# Performance Analysis and Power Allocation for M-QAM Cooperative Diversity Systems

Veluppillai Mahinthan, *Member, IEEE*, Jon W. Mark, *Life Fellow, IEEE*, and Xuemin (Sherman) Shen, *Fellow, IEEE*

***Abstract*—An adaptive *regenerate and forward* cooperative diversity (CD) system using quadrature amplitude modulation (QAM) for a two-user cooperation is proposed. The proposed system can achieve a maximum diversity order of two. The bit error probability (BEP) of a CD system depends primarily on the quality of the inter-user channel and user-to-destination channels, and the transmitted power of the cooperating users. Expressions which characterize the asymptotic behavior of the BEP as a function of the received signal-to-noise ratio (SNR) at the relay and the destination are derived. The transmit power is allocated optimally between the source and the relay according to the channel qualities and the CD system employed to achieve a prescribed BEP. Three power allocation strategies, each for a specific wireless communication scenario, which optimize the power consumption of the proposed CD system are introduced. The cooperative region, corresponding to each power allocation scheme, within which the partners must be located in order to yield a specified cooperative energy gain, is determined.**

***Index Terms*—Cooperative diversity, maximum ratio combining, power allocation, quadrature amplitude modulation, bit error probability, cooperative region.**

## I. INTRODUCTION

COOPERATIVE diversity (CD), which emulates a multiple transmit-antennas situation in multiple input and multiple output (MIMO) systems, is used where the transmitting device is simple and can only have a single transmit antenna. The cooperation between two or more users in a wireless communication system can yield diversity gain [1]-[11]. By user cooperation, it is meant that when the sender transmits to the destination, it also sends the same signal to other users, called partners, for relaying to the destination. The antennas of the sender and the partners together form a multiple transmit antennas situation. In a CD system, the cooperating partners are geographically separated. Therefore, the spacing between any pair of users, and hence their antennas, is wider than that of a conventional MIMO system. CD systems are thus immuned not only against small scale channel fading but also against large scale channel fading, i.e., if the source experiences shadowing, the relay that do not have any shadowing can be selected to mitigate the shadowing. On the other hand, CD systems are more sensitive to inter-user (between sender and partner) transmission errors.

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V. Mahinthan, J. W. Mark, and X. Shen are with the Centre for Wireless Communications, Dept. of E&CE, University of Waterloo, Waterloo, ON, Canada, N2L 3G1 (e-mail: {mveluppi, jwmark, xshen}@bbcr.uwaterloo.ca).

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In general, CD systems can be divided into two categories based on the forwarding schemes employed by the partner: *amplify and forward* (relaying) and *regenerate and forward* (regenerative repeat). For *amplify and forward* schemes, the partner simply amplifies the signal received from the sender and retransmits it to the destination. For *regenerate and forward* schemes, the partner detects the received signal and transmits a regenerated version of the detected signal to the destination. CD systems can also be fixed or adaptive. In fixed CD systems, the partner always forwards the information to the destination. In contrast, the partner of an adaptive CD system decides whether or not to forward the received signal based on knowledge of the cyclic redundancy check (CRC) error detection of the received message, the inter-user channel quality, or the feedback from the destination. If the partner decides not to forward, then it may repeat transmitting the information sent in the earlier time slot, or remain silent. Thus, the term adaptive as applied to CD systems here refers to the adaptation whether or not the relay should forward the received information.

With the currently available radio frequency (RF) circuitry, simultaneous transmission and reception in the same frequency is not feasible. Therefore, the orthogonality of the transmitted signals should be maintained at the relay to handle the transmission and reception of the uplink signal. In the works reported in the literature, orthogonality is achieved by dividing the available channels into sub-channels. In the scheme discussed in [6] and [7], the channel is divided into two sub-channels in the time domain. The division into two sub-channels means that the transmission requires twice the bandwidth; otherwise the signal can only be transmitted at half the data rate of a non-cooperative diversity (NCD) scheme.

In [12], a bandwidth and energy efficient CD system using quadrature signaling for two-user cooperation is proposed. Quadrature signaling is achieved by transmitting in the in-phase and quadrature components of a quadrature amplitude modulation scheme. Therefore, it can be applied to fixed multiple access schemes such as TDMA and FDMA. Since each user employs its own multiple access channel to send both its and the partner's signals to the destination, quadrature signaling eliminates the need to use the partners' multiple access channels, thereby reduces the additional synchronization normally required in schemes such as distributed space-time block coding (DSTBC) in which the signal from both users should be received coherently [2].

With quadrature signaling, instead of expanding the bandwidth or reducing the effective data rate of the cooperative system, the signal constellation of the modulation scheme is expanded to accommodate the transmissions by the partner. The CD system in [12] employs fixed relaying at the partner, which means that the relay forwards whatever information received from the source to the destination without any error checking. It is shown that the CD system in [12] can achieve diversity order of two if the inter-user channel is better than that of source-to-destination channel. In addition, the cooperative region within which cooperation can achieve a specified amount of energy gain over an NCD system is determined.

However, employing adaptive relaying at the partner can provide more diversity gain without the constraint that the inter-user channel should be better than the source-to-destination channel at the expense of increased device and signaling complexity. In this paper, we consider an adaptive *regenerate and forward* CD system which employs cyclic redundancy check (CRC) in the partner's device to check the correctness of the received frame before making a decision whether or not to regenerate and forward. The asymptotic behavior of the bit error probability (BEP) of the proposed scheme is derived for QAM modulation as a function of the received SNR at the relay and at the destination. The tightness of the derived analytical upper bound of BEP is validated using simulation. Advantages of the proposed quadrature signaling based CD system include minimal modification of existing point-to-point systems, implementability using selection relaying [1] and bandwidth efficiency in terms of spectral utilization. In addition, the performance analysis and the numerical results in this paper are unique to the proposed adaptive CD system and quite different from the fixed CD system of [12].

The quality of service (QoS) parameters such as BEP, frame error rate, packet error rate, etc. of a CD system primarily depend on the quality of the inter-user channel and user-to-destination channels, and the transmitted power of the cooperating users. To minimize the power needed for a guaranteed BEP, the transmit power should be allocated optimally between the source and the relay according to the channel qualities and the CD system employed. The bit energies of the source's and relaying bits of our proposed QAM CD system should be equal in order to reduce the I-Q imbalance in the RF circuitry. Since most of the CD systems in the literature do not have this constraint, previous works of power allocation for CD systems do not apply to the proposed system. Thus, we are motivated to study power allocation strategies in Section IV.

In this respect, we introduce three power allocation strategies for our proposed CD system, namely, optimal power allocation (OPA), equal power transmission (EPT) by users, and equal SNR reception (ESR) at the BS, to address wireless communication in different networks. The OPA strategy satisfies the guaranteed BEP with the lowest power for the symmetric QAM constellations. Even though optimally allocating the power between the source and the relay, it would not be enough for some applications that have some additional power allocation constraints. For example, in a sensor network, maximize the lifetime of the network is an objective. This can be achieved by allocating equal transmitting power to the cooperative users under the EPT strategy. On the other hand, system performance of cellular networks is limited by inter-cell interference. It can be mitigated by allocating power that provides equal SNR reception (ESR) at the BS.

The location of the cooperating partner who requires minimal energy to achieve a given BEP is unknown for a particular power allocation strategy. For a given energy gain over an NCD system, the relay's location can be specified by drawing contours in a two-dimensional plane and the shape of contours may differ for each power allocation strategy. In other words, for a given energy gain over no cooperation and a given BEP, a contour of the relay's location relative to those of the source and the destination, referred to as the cooperative region [4], [12], [13], can be determined. The cooperative region gives a clear picture of the relative location of the partner to achieve a guaranteed energy gain by user cooperation for a given power allocation strategy.

The main concern of this paper is to attain cooperative diversity gain of the bandwidth efficient quadrature signaling based CD system and to optimally allocate the power between the source and the relay by minimizing the total requested power. The major contributions of this paper are (i) the proposal of bandwidth efficient adaptive relaying of the sender's signal by the cooperating partner in a quadrature signaling regenerate and forward CD system, (ii) the derivation of an analytical upper bound of the BEP that shows its degree of tightness and (iii) a framework of power allocation that minimizes the total energy consumption by considering the physical layer aspects and operating environment of the CD system.

The remainder of the paper is organized as follows. Section II describes the system model under consideration. Section III analyzes the performance of the proposed CD system. In section IV, optimal power allocation strategies, applied to different wireless communication scenarios, are introduced. These power allocation strategies are assessed based on contours of cooperative regions. Finally, concluding remarks are given in section V.

