Flow-level Performance of Opportunistic OFDM-TDMA and OFDMA Networks

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***Abstract*—In this paper, the flow-level performance of opportunistic scheduling in orthogonal frequency division multiplexing (OFDM) networks is studied. The analysis accounts for the applications with a dynamic number of competing flows, such as continuous transfers of file transport protocol (FTP) or web browsing sessions. An analytical model is developed to extend the multi-class Processor-Sharing model in single-carrier networks to multi-carrier OFDM networks, where the total service rate varies with the number of flows. Based on the analytical model, the scheduling gains in both OFDM-TDMA (time division multiple access) and OFDMA (orthogonal frequency division multiple access) networks are evaluated for low and moderate signal-to-noise ratio (SNR). Different from previous works, we focus on the scheduling performance at the flow level and consider a dynamic network setting with random sized service demands. Furthermore, we use stochastic comparison techniques to examine the effects of physical-layer characteristics, such as fading speed and channel frequency selectivity, on flow-level performance. Simulations are performed to verify the analytical results.**

***Index Terms*—Opportunistic scheduling; OFDM-TDMA; OFDMA; processor-sharing model.**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) is a physical-layer multi-carrier technology, which has been successfully applied in a wide variety of wireless communication systems such as wireless local area networks (WLANs). The major advantages of OFDM exist in its capability of effectively combating inter-symbol interference (ISI) and its high spectral efficiency due to spectrum overlapping. OFDM can be combined with multiple access schemes, such as time division multiple access (TDMA), to achieve efficient bandwidth utilization in presence of multiple users. In IEEE 802.16 standard, for instance, both OFDM-TDMA and orthogonal frequency division multiple access (OFDMA) have been adopted at 2–11 GHz band [1].

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In multiuser wireless communication networks, opportunistic scheduling (OS) provides an effective mechanism to improve throughput performance by exploiting channel fluctuations. The concept of OS is first applied for the third-generation (3G) wireless systems such as Code Division Multiple Access (CDMA) 2000 1xEV-DO [2] and Universal Mobile Telecommunications System (UMTS) High Speed Downlink Packet Access (HSDPA) [3]. Performance analysis of OS algorithms provides guidelines for comparing and optimizing these algorithms. Furthermore, it can also be used for radio network planning and other radio resource management strategies, e.g., admission control, to achieve the network quality goals. The research work in this area belongs to two broad categories. The first category focuses on the investigation of OS algorithms at the packet level with an assumption of a static user population [4]–[10]. The traffic pattern is usually assumed to be saturated with infinite backlogs (i.e., each user always has data to transmit) or features dynamic packet arrivals [11]. For the saturated model, a common objective is to optimize some utility functions of the throughput; while for dynamic packet arrivals, the focus is on network stability, i.e., the queue occupancy can be bounded whenever feasible. The second category investigates OS algorithms at the flow level with time-variant user population [12]–[14]. In the flow-level analysis, new users arrive according to a stochastic process, and each user has a finite-length file for transmission. A user[1] leaves the system when the entire file is transmitted. Important flow-level performance metrics include the distribution of the number of flows, flow throughput and mean response time. Compared to the first category, the flow-level analysis is based on more practical traffic patterns, which consider the dependence of the throughput on both the user population and the scheduling algorithms [12].

Due to its effectiveness, OS in sophisticated OFDM-based beyond 3G (B3G) or fourth-generation (4G) wireless systems has been attracting more and more interests. In [15], OS algorithms in OFDM systems under infinite backlogs and dynamic packet arrivals have been investigated; in [16], a generalized processor sharing (GPS) based scheduler integrated with power and subcarrier allocation is proposed to maximize the system throughput; and in [17], the OS performance of OFDM-TDMA systems has been compared with that of OFDMA systems at the packet level. So far, research on OS for OFDM systems has mainly focused on the packet level. The performance of OS at the flow level has not been well addressed.

[1]For the flow-level analysis, the terms "user" and "flow" are usually used exchangeably. Therefore, for the purpose of unification, only "flow" will be used in the rest of the paper.

The full analysis of flow-level performance can be very complicated, as it may involve the analysis of a Markov chain with a large state space. This complexity mainly results from the following two aspects: 1) the service process is stochastic, i.e., the service rate varies with time; and 2) the number of users in the system is dynamic. To address these challenges, an approximation technique is developed in [14] to simplify the analysis at the flow level. It is shown that an approximate analysis can be performed in a time-scale decomposable regime, where the time scale of the data file transmission time is much longer than that of the service process fluctuation. Another equivalent way of stating the above condition is that the channel fading is much faster than the flow dynamics, since the channel fading determines the service process fluctuation and the flow dynamics result from the arrival rate and the size of the data files. In this case, the random fluctuations in the service rate become negligible, and a simple constant-rate service process can be applied. A rigorous justification is given for the above approximation method. This approximation has also been used in [12], which is different from [14] in that the time-slotted system is represented by a Processor-Sharing (PS) model in continuous time, based on the assumption that the duration of the time slot is much shorter than that of the data file transmission. Applying the approximation method in [14], the service rate of the PS model is deterministic as the random fluctuations become negligible, but it is state-dependent based on the number of flows.

In this paper, an analytical model for performance evaluation of an OFDM system with OS at the flow level is proposed. It extends the multi-class Processor-Sharing (PS) model for single-carrier system to the multi-carrier OFDM system, where the total service rate varies with the total number of flows. Although the queueing model for OFDM system has a multi-server nature at packet-level, we show for the first time that the single-server PS model can still be applied for flow-level performance analysis if the duration of the time slot is much shorter than that of the data file transmission. Based on this model, the scheduling gains achieved by proportional fair schedulers in both OFDMA and OFDM-TDMA systems are analyzed for low and moderate signal-to-noise ratio (SNR), where a linear relationship between the feasible rate and SNR holds. It is shown that the scheduling gain achieved in the OFDMA system is larger than that of the OFDM-TDMA system. Furthermore, stochastic comparison techniques are used to evaluate the impact of physical-layer characteristics on flow-level performance of OFDM systems, which apply OFDMA or OFDM-TDMA. The analytical results demonstrate that fast fading helps to improve performance as that in single-carrier system [13]. Moreover, the performance of the OFDM system can also be improved by high channel frequency selectivity. Fading variation has a less impact on performance in case of a higher channel frequency selectivity. In order to describe the channel variance in the frequency domain, we introduce two limit regimes referred to as *fully-selective* and *flat*, which indicate that the channel presents the same statistics but varies in an infinitely fast and an infinitely slow scale in the frequency domain, respectively. By combining the limit regimes in the time domain [13], i.e., *fluid* and *quasi-stationary*, we show that *fluid* and (*flat*, *quasi-stationary*) limit regimes provide good performance estimates for OFDM systems. Finally, simulation results are given to verify the analytical results.

The remainder of this paper is organized as follows. The flow-level model is described in Section II. Section III analyzes and compares the scheduling gains of OFDMA and OFDM-TDMA systems, and examines the flow-level performance of OFDM systems under different physical-layer conditions. In Section IV, simulations are performed to verify the analytical results. Section V concludes this paper.

