertain. Even if we choose the RBF kernel, how to determine an appropriate parameter w_g is still a challenge. To alleviate the kernel selection and kernel parameter determination procedure, a strategy based upon the kernel alignment criterion is proposed in this subsection.

The kernel alignment criterion is a measurement of similarity between two Mercer kernels or between a kernel and a target matrix [20]. Let $K^*\in\mathbb{R}^{m\times m}$ be the target function, and $K_a\in\mathbb{R}^{m\times m}$ be the Gram matrix over all m anchors. The alignment between these two matrices is defined by

$$\widehat{A}(K_a,K^*)=\frac{\langle K_a,K^*\rangle_F}{\sqrt{\langle K_a,K_a\rangle_F\langle K^*,K^*\rangle_F}}, \tag{13}$$

where $\langle\cdot,\cdot\rangle_F$ is the Frobenius norm with $\langle K_a,K^*\rangle_F=\sum_{i,j=1}^{m}K_a(i,j)K^*(i,j)$. The target matrix K^* is defined in terms of the similarity between anchors. Here, we give its definition with respect to signal strength and pair-wise distance, respectively.

- Let the localization data be the signal strength, we define

$$K^*(i,j)=\begin{cases}1, & \text{if } X_j\in\mathcal{V}(i),\\ 0, & \text{otherwise.}\end{cases}$$

for all $i,j=1,\ldots,m$, where $\mathcal{V}(i)$ is the set of anchor-neighbors of anchor X_i, the distance of which can be measured by the Euclidean norm of the signal strength vectors.

- Let the localization data be the pair-wise distance, we define

$$K^*(i,j)=\exp\left(-\frac{\|\mathbf{x}_i-\mathbf{x}_j\|^2}{2w_g^2}\right),$$

for all $i,j=1,\ldots,m$.

Note that $\langle K^*,K^*\rangle_F$ is constant in Eq. (13), hence the alignment criterion can be rewritten as

$$\widehat{A}(K_a)=\frac{\langle K_a,K^*\rangle_F}{\sqrt{\langle K_a,K_a\rangle_F}}.$$

Let $\mathcal{K}$ be a convex set of kernel functions, the problem of finding the optimal kernel over $\mathcal{K}$ in terms of maximum alignment can be written as

$$\begin{aligned}&\max\quad \widehat{A}(K_a)\\ &\text{s.t.}\quad K_a\in\mathcal{K}.\end{aligned} \tag{14}$$

In this paper, we study the case in which $\mathcal{K}$ consists of convex combinations of given kernel functions $\kappa_1,\ldots,\kappa_q$:

$$\mathcal{K}=\left\{\kappa:\mathcal{X}\times\mathcal{X}\to\mathbb{R}\,\middle|\,\kappa=\sum_{i=1}^{q}\theta_i\kappa_i,\theta_i\geqslant 0\right\}.$$

Let $K_i^{(a)}$ be the Gram matrix of κ_i over all anchors, *i.e.*, $K_i^{(a)}(j,k) = \kappa_i(\mathbf{x}_j, \mathbf{x}_k)$ for all j, $k = 1,\ldots,m$, where $\kappa_i(\mathbf{x}_j, \mathbf{x}_k)$ can be defined base on the localization data (as shown in Eqs. (6) and (7)). The Gram matrix of anchors is

$$K_a = \sum_{i=1}^{q} \theta_i K_i^{(a)},$$

where $\theta_i \geqslant 0$. The optimization problem (14) can be rewritten as

$$\begin{aligned} \max_{\theta} \quad & \left\langle \sum_{i=1}^{q} \theta_i K_i^{(a)}, K^* \right\rangle_F, \\ \text{s.t.} \quad & \left\langle \sum_{j=1}^{q} \theta_j K_j^{(a)}, \sum_{i=1}^{q} \theta_i K_i^{(a)} \right\rangle_F \leqslant 1, \\ & \theta_i \geqslant 0, \quad i = 1,\ldots,q. \end{aligned} \tag{15}$$