## II. System Model

We consider an M-QAM signaling cooperative diversity scheme in which a mobile user (the sender) cooperates with another mobile user (the partner) to transmit signals in the uplink of an infrastructure based network. In each cooperative mobile device, both its own and its partner's signals are simultaneously transmitted using quadrature signaling[1] [12]. In this paper, we are concerned with the transmissions of M-QAM signals with adaptive relaying at the partner using the quadrature signaling based CD system described in Fig. 1. The base station (BS) and each of the mobile devices has a single antenna, so the cooperative diversity scheme emulates a "two-input one-output" (2I1O) situation. The signals transmitted by the sender and relayed by the partner are combined at the BS receiver using maximal ratio combining.

The transmission frame format of the M-QAM modulation scheme is shown in Fig. 1(a), with the signal constellation of 4PAM/16-QAM modulation shown in 1(b). In quadrature signaling, we assign the in-phase channel to *user 1* and the quadrature channel to *user 2*. Fig. 1(a) shows frames of information symbols transmitted by *user 1* in multiple access

[1]With quadrature signaling, a user transmits the partner's signal as well as its own. Thus the roles of sender and partner are interchangeable.

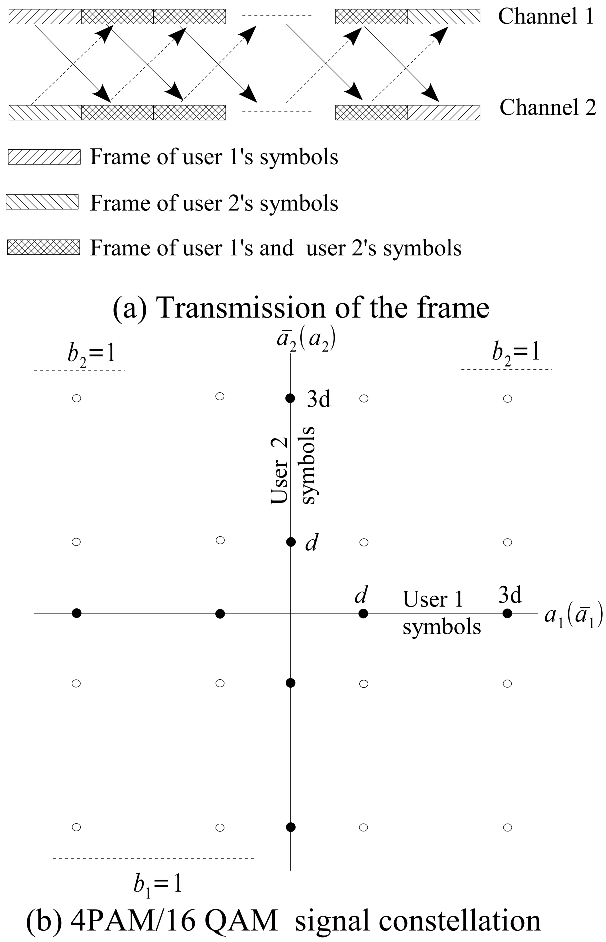


Fig. 1. (a) Frame transmission format and (b) signal constellation of 4-PAM/16-QAM using quadrature signaling.

*channel 1* and by *user 2* in multiple access *channel 2*. In the first frame interval of a cooperative session, each user transmits its own information only. In successive frame intervals, each user transmits not only its own information but also the regenerated version of the partner's information received in the previous frame interval. But, in the last frame interval of the cooperation session, each user transmits the regenerated version of the partner's information only. Since *user 1* and *user 2* use the in-phase and the quadrature components of the M-QAM modulation, respectively, each user equivalently employs I-PAM modulation, where $I = \sqrt{M}$. In the uplink receiver of both users, the in-phase and quadrature components are demodulated separately and the detected and regenerated partner's frame is forwarded to the BS if the frame is detected error free.

Throughout the paper, we assume that the inter-user and the user to BS channels exhibit flat Rayleigh fading, and that the two channel are independent of each other, static over a frame interval, and change independently from frame to frame. In this case, each frame is comprised of $N$ symbols. In addition, it is assumed that channel state information (channel fading coefficient) is available at the respective receivers to facilitate demodulation and that proper synchronization has been established.

At the BS, the signals received from both users are combined using a maximal ratio combiner (MRC). To decode the information of *user 1*, the received signal from *channel 1* is delayed by one frame interval and combined with the received signal from *channel 2* at the MRC. Similarly, to decode the information of *user 2*, the received signal from *channel 2* is delayed by one frame interval and combined with the received signal from *channel 1*. The output of the MRC is sent to a decision device which converts it to a binary format.

Let the baseband equivalent received signals from *user i*, $i = 1, 2$, at the BS be denoted as $r_{i,B}(t)$. Similarly, *user i*'s uplink signal received by *user j*, $j = 1, 2$ and $i \neq j$, is denoted as $r_{i,j}(t)$. From the system model, $r_{i,B}(t)$ and $r_{i,j}(t)$ can be written as

$$\begin{aligned} r_{i,B}(t) &= h_{i,B}(t)s_i(t) + \eta_{i,B}(t) \\ r_{i,j}(t) &= h_{i,j}(t)s_i(t) + \eta_{i,j}(t), \end{aligned} \tag{1}$$

where the channel fading coefficient between user $i$ and the BS is denoted by $h_{i,B}(t)$ and that from user $i$ to user $j$ by $h_{i,j}(t)$. When the inter-user channel is symmetric, $h_{i,j}(t) = h_{j,i}(t)$. The processes $\eta_{i,B}(t)$ and $\eta_{i,j}(t)$ are additive noise at the respective receivers and modeled as zero mean circularly symmetric, complex Gaussian distributed with variance $N_0/2$ per dimension, where $N_0$ is the one-sided power spectral density of white Gaussian noise. $s_i(t)$, chosen from the M-QAM signal constellation similar to that shown in Fig. 1(b), is the transmitted signal from *user i*. Therefore, $s_1(t)$ and $s_2(t)$ can be written as $s_1(t) = \sqrt{log_2 M \cdot E_{b1}/2}(a_1(t) + j\bar{a}_2(t - T_f))$ and $s_2(t) = \sqrt{log_2 M \cdot E_{b2}/2}(\bar{a}_1(t - T_f) + ja_2(t))$, respectively, where $a_i(t)$ is the I-PAM information symbol of *user i* and $\bar{a}_i(.)$ is the corresponding reproduced symbol at the partner. $T_f$ is the frame duration and $E_{bi}$ is the energy spent for a bit using NCD transmission at user $i$. To have a power consumption equal to that of an NCD system, the CD system equally shares the power in transmitting the user's and the partner's information bits at each mobile device.

At the BS, if each user relays the partner's information, the signals received from both user channels are combined using MRC and decoded using the maximum likelihood (ML) rule [14]. The decoded symbols are given by

$$\begin{aligned} \tilde{a}_1 = \arg \min_{\bar{a}_1 \epsilon S_I} \Big( & \Re\{h^*_{1,B}(t - T_f)r_{1,B}(t - T_f) \\ & + h^*_{2,B}(t)r_{2,B}(t)\} - (|h_{1,B}(t - T_f)|^2 + |h_{2,B}(t)|^2)\bar{a}_1 \Big) \\ \tilde{a}_2 = \arg \min_{\bar{a}_1 \epsilon S_I} \Big( & \Im\{h^*_{2,B}(t - T_f)r_{2,B}(t - T_f) \\ & + h^*_{1,B}(t)r_{1,B}(t)\} - (|h_{1,B}(t)|^2 + |h_{2,B}(t - T_f)|^2)\bar{a}_2 \Big), \end{aligned} \tag{2}$$

where the notations $\Re\{Z\}$ and $\Im\{Z\}$ denote the real and the imaginary parts of the complex number $Z$, respectively. $S_I$ is the set of I-PAM information symbols.

## III. Performance Analysis

The performance measures of the CD system described in Section II are the inter-user frame error probability ($P_{FEP}$), the BEP at the output of the MRC ($P_{MRC}$), when the partner helps, and the BEP of the MRC output ($P_{NH}$), when the partner does not help. In addition, we also present the BEP of the NCD (conventional I-PAM) system from [17]. Finally, analytical and simulation results are presented to demonstrate the tightness of the analytical upper bound and to study the performance characteristics of the proposed CD system.

Since the roles of sender and partner in quadrature signaling are interchangeable, in the derivations to follow, we consider *user 1* as the sender and *user 2* as the relay. For quasi-static fading channels among the users and the BS, $h_{1,2}(n) = h_{1,2}$, $h_1(n) = h_1$ and $h_2(n) = h_2$ within a frame of $N$ symbols and varying independently between frames. Let $\bar{\gamma}_{1,2} = \frac{E_{b1}}{N_0}\mathrm{E}\{|h_{1,2}|^2\}$, $\bar{\gamma}_1 = \frac{E_{b1}}{N_0}\mathrm{E}\{|h_1|^2\}$ and $\bar{\gamma}_2 = \frac{E_{b2}}{N_0}\mathrm{E}\{|h_2|^2\}$ be the inter-user signal-to-noise ratio (ISNR), *user 1*-to-BS SNR and *user 2*-to-BS SNR, respectively, where $\mathrm{E}\{.\}$ is the expectation operator.

