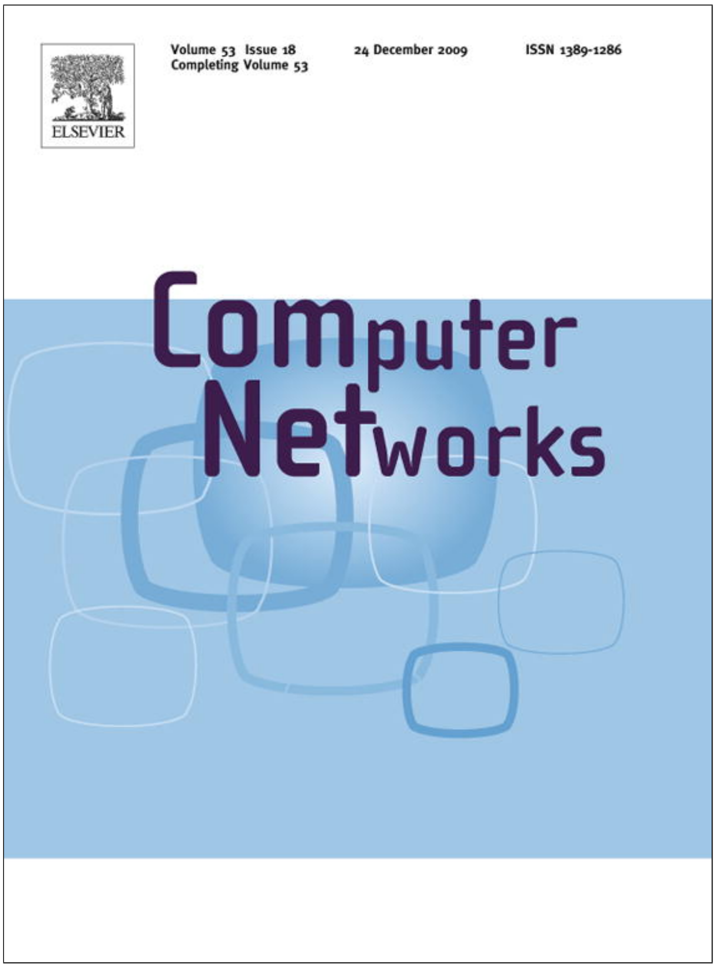
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Optimal flow control for utility-lifetime tradeoff in wireless sensor networks ☆

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ABSTRACT

In this paper, we study the utility-lifetime tradeoff in wireless sensor networks (WSNs) by optimal flow control. We consider the flow control in a more practical way by taking into account link congestion and energy efficiency in our network model, and formulate it as a constrained multi-objective optimization problem. Because of the variable coupling in the objective function, auxiliary variables are introduced to decouple it. We introduce the concept of inconsistent coordination price to balance the energy consumption of the sensor nodes. Based on the congestion price and inconsistent coordination prices, a distributed algorithm using gradient projection is proposed to solve the optimization problem. The convergence of the algorithm is also proved. Numerical results show the convergence of our algorithm, the tradeoff of utility and lifetime, as well as the necessity of considering link congestion in WSNs.

1. Introduction

Flow control is to deal with designing distributed algorithms to regulate traffic rate from the users in order to maximize the total network utility. A general utility function is defined to characterize the network performance under the Network Utility Maximization (NUM) framework [2]. The objectives of the flow control is to avoid the congestion of links, and to guarantee fairness among users, since only maximizing the total utility of a network may result in unfairness, starving some users all the time. In this paper, we confine our interest to Wireless Sensor Networks (WSNs), which are usually composed of many battery-driven sensors. A WSN can only operate in a finite time interval, making energy management one of the fundamental challenges. When addressing the flow control problem in WSNs, it is necessary to simultaneously take into account the energy constraint of the sensor nodes. This makes the flow control approach for WSNs different from those for traditional wired networks. For example, traditional flow control mainly focuses on two components [3,4]: a source algorithm that dynamically adjusts the node rate in response to the congestion price defined by some congestion metric and a link algorithm that updates the link price, implicitly or explicitly. Transmission Control Protocol (TCP) is mainly used for source rate control and Active Queue Management (AQM) is adopted to deal with link update [5]. When energy management is required, in addition to link prices, new measures need to be introduced to coordinate the energy consumption among the sensor nodes. Thus traditional flow control mechanisms may no longer be effective and we should resort to new approaches to effectively regulate the rates of the sensors and prolong the network lifetime.

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When energy management is also considered with the flow control, the problem becomes more complex. If the rates of the network are large, the total utility achieved is correspondingly large. However, the sensor network may die quickly. There is a tradeoff between total utility maximization and network lifetime maximization. Therefore, each sensor node needs to regulate its rate according to not only the congestion condition of the links but also the energy consumption of other sensor nodes.

In this paper, we study the utility-lifetime tradeoff in WSNs with link capacity constraint and energy constraint by optimal flow control and formulate it as a constrained multi-objective optimization problem under the NUM framework. The comprehensive formulation of the utility-lifetime tradeoff in the transport layer, is particularly important with the development and integration of video technology WSNs, such as multimedia application in sensor networks [6,7]. Most of existing works under the NUM framework assume each source has an independent utility function, while the adopted objective function is generally coupled and can not be separable directly, thus making a fully distributed algorithm extremely difficult. To the best of our knowledge, this is the first study that provides a practical solution to decouple the coupling in the objective function by introducing auxiliary variables and present a systematic distributed approach based on inconsistent coordination price for the formulated constrained multi-objective optimization problem. We first introduce two system parameters: scaled parameter ω, which is used to map the two performance metrics (utility and lifetime) into the same order of magnitude, and weight parameter γ, which is used to combine the two objective functions into a single one. We show that the combined objective function is strictly concave and the global optimal solution exists. γ is also called a tradeoff parameter and we can change its value to achieve different tradeoffs between network lifetime and total utility. We adopt Lagrange duality method to decompose the problem. We can interpret the Lagrange multipliers, $\boldsymbol{\lambda}$, as link congestion price. To coordinate the energy consumption among the sensor nodes, we also introduce Lagrange multipliers $\boldsymbol{\mu}$, which can be interpreted as the inconsistent coordination price. Based on the link congestion price and inconsistent coordination price, we propose a distributed algorithm to obtain the optimal solution by using gradient projection. We also prove the convergence of the algorithm by using the knowledge of convex optimization. Numerical investigations are conducted to demonstrate the following three aspects: (1) the convergence of our distributed algorithm; (2) the need for considering link congestion, which is left aside in [8]; and (3) the affects of weight parameter γ on total utility and network lifetime.

The remainder of the paper is organized as follows. In Section 2, we discuss the related work about flow control and energy management. We establish our mathematical model in Section 3, and propose a distributed algorithm to solve the problem in Section 4, followed by the proof of convergence in Section 5. Numerical results are given to verify the analysis in Section 6. Finally, we conclude our paper in Section 7.

2. Related work

Flow control is a fundamental problem in the traditional wired network and has been extensively studied [9,10,2]. Two important aspects in flow control are congestion avoidance and fairness [11], apart from the stability of both homogeneous and heterogeneous flow control with/without feedback delay [12,13]. Mo et al. demonstrate the existence of fair end-to-end congestion control protocols for packet-switched networks [14]. In [15,16], Kelly proposes a novel way to solve the problem and converts the flow control problem with fairness requirement into a convex optimization problem. In this way, the design of flow control algorithms can be systematically investigated. In [2], Low shows that this optimization problem for the single-path case is strictly concave under the assumption that the utility function is strictly concave, thus a global optimal solution exists. Gradient projection or subgradient projection are often adopted to design a distributed algorithm for obtaining the optimal solution. The congestion avoidance functionality of TCP has been recently reverse-engineered to implicitly solve the basic Network Utilization Maximum (NUM) problem [17]. Due to its advantages, the methods for NUM dominate solutions to the flow control problem. Current work on flow control can be differentiated from each other in (1) the types of networks, e.g., single or multi-path networks [2,18]; (2) the choices of utility functions, e.g., fairness utility function or other metrics; and (3) the approaches to solving the problem, e.g., primal decomposition or dual decomposition by gradient projection or penalty function-based method [19,20]. However, most of the research on this problem mentioned above does not take the energy constraint into account, which is one of the fundamental challenges in WSNs.