II. Formulation of Flow-Level Model

A. System Model

Consider the downlink of an OFDM system, where a single base station (BS) communicates with multiple mobile stations (MSs). At the base station, signal modulation is carried out by N_c points inverse fast Fourier transform (IFFT), where $N_c \geq K$ and K denotes the number of subcarriers. For ISI elimination, a cyclic prefix (CP) of length N_{cp} is also added before transmission.

For each MS i, the channel is a frequency-selective Rayleigh fading channel with L_i non-zero taps. The channel impulse response (CIR) remains unchanged during at least one OFDM symbol interval and can be expressed as

$$h_i(t,\tau) = \sum_{l=0}^{L_i-1} \alpha_{l,i}(t)\delta(\tau-\tau_{l,i}) \tag{1}$$

where the lth tap gain $\alpha_{l,i}(t)$ with propagation delay $\tau_{l,i}$ is complex Gaussian random variable with zero mean and variance of $\sigma_{l,i}^2$. If the cyclic prefix is larger than the channel delay spread, it is reasonable to assume that the narrow-band signal transmitted through each subcarrier experiences a flat Rayleigh fading channel. The channel frequency response (CFR) with respect to the kth subcarrier for MS i can be expressed as

$$H_{k,i}(t) = \sum_{l=0}^{L_i-1} \alpha_{l,i}(t) e^{-j2\pi\tau_{l,i}k/N_c}. \tag{2}$$

For OFDM, the cross covariance function of $H_{k,i}(t)$ has the following factorable form [18]

$$\Phi_{H_{k,i},H_{l,i}}(\tau) = \Phi_{\mathrm{T}}^i(\tau)\Phi_{\mathrm{F}}^i(k-l), \quad k,l=1,\ldots,K \tag{3}$$

where $\Phi_{H_{k,i},H_{l,i}}(\tau) = \mathbf{E}[H_{k,i}(t)H_{l,i}^*(t+\tau)]$. Here, $\Phi_{\mathrm{T}}^i(\tau) = \Phi_{H_{k,i},H_{k,i}}(\tau)$ gives the temporal correlation for $H_{k,i}(t)$, which is seen to be identical for all $k=1,\ldots,K$, and $\Phi_{\mathrm{F}}^i(k-l) = \Phi_{H_{k,i},H_{l,i}}(0)$ represents the correlation in frequency across subcarriers.

At the receiver end, the CP is removed first and the received signals are demodulated by fast Fourier transform (FFT). The received signal on the kth subcarrier of MS i can be expressed as

$$Z_{k,i}(t) = X_{k,i}(t)H_{k,i}(t) + W_{k,i}(t) \tag{4}$$

where $X_{k,i}(t)$ is the transmitted signal on the kth subcarrier of MS i, and $W_{k,i}(t)$ is the complex additive white Gaussian noise (AWGN) with zero mean and variance of σ_i^2.

Ideally, the MS adaptively determines the appropriate transmission rate with the proper modulation and coding selection (MCS) at each subcarrier based on the received SNR $\gamma_{k,i}(t) = |H_{k,i}(t)|^2/\sigma_i^2$, and feeds back the selection to the BS through a uplink control channel.

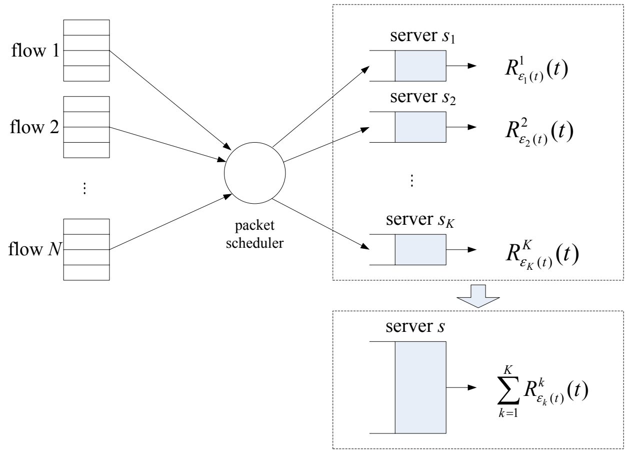

Fig. 1. Queueing model for multi-user scheduling in the OFDM system.

B. Channel Rate Process Model

The base station transmits information to $n(t)$ MSs in equal-length time slots. Let $R_i^k(t)$, a stationary and ergodic process, be the feasible rate for MS i at subcarrier k in time slot t. For low and moderate SNR, $R_i^k(t)$ can be approximated as a linear function of the instantaneous SNR, $\gamma_{k,i}(t)$ [12].

The random processes $\{R_i^k(t)\}_{i=1,\ldots,n(t);k=1,\ldots,K}$ have the following properties:

1) **For different MSs**: $\forall i_1, i_2 \in \{1,\ldots,n(t)\}$, $R_{i_1}^k(t)$ and $R_{i_2}^k(t)$ for different MSs $i_1 \neq i_2$ are mutually independent.
2) **For a same MS**:
   a) $\forall i \in \{1,\ldots,n(t)\}$, $\{R_i^k(t)\}_{k=1,\ldots,K}$ for the same MS i are identically distributed.
   b) **Different Time Slots**: $\forall t_1, t_2 \in \mathbb{Z}^+$, let $\Delta t = |t_1 - t_2|$. If Δt is small or large enough, $R_i^k(t_1)$ and $R_i^k(t_2)$ are strongly correlated and approximately independent, respectively, with the temporal correlation function $R_{\mathrm{T}}^i(\Delta t) \approx 1 \text{ and } 0$.
   c) **Different Subcarriers**: $\forall k_1, k_2 \in \{1,\ldots,K\}$, let $\Delta k = |k_1 - k_2|$. For small or large enough Δk, $R_i^{k_1}(t)$ and $R_i^{k_2}(t)$ are strongly correlated and approximately independent, respectively, with the frequency correlation function $R_{\mathrm{F}}^i(\Delta k) \approx 1 \text{ and } 0$.

Let $Y_i^k(t) = R_i^k(t)/C_i$, where $C_i = \mathbf{E}[R_i^k(t)]$ is the time-average rate of MS i at any subcarrier. $Y_i^1(t),\ldots,Y_i^K(t)$ represent the relative rate fluctuations for subcarriers $1,\ldots,K$ of MS i. According to property 1), they are identically distributed.