### A. Average BEP of the CD system

The average BEP of the CD system, $P_b$, can be written as

$$P_b = (1 - P_{FEP})P_{MRC} + P_{FEP}P_{NH} \tag{3}$$

where the derivations of $P_{FEP}$, $P_{MRC}$ and $P_{NH}$ are given in the Appendix. From (27), $P_{FEP}$ can be written as

$$P_{FEP} \leq \frac{1}{I\log_2 I}\sum_{m=1}^{\log_2 I}\sum_{i=0}^{(1-2^{-m})I-1}\lambda(I,m,i)\sum_{N_{e|m}=1}^{N}\Bigg(1 - \sqrt{\frac{3\log_2 I.(2i+1)^2 N_{e|m}\bar{\gamma}_{1,2}}{2(I^2-1)+3\log_2 I.(2i+1)^2 N_{e|m}\bar{\gamma}_{1,2}}}\Bigg). \tag{4}$$

From (31) in the Appendix, the $P_{MRC}$ for similar ($\bar{\gamma}_1 = \bar{\gamma}_2$) and dissimilar ($\bar{\gamma}_1 \neq \bar{\gamma}_2$) channel towards the BS can be written as

$$P_{MRC} = \frac{1}{I\log_2 I}\sum_{m=1}^{\log_2 I}\sum_{i=0}^{(1-2^{-m})I-1}\lambda(I,m,i)\left(\frac{1-\mu_1(I,i)}{2}\right)^2(2+\mu_1(I,i)) \tag{5}$$

and

$$P_{MRC} = \frac{1}{I\log_2 I}\sum_{m=1}^{\log_2 I}\sum_{i=0}^{(1-2^{-m})I-1}\lambda(I,m,i)\left[\frac{\bar{\gamma}_1}{\bar{\gamma}_1-\bar{\gamma}_2}(1-\mu_1(I,i)) + \frac{\bar{\gamma}_1}{\bar{\gamma}_1-\bar{\gamma}_2}(1-\mu_2(I,i))\right] \tag{6}$$

respectively, where $\mu_1(I,i) = \sqrt{\frac{3\log_2 I.(2i+1)^2\bar{\gamma}_1}{2(I^2-1)+3\log_2 I.(2i+1)^2\bar{\gamma}_1}}$ and $\mu_2(I,i) = \sqrt{\frac{3\log_2 I.(2i+1)^2\bar{\gamma}_2}{2(I^2-1)+3\log_2 I.(2i+1)^2\bar{\gamma}_2}}$. The parameter $\lambda(I,m,i) = (-1)^{\lfloor\frac{i.2^{m-1}}{I}\rfloor}(2^{m-1} - \lfloor\frac{i.2^{m-1}}{I} + \frac{1}{2}\rfloor)$, where the symbol $\lfloor\cdot\rfloor$ denotes the integer greater than or equal to its argument. $P_{NH}$ can be written as

$$P_{NH} = \frac{1}{I\log_2 I}\sum_{m=1}^{\log_2 I}\sum_{i=0}^{(1-2^{-m})I-1}\lambda(I,m,i)(1-\mu_1(I,i)). \tag{7}$$

By substituting (4), (5) or (6) and (7) into (3), BEP can be written for similar and dissimilar channel towards the BS.

For the CD system with 4-QAM signaling, i.e., 2-PAM per user, by substituting (32) or (34) and (36) into (3), the BEP can be written for similar and dissimilar channel towards the BS as

$$P_b = (1-P_{FEP})\left(2+\sqrt{\frac{\bar{\gamma}_1}{2+\bar{\gamma}_1}}\right)\left[\frac{1}{2}\left(1-\sqrt{\frac{\bar{\gamma}_1}{2+\bar{\gamma}_1}}\right)\right]^2 + P_{FEP}\frac{1}{2}\left(1-\sqrt{\frac{\bar{\gamma}_1}{2+\bar{\gamma}_1}}\right) \tag{8}$$

and

$$P_b = (1-P_{FEP})\frac{1}{2}\Bigg[\frac{\bar{\gamma}_1}{\bar{\gamma}_1-\bar{\gamma}_2}\left(1-\sqrt{\frac{\bar{\gamma}_1}{2+\bar{\gamma}_1}}\right) + \frac{\bar{\gamma}_2}{\bar{\gamma}_2-\bar{\gamma}_1}\left(1-\sqrt{\frac{\bar{\gamma}_2}{2+\bar{\gamma}_2}}\right)\Bigg] + P_{FEP}\frac{1}{2}\left(1-\sqrt{\frac{\bar{\gamma}_1}{2+\bar{\gamma}_1}}\right), \tag{9}$$

respectively. By substituting (33) or (35) and (37) into (3), a closed-form solution can be derived for BEP of the 16-QAM adaptive CD system for similar and dissimilar channels towards the BS as

$$\begin{aligned} P_b = (1-P_{FEP})\frac{1}{4}\Bigg\{&\frac{3}{4}(1-\mu_1(4,0))^2(2+\mu_1(4,0)) \\ &+ \frac{1}{2}(1-\mu_1(4,1))^2(2+\mu_1(4,1)) \\ &- \frac{1}{4}(1-\mu_1(4,2))^2(2+\mu_1(4,2))\Bigg\} \\ &+ P_{FEP}\frac{1}{8}(4-3\mu_1(4,0)-2\mu_1(4,1)+\mu_1(4,2)), \end{aligned} \tag{10}$$

and

$$\begin{aligned} P_b = (1-P_{FEP})& \\ \times\frac{1}{8}\Bigg\{&\frac{\bar{\gamma}_1}{\bar{\gamma}_1-\bar{\gamma}_2}\Big(4-3\mu_1(4,0)-2\mu_1(4,1)+\mu_1(4,2)\Big) \\ &+ \frac{\bar{\gamma}_2}{\bar{\gamma}_2-\bar{\gamma}_1}\Big(4-3\mu_2(4,0)-2\mu_2(4,1)+\mu_2(4,2)\Big)\Bigg\} \\ &+ P_{FEP}\frac{1}{8}\Big(4-3\mu_1(4,0)-2\mu_1(4,1)+\mu_1(4,2)\Big), \end{aligned} \tag{11}$$

respectively.

### B. Average BEP of NCD system

The average BEP of the NCD system is given by [17]:

$$P_{NCD} = \frac{1}{I\log_2 I}\sum_{m=1}^{\log_2 I}\sum_{i=0}^{(1-2^{-m})I-1}\lambda(I,m,i)(1-\hat{\mu}_1(I,i)), \tag{12}$$

where $\hat{\mu}_1(I,i) = \sqrt{\frac{3\log_2 I.(2i+1)^2\bar{\gamma}_1}{(I^2-1)+3\log_2 I.(2i+1)^2\bar{\gamma}_1}}$.

### C. Tradeoff between power and data rate

In any cooperative system, the cooperative diversity gain is achieved at the expense of radio resources, i.e., time, frequency, data rate, etc. In our system, we expand the signal constellations which indirectly reduces the rate. At this point, a fair comparison can be done in two ways. (i) the CD and NCD transmissions are compared for the same rate (spectral efficiency) for each user. For example, the resultant modulation for the CD and NCD systems with 2 bits/Hz/s are 16-QAM

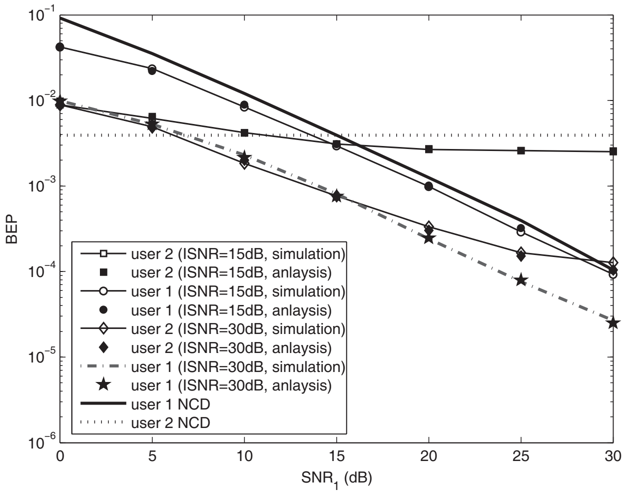


Fig. 2. Bit error probability of the CD system with 4-QAM/QPSK for dissimilar channel towards BS ($SNR_2 = 15dB$).