Expending Eq. (15) yields

$$\left\langle \sum_{i=1}^{q} \theta_i K_i^{(a)}, K^* \right\rangle_F = \sum_{i=1}^{q} \theta_i \left\langle K_i^{(a)}, K^* \right\rangle_F = \mathbf{c}^T \theta,$$

$$\left\langle \sum_{j=1}^{q} \theta_j K_j^{(a)}, \sum_{i=1}^{q} \theta_i K_i^{(a)} \right\rangle_F = \sum_{i,j=1}^{q} \theta_i \theta_j \left\langle K_i^{(a)}, K_j^{(a)} \right\rangle_F = \theta^T Q \theta$$

with $c_i = \left\langle K_i^{(a)}, K^* \right\rangle_F = \mathrm{trace}\left(\left(K_i^{(a)}\right)^T K^* \right)$ and $Q_{ij} = \left\langle K_i^{(a)}, K_j^{(a)} \right\rangle_F$. We obtain the final learning problem

$$\begin{aligned} \max_{\theta} \quad & \mathbf{c}^T \theta, \\ \text{s.t.} \quad & \theta^T Q \theta \leqslant 1, \\ & \theta_i \geqslant 0, \quad i = 1,\ldots,q. \end{aligned} \tag{16}$$

Note that this is a QCQP problem, which can be solved efficiently with optimization tools such as SeDuMi or Mosek. The worst-case time complexity is $O(qm^3)$ [29].

After obtaining the optimal kernel coefficient θ^*, the optimal f^* (in Eq. (2)) in $\mathcal{H}_\kappa$ can be rewritten as

$$f^*(\mathbf{x}) = \sum_{i=1}^{n} \alpha_i \sum_{j=1}^{q} \theta_j^* \kappa_j(\mathbf{x}_i, \mathbf{x}). \tag{17}$$

Let K_i be the Gram matrix of κ_i over all sensor nodes. The optimal solution in Eq. (11) of the localization problem is given by

$$A^* = \left(J \sum_{i=1}^{q} \theta_i^* K_i + CI + qL \sum_{i=1}^{q} \theta_i^* K_i \right)^{-1} J X_A, \tag{18}$$

while the estimated location of non-anchor X_j can be obtained by

$$\tilde{\mathbf{x}}_j^{(k)} = \sum_{i=1}^{n} A_{ik}^* \sum_{k=1}^{q} \theta_k^* \kappa_k(\mathbf{x}_j, \mathbf{x}_i), \quad k = 1,\ldots,p, \tag{19}$$

where $\tilde{\mathbf{x}}_j^{(k)}$ is the kth column of $\tilde{\mathbf{x}}_j$.

A summary of our algorithm is provided in Algorithm 1.

Algorithm 1: Node location estimation algorithm

Input: $X_a = [\mathbf{x}_1,\ldots,\mathbf{x}_m]$: location matrix of the anchors. $\{\mathbf{s}_i\}_{i=1}^n$: signal strength vectors, or $\{\mathbf{d}_i\}_{i=1}^n$: pair-wise distance vectors.

Outpur: $\{\tilde{\mathbf{x}}_i\}_{i=m+1}^n$: estimated location of the non-anchors.

1: For $i = 1,\ldots,n$, set $s_{ii} = 1$ or $d_{ii} = 0$.
2: Compute the basic Gram matrix $\{K_i\}_{i=1}^q$ based upon $\{\mathbf{s}_i\}_{i=1}^n$ or $\{\mathbf{d}_i\}_{i=1}^n$ (as shown in Eqs. (6) and (7)), then $K_i^{(a)} = K_i(1:m, 1:m)$.
3: Calculate the vector $\mathbf{c}$ and the matrix Q in terms of $\{K_i^{(a)}\}_{i=1}^q$.
4: Solve the optimization problem (16) for optimum θ^*.
5: Define the weight matrix W based upon $\{\mathbf{s}_i\}_{i=1}^n$ or $\{\mathbf{d}_i\}_{i=1}^n$.
6: Calculate the graph Laplacian matrix L by Eq. (8).
7: Find the optimal coefficient matrix A^* by Eq. (18).
8: For $j = m+1,\ldots,n$, the estimated location $\{\tilde{\mathbf{x}}_i\}_{i=m+1}^n$ is obtained by Eq. (19).

4. Performance evaluation

We simulate the localization algorithms with Matlab. To evaluate the performance of the proposed algorithm S^2LapRLS, we suppose the sensor nodes are randomly placed in 2-dimensional (2D) and 3-dimensional (3D) environment. We consider two kinds of localization data, signal strength and pair-wise distance, respectively. We choose RBF for the basis kernel functions with different parameters. For the signal strength localization data, we choose 14 RBF basis kernel functions with w_g varying from 0.0001 to 1.0. For the pair-wise distance, we choose 20 RBF basis kernel functions with w_g varying from 0.8 to 10.0.