C. Dynamic Flow Model

We define a dynamic flow model, where a new flow arrives into the system with a finite-length file request, and leaves the system when the file is transmitted. Without loss of generality, each MS is assumed to start a new transmission only after the old one is finished, and each new transmission by the same MS is treated as a new flow. The scheduler at the BS allocates each subcarrier k to a flow $\epsilon_k(t)$ at a given time slot t, according to different scheduling strategies. The queueing model of multi-user scheduling in the OFDM system is shown in Fig.1. This is a multi-server scheduling problem, and the actual service rate of each server S_k is $R^k_{\epsilon_k(t)}(t)$, which depends on the scheduling strategy and the number of flows. For the packetized multi-server system where a packet of any flow can be serviced at any of the servers, [19] shows that compared with a single Generalized Processor Sharing (GPS) server whose rate equals to the sum rate of all servers, performance differences exist because the flow in the packetized system is not infinitely divisible. Since the duration of the time slots is relatively short with respect to the size and arrival frequency of the service demands (e.g., the minimum scheduling time unit is 1ms in 3G Long-Term Evolution (LTE) [20], while it usually takes at least several seconds to transmit a file), the flow-level performance can be analyzed in continuous rather than discrete time, and it can be assumed that the flows are served simultaneously by a single server with a service rate $\sum_{k=1}^K R^k_{\epsilon_k(t)}(t)$, rather than by K servers in a time-slotted fashion.

For comparison purpose, we consider the following three scenarios.

1) Consider the situation that there is only a single flow i in the system. Obviously, its transmission rate is $T_i^{\mathrm{sg}}(t) = \sum_{k=1}^K R_i^k(t)$. When fading is relatively fast compared to flow dynamics, $T_i^{\mathrm{sg}}(t)$ can be replaced by a constant value $\mathbf{E}[\sum_{k=1}^K R_i^k(t)] = KC_i$.
2) Consider a simple round-robin (RR) scheduler. In OFDM-TDMA systems, the scheduler assigns all the subcarriers to one of the $n(t)$ flows at each time slot t in a round-robin fashion, where $n(t)$ denotes the total number of flows present at time slot t, while in OFDMA systems, each subcarrier is assigned to one of the $n(t)$ flows in a round-robin fashion at each time slot t. In both systems under Processor-Sharing model, the transmission rate of flow i can be represented as $T_i^{\mathrm{rr}}(t) = T_i^{\mathrm{sg}}(t)/n(t)$, where $T_i^{\mathrm{sg}}(t)$ represents the transmission rate of user i if it was the only user in the system at time t. This idealization is a typical use of the Processor-Sharing discipline as a theoretical abstraction of round-robin scheduling. Furthermore, if the fading speed is relatively fast compared to flow dynamics, $T_i^{\mathrm{rr}}(t)$ can be replaced by a constant value $KC_i/n(t)$. Notice that the value depends on the number of flows.
3) Consider an opportunistic scheduler to achieve fair sharing. Let $G\big(n(t)\big)$ denote the scheduling gain of the opportunistic scheduler, which accounts for the throughput gain it achieves with respect to the simple round-robin scheduling. Obviously, for $n(t) = 1$, $G(1) = 1$. It is natural to represent the transmission rate of flow i as

$$T_i^{\mathrm{os}}(t) = T_i^{\mathrm{sg}}(t)\frac{G\big(n(t)\big)}{n(t)} = KC_i\frac{G\big(n(t)\big)}{n(t)} \qquad (5)$$

where the second equality holds when the fading speed is relatively fast compared to flow dynamics.

The flow-level model defined by (5) corresponds to a Processor-Sharing type queue where the service rate of each flow varies with the number of flows in the system. The model belongs to the class of product-form queueing networks and is analytically tractable [21]. We consider a scenario with P flow classes. Class-p flows submit file transfer requests as a Poisson process of rate λ_p. Let F_p be a random variable representing the file size of an arbitrary class-p flow. Let $(N_1, \ldots, N_P)$ be a random vector representing the number of flows of the various classes in the system at an arbitrary epoch in statistical equilibrium. The joint stationary distribution of $(N_1, \ldots, N_P)$ can be obtained as [13]

$$\begin{aligned} & \mathbf{Pr}(N_1 = n_1, \ldots, N_P = n_P) \\ = \; & \mathbf{Pr}(0)(n_1 + \ldots + n_P)! \prod_{p=1}^{P} \frac{(\rho_p)^{n_p}}{n_p!} \end{aligned} \tag{6}$$

where $\mathbf{Pr}(0)$ is determined by the normalizing condition. ρ_p is the class-p traffic loads defined as $\rho_p := \lambda_p \mathbf{E}[F_p] / \big(KC_p \sqrt[n]{\prod_{i=1}^{n} G(i)}\big)$, where $n = n_1 + \ldots + n_P$. Then, the total traffic load equals $\rho = \sum_{p=1}^{P} \rho_p$. By the Little's law, we obtain the mean response time T_p and the flow throughput η_p of class-p flows as

$$\mathbf{E}[T_p] = \frac{\mathbf{E}[N_p]}{\lambda_p}, \quad \eta_p = \frac{\mathbf{E}[F_p]}{\mathbf{E}[T_p]}. \tag{7}$$

When $G(n) \equiv 1$, we have

$$\eta_p = KC_p(1 - \rho). \tag{8}$$

III. Performance Analysis and Comparison

In this section, we analyze and compare the flow-level performance based on the model in Section II. Section III(A) focuses on the scheduling gains of the proportional fair (PF) scheduler in OFDMA and OFDM-TDMA systems, respectively. The impact of physical-layer factors on flow-level performance is analyzed in Section III(B).

The PF algorithm in single-carrier systems has been extended to multi-carrier systems in [22]. In an OFDMA system, subcarrier k is assigned to the flow that satisfies the following condition at time slot t

$$\epsilon_k(t) = \arg \max_{i=1,\ldots,n} R_i^k(t)/T_i(t), k = 1, \ldots, K \tag{9}$$

where $T_i(t)$ is the exponential filtered average throughout of flow i at time slot t.

In OFDM-TDMA systems, all the subcarriers are assigned to the same flow that satisfies the following condition at time slot t

$$\epsilon(t) = \arg \max_{i=1,\ldots,n} \sum_{k=1}^{K} R_i^k(t)/T_i(t), k = 1, \ldots, K. \tag{10}$$

A. Scheduling Gain

As indicated in Section II(B), $\{Y_i^k(t)\}_{i=1,\ldots n(t)}$ are independent identically distributed (i.i.d.) r.v.'s, which means that the fluctuations of flow feasible rates around the respective time-average values are statistically identical. In this case, the instantaneous rate $R_i^k(t)$ and the exponential smoothed average throughput $T_i(t)$ of PF algorithm scales linearly with the time average rate $\mathbf{E}[\sum_{k=1}^{K} R_i^k(t)] = KC_i$. A rigorous justification of this claim is provided in [23]. In addition, $T_i(t)$ will not show any significant variation when the time constant in the exponential smoothing is large. Therefore, we may write $T_i(t) \approx VKC_i$, where V is some constant value and $T_i(t)$ is approximated as a constant independent of t [12]. As a result, the allocation of time slots and subcarriers only depends on the relative rate fluctuations instead of the time-average rates. Thus, PF algorithm results in fair sharing, since the relative rate fluctuations are statistically identical. This means that (5) is valid for PF algorithm.