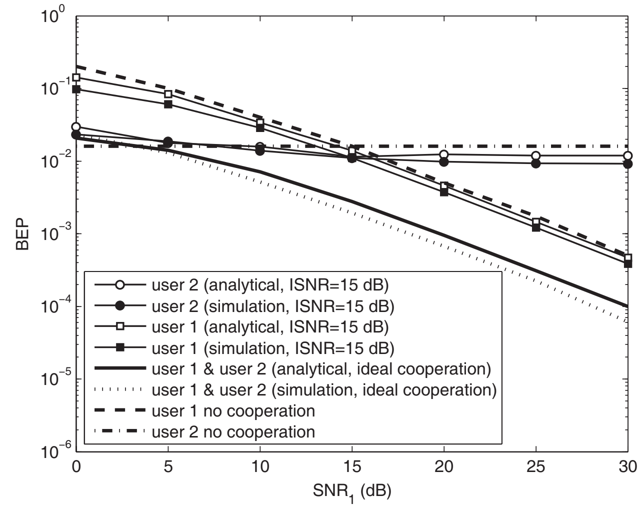


Fig. 3. Bit error probability of the CD system with 16-QAM for dissimilar channel towards BS ($SNR_2 = 15dB$).

and 4-PAM, respectively, where the same bit energy of the 4-PAM transmission is used for 16-QAM transmission as well. That means the symbol energy required for the NCD is shared for the CD transmission. (ii) The CD and NCD systems are compared with the same signal constellations. For example, the resultant modulation for CD and NCD systems are 16-QAM where the systems have different rate but same symbol energy. To have a fair comparison, we have to normalize the SNR by the minimum SNR required for each system to achieve a given rate $R$ per dimension. The minimum SNR is given by $2^R - 1$ where $R = 2$ for 16 QAM NCD and $R = 1$ for 16 QAM CD. In this paper, we consider the first comparison method. The second method is utilized in [12] for fixed CD systems.

### *D. Numerical Results*

In this subsection, we present numerical results to demonstrate the performance of the proposed CD system. The BEP is evaluated analytically and via simulation. In our simulation, a frame consisting of 128 information symbols is transmitted through a Rayleigh flat fading channel. In a cooperative session, 100 frames are considered. The noise is assumed to be a zero mean complex Gaussian process with variance $N_0/2$ per dimension. The channel is also considered as slow fading that remains constant over a frame interval. We assume that channel state information (CSI) is available at the respective receivers for decoding the received signal.

*1) BEP of the CD system with dissimilar channel towards the BS:* The BEPs as a function of *user 1*'s SNR of the proposed CD system with 4-QAM/QPSK for dissimilar channels are plotted in Fig. 2. Here, $\bar{\gamma}_2$ is fixed at 15dB and $\bar{\gamma}_1$ varies from 0 to 30 dB for the cases in which the inter-user SNR (ISNR) is $\bar{\gamma}_{1,2}$=15dB and 30dB. The proposed CD system provides better bit error performance not only for *user 1* but also for the stronger *user 2* over the NCD system (standard BPSK) for ISNR=15dB. The BEP of the CD system gradually decreases when ISNR increases because the partner's help tends to increase with ISNR. For clarity of the figure, we only show two cases such as ISNR=15dB and ISNR=30dB in Fig. 2. Furthermore, the figure shows that the analytical results for upper bound virtually coincide with the simulation results. This is due to the fact that the difference between the exact FEP and the upper bound of FEP is insignificant in the calculation of the overall BEP by (3).

The simulation and analytical results of the proposed CD system for 16-QAM are given for ISNR=15dB and 30dB in Figs. 3 and 4, respectively. For the curve shown in Fig. 3, $\bar{\gamma}_2$ is fixed at 15dB and $\bar{\gamma}_1$ varies from 0 to 30 dB. It can be seen that the BEP of both users is enhanced significantly by the proposed scheme. This means that cooperation enhances not only the performance of the user far away from the BS but also the user near to the BS. In addition, the BEP improves with the inter-user channel quality, as shown in Fig. 4 for ISNR=30dB. Thus, the BEP performance of the proposed scheme improves as the ISNR increases. For ideal cooperation (assuming that the partner's information is perfectly decoded), the performance of the proposed scheme is the same as that of the case with one transmit antenna and two receive antennas using MRC. For the CD system with 16-QAM, the BEP upper bound is very close to the simulation results.

From all of the above figures, we can observe that the BEP of *user 2* is saturated at one point because $\bar{\gamma}_2$ is fixed at 15dB. The level of saturation is a function of the inter-user channel quality.

*2) BEP of the CD system with similar channel towards the BS:* In the case of similar channel towards the BS ($\bar{\gamma}_1 = \bar{\gamma}_2$), due to symmetric nature, both users experience the same error performance as shown in Figs. 5 and 6 for 4-QAM/QPSK and 16-QAM, respectively. In Fig. 5, the BEPs of the proposed scheme with 4-QAM/QPSK for three different ISNR values are compared with those of the non-cooperative diversity scheme and the ideal cooperation scheme. Since the power allocated for cooperative transmission is half of the non-cooperative transmission, the CD system performs similar to the NCD system when ISNR=10 dB. The BEP performance of the CD system improves as ISNR increases, and approaches the ideal cooperation performance when ISNR is above 30 dB. In the ideal cooperation situation, the proposed

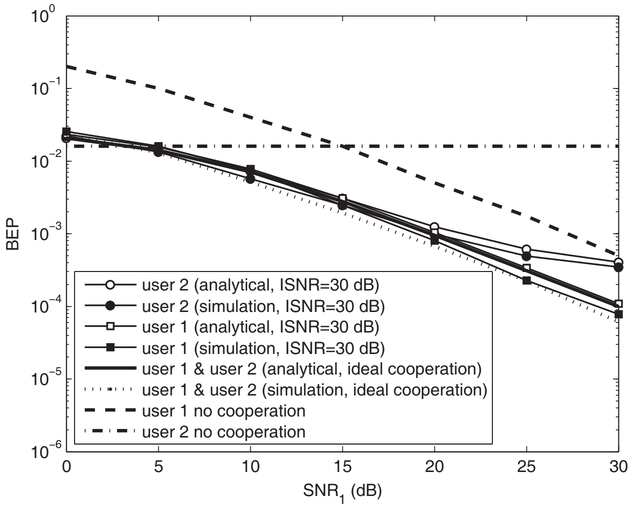


Fig. 4. Bit error probability of the CD system with 16-QAM for dissimilar channel towards BS ($SNR_2 = 30dB$).

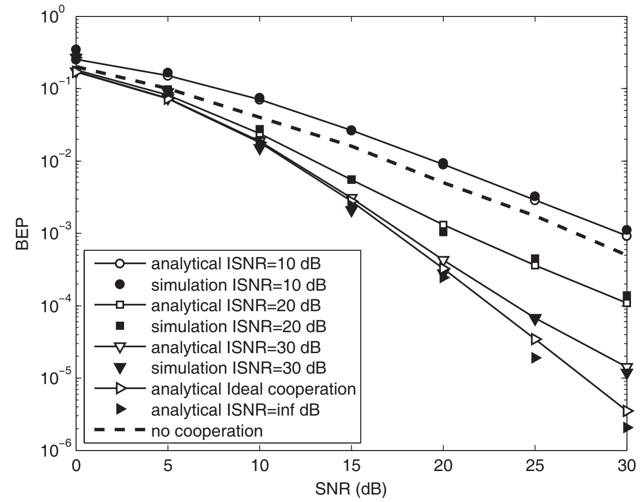


Fig. 6. Bit error probability of the CD system with 16-QAM for similar channel towards BS.

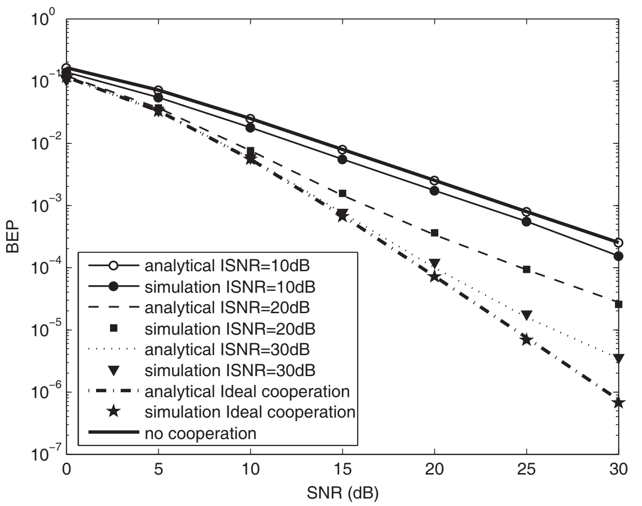


Fig. 5. Bit error probability of the CD system with 4-QAM/QPSK for similar channel towards BS.

CD system provides a diversity order of two. In addition, from the simulation results, we show that the BEP upperbound is tight.