As mentioned above, network lifetime is a critical performance metric in WSNs and should be involved when a rate allocation scheme is designed. In [19], Srinivasan et al. consider the optimal rate allocation with guaranteed lifetime in multi-path networks. They incorporate the energy dissipation as a constraint. In [8], Zhu et al. study the tradeoff between network lifetime and fair rate allocation in multi-path sensor networks. However, they do not consider the link congestion and formulate it as an unconstrained multi-objective optimization problem. In [21], the link capacity constraint is added in the cross-layer formulation, and the rate allocation and energy conservation problem is solved directly using gradient projection method. But they do not give detailed information about how to distributively implement the algorithm to solve the rate allocation and energy conservation problem in each layer. In [22], Nama et al. formulate a similar cross-layer model. They use penalty functions to regulate the rates to conserve energy and do not provide the distributed solution in each layer. To the best of our knowledge, most reported research work have not provided a distributed algorithm in the transport layer by transferring the coupling in the objective function to the coupling of the constraints for the tradeoff between total utility and network lifetime in WSNs, which are the focus of our study.

3. Problem statement

In this section, we model the utility-lifetime tradeoff in WSNs as a constrained multi-objective problem and show that the combined objective function is strictly concave, thus having a unique global optimal solution.

Throughout the paper, we denote sets and the cardinality of sets by capital letters, variables by lowercase letters, vectors by bold lowercase letters and matrices by bold capital letters. For a vector $\boldsymbol{x}$, its ith component is x_i, and its transpose is $\boldsymbol{x}^T$. Let $\|\boldsymbol{x}\|_1 = \sum_i |x_i|$, $\|\boldsymbol{x}\|_2 = \left(\sum_i |x_i|^2\right)^{\frac{1}{2}}$ and $\|\boldsymbol{x}\|_\infty = \max_i |x_i|$ denote the 1-norm, 2-norm and ∞-norm of $\boldsymbol{x}$, respectively. For matrix $\boldsymbol{A}$, denote its (i,j) component by a_{ij}, and its transpose by $\boldsymbol{A}^T$. Let $\|\boldsymbol{A}\|_1$, $\|\boldsymbol{A}\|_2$ and $\|\boldsymbol{A}\|_\infty$ denote the 1-norm, 2-norm and ∞-norm of the corresponding matrix.

We consider a WSN consisting of a set $S = \{1, 2, \ldots, S\}$ of sensor nodes and $N = \{1, 2, \ldots, N\}$ of sink nodes. The sensor nodes can transfer their sensing data to the sink nodes over a set $L = \{1, 2 \ldots, L\}$ of links, each of which has capacity $c_l, l \in L$. The single-path routing is assumed in this paper. Each sensor node is characterized by three parameters $(U_s(\cdot), \underline{m}_s, \bar{m}_s)$, where $U_s : \Re_+ \to \Re$ is a strictly concave utility function related to rate allocation and $\underline{m}_s \geqslant 0$ and $\bar{m}_s < \infty$ are the required minimum and maximum transmission rates, x_s, for each sensor node s, respectively. The notations used in this paper has been summarized in Table 1.

3.1. General flow control model

A general flow control problem is often concerned with how to allocate the rates to users to maximize the total utility of the network. Two important goals in flow control are congestion avoidance and rate fairness. There are several fairness definitions introduced in the literature, such as *max–min* or *proportional* rate fairness [16], *application-oriented* fairness [23] and so on. It is shown by Kelly [15,16] that each data flow issued from sensor node s can be associated with a utility function $U_s(\cdot)$ and achieve different kinds of fairness by maximizing the aggregate utility functions $\sum_s U_s(x_s)$. The sink nodes are responsible for receiving data from sensor nodes and they contribute nothing to the utility of the network. We establish the mathematical model for a general flow control problem in WSNs as follows.

$$\max \quad \sum_{s \in S} U_s(x_s), \tag{1}$$

$$\text{s.t} \begin{cases} \sum\limits_{s \in S(l)} x_s \leqslant c_l, l \in L, \\ \underline{m}_s \leqslant x_s \leqslant \bar{m}_s, \forall s \in S. \end{cases} \tag{2}$$

The feasible set of constraints in Eq. (2) is convex, $\sum_{s \in S(l)} x_s \leqslant c_l$, $l \in L$ is the constraint of link capacity, and the objective function in Eq. (1) is strictly concave in $\boldsymbol{x}$. According to the convex optimization theorem [24], the defined problem has a unique global optimal solution. Lagrange duality method is introduced to decompose the problem into several subproblems, which can be easily solved distributively at each individual sensor node.

3.2. Lifetime model

The energy of a sensor node is mainly used for sensing, processing, transmitting and receiving data. It is widely recognized that the process of transmitting and receiving data dominates the energy consumption [25]. Similar to [26], we only consider the energy consumption for communication. Assume that the sink nodes have enough energy.

The power depletion, p_{sl}^t, at sensor node s for transmitting unit data over link l can be stated as:

$$p_{sl}^t = \rho + \sigma d_{sl}^n, \tag{3}$$

where ρ and σ are constants related to the functionality of the physical layer and the environment factors, d_{sl} is the length of the logic link l, and $n (2 \leqslant n \leqslant 4)$ is the path loss constant. The power consumption, p_{sl}^r, for receiving a unit of data from link l at a sensor node s is generally assumed to be a constant. Then the total power dissipation, p_s, at sensor node s equals

$$p_s = \sum_{l \in L_{in}(s)} \sum_{s' \in S(l)} p_{sl}^r x_{s'} + \sum_{l \in L_{out}(s)} \sum_{s' \in S(l)} p_{sl}^t x_{s'}. \tag{4}$$

The sensor node s is often powered with energy-constrained batteries and has a limited initial energy, e_s. Its lifetime, T_s, is then described as

$$T_s = \frac{e_s}{p_s}. \tag{5}$$

Generally, the network lifetime is defined as the lifetime of the sensor node whose energy is first drained [27], and maximizing network lifetime is equivalent to maximize the minimum lifetime of the sensor nodes in the network. let T denote the network lifetime: $T = \min_{s \in S} T_s$. The lifetime problem can be formulated as

$$\max \quad T \tag{6}$$

It is very difficult to solve Eq. (6) in a distributed manner because each sensor node needs to communicate with all other sensor nodes to know their energy consumption.

Table 1
Notation definitions.

Symbol	Definition
L	The set of links
$L(s)$	The subset of links L used by sensor node s
c_l	The capacity of link l
$S(l)$	$S(l) = \{s \in S \mid l \in L(s)\}$ is the set of nodes using link l
$L_{in}(s)$	The set of incoming links of sensor node s
$L_{out}(s)$	The outgoing link set of sensor node s
$S_{in}(s)$	The set of sensor nodes that use sensor node s as a relay
$S_t(s)$	$S_t(s) = \{s' \in S \mid s \in S_{in}(s')\}$ is the set of sensor nodes that sensor node s uses as relays
l_s	The outgoing link that sensor node s uses for transmitting its own data at rate x_s
U_s	The utility function at sensor node s
$\underline{m}_s$	The required minimum transmission rates for each sensor node s
$\overline{m}_s$	The required maximum transmission rates for each sensor node s
p_s	The total power dissipation at sensor node s
T_s	The lifetime of sensor node s

There are only partial distributed algorithms for this primal problem [26]. A viable approach is to substitute the objective function Eq. (6) with other separable objective functions with an approximation guarantee.