1) OFDMA system: Substituting $R_i^k(t) := C_i Y_i^k(t)$ and $T_i(t) \approx VKC_i$, we find that the expected rate of the selected flow i at each sub-channel k approximately equals

$$\mathbf{E}[C_i Y_i^k(t) | Y_i^k(t) = \max_{j=1,\ldots,n} Y_j^k(t)] = C_i \mathbf{E}[\max_{j=1,\ldots,n} Y_j^k(t)]. \tag{11}$$

Therefore, the transmission rate of flow i is $T_i^{\mathrm{os}}(t) = KC_i \mathbf{E}[\max_{j=1,\ldots,n} Y_j^k(t)]/n(t)$. Compared with (5), the scheduling gain of the OFDMA system is

$$G^{\mathrm{OFDMA}}(n) = \mathbf{E}[\max_{j=1,\ldots,n} Y_j^k(t)]. \tag{12}$$

Assume Rayleigh fading and the data rate are linear functions of SNR, $Y_j^k(t), j = 1, \ldots, n$ are exponentially distributed with unit mean. According to the property 1) in Section II.B, $Y_j^k(t), j = 1, \ldots, n$ are mutually independent. We then obtain [24]

$$\begin{aligned} G^{\mathrm{OFDMA}}(n) &= \int_0^\infty 1 - \Big(1 - \mathbf{Pr}\big(Y_i^k(t) > x\big)\Big)^n dx \\ &= 1 + \frac{1}{2} + \ldots + \frac{1}{n}. \end{aligned} \tag{13}$$

2) OFDM-TDMA system: Substituting $R_i^k(t) := C_i Y_i^k(t)$, the expected rate of the selected flow i can be approximated as

$$\begin{aligned} & \mathbf{E}[\sum_{k=1}^{K} C_i Y_i^k(t) | \sum_{k=1}^{K} Y_i^k(t) = \max_{j=1,\ldots,n} \sum_{k=1}^{K} Y_j^k(t)] \\ = \; & C_i \mathbf{E}[\max_{j=1,\ldots,n} \sum_{k=1}^{K} Y_j^k(t)]. \end{aligned} \tag{14}$$

Therefore, the transmission rate of flow i is $T_i^{\mathrm{os}}(t) = C_i \mathbf{E}[\max_{j=1,\ldots,n} \sum_{k=1}^{K} Y_j^k(t)]/n(t)$. Compared with (5), the scheduling gain of OFDM-TDMA system is

$$G^{\mathrm{OFDM-TDMA}}(n, K) = \frac{1}{K} \mathbf{E}[\max_{j=1,\ldots,n} \sum_{k=1}^{K} Y_j^k(t)] \tag{15}$$

which is a function of both n and K.

With Rayleigh fading, $\sum_{k=1}^{K} Y_j^k(t), j = 1, \ldots, n$ are the sum of K identically distributed (but not necessarily independent according to the property 2c) in Section II.B) exponential r.v.'s at any time t, which has no closed-form expression for the probability distribution function. However, an upper and lower bound for $G^{\mathrm{OFDM-TDMA}}(n, K)$ can be derived using stochastic comparison technique. According to the result of convex ordering, $\sum_{k=1}^{K} \overline{Y}_j^k(t) \leq_{\mathrm{cx}} \sum_{k=1}^{K} Y_j^k(t) \leq_{\mathrm{cx}} \sum_{k=1}^{K} Y_j^1(t)$, where $\{\overline{Y}_j^1(t), \ldots, \overline{Y}_j^K(t)\}$ are the independent version of $\{Y_j^1(t), \ldots, Y_j^K(t)\}$ [26] (some basic definitions of stochastic comparison are given in Appendix (A)). The above inequality means that $\sum_{k=1}^{K} Y_j^k(t)$ is most variable when the data rates of all subcarriers are the same, and least variable when the data rates are stochastically independent. Since maximization is a convex function, the upper bound $G_{\mathrm{UB}}^{\mathrm{OFDM-TDMA}}(n, K)$ and the lower bound $G_{\mathrm{LB}}^{\mathrm{OFDM-TDMA}}(n, K)$ can be derived by replacing

$\sum_{k=1}^{K} Y_j^k(t)$ in (15) with $\sum_{k=1}^{K} Y_j^1(t)$ and $\sum_{k=1}^{K} \overline{Y}_j^k(t)$, respectively. In obtaining the upper bound, the wireless channel reduces to flat fading and $\sum_{k=1}^{K} Y_j^1(t)$ is exponentially distributed with expectation K. Therefore, $G_{\mathrm{UB}}^{\mathrm{OFDM-TDMA}}(n,K)$ equals $G^{\mathrm{OFDMA}}(n)$ given in (13), i.e., $G^{\mathrm{OFDM-TDMA}}(n,K)$ is always smaller than or equal to $G^{\mathrm{OFDMA}}(n)$.

To obtain the lower bound, the data rates over any two subcarriers are considered to be independent, so that $\sum_{k=1}^{K} \overline{Y}_j^k(t)$ is an Erlang-K r.v. at any time t. In this case, it is difficult to obtain a closed-form solution as in (13). Therefore, we derive another upper bound for the low bound based on the result in [25], i.e.,

$$\begin{aligned} & G_{\mathrm{LB}}^{\mathrm{OFDM-TDMA}}(n,K) \\ \leq\ & \frac{1}{K}\Big(m_n + n\int_{m_n}^{\infty} \mathbf{Pr}\big(\sum_{k=1}^{K} Y_j^k(t) > x\big)dx\Big) \\ =\ & 1 + ne^{-m_n}(m_n)^K/K! \end{aligned} \tag{16}$$

where m_n is the smallest positive solution of the equation

$$\mathbf{Pr}\big(\sum_{k=1}^{K} Y_j^k(t) > x\big) = ne^{-m_n}\sum_{i=0}^{K-1}(m_n)^i/i! = 1. \tag{17}$$

It has been shown in [25] that the upper bound is relatively tight. For example, the upper bound for $G^{\mathrm{OFDMA}}(n)$ is $1+\log(n)$ using the above method.

The upper and lower bounds derived for $G^{\mathrm{OFDM-TDMA}}(n,K)$ above are generally not very tight. However, by letting K' represent the number of *independent* subcarriers instead of the total number of subcarriers, $G_{\mathrm{LB}}^{\mathrm{OFDM-TDMA}}(n,K')$ provides a close approximation of $G^{\mathrm{OFDM-TDMA}}(n,K')$, and a tight upper bound of $G^{\mathrm{OFDM-TDMA}}(n,K')$ can be derived based on the solution of (16) and (17). Note that the number of independent subcarriers K' equals to $\lfloor K/\Delta k^* \rfloor$, where $\lfloor x \rfloor$ denotes the largest integer which is less than or equal to x and Δk^* is the smallest solution of the equation for frequency correlation function $R_{\mathrm{F}}^i(\Delta k^*) = 0$. The theoretical gains of PF algorithm in OFDM-TDMA and OFDMA systems with different numbers of *independent* subcarriers are shown in Fig.2, where UB stands for "upper bound". Since $K' = 2, 10, 100, 1000$ represents the number of *independent* subcarriers instead of the total number of subcarriers in the system, the four curves obtained from (16) show the upper bounds of $G^{\mathrm{OFDM-TDMA}}(n,K')$.