A complete signal strength model characterizing the effect of shadowing and fading is given in [11]. For simplicity, we use the model proposed in [17], the signal strength between two sensor nodes is inversely proportional to their distance, and can be simulated by the noisy Gaussian function $s(\mathbf{x}_i, \mathbf{x}_j) = \exp(-w_s\|\mathbf{x}_i - \mathbf{x}_j\|^2 + N(0,\sigma^2))$, where $N(0,\sigma^2)$ denotes an independently generated normal random variable with standard deviation σ, and w_s is a constant proportional to determine the fading of the signal strength. We set $w_s = 1$ in our simulations.

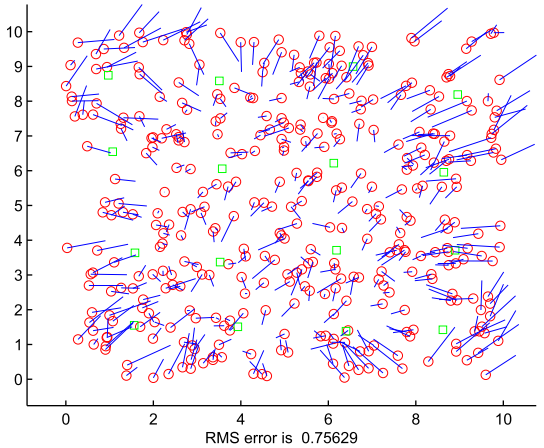
For pair-wise distance between sensor nodes, in order to simplify the ranging process, we assume that the measured pair-wise distance is $\tilde{d}(\mathbf{x}_i, \mathbf{x}_j) = d(\mathbf{x}_i, \mathbf{x}_j)(1 + N(0,\sigma^2))$, where $d(\mathbf{x}_i, \mathbf{x}_j)$ is the real distance and σ is the standard deviation of noise. If X_i is out of the communication range of X_j, we set $\tilde{d}(\mathbf{x}_i, \mathbf{x}_j) = \infty$. Note that the sensor nodes are placed in complicated environments for most real applications. The localization data are always affected by various noises. For example, in most urban environments, the buildings or cars may block the sensor nodes, which in turn makes the obtained signal strengths or pair-wise distances

inaccurate. Hence, we assume that the standard deviation σ is greater than or equal to 0.05 in all of our simulations.

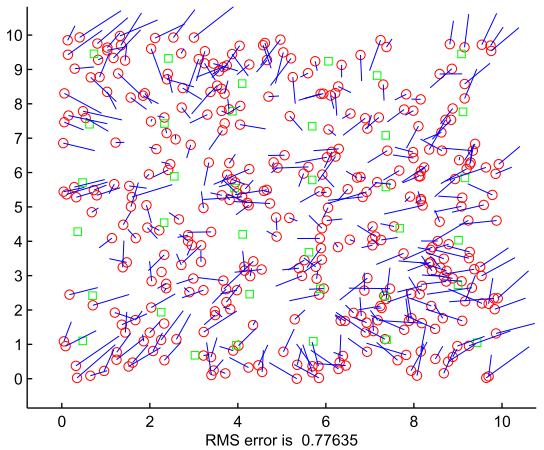
In the simulations, when the localization data is signal strength, we compare S^{2}LapRLS with three related kernel-based localization algorithms: (1) Least squares kernel regression (LSKR) proposed in [17]; (2) Kernel matrix regression (KMR) method proposed in [30]; and (3) SVR, a simplified variant of a kernel-based method used for location estimation [18]. If the localization data is pair-wise distance, we compare our algorithm with the two related range-based localization algorithms: (1) MDS-MAP, which is proposed in [12] and (2) ISOMAP localization algorithm proposed in [22]. All of the reported simulation results are the average over 50 trials.

4.1. 2D Results

Let the geographical region be marked by a 10×10 grid. There are L^2 anchors placed approximately on the grid and the locations are perturbed by additive Gaussian noise. 400 nodes are placed randomly in the area.

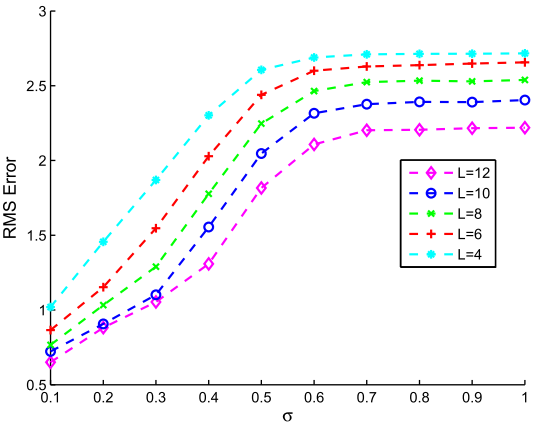


(a) $L = 4$ and $\sigma = 0.05$.