An effective approximation approach to solve Eq. (6) is proposed by in [14]. It is shown that maximizing the aggregate utility can achieve *max–min* rate allocation for each source when the utility functions are given by $V^{\beta}(\cdot)$ and $\beta \to \infty$, where $V^{\beta}(\cdot)$ is defined as follows:

$$V^{\beta}(x) = \begin{cases} \log x, & \beta = 1, \\ \frac{1}{1-\beta} x^{1-\beta}, & \beta > 1. \end{cases} \tag{7}$$

Notice that in Eq. (6) we have to maximize the minimum lifetime of the sensor nodes, we take the lifetime of each sensor node as its profit, which is similar to the *max–min* rate allocation problem. If we introduce new utility functions, $V_s^{\beta}(\cdot)$, for each sensor node and maximize the aggregate utility, the lifetime model of Eq. (6) is approximated. The problem is then transformed into

$$\max \sum_{s \in S} V_s^{\beta}(T_s), \tag{8}$$

where $V_s^{\beta}(T_s) = \frac{1}{1-\beta} T_s^{1-\beta}$.

To simplify the problem, let $z_s = 1/T_s$, where z_s is defined as the normalized power dissipation of the sensor node s. Then Eq. (8) becomes

$$\max \quad \sum_{s \in S} \frac{1}{1-\beta} z_s^{\beta-1}, \tag{9}$$

$$\text{s.t.} \quad p_s = e_s z_s, \quad s \in S. \tag{10}$$

Eq. (9) is called the lifetime model of the WSN.

3.3. Utility-lifetime tradeoff model

Generally, unattended operation of the WSNs is often desirable or required for area monitoring applications, which makes the sensor nodes cannot be recharged due to the inaccessibility of the area of interest. So there is a need to maximize the network lifetime as long as possible by balancing the energy consumption of each node, meanwhile maximizing the total monitoring information gained.

We consider a more practical model by considering fairness and link congestion into utility-lifetime problem, which differentiates our work from that in [8]. Eq. (1) tries to maximize the whole utility under link capacity constraints and fairness guarantees. While, Eq. (9) tries to prolong the lifetime of the sensor network. One extreme situation is not to let any sensor node transmit data, i.e., $x_s = 0, s \in S$. Under this situation, the WSN is useless and impractical although it may have a longer lifetime. Thus, the utility-lifetime problem is important but the conflicting network performance metrics generate tradeoff. This is a constrained multi-objective optimization problem. To solve the problem, we introduce two system parameters, ω, a scaled constant to transform two objective functions into the same order of magnitude, and γ, a system weight constant to combine two objective functions into a single one. Then, the optimal flow control problem for the utility-lifetime tradeoff can be formulated as follows.

$$\max \quad \sum_{s} \left(\gamma U_s(x_s) - (1-\gamma) \frac{\omega}{\beta - 1} z_s^{\beta-1} \right), \tag{11}$$

$$\text{s.t} \begin{cases} \sum_{s \in S(l)} x_s \leqslant c_l, & l \in L, \\ p_s = e_s z_s, & s \in S, \\ \underline{m}_s \leqslant x_s \leqslant \overline{m}_s, & \forall s \in S, \\ p_s = \sum_{l \in L_{in}(s)} \sum_{s' \in S(l)} p_{sl}^r x_{s'} + \sum_{l \in L_{out}(s)} \sum_{s' \in S(l)} p_{sl}^t x_{s'}. \end{cases} \tag{12}$$

In the above formulation, the two metrics are combined into a single one and existing optimization methods in [24,28] can be applied to solve it. ω is a mapping parameter which can transform the values of the two objective functions into the same level. This is important because if the two metrics are not at the same level, it is hard to get a right and clear understanding about the tradeoff problem. γ can be interpreted as a tradeoff parameter and used to evaluate the importance of the two performance metrics. Both ω and γ are application-dependent. For example, when preferring total utility to the network lifetime, we can have a large γ and vice versa. In Eqs. (11) and (12), z_s can be expressed by $\boldsymbol{x}$, thus they are dummy variables. Since the constraints for $\boldsymbol{x}$ are linear, the feasible set of Eq. (12) is convex. Under the assumption that $U_s(\cdot), V_s^{\beta}(\cdot), s \in S$ are strictly concave, the objective function Eq. (11) is strictly concave in $\boldsymbol{x}$. Thus, there exists a unique global optimal solution for the problem Eq. (11) [28]. In the next section we will utilize this property to design a distributed algorithm.

4. Distributed algorithm

In this section, we will adopt the gradient projection to design a distributed algorithm to solve the problem Eq. (11) and prove its convergence.

Notice that the objective function Eq. (11) is coupled in $\boldsymbol{x}$. To make the problem solvable in a distributed manner, we solve the coupled objective function by introducing auxiliary variables and additional equality constraints. We can then transform the coupling in the function to the coupling in the constraints, which can be decoupled by Lagrangian dual decomposition [20,29]. We introduce auxiliary variables, $\{y_{ss'}\}_{s' \in S_{in}(s)}, s \in S$, and transform Eq. (11) into the problem **P**.

$$\mathbf{P} : \max \quad \sum_{s} \widehat{U}_s(x_s, \{y_{ss'}\}_{s' \in S_{in}(s)}), \tag{13}$$

$$\text{s.t} \begin{cases} \sum_{s \in S(l)} x_s \leqslant c_l, & l \in L, \\ \sum_{l \in L_{in}(s)} \sum_{s' \in S(l)} (p_{sl}^r + p_{sl}^t) y_{ss'} + x_s p_{sl_s}^t = e_s z_s, & s \in S, \\ y_{ss'} = x_{s'}, & s' \in S_{in}(s), \\ \underline{m}_s \leqslant x_s \leqslant \bar{m}_s, \\ y_{ss'} \geqslant 0, & s \in S, \quad s' \in S_{in}(s), \end{cases} \tag{14}$$

where $\widehat{U}_s(x_s, \{y_{ss'}\}_{s' \in S_{in}(s)}) = \gamma U_s(x_s) - (1-\gamma) \frac{\omega}{\beta - 1} z_s^{\beta-1}$.

Define the Lagrangian as [28, Section 6]

$$L(\boldsymbol{x},\boldsymbol{y},\boldsymbol{\lambda},\boldsymbol{\mu}) = \sum_{s\in S}\widehat{U}_s(x_s,\{y_{ss'}\}_{s'\in S_{in}(s)}) - \sum_{l\in L}\lambda_l\left(\sum_{s\in S(l)}x_s - c_l\right) + \sum_{s'\in S_{in}(s),s\in S} u_{ss'}(x_{s'} - y_{ss'})$$
$$= \sum_{s\in S}\left\{\widehat{U}_s(x_s,\{y_{ss'}\}_{s'\in S_{in}(s)}) - x_s\sum_{l\in L(s)}\lambda_l + x_s\sum_{s'\in S_t(s)}u_{s's} - \sum_{s'\in S_{in}(s)}u_{ss'}y_{ss'}\right\} + \sum_{l\in L}\lambda_l c_l. \quad (15)$$

With this formulation, the first term is separable in each sensor node s. Let $B_s(\boldsymbol{\lambda},\boldsymbol{\mu})$ be the maximum of the following optimization problem $\mathbf{DP}_s$:

$$\max \quad \widehat{U}_s(x_s,\{y_{ss'}\}_{s'\in S_{in}(s)}) - x_s(\lambda^s - \mu^s) - \sum_{s'\in S_{in}(s)}u_{ss'}y_{ss'}, \quad (16)$$

$$\text{s.t.}\begin{cases}\sum_{l\in L_{in}(s)}\sum_{s'\in S(l)}(p^r_{sl}+p^t_{sl})y_{ss'} + x_s p^t_{sl_s} = e_s z_s,\\ \underline{m}_s \leqslant x_s \leqslant \bar{m}_s,\\ y_{ss'} \geqslant 0, s'\in S_{in}(s).\end{cases} \quad (17)$$

where $\lambda^s = \sum_{l\in L(s)}\lambda_l, \mu^s = \sum_{s'\in S_t(s)}\mu_{s's}$. Then the objective function of the dual problem is [28, Section 6]

$$\mathbf{DP}: D(\boldsymbol{\lambda},\boldsymbol{\mu}) = \sum_{s\in S}B_s(\boldsymbol{\lambda},\boldsymbol{\mu}) + \sum_{l\in L}\lambda_l c_l, \quad (18)$$

and the dual problem is

$$\min_{\boldsymbol{\lambda}\succeq 0,\boldsymbol{\mu}} D(\boldsymbol{\lambda},\boldsymbol{\mu}). \quad (19)$$