From the above discussion, it can be seen that the scheduling gain of the PF algorithm in OFDMA systems is always larger than or equal to that of the PF algorithm in OFDM-TDMA systems, and the scheduling gain of the latter decreases with the increase of the number of independent subcarriers. This is because by the law of large numbers, the variation of the average rate of all subcarriers becomes smaller with the increasing number of subcarriers in OFDM-TDMA systems. By (15), the scheduling gain tends to 1 when there are infinite independent subcarriers.

B. Impact of Physical-Layer Characteristics on Flow-Level Performance

In this section, the impact of several physical-layer characteristics, including fading speed and frequency selectivity,

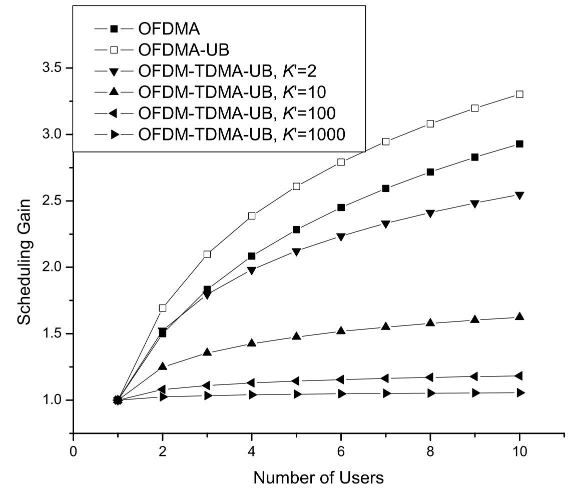

Fig. 2. Theoretical scheduling gain of OFDMA and OFDM-TDMA systems.

on the flow-level performance is examined by assuming a fixed scheduling gain function from the OS algorithm. It will be shown below that the flow-level performance measures behave as convex and supermodular functions of the rate process, which is only impacted by the physical-layer characteristics and independent of the specific scheduling algorithm. Therefore, the OFDMA and OFDM-TDMA systems are not differentiated in the following analysis based on stochastic ordering [26].

We consider a finite-length duration, which is divided into T slots such that the feasible rate remains constant during each time slot. Let $N_p(t)$ be the number of class-p flows at the end of time slot $t \in \{1,\ldots,T\}$. Assume that there is no flow at the beginning of this observation period and the set of flows that arrive at the system during this period is denoted by $\mathcal{N}$. Obviously, $N_p(T)$ is a function of the following r.v.'s : the file size F_i and the feasible rate $\sum_{k=1}^{K} R_i^k(t), t \in \{1,\ldots,T\}$ of each flow $i \in \mathcal{N}$. Note that the variation of physical-layer characteristics for any flow only affects its own rate process, and has no impact on the rate processes of other flows. Without loss of generality, we fix the file sizes and feasible rates of all the flows except flow j to focus on the variation of physical-layer characteristics for only one flow, and denote the rate process of flow j as

$$\boldsymbol{S}_j := \{\sum_{k=1}^{K} R_j^k(1),\ldots,\sum_{k=1}^{K} R_j^k(T)\}.$$

Therefore, $N_p(T)$ is a function of F_j and $\boldsymbol{S}_j$ only. Denote the conditional expectation of $N_p(T)$ given $\boldsymbol{S}_j$ by

$$\mathrm{E}_{F_j}[N_p(T)] := \mathbf{E}\Big[N_p(T)\big|\big(\sum_{k=1}^{K} R_j^k(1),\ldots,\sum_{k=1}^{K} R_j^k(T)\big)\Big]$$

which is a function of the rate process $\boldsymbol{S}_j$. Note that E_{F_j} means that the expectation is taken over the r.v. F_j.

The following theorem gives a formal statement that *the flow-level performance measures behave as convex and supermodular functions of the rate process*, which can be easily derived from the Lemma 2 of [13]. Although the study below only considers the *flow number*, similar results can be extended to other flow-level performance measures such as the *response time* and *flow throughput*.

Theorem 1: Assume the cumulative distribution function (c.d.f.) associated with the random flow size F_j is concave. For all increasing functions $f(\cdot)$, the conditional expectation of the number of flows $\mathrm{E}_{F_j}\big[f\big(N_p(T)\big)\big]$ is a supermodular and convex function of the rate process $\boldsymbol{S}_j$ of flow j.

The assumption on the flow size distribution is satisfied by a broad class of distributions, e.g., exponential and Weibull, etc. The definition of supermodular function is given in Appendix A. Theorem 1 is proved by expressing $\mathrm{E}_{F_j}\big[f\big(N_p(T)\big)\big]$ as the sum of T supermodular and convex functions, where each supermodular and convex function includes the composition of an affine function and a convex function, which denotes the probability that flow j leaves the system at the end of slot t, $t = 1, \ldots, T$.

Next, we show that the physical-layer characteristics, such as fading speed and frequency selectivity, impact the flow-level performance through the change of stochastic orders of the rate process $\boldsymbol{S}_j$. We first arrange the feasible rate of flow j at each subcarrier and time slot into a random matrix

$$\boldsymbol{R}_j = \begin{pmatrix} R_j^1(1) & \ldots & R_j^1(T) \\ \vdots & \ddots & \vdots \\ R_j^K(1) & \ldots & R_j^K(T) \end{pmatrix}.$$

Denote the covariance of any two elements in the same row or column of $\boldsymbol{R}_j$ by $\Psi_r(\Delta t) := \mathrm{cov}\big(R_j^k(t), R_j^k(t+\Delta t)\big)$ and $\Psi_c(\Delta k) := \mathrm{cov}\big(R_j^k(t), R_j^{k+\Delta k}(t)\big)$, respectively.

1) Impact of fading speed: We assume that the channel frequency selectivity is fixed, and examine the impact of fading speed on the flow-level performance only. Let a random matrix $\boldsymbol{R}_j$ representing the feasible rate of flow j be replaced by another random matrix $\widetilde{\boldsymbol{R}}_j = \{\widetilde{R}_j^k(t)\}$, $k = 1, \ldots, K$, $t = 1, \ldots, T$, when the fading speed of flow j is increased while all other conditions are the same. Denote the covariance of any two elements in the same row or column of $\widetilde{\boldsymbol{R}}_j$ by $\widetilde{\Psi}_r(\Delta t)$ and $\widetilde{\Psi}_c(\Delta k)$, respectively.

The two random matrices $\boldsymbol{R}_j$ and $\widetilde{\boldsymbol{R}}_j$ have the following properties:

(i) All the elements of $\boldsymbol{R}_j$ and $\widetilde{\boldsymbol{R}}_j$ are identically distributed;
(ii) Since the fading speed is higher in the latter scenario, $\Psi_r(\Delta t) \geq \widetilde{\Psi}_r(\Delta t)$;
(iii) Since the channel frequency selectivity is the same for both scenarios, $\Psi_c(\Delta k) = \widetilde{\Psi}_c(\Delta k)$.