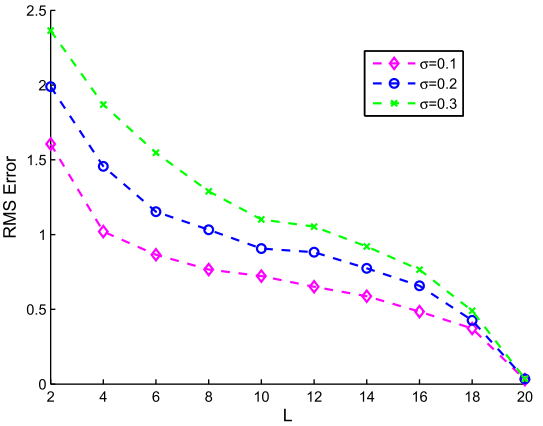


(b) $L = 6$ and $\sigma = 0.1$.

Fig. 1. Localization results with signal strength in 2D environment.



(a) RMS error under different σ.



(b) RMS error under different L.

Fig. 2. Root-mean-square error on locations based upon signal strength in 2D environment.

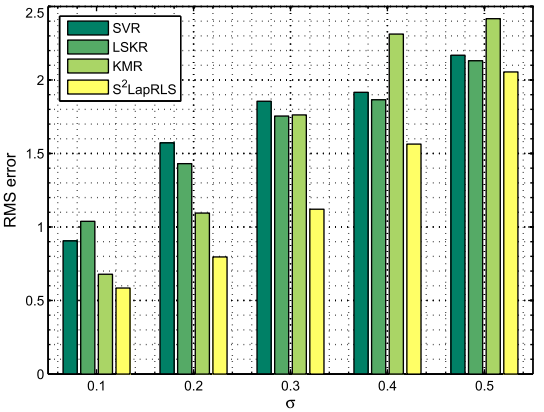


Fig. 3. Performance of different algorithms by using signal strength in 2D environment.

Fig. 1 shows the location results of first group of 2D environment using signal strength. The squares are an-

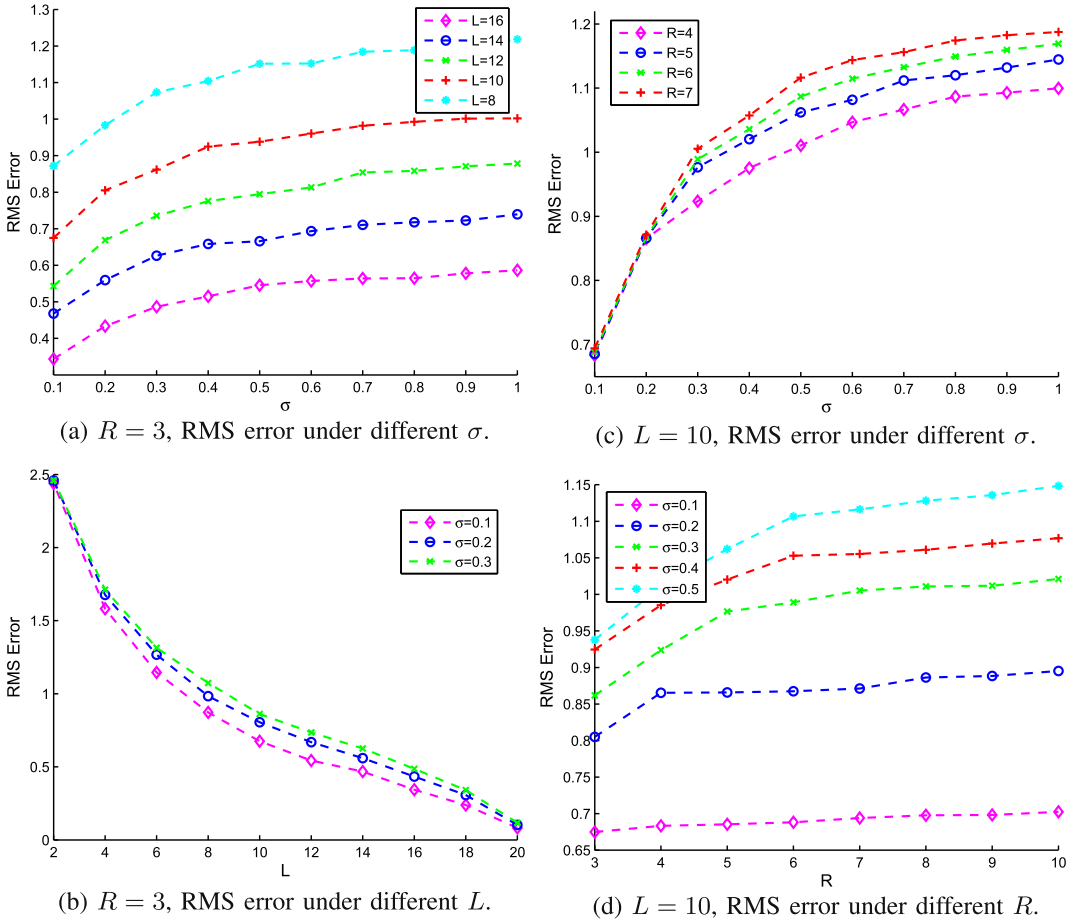
chors and the circles denote the non-anchors. Each line connects a true node location and its estimation. The length of each line denotes the estimation error. In this group of simulation, we suppose the maximum communication of each sensor node is fixed $R = 3$. We set $L = 4$, $\sigma = 0.05$, and plot the location result of each sensor node in Fig. 1(a). The root-mean-square (RMS) error is about 0.76. We also take $L = 6$, $\sigma = 0.1$, and show the final estimation of each sensor node in Fig. 1(b). The final RMS error is about 0.78.