The first term of the dual function, $D(\boldsymbol{\lambda},\boldsymbol{\mu})$, is decomposed into S subproblems $\mathbf{DP}_s$. Notice that each sensor node s maintains x_s and $\{y_{ss'}\}_{s'\in S_{in}(s)}$ in its memory which facilitates the implementation of a distributed algorithm. If we interpret λ_l as the price per unit bandwidth at link l, then λ^s is the total price per unit bandwidth for all links paid by sensor node s. Different from the existing methods, we introduce the auxiliary variables $\{y_{ss'}\}_{s'\in S_{in}(s)}$, and the Lagrange multipliers $\{\mu_{ss'}\}_{s'\in S_{in}(s)}$ to coordinate the values of $x_{s'}$ in sensor node s' and $y_{ss'}$ in sensor node s. We can interpret $\mu_{ss'}$ as the inconsistent coordination price between $x_{s'}$ and the auxiliary variable $y_{ss'}$ for sensor node s, and μ^s as the total inconsistent coordination price of all the sensor nodes that the sensor node s uses along its route. Since the problem $\mathbf{P}$ is strictly concave the dual problem $\mathbf{DP}$ is continuously differentiable with derivatives given by [28, Section 6]

$$\begin{cases}\frac{\partial D}{\partial\lambda_l}(\boldsymbol{\lambda},\boldsymbol{\mu}) = c_l - x^l, \quad l\in L,\\ \frac{\partial D}{\partial\mu_{ss'}}(\boldsymbol{\lambda},\boldsymbol{\mu}) = x_{s'} - y_{ss'}, \quad s'\in S_{in}(s), \quad s\in S.\end{cases} \quad (20)$$

where $x^l = \sum_{s\in S(l)}x_s$ is the aggregate rate on link l. We adopt the gradient projection method [28, Section 6,30] to solve the dual problem where the link price and the inconsistent coordination price are updated in the opposite direction to the gradient $\nabla D(\boldsymbol{\lambda},\boldsymbol{\mu})$:

$$\begin{cases}\lambda_l(t+1) = [\lambda_l(t) - \delta(c_l - x^l(t))]^+, \quad l\in L,\\ \mu_{ss'}(t+1) = \mu_{ss'}(t) - \delta(x_{s'}(t) - y_{ss'}(t)), \quad s'\in S_{in}(s), \quad s\in S.\end{cases} \quad (21)$$

where δ is a stepsize, and $[z]^+ = \max\{0,z\}$.

After both the link price and the inconsistent coordination price are updated, each sensor node s collects λ^s, and updates its rate x_s and the auxiliary variables $\{y_{ss'}\}_{s'\in S_{in}(s)}$ by solving the problem $\mathbf{DP}_s$. We summarize the distributed algorithm for utility-lifetime tradeoff (*DAULT*) as follows.

4.1. Algorithm DAULT

Link l's Algorithm for $t = 1,2,\ldots$:

(1) Receives data at rate $x_s(t)$ from each sensor node $s\in S(l)$ that uses link l and computes $x^l(t)$.
(2) Computes a new price.

$$\lambda_l(t+1) = [\lambda_l(t) - \delta(c_l - x^l(t))]^+.$$

(3) Communicates the new link price $\lambda_l(t+1)$ to all sensor nodes $s\in S(l)$, which use the link l.

Sensor node s's Algorithm for time $t = 1,2,\ldots$:

(A) Inconsistent coordination price update:
  (1) Receives from the network $x_{s'}(t), s'\in S_{in}(s)$ of sensor nodes that use sensor node s in its transport route.
  (2) Updates the inconsistent coordination price $\{\mu_{ss'}(t+1)\}_{s'\in S_{in}(s)}$ according to the following equation

$$\mu_{ss'}(t+1) = \mu_{ss'}(t) - \delta(x_{s'}(t) - y_{ss'}(t)).$$

  (3) Communicates its inconsistent coordination price $\{\mu_{ss'}(t)\}, s'\in S_{in}(s)$, to the sensor nodes that use it as a relay node.

(B) Rate and auxiliary variables update:
  (1) Receives from the network the link price $\lambda_l(t)$ of the links $l\in L(s)$ that are used by sensor node s and computes $\lambda^s(t)$.
  (2) Receives from the network the inconsistent coordination price $\mu_{s's}(t)$ of the sensor nodes s' that are used as relays by node s and computes $\mu^s(t)$.
  (3) Updates the internal variables in the sensor node $s, x_s(t+1), \{y_{ss'}(t+1)\}_{s'\in S_{in}(s)}$, by solving the dual problem $\mathbf{DP}_s$ (i.e., Eqs. (16) and (17)) for the given $\lambda^s(t), \mu^s(t), \{\mu_{ss'}(t)\}_{s'\in S_{in}(s)}$.
  (4) Communicates new rate $x_s(t+1)$ to links that sensor node s uses.

The following are remarks on the*Algorithm DAULT* :

- Link prices $\lambda_l, l\in L$, are updated by collecting the corresponding rate information $x^l(t), l\in L$, which can be implemented through active queue management (AQM). The inconsistent coordination prices

$\{\mu_{ss'}\}_{s'\in S_{in}(s)}, s \in S$, are updated in each sensor node s, which can also be easily implemented. The parameters $\lambda_l, l \in L$ are the measures of how the links are congested. The larger the parameters $\lambda_l, l \in L$, the more congested the corresponding link will be. Correspondingly, parameters $\{\mu_{ss'}\}_{s'\in S_{in}(s)}, s \in S$, are used to evaluate the consistency between the rate variables and their auxiliary variables.

- In each sensor node's algorithm, sensor node s has to solve the dual problem $\mathbf{DP}_s$ for the given λ^s, μ^s and $\{\mu_{ss'}\}_{s'\in S_{in}(s)}$, which can be solved in a centralized manner. There are many effective algorithms such as Newton–Raphson method to achieve this. As the process of transmitting and receiving data dominates the energy consumption [25], the energy cost of computation can be neglectable and is excluded from our energy model.
- Each sensor node, s, has to communicate with other nodes to collect the aggregate link price λ^s, which is similar to the approach in [2]. Also, each sensor node has to exchange message packets to collect the aggregate inconsistent coordination price μ^s and the rate of sensor nodes, $x_{s'}, s' \in S_{in}(s)$. A similar mechanism for link price can be exploited to solve this problem. The updates are necessary, because we have to balance the energy consumption among sensor nodes. As shown in *Algorithm DAULT*, each sensor node s only needs to communicate with those sensor nodes lying along its route. So the message exchange in each iteration is tolerable, which is very important for implementation of the algorithm.
- There are two time scales, one for link and inconsistent coordination price updates and the other for deciding the rate of each sensor node by solving $\mathbf{DP}_s$. It is reasonable to assume that the time scale for the rate update is much smaller than that of link and inconsistent coordination price updates, since each sensor node s can solve $\mathbf{DP}_s$ in a centralized manner by using local information.
- We adopt the gradient projection method to distributively solve the optimal flow control problem, which is a strongly convex problem as discussed in Section 3. Hence, according to [30], the *algorithm DAULT* converges geometrically.

Using the concepts of control theory, the close-loop system framework of *Algorithm DAULT* is illustrated in Fig. 1. The matrices $\boldsymbol{A}$ and $\boldsymbol{H}$ in the figure are defined in Section 5.

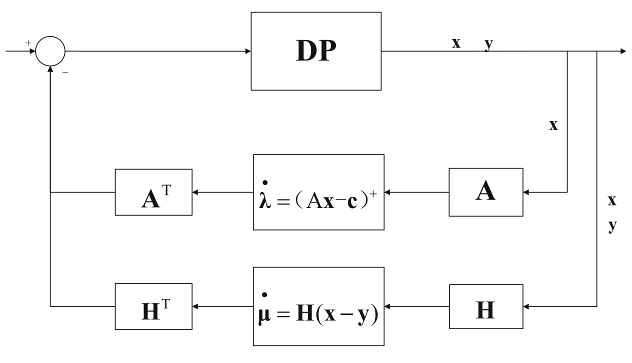

Fig. 1. Close-loop system framework of algorithm DAULT.