The BEP performance of the CD system with 16-QAM and similar channel towards the BS is plotted in Fig. 6. In contrast to 4-QAM/QPSK, the performance of the CD system with 16-QAM is 2 dB worse than that of the NCD scheme when ISNR is 10 dB. This is due to the fact that expanding the signal constellation reduces the Euclidean distance between constellation points hence more susceptible to symbol error. On the other hand, the BEP decreases when ISNR increases. Furthermore, the CD system achieves a diversity order of two when ideal cooperation takes place; similar performance can be achieved when ISNR is 30 dB. In contrast to the fixed *regenerate and forward* CD system in [12], the proposed CD systems shows that cooperation is beneficial even when the inter-user channel is worse than the channel between the user and the BS.

In summary, the derived analytical upper bound of the BEP follows the simulated BEP and the proposed CD system shows a similar behavior regardless of the size of the QAM signal constellation. It is conjectured that the proposed CD system can achieve similar BEP performance for higher order QAM modulation.

## IV. Power Allocation

From Section III, the BEP of a CD system primarily depends on the quality of the inter-user channel and user-to-destination channels, and the transmitted power of the cooperating users. As stated in Section I, to minimize the total requested power for a guaranteed BEP, the transmitted power should be allocated optimally between the source and the relay. In this section, we consider power allocation strategies to achieve a prescribed BEP.

Although optimally allocating transmit power to the sender and the relay can enhance system performance, it is done at the expense of increased system complexity. It is thus important to consider power allocation strategies that offer a good performance-complexity tradeoff by imposing certain suitable constraints. Implementation complexity of a power allocation strategy is a function of the operational environment of the CD system. For example, in a sensor network, the objective is to maximize the lifetime of the network. It is therefore necessary to minimize the energy consumption of those nodes that consume energy at the maximum rate. Since all nodes start with the same initial energy, equal power transmission can prolong the lifetime of a pair of nodes, subject to a BEP constraint. On the other hand, for a cellular network using either TDMA or FDMA as the multiple access technology, the system performance is limited by the inter-cell interference, which can be reduced by power control. In this case, a power allocation strategy that constrains the BS receiver to have equal SNR reception would have the effect of reducing the inter-cell interference.

With regard to power allocation, our concern is with minimizing the combined energy consumption at the sender and the relay, subject to BEP and other appropriate constraints. Accordingly, we seek expressions for the energy consumption at the sender and the relay as a function of BEP.

### *A. Energy as a function of BEP*

For high SNR ($\bar{\gamma}_1 >> 1$, $\bar{\gamma}_2 >> 1$ and $\bar{\gamma}_{1,2} >> 1$) i.e., greater than 8 dB, the approximate bit error probability of *user 1* from (9), using 4-QAM/QPSK modulation, can be written as

$$P_{b1} = \frac{K_N}{4\bar{\gamma}_1\bar{\gamma}_{1,2}} + \frac{3}{4\bar{\gamma}_1\bar{\gamma}_2} - \frac{3K_N}{8\bar{\gamma}_1\bar{\gamma}_2\bar{\gamma}_{1,2}} \tag{13}$$

where $K_N = \sum_{e_b=1}^{N} \frac{1}{e_b}$. Similar to the 4-QAM/QPSK modulation, for high SNR, the approximate bit error probability of *user 1*, using 16-QAM modulation from (11), can be written as

$$P_{b1} = \frac{K_N}{\bar{\gamma}_1\bar{\gamma}_{1,2}} + \frac{4.52}{\bar{\gamma}_1\bar{\gamma}_2} - \frac{4.52K_N}{\bar{\gamma}_1\bar{\gamma}_2\bar{\gamma}_{1,2}}. \tag{14}$$

It can be seen that the BEP is asymptotically dominated by the first and second terms which provide a diversity of order two. Furthermore, we can write the general form of approximate bit error probability of *user 1*[2] as

$$P_{b1} = \frac{A}{\bar{\gamma}_1\bar{\gamma}_{1,2}} + \frac{B}{\bar{\gamma}_1\bar{\gamma}_2} - \frac{C}{\bar{\gamma}_1\bar{\gamma}_2\bar{\gamma}_{1,2}}. \tag{15}$$

Let $k_1 = \frac{\sigma_{1,2}^2}{\sigma_1^2}$ and $k_2 = \frac{\sigma_{2,1}^2}{\sigma_2^2}$. Due to the broadcast nature of the channel, $\bar{\gamma}_{1,2} = k_1\bar{\gamma}_1$ and $\bar{\gamma}_{2,1} = k_2\bar{\gamma}_2$. Further, $\sigma_2^2 = \frac{k1}{k2}\sigma_1^2$ because of the reciprocity of the inter-user channel ($\sigma_{1,2}^2 = \sigma_{2,1}^2$). By assuming $\sigma_1^2 = 1$ and substituting for the $\bar{\gamma}$'s in terms of the bit energy-to-noise spectral density ratio, (13) can be re-arranged and written as

$$E_{b2} = \frac{\left(Bk_1k_2E_{b1}N_0^2 - Ck_2N_0^3\right)}{\left(k_1^2P_{b1}E_{b1}^2 - Ak_1N_0^2\right)}. \tag{16}$$

Similarly for *user 2*,

$$E_{b1} = \frac{\left(Bk_1k_2E_{b2}N_0^2 - Ck_2N_0^3\right)}{\left(k_1^2P_{b2}E_{b2}^2 - Ak_2N_0^2\right)}. \tag{17}$$

The reasonableness of the high SNR approximation is demonstrated by the numerical results in subsection IV-D.

### *B. Power Allocation Strategies*

*1) Optimal Power Allocation (OPA):* To minimize the power consumption for the source-relay pair, we formulate the optimization problem as the following constrained minimization problem:

$$\min\left(E_{b1} + E_{b2}\right) \tag{18}$$
$$s.t.\ P_{b1} \le e_1, \text{ and } P_{b2} \le e_2.$$

Minimizing (18) does not admit a closed-form solution. As such, we will solve the power allocation problems numerically.

According to (16) and (17), the conditions $P_{b1} \le e_1$ and $P_{b2} \le e_2$ correspond to two convex areas of $E_{b1}$ and $E_{b2}$ where $e_1$ and $e_2$ are the BEP of *user 1* and *user 2*, respectively. The optimal $E_{b1}$ and $E_{b2}$ correspond to the intersecting points of the curves drawn from (16) and (17), as shown in Fig. 7. The coordinates of the intersection point of $P_{b1} = e_1$ and $P_{b2} = e_2$, point **A** (for $k_1 = 1$ and $k_2 = 1$) and point **B** (for $k_1 = 1$ and $k_2 = 2$), can be obtained from the solutions of the fifth order polynomial which is given by substituting $E_{b1}$ in (16) by the RHS of (17). The coordinates of point **A** and point **B** can be solved numerically (e.g., using Matlab).

[2]By neglecting the third term and optimizing the power allocation to minimize BEP, we also come to a similar conclusion as in [6].

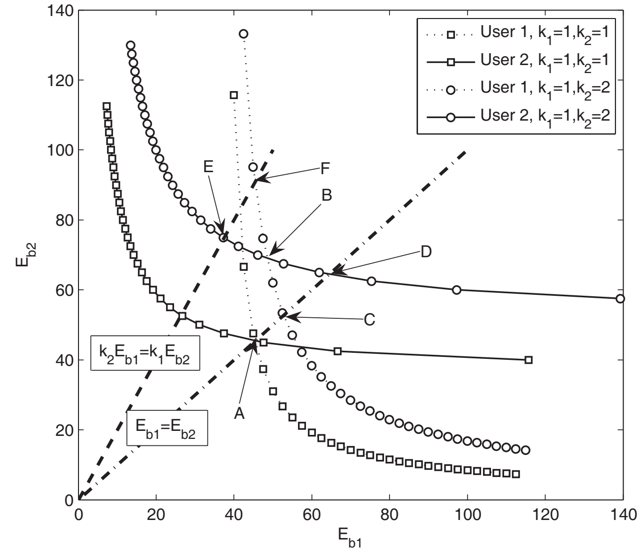


Fig. 7. $E_{b1}$ vs $E_{b2}$ of both *user 1* and *user 2* for various channel conditions ($e_1 = e_2 = 1 \times 10^{-3}$).

*2) Equal Power Transmission (EPT) by Users:* To minimize the power consumption for the pair with EPT, the power allocation problem can be formulated as

$$\min\left(E_{b1} + E_{b2}\right) \tag{19}$$
$$s.t.\ E_{b1} = E_{b2}, P_{b1} \le e_1, \text{ and } P_{b2} \le e_2.$$

By substituting $E_{b1} = E_{b2}$ into (16) and (17), the coordinates of points **C** and **D** in Fig. 7 can be obtained for a dissimilar channel towards the BS ($k_1 = 1$ and $k_2 = 2$). Between the points **C** and **D**, the point with the higher power level can satisfy both conditions $P_{b1} \le e_1$ and $P_{b2} \le e_2$, and it will be the optimal equal-power solution for the CD schemes. For the case of a similar channel towards the BS ($k_1 = 1$ and $k_2 = 1$), point A is selected as the operating point.