According to Appendix (B), we have the following theorem.

Theorem 2: The flow-level performance is improved when the fading speed of flow j is accelerated, i.e.,

$$\widetilde{N}_p(T) \leq_{\mathrm{st}} N_p(T), \quad p = 1, \ldots, P \tag{18}$$

where $\widetilde{N}_p(T)$ is the number of class-p flows when the rate process of flow j is

$$\widetilde{\boldsymbol{S}}_j := \{\sum_{k=1}^{K} \widetilde{R}_j^k(1), \ldots, \sum_{k=1}^{K} \widetilde{R}_j^k(T)\}.$$

Under each channel frequency selectivity condition, we define two limit regimes referred to as *fluid* and *quasi-stationary*, where the rate variation speed remains unchanged in the frequency domain, while it is infinitely fast and infinitely slow in the time domain, respectively. Two similar limits have been given in [13] for single-carrier systems.

In the *fluid* limit regime, the rate process of flow j ($\boldsymbol{S}_j$) completely averages out over the time scale of the transmission of data file. Therefore, it can be replaced by a constant according to [14], which is defined as

$$\boldsymbol{S}_j^{\mathrm{fl}} := \{\sum_{k=1}^{K} \mathbf{E}[R_j^k(1)], \ldots, \sum_{k=1}^{K} \mathbf{E}[R_j^k(T)]\}.$$

In the *quasi-stationary* limit regime, the rate process of flow j remains in the initial state during the transmission of data file. Therefore, it also reduces to a constant, which is defined as

$$\boldsymbol{S}_j^{\mathrm{qs}} := \{\sum_{k=1}^{K} R_j^k(1), \ldots, \sum_{k=1}^{K} R_j^k(1)\}.$$

According to the definitions of the two limit regimes, we have

Theorem 3: The flow-level performance is improved or deteriorated when the rate process of flow j is replaced by the corresponding *fluid* and *quasi-stationary* versions, respectively, i.e.,

$$N_p^{\mathrm{fl}}(T) \leq_{\mathrm{st}} N_p(T) \leq_{\mathrm{st}} N_p^{\mathrm{qs}}(T), \quad p = 1, \ldots, P \tag{19}$$

where the superscripts $^{\mathrm{fl}}$ and $^{\mathrm{qs}}$ refer to the system in the *fluid* and *quasi-stationary* limit regimes, respectively.

Similar results from the above Theorems 2 and 3 have been observed in the single-carrier system [13]. However, the proofs of these comparison results are more complex in OFDM systems, which have been given in Appendix (B).

2) Impact of channel frequency selectivity: Let the fading speed be fixed. We investigate how the performance varies with the frequency selectivity of the multipath fading channel. In order to do so, we define a new random matrix $\widehat{\boldsymbol{R}}_j$ to represent the feasible rate of flow j when the frequency selectivity is increased while all other conditions are the same. Denote the covariance of any two elements in the same row or column of $\widehat{\boldsymbol{R}}_j$ by $\widehat{\Psi}_r(\Delta t)$ and $\widehat{\Psi}_c(\Delta k)$, respectively. Then the two random matrices $\boldsymbol{R}_j$ and $\widehat{\boldsymbol{R}}_j$ have the following properties:

(i) All the elements of $\boldsymbol{R}_j$ and $\widehat{\boldsymbol{R}}_j$ are identically distributed;
(ii) Since the fading speed is the same for both scenarios, $\Psi_r(\Delta t) = \widehat{\Psi}_r(\Delta t)$;
(iii) Since the channel frequency selectivity is higher in the latter scenario, $\Psi_c(\Delta k) \geq \widehat{\Psi}_c(\Delta k)$.

Theorem 4: The flow-level performance is improved when the frequency selectivity of flow j is increased, i.e.,

$$\widehat{N}_p(T) \leq_{\mathrm{st}} N_p(T), \quad p = 1, \ldots, P \tag{20}$$

where $\widehat{N}_p(T)$ is the number of class-p flows when the rate process of flow j is

$$\widehat{\boldsymbol{S}}_j := \{\sum_{k=1}^{K} \widehat{R}_j^k(1), \ldots, \sum_{k=1}^{K} \widehat{R}_j^k(T)\}.$$

Similarly, we define two limit regimes, termed *fully-selective* and *flat*, where the rate variation speed remains unchanged in the time domain, while it is infinitely fast and infinitely slow in the frequency domain, respectively. The elements in the same column of $\boldsymbol{R}_j$ are independent in the former regime, and reduces to flat fading where $\boldsymbol{R}_j(t) = (R_j^1(t), \ldots, R_j^1(t))$ in the latter regime. Therefore, the rate processes in the *fully-selective* and *flat* limit regimes can be denoted as

$$\boldsymbol{S}_j^{\text{fsel}} := \{\sum_{k=1}^{K} \overline{R}_j^k(1), \ldots, \sum_{k=1}^{K} \overline{R}_j^k(T)\}$$
$$\boldsymbol{S}_j^{\text{flat}} := \{KR_j^1(1), \ldots, KR_j^1(T)\}$$

where the random vector $\{\overline{R}_j^k(t)\}_{k=1,\ldots,K}$ is the independent version of $\{R_j^k(t)\}_{k=1,\ldots,K}$.

Theorem 5: The flow-level performance can be improved or deteriorated when the rate process of flow j is replaced by the corresponding *fully-selective* and *flat* versions, respectively, i.e.,

$$N_p^{\text{fsel}}(t) \leq_{\text{st}} N_p(t) \leq_{\text{st}} N_p^{\text{flat}}(t), \quad p = 1, \ldots, P \tag{21}$$

where the superscripts $^{\text{fsel}}$ and $^{\text{flat}}$ refer to the system in the *fully-selective* and *flat* limit regimes, respectively.

Now we have four limit regimes for flow-level performance termed *fluid*, *quasi-stationary*, *fully-selective* and *flat*, when the rate process of flow j is replaced by its corresponding limit versions, respectively. Combining these four limit regimes, we can derive simple upper and lower bounds for the flow-level performance, which only depend on easily calculated load factors. The rate processes in the four limit regimes are denoted as follows

$$\boldsymbol{S}_j^{\text{fl,fsel}} := \{\mathbf{E}[\sum_{k=1}^{K} \overline{R}_j^k(1)], \ldots, \mathbf{E}[\sum_{k=1}^{K} \overline{R}_j^k(T)]\}$$
$$\boldsymbol{S}_j^{\text{fl,flat}} := \{\mathbf{E}[KR_j^1(1)], \ldots, \mathbf{E}[KR_j^1(T)]\}$$
$$\boldsymbol{S}_j^{\text{qs,fsel}} := \{\sum_{k=1}^{K} \overline{R}_j^k(1), \ldots, \sum_{k=1}^{K} \overline{R}_j^k(1)\}$$
$$\boldsymbol{S}_j^{\text{qs,flat}} := \{KR_j^1(1), \ldots, KR_j^1(1)\}.$$

According to Theorems 3 and 5, we can derive the following theorem, which compares the performance of different combinations of the limit regimes.