From the above discussion, we can conclude that *Algorithm DAULT* can be implemented and the message exchanges for the global information are tolerable, which makes *DAULT* a practical algorithm.

5. Convergence performance of the distributed algorithm

In this section, we will establish the convergence of the distributed algorithm designed in the previous section. Some assumptions are given as follows:

- A1: The utility function $U_s(\cdot), V_s^{\beta}(\cdot), s \in S$, for each sensor node s is strictly concave and twice continuously differentiable. Hence $\widehat{U}_s(\cdot), s \in S$ is strictly concave and twice continuously differentiable.
- A2: If every sensor node s transmits its data at the minimum required rate $\underline{m}_s$, then the aggregate rates on link l are less than its capacity, which makes the feasible constraint set Eq. (14) a nonempty set.
- A3: The curvature of $\widehat{U}_s(\cdot)$ is bounded away from zero in the feasible set $\{(x_s, y_{ss'}), \underline{m}_s \leqslant x_s \leqslant \bar{m}_s, x_{s'} = y_{ss'}, .s' \in S_{in}(s)\}$, i.e., $-\widehat{U}_s''(x_s, \{y_{ss'}\}_{s'\in S_{in}(s)}) \geqslant \frac{1}{\alpha_s}$.

Define $\overline{R} = \max_{s\in S}|L(s)|$ as the maximum number of links that a sensor node uses. Let $\bar{\alpha} = \max_{s\in S}\alpha_s$, the maximum of α_s. Let $\overline{S} = \max_{l\in L}|S(l)|$, the maximum number of sensor nodes that use link l, and $\widehat{S} = \max\{2, \overline{S}\}$. Define $\overline{R}_{in} = \max_{s\in S}|S_{in}(s)|$ as the maximum number of sensor nodes that the sensor node s uses as interim relays. Let $\overline{L} = \overline{R} + \overline{R}_{in}$.

For the convenience of the proof, we first introduce some vectors and matrices. Let $\boldsymbol{p} = (\boldsymbol{\lambda}^T, \boldsymbol{\mu}^T)^T$ be the vector of Lagrangian multipliers, $\boldsymbol{z} = (z_1, z_2, \ldots, z_S)^T$ be the vector of power dissipation and $\boldsymbol{y} = (y_{11}, \ldots, y_{1n_1}, \ldots, y_{s1}, \ldots, y_{sn_2}, \ldots, \ldots, y_{S1}, \ldots, y_{Sn_S})^T$ be the vector of auxiliary variables. Here $y_{s1}, y_{s2}, \ldots, y_{sn_s}$ are the auxiliary variables that the sensor node s maintains in its memory, with the total number of auxiliary variables in the sensor node s being n_s. Let $\boldsymbol{\chi} = (\boldsymbol{x}^T, \boldsymbol{y}^T)^T$. Let $\boldsymbol{d}_i$ be the vector whose ith component is 1 and other components are 0. Define matrix $\boldsymbol{H}_s$ as $n_s \times S$ whose ith row is set to be $\boldsymbol{d}_{s'}$, if y_{si} is an auxiliary variable corresponding to $x_{s'}$. Then define $(n_1 + n_2 + \cdots + n_S) \times S$ matrix $\boldsymbol{H}$ as

$$\boldsymbol{H} = \begin{pmatrix} \boldsymbol{H}_1 \\ \boldsymbol{H}_2 \\ \vdots \\ \boldsymbol{H}_S \end{pmatrix}. \tag{22}$$

Also we define $\boldsymbol{A}$ as $L \times S$ routing matrix, with its elements being

$$a_{ls} = \begin{cases} 1, & s \in S(l), \\ 0, & \text{otherwise}. \end{cases} \tag{23}$$

With the above definition of the vectors and matrices, the constraints given in Eq. (14) of the objective function (13) can be transformed in matrix form as

$$\text{s.t.}\begin{cases} \boldsymbol{Ax} \leqslant \boldsymbol{c}, \\ \boldsymbol{y} = \boldsymbol{Hx}, \\ \underline{\boldsymbol{m}} \leqslant \boldsymbol{Ix} \leqslant \overline{\boldsymbol{m}}, \\ \sum_{l \in L_{in}(s)} \sum_{s' \in S(l)} (p^r_{sl} + p^t_{sl}) y_{ss'} + x_s e^t_{sl_s} = e_s z_s, \quad s \in S, \\ \boldsymbol{x} \succeq 0, \quad \boldsymbol{y} \succeq 0, \end{cases} \tag{24}$$

where $\boldsymbol{I}$ is an identity matrix. Then the objective function of Eq. (15) can be described as

$$L(\boldsymbol{x},\boldsymbol{y},\boldsymbol{p}) = \sum_s \widehat{U}_s(x_s, \{y_{ss'}\}_{s' \in S_{in}(s)}) - \boldsymbol{p}\begin{pmatrix} \boldsymbol{A} & \boldsymbol{0} \\ -\boldsymbol{H} & \boldsymbol{I} \end{pmatrix}\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{pmatrix} + \boldsymbol{p}\begin{pmatrix} \boldsymbol{c} \\ \boldsymbol{0} \end{pmatrix}. \tag{25}$$

In the sensor node s's algorithm, we need to solve the problem $\mathbf{DP}_s$ to get $(x_s, \{y_{ss'}\}_{s' \in S_{in}(s)})$, which can be carried out by taking the derivative again of Eq. (16)

$$\begin{cases} \frac{\partial \widehat{U}_s}{\partial x_s} = \lambda^s - \mu^s, \quad s \in S, \\ \frac{\partial \widehat{U}_s}{\partial y_{ss'}} = \mu_{ss'}, \quad s' \in S_{in}(s). \end{cases} \tag{26}$$

The relationship between $x_s, \{y_{ss'}\}_{s' \in S_{in}(s)}$ and λ^s, μ^s and $\{\mu_{ss'}\}_{s' \in S_{in}(s)}$ is implicit. Since Eq. (16) is strictly concave and continuously differentiable, x_s and $\{y_{ss'}\}_{s' \in S_{in}(s)}$ are uniquely decided by λ^s, μ^s and $\{\mu_{ss'}\}_{s' \in S_{in}(s)}$. By taking the derivative again of Eq. (26), we get

$$\begin{cases} \frac{\partial^2 \widehat{U}_s}{\partial x_s^2} \frac{\partial x_s}{\partial \lambda_l} = A_{sl}, \quad \frac{\partial^2 \widehat{U}_s}{\partial x_s^2} \frac{\partial x_s}{\partial \mu_{ss'}} = -H_{ss'}, \\ \frac{\partial^2 \widehat{U}_s}{\partial y_{ss'}^2} \frac{\partial y_{ss'}}{\partial \lambda_l} = 0, \quad \frac{\partial^2 \widehat{U}_s}{\partial y_{ss'}^2} \frac{\partial y_{ss'}}{\partial \mu_{ss'}} = 1. \end{cases} \tag{27}$$

Let $\boldsymbol{\chi} = (\boldsymbol{x}, \boldsymbol{y})$, and n_χ is the number of its elements. We define $\theta_i(\boldsymbol{p}) = -\frac{1}{\frac{\partial^2 \widehat{U}_s}{\partial \chi_i^2}}$, if $\frac{\partial^2 \widehat{U}_s}{\partial \chi_i^2}$ is nonzero and the implicit solution of Eq. (26) is within the region of the feasible set of constraints (17); $\theta_i(\boldsymbol{p}) = 0$, otherwise. Let $\boldsymbol{\theta}(\boldsymbol{p}) = \mathrm{diag}(\theta_i(\boldsymbol{p}), i = 1, 2, \ldots, n_\chi)$. Using this, we get the Lemmas shown below.