We present a quantitative analysis of the effect of σ and the number of anchors in Fig. 2. Fig. 2(a) plots the RMS error as a function of σ under different values of L, while Fig. 2(b) shows the resulting RMS error as a function of L under different values of σ. It can be seen that the localization error decreases with the number of anchors and increases with the value of σ. The localization accuracy is high, especially when the noise is small. For instance, when $L = 6$ (36 anchors, about 9% of the nodes) and $\sigma = 0.1$, the RMS error is about 0.78. Even for $L = 2$ (4 anchors, about 1% of the nodes) and $\sigma = 0.1$, the RMS error is only about 1.60.

We set $L = 10$ and $\sigma = 0.1, 0.2, 0.3, 0.4, 0.5$, respectively, and obtain the RMS error by using various kernel-based localization algorithms with Matlab. The results are shown in Fig. 3. We can see that S^{2}LapRLS always obtains the best localization accuracy.

In the second group of 2D environment, we use the measured pair-wise distance as the localization data. Let the maximum communication range be R. Similar to the first group of experiments, we show location results of each sensor node in Fig. 5. We set $R = 3$, $L = 6$, $\sigma = 0.05$, and depict the estimated location of each sensor node in Fig. 5(a). The RMS error is about 1.05. We take $R = 3.5$, $L = 8$, $\sigma = 1.0$ and plot the final estimated location in Fig. 5(b). The final RMS error is about 1.25.

We also present a quantitative analysis of the effects of σ, L and R in Fig. 4. We set $R = 3$ and $L = 8, 10, 12, 14, 16$, respectively, and plot the RMS error under different values of σ in Fig. 4(a). Meanwhile, we set $R = 3$ and $\sigma = 0.1, 0.2, 0.3$, respectively, and show the RMS error with different L in Fig. 4(b). It can be seen that the RMS error increases with larger σ and decreases with larger L. These two observations are similar to the results plotted in Fig. 2. However, when σ is large, using pair-wise distance will result in better localization accuracy. Take $\sigma = 0.4$ and $L = 10$ for instance, the RMS error is about 0.91 by using pair-wise distance, while the RMS error is about 1.51 with the signal strength.



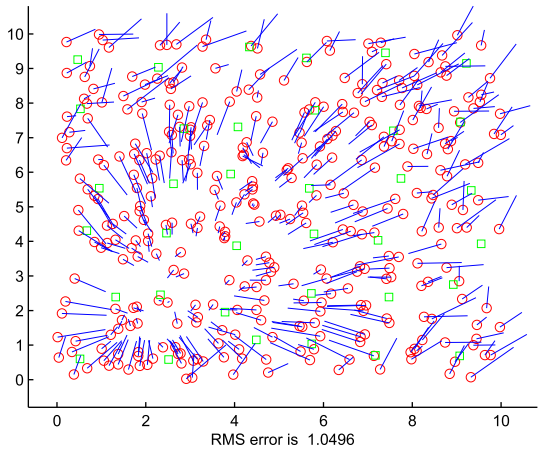
(a) $R = 3$, RMS error under different σ.