*3) Equal SNR Reception (ESR) at the BS:* To minimize the power consumption for ESR at the BS, the power allocation problem can be formulated as follows:

$$\min\left(E_{b1} + E_{b2}\right) \tag{20}$$
$$s.t.\ E_{b1} = \frac{k_1}{k_2}E_{b2}, P_{b1} \le e_1, \text{ and } P_{b2} \le e_2.$$

By substituting $E_{b1} = \frac{k_1}{k_2}E_{b2}$ into (16) and (17), the coordinates of points **E** and **F** in Fig. 7 can be obtained for a dissimilar channel towards the BS ($k_1 = 1$ and $k_2 = 2$). Between the points **E** and **F**, the point with the higher power level can satisfy both conditions of $P_{b1} \le e_1$ and $P_{b2} \le e_2$, and it will be the optimal solution for ESR at the BS. For the case of similar channel towards the BS ($k_1 = 1$ and $k_2 = 1$), point **A** is selected as the operating point.

### *C. Cooperative Region*

The optimal operating point of each strategy was derived for given channel conditions ($k_1$ and $k_2$). The location of the cooperating partner who requires minimal energy to achieve a given BEP in a cell is unknown. This can be solved by drawing contours of the relay's location for a given energy gain over the NCD system. The contour is called the cooperative region

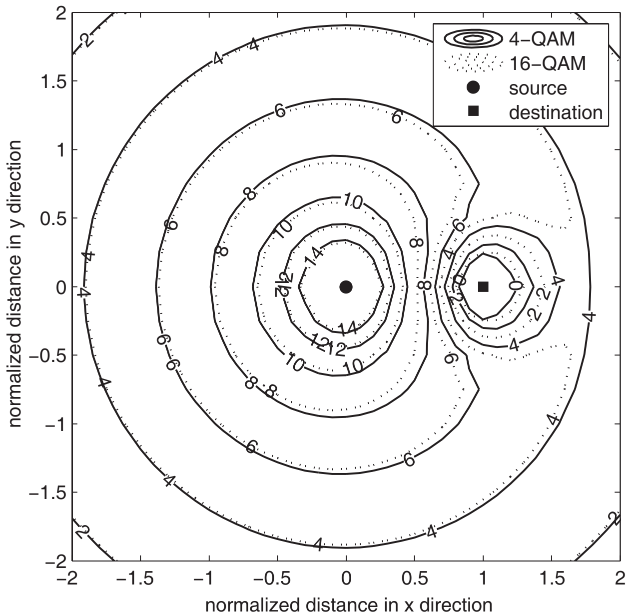


Fig. 8. Cooperative regions of the CD system with OPA mechanism for various $G_{cd}$ of the sender ($e_1 = e_2 = 1 \times 10^{-3}$).

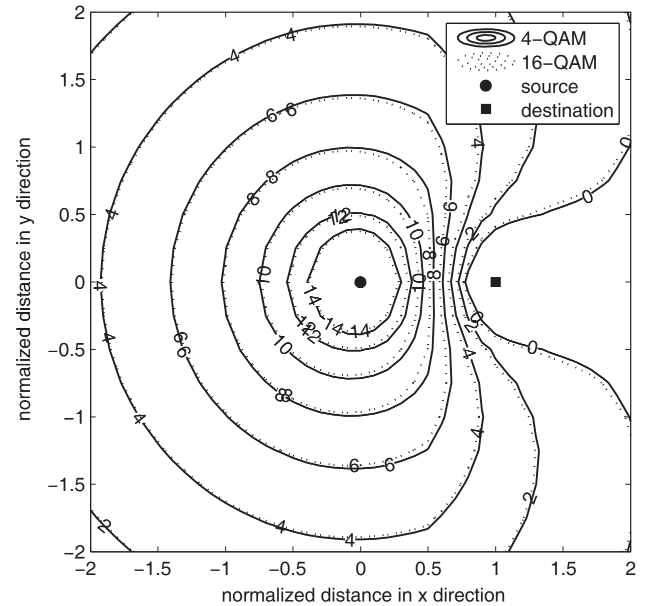


Fig. 9. Cooperative regions of the CD system with EPT for various $G_{cd}$ of the sender($e_1 = e_2 = 1 \times 10^{-3}$).

which should be known to all users as well as to the BS. In this subsection, we derive the cooperative region of the proposed CD system using the power allocation strategy that yields a specified SNR gain over a non-cooperative scheme (direct transmission scheme).

The path losses are dependent on the distance between the transmitter and the receiver as well as on the propagation environment. There are many path loss models reported in the literature and several of these are given in [18]. We consider the log-distance path loss model:

$$\sigma_{i,j}^2 \propto d_{i,j}^{-\alpha}, \tag{21}$$

where $d_{i,j}$ is the distance between $i$ and $j$ and $\alpha$ is the path loss exponent. Without loss of generality, we consider *user 1* located at the reference point, which is given by $\sigma_{1,b}^2 = 1$, and $d_{1,b} = 1$,

$$\begin{aligned} k_1 &= (d_{1,2})^{-\alpha} \\ k_2 &= \left(\frac{d_{2,b}}{d_{1,2}}\right)^{-\alpha} \end{aligned} \tag{22}$$

For a given BEP, define the cooperative SNR gain, $G_{cd}$, as

$$G_{cd} = \frac{E_{bnc}}{E_{b1}} \tag{23}$$

where $E_{bnc}$ is the bit energy needed for the non-cooperative diversity scheme to achieve the same bit error probability as the cooperative diversity scheme.

By substituting (22) and (23) into (16) and (17), the contour of cooperative region can be drawn for given $G_{cd}$ and power allocation strategy.

To plot the cooperative region, we fix the location of the source and destination at a unit distance apart. Furthermore, the Cartesian coordinate of the source, destination and the partner are represented by (0,0), (1,0) and ($x$, $y$), respectively. Therefore, to achieve the given $G_{cd}$, the contour of the partner is bounded by

$$\begin{aligned} d_{2,b} &= \sqrt{(x-1)^2 + y^2} \\ d_{1,2} &= \sqrt{x^2 + y^2}. \end{aligned} \tag{24}$$

The contour is a function of $x$ and $y$ with $e_1$, $e_2$, $G_{cd}$ and $\alpha$ as parameters (for a given power allocation strategy). We consider $e_1 = e_2 = 10^{-3}$ and $\alpha = 3$ in our numerical evaluation.

### D. Numerical Results

In Figs. 8 and 9, cooperative regions are given for 4-QAM and 16-QAM with the OPA strategy and EPT strategy, respectively. In both figures, the cooperative regions of 4-QAM and 16-QAM CD systems almost coincide. This is due to the fact that 4-QAM and 16-QAM CD schemes are compared with the respective NCD schemes for the calculation of energy gain (Eq. (35)). The energy gained by cooperation is almost equal at a particular location of the relay. The contours are symmetric along $y = 0$. The cooperative region shrinks when $G_{cd}$ increases. Both the OPA and EPT strategies can provide a $G_{cd}$ of about 14 dB independent of the signal constellation size of the QAM modulation used, which corresponds to the region close to the source. Further, $G_{cd}$ decreases when the partner approaches the destination. When the x coordinate of the partner is less than 0.5, EPT performs as well as OPA. This is due to the fact that the optimal operating point of the EPT is very close to the optimal operating point of the OPA. On the other hand, the cooperative regions for the EPT shrink faster than that for the OPA when the x coordinate of the partner is greater than 0.5. In contrast to OPA, EPT does not provide any positive gain when the partner is located to the right side of the destination. In such a situation, cooperation is not beneficial.