Lemma 1. *Under assumption A1, the Hessian of D is given by*

$$\nabla^2 D(\boldsymbol{p}) = \begin{pmatrix} \boldsymbol{A} & \boldsymbol{0} \\ -\boldsymbol{H} & \boldsymbol{I} \end{pmatrix}\theta(\boldsymbol{p})\begin{pmatrix} \boldsymbol{A} & \boldsymbol{0} \\ -\boldsymbol{H} & \boldsymbol{I} \end{pmatrix}^T. \tag{28}$$

Proof. From Eq. (27), we have

$$\left[\frac{\partial \boldsymbol{\chi}}{\partial \boldsymbol{p}}(\boldsymbol{p})\right] = -\boldsymbol{\theta}(\boldsymbol{p})\begin{pmatrix} \boldsymbol{A} & \boldsymbol{0} \\ -\boldsymbol{H} & \boldsymbol{I} \end{pmatrix}^T. \tag{29}$$

From Eq. (25), we have

$$\nabla D(\boldsymbol{p}) = \begin{pmatrix} \boldsymbol{c} \\ \boldsymbol{0} \end{pmatrix} - \begin{pmatrix} \boldsymbol{A} & \boldsymbol{0} \\ -\boldsymbol{H} & \boldsymbol{I} \end{pmatrix}\chi. \tag{30}$$

Then, the Hessian of D is given by

$$\nabla^2 D(\boldsymbol{p}) = -\begin{pmatrix} \boldsymbol{A} & \boldsymbol{0} \\ -\boldsymbol{H} & \boldsymbol{I} \end{pmatrix}\left[\frac{\partial \boldsymbol{\chi}}{\partial \boldsymbol{p}}(\boldsymbol{p})\right], \tag{31}$$

$$= \begin{pmatrix} \boldsymbol{A} & \boldsymbol{0} \\ -\boldsymbol{H} & \boldsymbol{I} \end{pmatrix}\theta(\boldsymbol{p})\begin{pmatrix} \boldsymbol{A} & \boldsymbol{0} \\ -\boldsymbol{H} & \boldsymbol{I} \end{pmatrix}^T, \tag{32}$$

which yields the conclusion. □

Let $\boldsymbol{K} = \begin{pmatrix} \boldsymbol{A} & \boldsymbol{0} \\ -\boldsymbol{H} & \boldsymbol{I} \end{pmatrix}$; then $D(\boldsymbol{p}) = \boldsymbol{K}\theta(\boldsymbol{p})\boldsymbol{K}^T$ and we have the following Lemma.

Lemma 2. *Under assumptions A1 and A2, ∇D is Lipschitz with*

$$\|\nabla D(\boldsymbol{p}^{(1)}) - \nabla D(\boldsymbol{p}^{(2)})\|_2 \leqslant \bar{\alpha}\overline{LS}\|\boldsymbol{p}^{(1)} - \boldsymbol{p}^{(2)}\|_2, \tag{33}$$

where $\boldsymbol{p}^{(1)} = (\boldsymbol{\lambda}^{(1)}, \boldsymbol{\mu}^{(1)}), \boldsymbol{p}^{(2)} = (\boldsymbol{\lambda}^{(2)}, \boldsymbol{\mu}^{(2)})$ and for all $\boldsymbol{\lambda}^{(1)}, \boldsymbol{\lambda}^{(2)} \succeq 0$.

Proof. For any given $\boldsymbol{p}^{(1)} = (\boldsymbol{\lambda}^{(1)}, \boldsymbol{\mu}^{(1)}), \boldsymbol{p}^{(2)} = (\boldsymbol{\lambda}^{(2)}, \boldsymbol{\mu}^{(2)})$, and $\boldsymbol{\lambda}^{(1)}, \boldsymbol{\lambda}^{(2)} \succeq 0$, by adopting Taylor Theorem and Lemma 1, we have

$$\begin{aligned} \nabla D(\boldsymbol{p}^{(1)}) - \nabla D(\boldsymbol{p}^{(2)}) &= \nabla^2 D(\boldsymbol{\varepsilon})(\boldsymbol{p}^{(1)} - \boldsymbol{p}^{(2)}) \\ &= \boldsymbol{K}\theta(\boldsymbol{\varepsilon})\boldsymbol{K}^T(\boldsymbol{p}^{(1)} - \boldsymbol{p}^{(2)}), \end{aligned}$$

where $\boldsymbol{\varepsilon} = t\boldsymbol{p}^{(1)} + (1-t)\boldsymbol{p}^{(2)}, t \in [0,1]$. Then

$$\|\nabla D(\boldsymbol{p}^{(1)}) - \nabla D(\boldsymbol{p}^{(2)})\|_2, \tag{34}$$

$$\leqslant (\|\boldsymbol{K}\theta(\boldsymbol{\varepsilon})\boldsymbol{K}^T\|_2^2 \|\boldsymbol{p}^{(1)} - \boldsymbol{p}^{(2)}\|_2^2)^{\frac{1}{2}},$$

$$\leqslant (\|\boldsymbol{K}\theta(\boldsymbol{\varepsilon})\boldsymbol{K}^T\|_\infty \|\boldsymbol{K}\theta(\boldsymbol{p})\boldsymbol{K}^T\|_1)^{\frac{1}{2}} \|\boldsymbol{p}^{(1)} - \boldsymbol{p}^{(2)}\|_2,$$

$$= \|\boldsymbol{K}\theta(\boldsymbol{\varepsilon})\boldsymbol{K}^T\|_\infty \|\boldsymbol{p}^{(1)} - \boldsymbol{p}^{(2)}\|_2, \tag{35}$$

$$= \|\boldsymbol{p}^{(1)} - \boldsymbol{p}^{(2)}\|_2 \max_i \sum_j |\boldsymbol{K}\theta(\boldsymbol{\varepsilon})\boldsymbol{K}^T|_{ij}, \tag{36}$$

$$= \|\boldsymbol{p}^{(1)} - \boldsymbol{p}^{(2)}\|_2 \max_i \sum_j |\sum_{i'} \theta_j(\boldsymbol{\varepsilon}) K_{ij} K_{i'j}|, \tag{37}$$

$$\leqslant \|\boldsymbol{p}^{(1)} - \boldsymbol{p}^{(2)}\|_2 \bar{L} \max_i \sum_j |\theta_j(\boldsymbol{\varepsilon}) K_{ij}|, \tag{38}$$

$$\leqslant \|\boldsymbol{p}^{(1)} - \boldsymbol{p}^{(2)}\|_2 \bar{L}\bar{\alpha} \max_i \sum_j |K_{ij}|, \tag{39}$$

$$\leqslant \|\boldsymbol{p}^{(1)} - \boldsymbol{p}^{(2)}\|_2 \bar{L}\bar{\alpha}\widehat{S}. \tag{40}$$

The Eq. (35) is correct because of the symmetry of $\boldsymbol{K}\theta(\boldsymbol{\varepsilon})\boldsymbol{K}^T$. Let $\bar{k} = \bar{L}\bar{\alpha}\widehat{S}$, then $\|\nabla D(\boldsymbol{p}^{(1)} - \nabla D(\boldsymbol{p}^{(2)}))\|_2 \leqslant \bar{k}\|\boldsymbol{p}^{(1)} - \boldsymbol{p}^{(2)}\|_2$, which yields the conclusion. □

The above discussion and Lemmas establish the convergence of the sequence that is generated by the *Algorithm DAULT*, provided that the assumptions A1–A3 are satisfied. We get the following result.