(c) $L = 10$, RMS error under different σ.

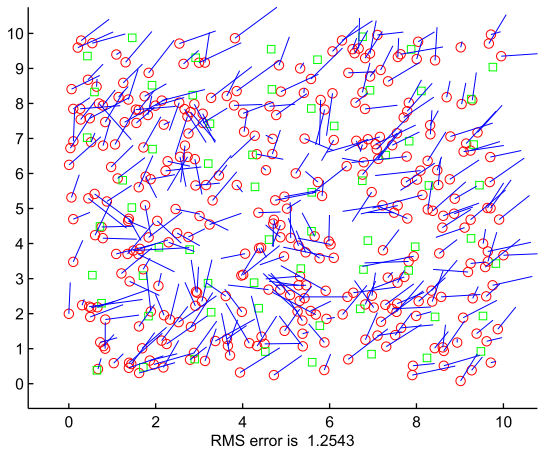
(b) $R = 3$, RMS error under different L.

(d) $L = 10$, RMS error under different R.

Fig. 4. Root-mean-square error on locations based upon measured pair-wise distance in 2D environment.



(a) $R = 3$, $L = 6$ and $\sigma = 0.05$.



(b) $R = 3.5$, $L = 8$ and $\sigma = 1.0$.

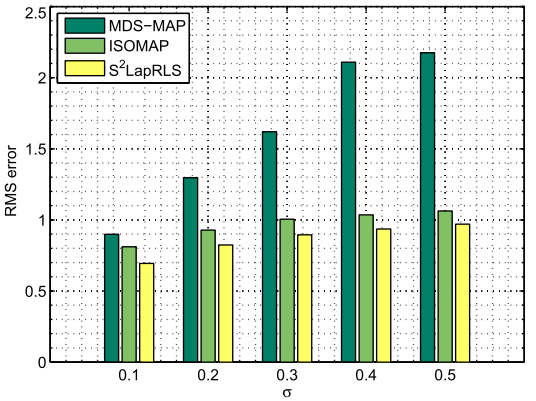
Fig. 5. Localization results with pair-wise distance in 2D environment.

We set $L = 10$ and choose $R = 3, 4, 5, 6, 7$, respectively and plot the resulting RMS error as a function of σ in Fig. 4(c). We also set $L = 10$ and choose $\sigma = 0.1, 0.2, 0.3, 0.4, 0.5$, and depict the resulting RMS error as a function of R in Fig. 4(d). For a given value of R, it can seen that the RMS error is increasing with the value of σ. For a fixed σ, appropriately reducing the communication range R will not drop the localization accuracy. This property is very helpful since reducing R means reducing the transmission power, which in turn conserves the energy of the sensor nodes and extends the life cycle of the whole WSNs.

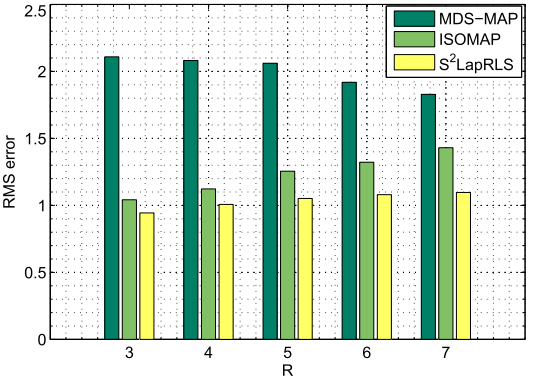
We also compare S^{2}LapRLS with the two related range-based algorithms. We set $R = 3$, $L = 10$, and report their RMS error under different σ in Fig. 6(a). We set $L = 10$, $\sigma = 0.4$ and plot their RMS error under different R in Fig. 6(b). As expected, S^{2}LapRLS always achieves the best results. The key is that S^{2}LapRLS used non-anchor information in addition to anchor information to train the estimator.

4.2. 3D Results

The simulation setup is similar to that for the 2D environment except that the sensor nodes are randomly placed



(a) $R = 3$, $L = 10$, RMS error under different σ.



(b) $L = 10$, $\sigma = 0.4$, RMS error under different R.

Fig. 6. Performance of different algorithms by using pair-wise distance in 2D environment.