In Fig. 10, cooperative regions are given for 4-QAM and 16-QAM for the ESR power allocation strategy. The cooperative

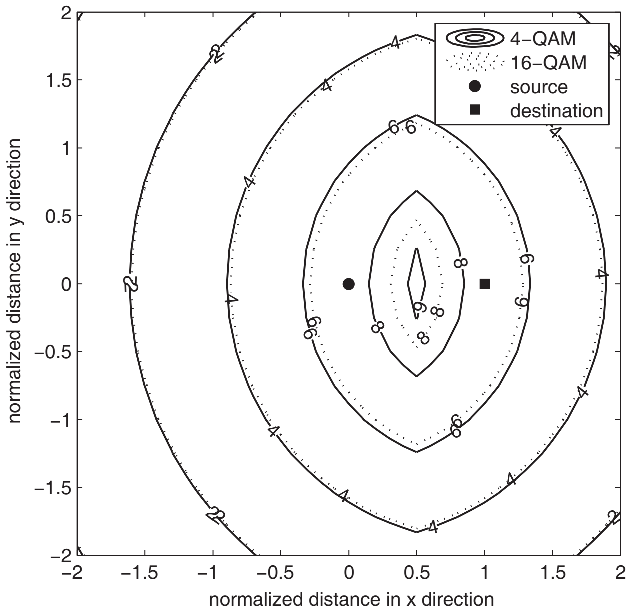


Fig. 10. Cooperative regions of the CD system with ESR at BS for various $G_{cd}$ of the sender ($e_1 = e_2 = 1 \times 10^{-3}$).

regions for 16-QAM are similar to those of 4-QAM at lower $G_{cd}$ regions and deviate only a little bit in the high $G_{cd}$ regions. The cooperative regions of the ESR strategy are significantly different from those of the OPA strategy. The contours of the cooperative regions are symmetric along $y = 0$ and $x = 0.5$. Furthermore, it can be observed that if the relay is located at (0.5,0), the ESR power allocation strategy provides the maximum cooperative diversity gain which is approximately 9dB, far less than those of the OPA and the EPT strategies. This is due to the fact that the unequal BEP of the cooperating users limits the maximum available cooperative diversity gain. This means that the optimal operating point of the ESR is far from that of the OPA.

On the other hand, the NCD 2-PAM system could maintain the received SNR at the BS around 22dB to achieve $P_b = 10^{-3}$ (Fig. 2). But, the maximum achievable energy gain ($G_{CD}$) by cooperation is around 14dB. So, the received SNR by cooperation at the BS is around 8dB. The inter-user SNR is higher than 8dB since they are close to each other. On the other hand, the BER performance of the high SNR approximation only deviates from the exact curve when SNR is below 5dB. Therefore, our high SNR approximation is valid for the power allocation and cooperative region studied.

However, shadowing does affect the shape of the contours of the cooperative regions. That is, the contours of the cooperative regions are affected by the degree of shadowing in each cell; this aspect will be considered in our future work.

## V. Conclusions

An adaptive *regenerate and forward* CD system using quadrature signaling to send M-QAM signals, with suitable power allocation, has been proposed and analyzed. The bit error performance of the scheme has been evaluated analytically and via simulation for 4-QAM/QPSK and 16-QAM modulations. Numerical results show that the derived analytical upper bound is quite tight. The bit error performance of the proposed scheme can be improved when the inter-user channel signal strength increases and the proposed scheme can achieve a maximum diversity order of two. In addition, the cooperation is beneficial not only for a user far away from the BS but also for a user near the BS. In contrast to a fixed CD system, the proposed adaptive CD system performs better than an NCD system even when the inter-user channel is worse than the user to BS channel.

To optimize the power utilization by provisioning the guaranteed QoS, we have formulated a power allocation framework. First, we study optimal power allocation and show that it can achieve maximal cooperative diversity gain. Second, for sensor networks, an equal power transmission strategy is analyzed and the corresponding cooperative regions are derived. Finally, an equal SNR reception power allocation strategy is introduced and its cooperative regions are analyzed. The region that defines the relative location of the partners to achieve cooperative diversity gain are presented for each power allocation strategy. The cooperative regions provide information concerning the location of the partner to achieve a given cooperative diversity gain. The study presented in this paper can facilitate the development of partner selection using the matching algorithms described in [19].

## Appendix

### Derivation of $P_{FEP}$ and $P_{MRC}$ and $P_{NH}$

### *A. Average Inter-user frame error probability ($P_{FEP}$)*

Consider the pairwise error probability (PEP) of a frame transmitted by the sender and received by the partner where each frame consists of $N$ symbols. Pairwise error occurs if there is symbol error in a frame. Using the approach proposed in [17], the PEP between the transmitted frame $X$ and the received frame $\hat{X}$, conditional on the channel fading coefficient, $H = [h_{1,2}(1)h_{1,2}(2)....h_{1,2}(N)]^T$, of I-PAM transmission is given by

$$P(X \longrightarrow \hat{X}|H) = \frac{1}{I\log_2 I}\sum_{m=1}^{\log_2 I}\sum_{i=0}^{(1-2^{-m})I-1}\lambda(I,m,i)\times Q\left((2i+1)\sqrt{\frac{3\log_2 I.E_{b1}}{4(I^2-1)N_0}} \times\sqrt{\sum_{n=1}^{N}|h_{1,2}(n)\left(x(n)|_m - \hat{x}(n)|_m\right)|^2}\right) \tag{25}$$

where $x(n)|_m$ and $\hat{x}(n)|_m$ are respectively the transmitted and received $m$th bit of the $n$th I-PAM modulated symbols of a frame. The PEP can be derived by averaging the conditional PEP over $h_{1,2}$ [15]:

$$P(X \longrightarrow \hat{X}) = \frac{1}{I\log_2 I}\sum_{m=1}^{\log_2 I}\sum_{i=0}^{(1-2^{-m})I-1}\lambda(I,m,i) \times\left(1-\sqrt{\frac{3\log_2 I.(2i+1)^2N_{e|m}\bar{\gamma}_{1,2}}{2(I^2-1)+3\log_2 I.(2i+1)^2N_{e|m}\bar{\gamma}_{1,2}}}\right) \tag{26}$$

where $N_{e|m}$ is the number of symbol errors in a frame due to error in the $m$th bit of the symbol. Therefore, the frame error probability, $P_{FEP}$, can be written as

$$P_{FEP} \leq \sum_{X} P(X) \sum_{X \neq \hat{X}} P(X \longrightarrow \hat{X}). \quad (27)$$

Letting $I = 2$ in (4), $P_{FEP}$ of a 2-PAM system reduces to ( [20],(5))

$$P_{FEP} \leq \sum_{N_{e|m}=1}^{N} \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}_{1,2}N_{e|m}}{2+\bar{\gamma}_{1,2}N_{e|m}}}\right). \quad (28)$$

Similarly, the inter-user FEP for 4-PAM systems can be written as

$$\begin{aligned} P_{FEP} \leq \sum_{N_{e|m}=1}^{N} \frac{1}{8}\Bigg(& 4 - 3\sqrt{\frac{\bar{\gamma}_{1,2}N_{e|m}}{5+\bar{\gamma}_{1,2}N_{e|m}}} \\ & - 2\sqrt{\frac{9\bar{\gamma}_{1,2}N_{e|m}}{5+9\bar{\gamma}_{1,2}N_{e|m}}} + \sqrt{\frac{25\bar{\gamma}_{1,2}N_{e|m}}{5+25\bar{\gamma}_{1,2}N_{e|m}}}\Bigg). \end{aligned} \quad (29)$$

### B. Average BEP of the CD system when the partner forwards the regenerated signal

In this subsection, we derive the BEP, $P_{MRC}$, when the relay forwards the information. First, we derive the analytical expression for the probability of error, $P_m$, for bit $m$, $m = 1, 2, ..., \log_2 I$ for I-PAM when the partner forwards the signal. The conditional probability of error for bit $m$, $P_m(h_{1,B}, h_{2,B})$, can be written using the Log-likelihood ratio (LLR) method of [16]:

$$\begin{aligned} P_m(h_{1,B}, h_{2,B}) = & \sum_{i=0}^{(1-2^{-m})I-1} \frac{\lambda(I,m,i)}{I} \times \\ & Q\left((2i+1)\sqrt{\frac{3\log_2 I}{4(I^2-1)N_0}(E_{b1}|h_{1,B}|^2 + E_{b2}|h_{2,B}|^2)}\right). \end{aligned} \quad (30)$$

So, the conditional bit error probability of the CD system, which is equivalent to maximal ratio combining for an I-PAM system, can be written as

$$\begin{aligned} P_{MRC}(h_{1,B}, h_{2,B}) = & \sum_{m=1}^{\log_2 I} \sum_{i=0}^{(1-2^{-m})I-1} \frac{\lambda(I,m,i)}{I\log_2 I} \times \\ & Q\left((2i+1)\sqrt{\frac{3\log_2 I}{4(I^2-1)N_0}(E_{b1}|h_{1,B}|^2 + E_{b2}|h_{2,B}|^2)}\right). \end{aligned} \quad (31)$$

Define $\bar{\gamma}_1 = \sigma_{1,B}^2 \frac{E_{b1}}{N_0}$ and $\bar{\gamma}_2 = \sigma_{2,B}^2 \frac{E_{b2}}{N_0}$, where $\sigma_{1,B}^2 = \mathrm{E}\{|h_{1,B}|^2\}$ and $\sigma_{2,B}^2 = \mathrm{E}\{|h_{2,B}|^2\}$. Based on the signal-to-noise ratios $\bar{\gamma}_1$ and $\bar{\gamma}_2$, we can derive the unconditional probability of error $P_{MRC}$.