Theorem 6: The performance of different combinations of limit regimes are ranked as follows

$$N_p^{\text{fl}}(T) =_{\text{st}} N_p^{\text{fl,fsel/flat}}(T) \leq_{\text{st}} N_p^{\text{qs,fsel}}(T)$$
$$\leq_{\text{st}} N_p^{\text{qs,flat}}(T), p = 1, \ldots, P. \tag{22}$$

The theorem states that performance in the (*fluid*, *fully-selective*) and (*fluid*, *flat*) limit regimes are statistically the same, both providing an optimistic estimate of performance. The performance in (*quasi-stationary*, *fully-selective*) limit regime is better than that in the (*quasi-stationary*, *flat*) limit regime, while the latter provides a conservative estimate of performance. Therefore, the performance difference between the *fluid* limit regime and *quasi-stationary* limit regime is larger when they are combined with the *flat* limit regime than that when they are with *fully-selective* limit regime. The following corollary follows from the above observation.

Corollary 1: When the channel frequency selectivity is larger, the fading speed has relatively smaller impact on performance.

TABLE I
SIMULATION PARAMETERS

Carrier	2GHz
Bandwidth	10MHz
time slot duration (ms)	0.5
DFT size	1024
Subcarrier separation (kHz)	15
OFDM block duration (μs)	83.34
Number of OFDM symbols	7
Number of useful subcarriers	600
Fading channel model	TU, PA
Average SNR (dB)	0
Velocity (km/h)	3, 30

The above discussion didn't consider the transmission time of data files. The following theorem shows that the data file sizes (in transmission time) have an effect on the degree of impact of physical-layer characteristics on performance.

Theorem 7: When the transmission time of the data file is larger, the physical-layer characteristics, e.g., fading speed and channel frequency selectivity, have relatively smaller impact on performance.

Let the rate processes of all the P-class flows be replaced by the different combinations of limit regimes. The performance in these limit regimes can be easily derived by replacing C_p and ρ for $p = 1, \ldots, P$ in (6), (7), (8) with C_p^{fl}, $C_p^{\text{qs,fsel}}$, $C_p^{\text{qs,flat}}$ and ρ^{fl}, $\rho^{\text{qs,fsel}}$, $\rho^{\text{qs,flat}}$, respectively. From the above discussion, the instantaneous rate in the *fluid* regime can be replaced by the time average rate $\mathbf{E}[\sum_{k=1}^{K} R_p^k(t)]$, and the instantaneous rate in the quasi-stationary regime is the rate at the initial state $\sum_{k=1}^{K} R_p^k(0)$, where $R_p^k(0) =_{\text{st}} R_p^k(t)$. Therefore, C_p^{fl} and ρ^{fl} equal to C_p and ρ derived in Section III(B), respectively. On the other hand, $\rho^{\text{qs}} = \sum_{p=1}^{P} \lambda_p \mathbf{E}[F_p] / \big(KC_p^{\text{qs}} \sqrt[n]{\prod_{i=1}^{n} G(i)}\big)$, where $C_p^{\text{qs}} = \mathbf{E}[1/\sum_{k=1}^{K} R_p^k(0)]^{-1}$. Therefore, $C_p^{\text{qs,flat}} = \mathbf{E}[1/KR_p^1(0)]^{-1}$ and $C_p^{\text{qs,fsel}} = \mathbf{E}[1/\sum_{k=1}^{K} \overline{R}_p^k(0)]^{-1}$.

The proofs of Theorems 4-7 are given in Appendix (B).

IV. SIMULATION RESULTS

A. Parameter Setting

The analytical performance is illustrated and verified by simulations in this section. The simulation parameters are given in Table I [27]. The OFDM system has $N = 600$ available sub-carriers with DFT size of 1024. Multipath Rayleigh fading channels are considered, with each independent fading path generated by the Jakes Model using a U-shape Doppler power spectrum [28].

B. Scheduling Gain

This set of simulations compare the scheduling gains of the PF algorithm in OFDMA and OFDM-TDMA systems, and verify the analytical results in Section IV(A). The scheduling gain is calculated as the ratio between the average throughput of PF algorithm and RR algorithm. In order to examine the impact of the number of independent subcarriers on the scheduling gain of the OFDM-TDMA system, the 600 subcarriers are divided into 24 chunks, with each chunk consisting of

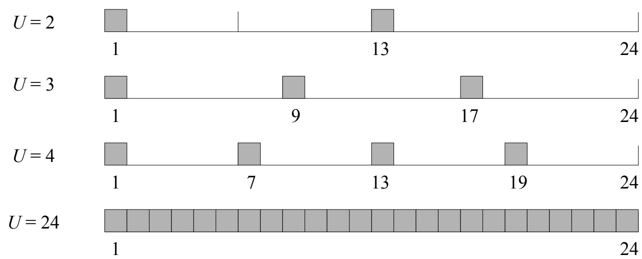

Fig. 3. Distribution of chunks in use with a total of 24 chunks.

25 subcarriers as defined in [27]. We assume that there exist circumstances when not all the chunks are in use, and perform simulations to compare the scheduling gains when the number of chunks in use (U) are increased from 2 to 24. The chunks in use are not selected continuously, but from chunk 1 with an increment of (total chunk number/U), as shown in Fig.3. For example, when U is 2, chunk 1 and chunk 13 are selected. In this way, it can be guaranteed that when the value of U is small, it can accurately represent the number of independent subcarriers in the system.

The simulation results in Fig. 4 show the scheduling gains of the PF algorithm in OFDMA and OFDM-TDMA systems when the numbers of chunks in use (U) are 2, 3, 4 and 24. In the figure, the theoretical results for OFDMA and the analytical upper bound for OFDM-TDMA are derived based on (13) and (16), respectively, where K' represents the number of independent subcarriers. It can be seen that scheduling gain for OFDMA system is approximately the same as the theoretical results. Furthermore, the variation in the number of chunks in use has little impact on the performance. The scheduling gain for the OFDM-TDMA system, on the other hand, decreases significantly when the number of chunks in use increases from 2 to 4. This is in accordance with the analytical results that the scheduling gain of the OFDM-TDMA system decreases with the increase of the number of independent subcarriers. The theoretical upper bounds are proved to be accurate when the number of chunks in use are 2, 3 and 4, respectively. Compared with Fig. 2, it can be seen that difference between the upper bound and simulation results of the scheduling gain in the OFDM-TDMA system is approximately the same as the difference between the upper bound and the theoretical result. Note that when $U = 24$ and $U = 4$, the scheduling gains of OFDM-TDMA system are nearly the same. Since the scheduling gains remain approximately constant when $U \geq 4$, the simulation results for $4 < U < 24$ are omitted.

C. Flow-Level Performance

This set of simulations evaluate the impact of different physical-layer characteristics on flow level performance of the OFDM system and verify the analytical results in Section IV(B).