Theorem 1. *If assumptions A1–A3 hold, and the stepsize satisfies $0 < \delta < \frac{2}{\bar{\alpha}\overline{LS}}$, then starting from any initial rates $\underline{\boldsymbol{m}} \preceq \boldsymbol{x} \preceq \overline{\boldsymbol{m}}, \boldsymbol{y} \succeq 0$ and link price $\boldsymbol{\lambda} \succeq 0$, each limit point $(\boldsymbol{x}^*, \boldsymbol{y}^*, \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*)$ of the sequence $(\boldsymbol{x}(t), \boldsymbol{y}(t), \boldsymbol{\lambda}, \boldsymbol{\mu})$ generated by Algorithm DAULT is primal-dual optimal.*

Proof. From the assumptions A1–A2, the dual objective function D is continuously differentiable and lower bounded. ∇D is Lipschitz from Lemmas 1 and 2. Then by following the process of [2, p. 871] [30, pp. 213–214], when $\delta < \frac{2}{\bar{\alpha}\overline{LS}}$, we can conclude that the algorithm is convergent. Because the objective function (11) is strictly concave, $\boldsymbol{x}^*(\boldsymbol{p}^*)$ is also primal optimal. □

6. Numerical results

In this section, we use numerical results from Matlab to evaluate the performance of *Algorithm DAULT*. Mainly we show the convergence of the algorithm and how the network performance depends on the system parameter γ.

We use 6 sensor nodes and 1 sink node in our numerical experiments and the positions of the nodes are randomly generated in an area of size 50×50, which is illustrated in Fig. 2. Nodes 1–6 are sensor nodes and node 7 is the sink node and there are 7 links (as shown in Fig. 2). The sensor nodes 1–6 will transmit their sensing data to the sink (node 7), which is only responsible for receiving data. The functionality of the network layer is beyond the scope of this paper, thus we just assume that there is a routing mechanism in place to find a route for each sensor node. We give the 6×7 routing matrix $\boldsymbol{A}$ as below.

$$\boldsymbol{A} = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{41}$$

In our experiments, we set $U_s(\cdot) = \xi_s \log x_s$ (i.e., the utility function is to guarantee the proportional fairness among the sensor nodes), where $\xi = (22, 24, 26, 28, 30, 32)$. As illustrated in [8,26], the function $\sum_s V_s^{\beta}(z_s)$ can have a ratio higher than 0.95 to approximate the original lifetime problem T (see Section 3.2) when $\beta \geqslant 8$. We use in our experiment $\beta = 9$. We set the capacity of links 1–7 to be $\boldsymbol{c} = (150, 180, 150, 280, 330, 180, 330)$ (bit/s). We set $\rho = 50$ nJ/bit, $\sigma = 0.0013$ pJ/b/m^4 and $n = 4$ for the parameters of the data transmitting power model, p_{sl}^{t}, in the formula (3). We set $p_{sl}^{r} = 50$ nJ/bit for the data receiving power model [8]. The initial energy of sensor nodes 1–6 are set to be $\boldsymbol{e} = (900, 910, 1100, 950, 950, 1100)$ (J) and the sink is assumed to have enough energy for all communications. In our numerical experiments, we set the scaled parameter $\omega = 3.2768 \times 10^{32}$ and we will show the network performance with different γ values.

First, we show the convergence of the *Algorithm DAULT*. The minimum and maximum rates of each sensor node are set to be $\underline{m}_s = 50$ and $\overline{m}_s = 250$, respectively. By randomly choosing the initial point $\lambda(0), \mu(0)$, we collect the rate information in the iteration and plot them in the figure. First, we set $\gamma = 0.8$ and the corresponding results are shown in Fig. 3. The rates of all nodes change sharply at the beginning of the iteration and then converge to the optimal solution quickly, which shows the effectiveness of our algorithm. From Fig. 3, we can also find something interesting. The rates of nodes 1, 2, 3 and 5 is relatively small and the rates of node 4 and node 6 are relatively large. In Fig. 2 we can see that node 4 and node 6 can transmit their data to the sink directly, so they transmit data in an efficient way, i.e., do not need other sensor nodes to relay their data. On the other hand, if node 1, 2, 3, 5 want to transmit data, they need other nodes to relay its data, thus consuming additional energy. So in the optimal rate allocation, the rates of node 4 and node 6 can be relatively large to obtain high utility, which shows the performance of

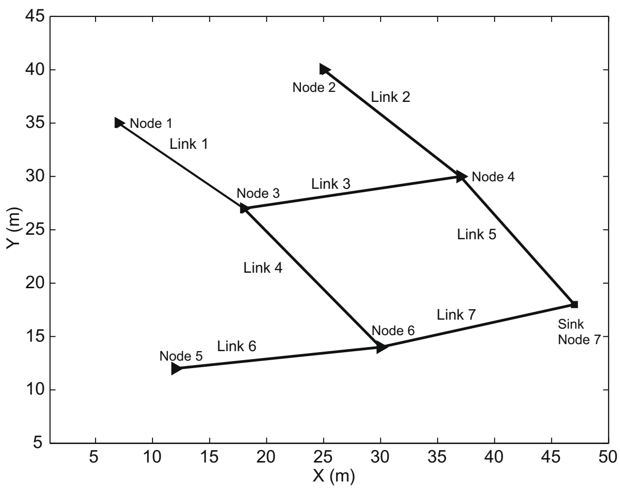

Fig. 2. Topology of a wireless sensor network.

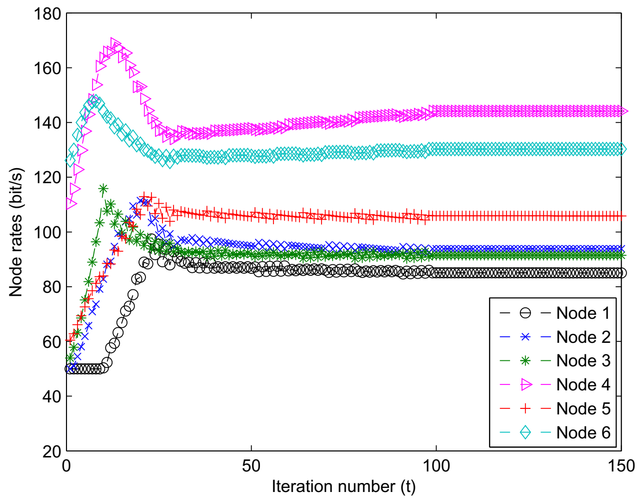

Fig. 3. The convergent performance of the *Algorithm DAULT*, $\gamma = 0.8$.

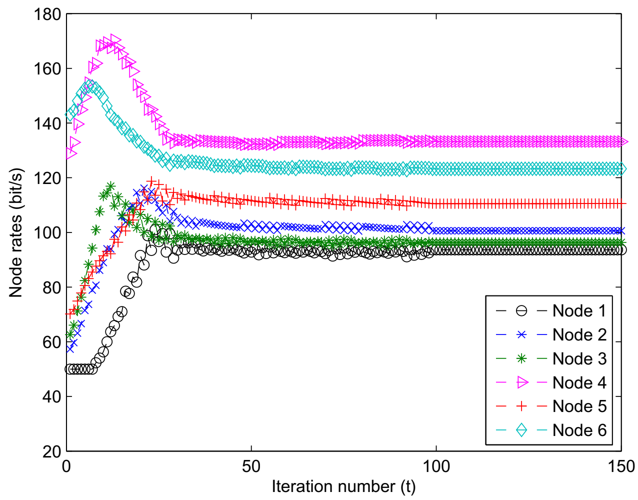

Fig. 4. The convergent performance of the *Algorithm DAULT*, $\gamma = 0.95$.

Algorithm DAULT is different from that of rate allocation without consideration of the lifetime of the whole network.

We then increase γ to a high value of 0.95, in which the network lifetime is almost out of consideration. The corresponding result is shown as in Fig. 4. As we increase γ to get more utility and reduce network lifetime, all the optimal rates shown in Fig. 4 are much larger than those in Fig. 3. Notice that the rates on link 5 and link 7 are almost equal to their capacity in this situation (the rates on link 5 and 7 are equal to $x^{l_5} = x_1 + x_2 + x_4 = 328.3$ bit/s, $x^{l_7} = 330$ bit/s, respectively), which is very similar to the link congestion problem. Thus, our mathematical model for the problem also covers the link congestion problem as a special case.