*1) Similar Uplink channels:* Assume that $\mathrm{E}\{|h_{1,B}|^2\}E_{b1}$ and $\mathrm{E}\{|h_{2,B}|^2\}E_{b2}$ are equal. The $P_{MRC}$ of (31) is given in (5). For a 2-PAM system, the $P_{MRC}$ can be written as

$$\begin{aligned} P_{MRC} &= (2 + \mu_1(2,0))\left[\frac{1}{2}(1 - \mu_1(2,0))\right]^2 \\ &= \left(2 + \sqrt{\frac{\bar{\gamma}_1}{2+\bar{\gamma}_1}}\right)\left[\frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}_1}{2+\bar{\gamma}_1}}\right)\right]^2. \end{aligned} \quad (32)$$

which is equivalent to ( [20], (8)). Similarly, for a 4-PAM system, $P_{MRC}$ can be obtained as

$$\begin{aligned} P_{MRC} = \frac{1}{4}\Bigg\{ & \frac{3}{4}(1 - \mu_1(4,0))^2(2 + \mu_1(4,0)) \\ & + \frac{1}{2}(1 - \mu_1(4,1))^2(2 + \mu_1(4,1)) \\ & - \frac{1}{4}(1 - \mu_1(4,2))^2(2 + \mu_1(4,2))\Bigg\}. \end{aligned} \quad (33)$$

*2) Dissimilar Uplink channels:* Assume that $\mathrm{E}\{|h_{1,B}|^2\}E_{b1}$ is not equal to $\mathrm{E}\{|h_{2,B}|^2\}E_{b2}$. The unconditional error probability of (31) is given in (6). By substituting I=2 for a 2-PAM system, the $P_{MRC}$ can be written as

$$\begin{aligned} P_{MRC} &= \frac{1}{2}\left[\frac{\bar{\gamma}_1}{\bar{\gamma}_1 - \bar{\gamma}_2}(1 - \mu_1(2,0)) + \frac{\bar{\gamma}_2}{\bar{\gamma}_2 - \bar{\gamma}_1}(1 - \mu_2(2,0))\right] \\ &= \frac{1}{2}\Bigg[\frac{\bar{\gamma}_1}{\bar{\gamma}_1 - \bar{\gamma}_2}\left(1 - \sqrt{\frac{\bar{\gamma}_1}{2+\bar{\gamma}_1}}\right) \\ &\quad + \frac{\bar{\gamma}_2}{\bar{\gamma}_2 - \bar{\gamma}_1}\left(1 - \sqrt{\frac{\bar{\gamma}_2}{2+\bar{\gamma}_2}}\right)\Bigg] \end{aligned} \quad (34)$$

which is equivalent to ( [20], (7)). Similarly for a 4-PAM system, the $P_{MRC}$ can be obtained as

$$\begin{aligned} P_{MRC} = \frac{1}{8}\Bigg\{ & \frac{\bar{\gamma}_1}{\bar{\gamma}_1 - \bar{\gamma}_2}(4 - 3\mu_1(4,0) - 2\mu_1(4,1) + \mu_1(4,2)) \\ & + \frac{\bar{\gamma}_2}{\bar{\gamma}_2 - \bar{\gamma}_1}(4 - 3\mu_2(4,0) - 2\mu_2(4,1) + \mu_2(4,2))\Bigg\}. \end{aligned} \quad (35)$$

### C. Average BEP of cooperation scheme when the partner does not forward the signal

In this subsection, we derive the analytical expression for the probability of error, $P_{NH}$, for I-PAM signaling when the partner does not forward the information. The $P_{NH}$ can be written as in (7).

For the case of 2-PAM and 4-PAM systems, the $P_{NH}$ can be written as

$$P_{NH} = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}_1}{2+\bar{\gamma}_1}}\right) \quad (36)$$

and

$$P_{NH} = \frac{1}{8}(4 - 3\mu_1(4,0) - 2\mu_1(4,1) + \mu_1(4,2)), \quad (37)$$

respectively.

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**Veluppillai Mahinthan** (S'02-GSM'06-M'08) was born in Jaffna, Sri Lanka. He received the B.Sc.Eng. degree in Electrical and Electronic Engineering from University of Peradeniya, Sri Lanka, in 2000, the M.Eng. degree and the Ph.D. degree in electrical and computer engineering from National University of Singapore (NUS), Singapore, in 2003 and University of Waterloo (UW), ON, Canada, in 2007, respectively. From 2000 to June 2001, he was an Instructor in the department of electrical and electronic engineering at the University of Peradeniya. From July 2001 to August 2003, he was a Research Scholar at the Institute for Infocom Research ($I^2R$), Singapore. He was a Research Assistant at the Centre for Wireless Communication, University of Waterloo from September 2003 to September 2007. Since October 2007, he has been a Postdoctoral Fellow and Adjunct Lecturer in the Department of Electrical and Computer Engineering at the University of Waterloo, ON, Canada. His current research interests are adhered to wireless communications including cooperative diversity, space-time coding, implementation aspects and performance analysis of cross layer communications, and channel estimation and signal processing for communications.

**Jon W. Mark** (M'62-SM'80-F'88-LF'03) received the B.A.Sc. degree from the University of Toronto in 1962, and the M.Eng. and Ph.D. degrees from McMaster University in 1968 and 1970, respectively, all in electrical engineering. From 1962 to 1970, he was an engineer and then a senior engineer at Canadian Westinghouse Co. Ltd., Hamilton, Ontario, Canada. In September 1970 he joined the Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, Ontario, where he is currently a Distinguished Professor Emeritus. He served as the Department Chairman during the period July 1984-June 1990. In 1996 he established the Centre for Wireless Communications (CWC) at the University of Waterloo and is currently serving as its founding Director. Dr. Mark had been on sabbatical leave at the following places: IBM Thomas J. Watson Research Center, Yorktown Heights, NY, as a Visiting Research Scientist (1976-77); AT&T Bell Laboratories, Murray Hill, NJ, as a Resident Consultant (1982-83): Laboratoire MASI, Universit Pierre et Marie Curie, Paris France, as an Invited Professor (1990-91); and Department of Electrical Engineering, National University of Singapore, as a Visiting Professor (1994-95). He has previously worked in the areas of adaptive equalization, image and video coding, spread spectrum communications, computer communication networks, ATM switch design and traffic management. His current research interests are in broadband wireless communications, resource and mobility management, and cross domain interworking. He is a co-author of the text entitled *Wireless Communications and Networking* (Prentice-Hall, 2003). A Life Fellow of IEEE, Dr. Mark is the recipient of the 2000 Canadian Award for Telecommunications Research and the 2000 Award of Merit of the Education Foundation of the Federation of Chinese Canadian Professionals. He was an editor of IEEE TRANSACTIONS ON COMMUNICATIONS (1983-1990), a member of the Inter-Society Steering Committee of the IEEE/ACM TRANSACTIONS ON NETWORKING (1992-2003), a member of the IEEE Communications Society Awards Committee (1995-1998), an editor of WIRELESS NETWORKS (1993-2004), and an associate editor of TELECOMMUNICATION SYSTEMS (1994-2004).

**Xuemin (Sherman) Shen** Xuemin (Sherman) Shen (M'97-SM'02-F'09) received the B.Sc.(1982) degree from Dalian Maritime University (China) and the M.Sc. (1987) and Ph.D. degrees (1990) from Rutgers University, New Jersey (USA), all in electrical engineering. He is a Professor and the Associate Chair for Graduate Studies, Department of Electrical and Computer Engineering, University of Waterloo, Canada. His research focuses on mobility and resource management in interconnected wireless/wired networks, UWB wireless communications systems, wireless security, and ad hoc and sensor networks. He is a co-author of three books, and has published more than 300 papers and book chapters in wireless communications and networks, control and filtering. Dr. Shen serves as the Technical Program Committee Chair for IEEE Globecom'07, General Co-Chair for Chinacom'07 and QShine'06, the Founding Chair for IEEE Communications Society Technical Committee on P2P Communications and Networking. He also serves as a Founding Area Editor for IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS; Editor-in-Chief for PEER-TO-PEER NETWORKING AND APPLICATION; Associate Editor for IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY; KICS/IEEE JOURNAL OF COMMUNICATIONS AND NETWORKS, COMPUTER NETWORKS; ACM/WIRELESS NETWORKS; and WIRELESS COMMUNICATIONS AND MOBILE COMPUTING (Wiley), etc. He has also served as Guest Editor for IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS, IEEE WIRELESS COMMUNICATIONS, and IEEE COMMUNICATIONS MAGAZINE. Dr. Shen received the Excellent Graduate Supervision Award in 2006, and the Outstanding Performance Award in 2004 from the University of Waterloo, the Premier's Research Excellence Award (PREA) in 2003 from the Province of Ontario, Canada, and the Distinguished Performance Award in 2002 from the Faculty of Engineering, University of Waterloo. Dr. Shen is a registered Professional Engineer of Ontario, Canada.