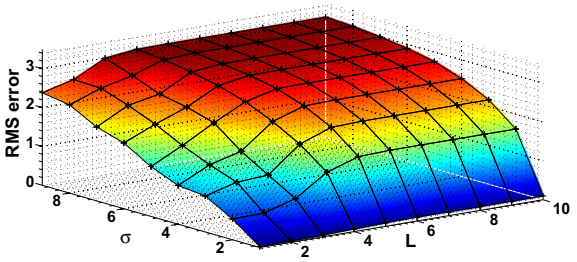


Fig. 7. Root-mean-square error on locations based upon signal strength, as a function of L and σ.

in $10 \times 10 \times 10$ grid and there are L^3 anchors. We also consider two kinds of localization data: signal strength and measured pair-wise distance.

In the first group of 3D environment, we choose signal strength as the localization data. We set the maximum communication range of each sensor node to be 3, and evaluate the influence of both the number of anchors and the standard deviation of the noise on the localization error by taking $L = 2, 3, \ldots, 10$ and $\sigma = 0.1, 0.2, \ldots, 1$. Fig. 7 shows the resulting RMS error as a function of L and σ. As expected, the localization error decreases with the increase of the number of anchors, as well as the decrease of the value of σ.

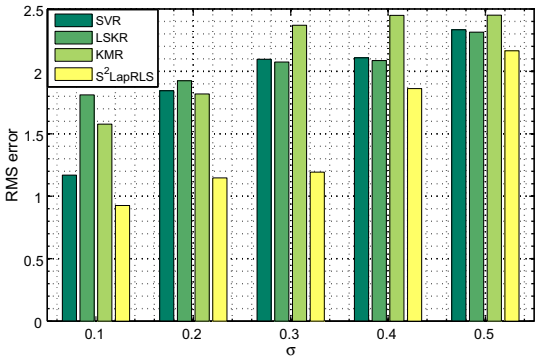
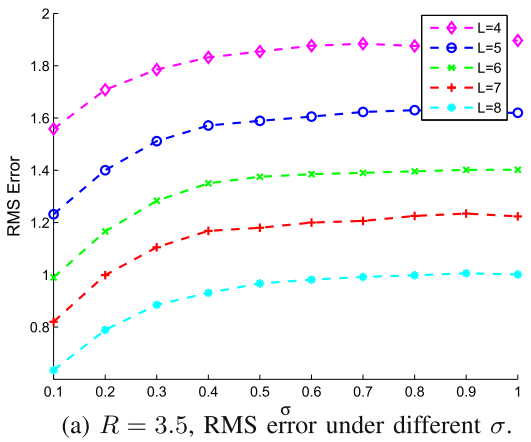


Fig. 8. Performance of different algorithms by using signal strength in 3D environment.

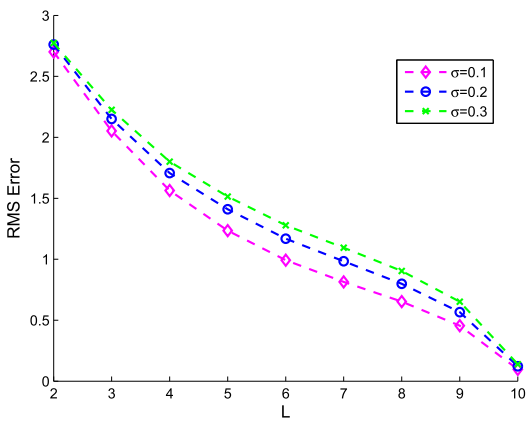
We also compare our algorithm with the three related kernel-based localization algorithms in the 3D environment. We take $L = 8$, and $\sigma = 0.1, 0.2, 0.3, 0.4, 0.5$, respectively. The RMS errors are shown in Fig. 8. It can be seen that our algorithm obtains the best results with various σ.

In the second group of 3D environment, we employ the measured pair-wise distance as the localization data. We consider the maximum communication range to be equal to a pre-given distance R, and show a quantitative analysis of the effects of σ, L and R, respectively (as shown in Fig. 9). Firstly, we set $R = 3.5$, and study the effects of σ under different L. The results are shown in Fig. 9(a). It can be seen that the RMS error increases with the value of σ. When $\sigma \geqslant 0.4$, the RMS error increases little and will reach a stable value. Secondly, we take $R = 3.5$ and exploit the influence of L under different σ. We plot the final RMS error in Fig. 9(b). As expected, the localization accuracy monotonously increases with the number of anchors. Thirdly, we study the effect of the communication range R. We set $L = 5$, $\sigma = 0.1, 0.2, \ldots, 1.0$ and $R = 3, 4, \ldots, 10$, respectively. We plot the resulting RMS error in Fig. 9(c) and (d). It can be seen that appropriately reducing the value of R will not cause significant drop of the localization accuracy. This observation is in consistence with the results in 2D environment.