Finally, we set $\gamma = 0.1$, a low value. In this case, we concern more about the network lifetime than the utility and the corresponding results are plotted in Fig. 5. Except for rates of node 4 and node 6, the rates of other nodes are very small (close to 50, which is the minimum rate that the sensor has to transmit). In such a manner, each sensor

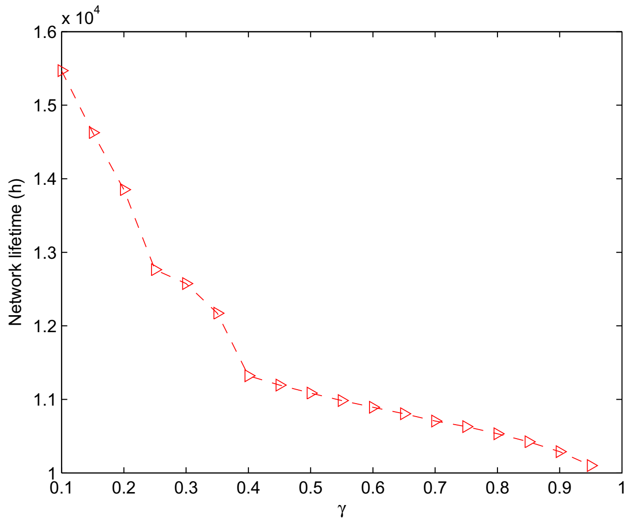

Fig. 7. Relationship between network lifetime and system parameter γ.

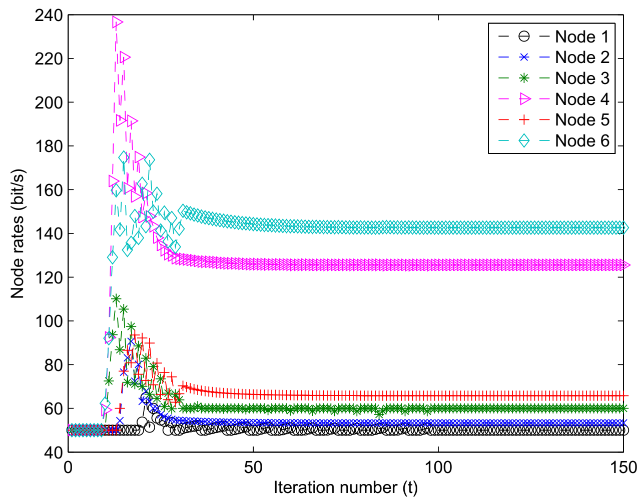

Fig. 5. The convergent performance of the *Algorithm DAULT*, $\gamma = 0.1$.

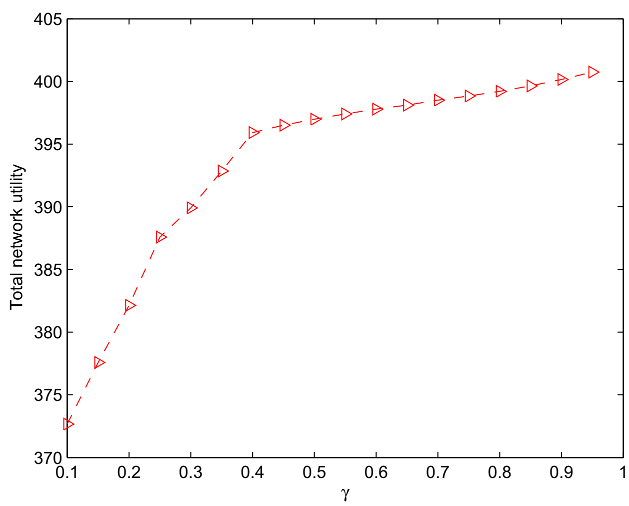

Fig. 6. Relationship between the total utility and system parameter γ.

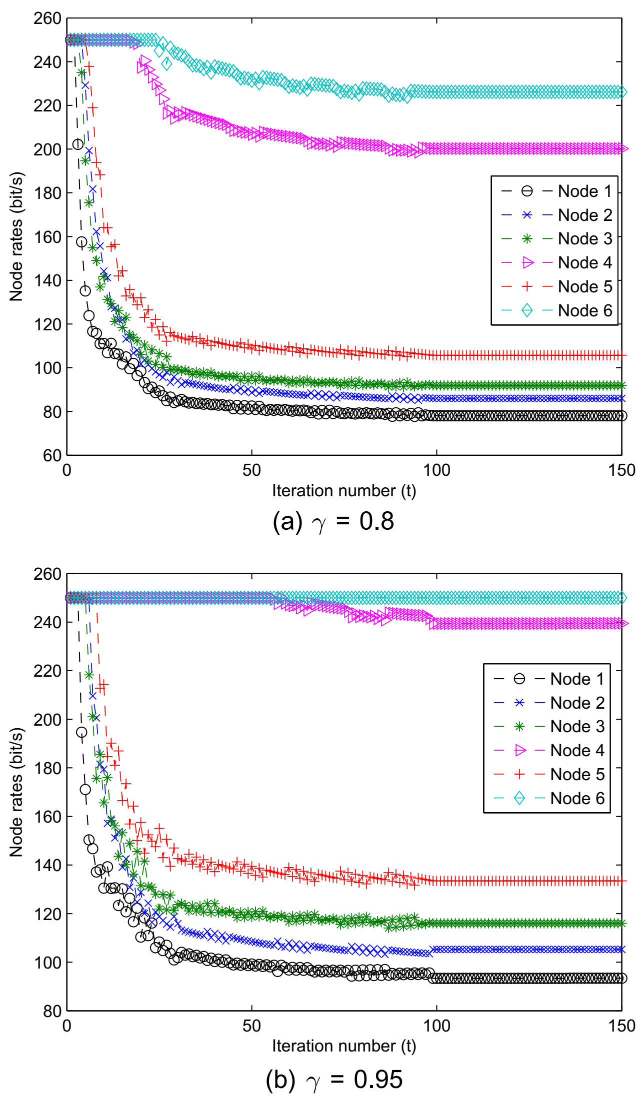

Fig. 8. Flow control for utility-lifetime tradeoff without link capacity constraint.

can conserve a lot of energy. From Figs. 3–5, it can be seen that there is a tradeoff between the network utility and network lifetime.

As discussed above, we show that the system parameter γ represents the tradeoff between the utility and network lifetime. Next we give some detailed information on how the total utility and network lifetime depend on γ. We use different γ and collect the corresponding results, which are shown in Figs. 6 and 7. From these two figures, we can see that as γ increases from 0 to 1, the whole utility of the network also increases; meanwhile the network lifetime decreases. There is apparently a tradeoff between utility and network lifetime. Thus, we can decide γ according to the actual requirements and make the network behave at a desired performance.

At last, we show the necessity of considering link congestion for the utility-lifetime tradeoff. In Fig. 8, we show the result of flow control for the utility-lifetime tradeoff without link capacity constraint. The rates are much larger than those by *Algorithm DAULT*. When $\gamma = 0.8$, the rates over links 5 and 7 are $x^{l_5} = 364, x^{l_7} = 424$ (bit/s), and when $\gamma = 0.95, x^{l_5} = 441$ bit/s, $x^{l_7} = 499$ (bit/s). The rates over links 5 and 7 exceed their capacities ($c_5 = 330, c_7 = 330$). Therefore, without link capacity constraint, the network will incur congestion, thus degrading the performance of the whole network.

7. Conclusions

In this paper, we have studied optimal flow control for utility-lifetime tradeoff in WSNs. First we characterize the tradeoff between utility and lifetime by introducing the system parameters, ω and γ, and demonstrate that the combined objective function is strictly concave and the global optimal solution exists. Then we introduce auxiliary variables to decouple the objective function, derive a distributed algorithm *DAULT*and prove its convergence. Further, we verify its fast convergence of *DAULT* as well as the necessity of considering link congestion by the numerical results.

Our future work will focus on cross-layer design, including quantifying the impacts of interference between links in the physical layer and the MAC layer, and the routing strategies in the network layer. We will also consider a general flow control algorithm for stochastic multi-objective optimization problems (e.g., cross-layer design) in multi-path routing.

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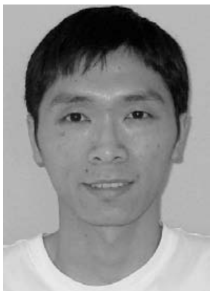

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