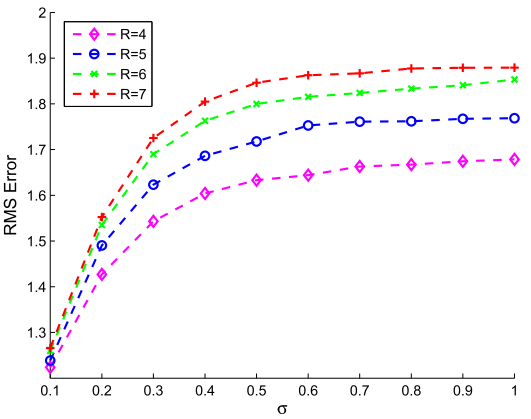
The comparison results of different algorithms by using the measured pair-wise distance are shown in Fig. 10. We first set $R = 3$ and $L = 5$, and compare S²LapRLS with MDS-MAP and ISOMAP under different σ. The resulting RMS error is plotted in Fig. 10(a). Secondly, we take $\sigma = 0.05$, $L = 5$, and $R = 3, \ldots, 7$, respectively. Fig. 10(b) shows the



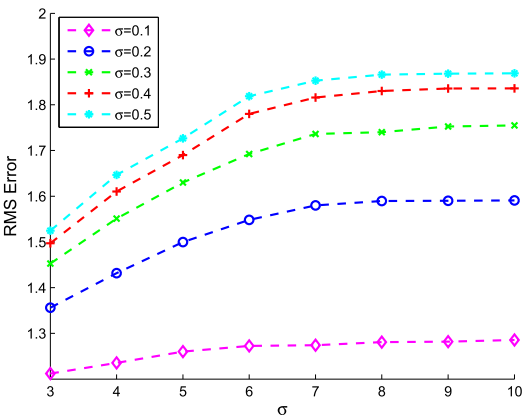
(a) $R = 3.5$, RMS error under different σ.



(b) $R = 3.5$, RMS error under different L.

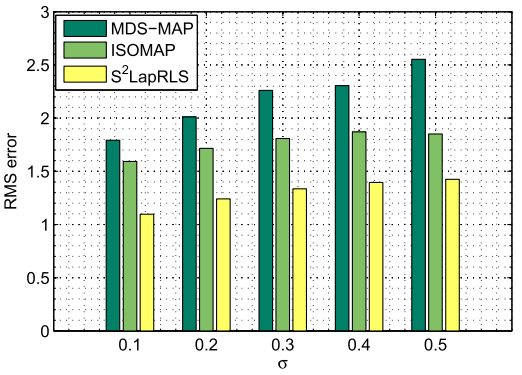


(c) $L = 5$, RMS error under different σ.

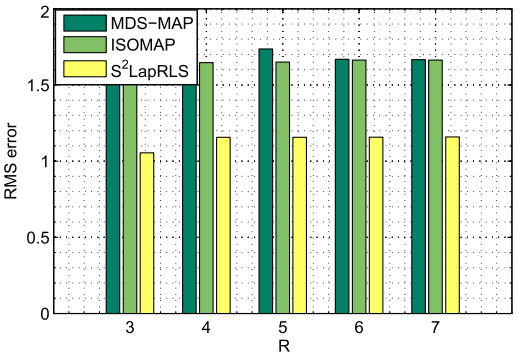


(d) $L = 5$, RMS error under different R.

Fig. 9. Root-mean-square error on locations based upon measured pair-wise distance in 3D environment.



(a) $R = 3,\ L = 5$, RMS error under different σ.



(b) $L = 5,\ \sigma = 0.05$, RMS error under different R.

Fig. 10. Performance of different algorithms by using pair-wise distance in 3D environment.

localization error. It can be seen that our algorithm has the lowest error in the 3D environment.

5. Conclusion

We have studied the location estimation issue for wireless sensor networks by using signal strength or measured pair-wise distance, and proposed a novel localization algorithm under the semi-supervised framework. We first choose kernel alignment criterion to select an optimal kernel function from a kernel conical hull with the help of the similarities between anchors. We show that the kernel selection can be formulated as a QCQP problem, which can be efficiently solved by standard optimization tools. Then the selected kernel function is employed by the semi-supervised Laplacian regularized least squares regression algorithm to train a model for estimation of the locations of non-anchors. Our future work will focus on the ad hoc network localization and cooperative mobile target tracking in wireless networks.

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