The physical-layer parameters of the OFDM system is described in Section V(A). The instantaneous rate in the simulation is logarithmic as the instantaneous SNR: $R = C \times \log_2(1 + \mathrm{SNR})$, where $C = 15$kbps is the subcarrier separation. The mean file sizes are set to be 48kbits and 480kbits, which can be considered as the sizes of HTTP objects and FTP files, respectively [29].

Fig. 5 compares the flow-level performance for varying arrival rates under different fading speed and channel frequency selectivity. The channel types are set to be PA (pedestrian A) and TU (Typical Urban), respectively, which belong to the tapped-delay-line channel models widely used in 3G LTE system evaluation [27]. The channel frequency selectivity is $\mathrm{TU} > \mathrm{PA}$, due to the difference in maximum multipath delays. A detailed treatment of the propagation models is given in Table II. The MS velocity is set to be 3 and 30, where increasing MS velocity leads to increased fading speed. As expected from the analytical results in Section IV(B), the fluid regime provides an optimistic estimate of the throughput. By observing Figs. 5(a) and 5(b), which respectively show the mean flow throughput and flow number when the files size is 48kbits, it can be seen that 1) increasing fading speed improves the performance when the frequency selectivity is fixed, and 2) increasing channel frequency selectivity improves the performance when the fading speed is fixed. Furthermore, the performance is less sensitive to the fading speed when the frequency selectivity is high. Since the throughput improvement under PA is larger than that under TU, when the MS velocity is increased from 3km/h to 30km/h. Finally, the impact of physical-layer characteristics on flow-level performance is less obvious when the file size is 480kbits, as shown in Figs. 5(c) and 5(d). This matches the analytical results given in Theorem 7.

V. Conclusions

In this paper, a flow-level model for performance analysis in OFDM systems has been proposed by extending the multi-class Processor-Sharing model for single-carrier systems to OFDM systems. Based on this model, we analyze and compare the scheduling gains achieved by proportional fair schedulers in both OFDMA and OFDM-TDMA systems. Moreover, the impacts of several physical-layer characteristics on the flow-level performance of OFDM systems are then evaluated using stochastic comparison, and the upper and lower bounds for the flow-level performance of OFDM systems have been derived. Both analytical and simulation results show that

- the scheduling gain achieved in the OFDMA system is larger than that of the OFDM-TDMA system;
- faster fading speed and higher channel frequency selectivity can both improve performance;
- fading speed variation has less impact on the performance in case of a higher channel frequency selectivity;
- *fluid* and (*flat*, *quasi-stationary*) limit regimes provide optimistic and conservative performance estimates for the OFDM system, respectively. The performance in both limit regimes only depends on appropriately defined traffic loads ρ^{fl} and $\rho^{\mathrm{qs,flat}}$.

Our future work will focus on quantifying the impacts of these physical-layer characteristics on the performance, and improving the accuracy of this first-order approximation by incorporating the effects of service variability more precisely.

Acknowledgement

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TABLE II
CHANNEL MODELS IN SIMULATION

Tap	PA		TU	
	Relative delay (ns)	Average power (dB)	Relative delay (ns)	Average power (dB)
1	0	0.0	0	0
2	0	-6.51	200	3
3	110	-16.21	600	1
4	190	-25.71	1600	-3
5	410	-29.31	2400	-5
6			5000	-7

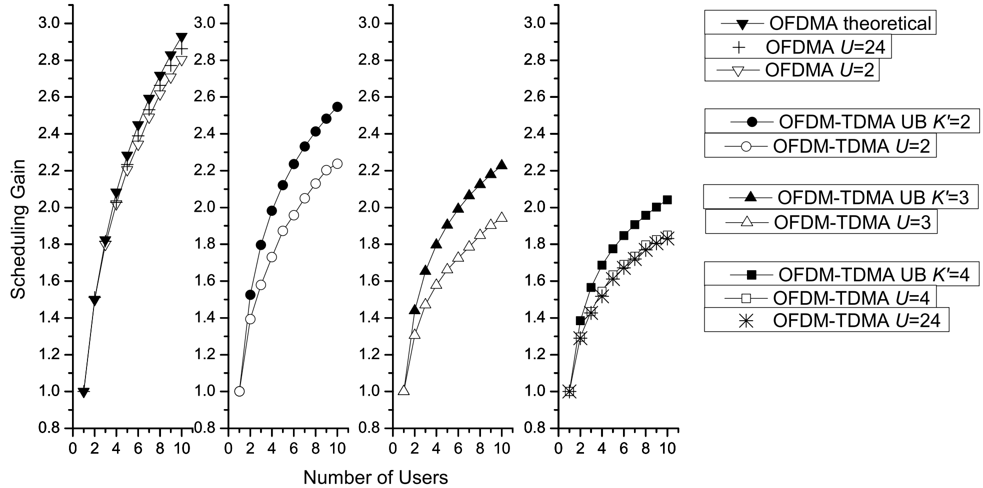


Fig. 4. Scheduling gain of OFDMA and OFDM-TDMA systems with different number of chunks in use.

Chao Yang from Beijing University of Posts & Telecommunications for the simulation code and the anonymous reviewers for their valuable comments.

APPENDIX

A. Basic Concepts of Stochastic Ordering

We introduce some basic definitions and properties of stochastic ordering from [26].

Definition 1: For random variables (vectors) $\mathbf{X}$ and $\mathbf{Y}$, define

$$\mathbf{X} \le_{\rm st} (\text{or} \le_{\rm cx})\mathbf{Y} \quad \text{iff} \quad \mathbf{E}\phi(\mathbf{X}) \le \mathbf{E}\phi(\mathbf{Y}), \\ \forall \text{ increasing (or convex) functions } \phi$$

provided the expectations exist.

The stochastic order $\le_{\rm st}$ and the convex order $\le_{\rm cx}$ compare the magnitude and variability of random variables (vectors), respectively. $\mathbf{X} \le_{\rm st} \mathbf{Y}$ means $\mathbf{X}$ is less likely than $\mathbf{Y}$ to take large values. On the other hand, $\mathbf{X} \le_{\rm cx} \mathbf{Y}$ means $\mathbf{X}$ is "less variable" than $\mathbf{Y}$, and we have

$$\mathbf{E}[\mathbf{X}] = \mathbf{E}[\mathbf{Y}], \quad \mathbf{Var}[\mathbf{X}] \le \mathbf{Var}[\mathbf{Y}]. \tag{23}$$

Let $\mathbf{e}_i$ denote the i-th n-dimensional unit vector. For $\mathbf{x} = (x_1, \ldots, x_n)$ and an arbitrary function $\phi : R^n \to R$, we define $\Delta_i^\varepsilon \phi = \phi(\mathbf{x} + \varepsilon \mathbf{e}_i) - \phi(\mathbf{x})$.

Definition 2: A function $\phi : R^n \to R$ is said to be supermodular if

$$\Delta_i^\varepsilon \Delta_j^\delta \phi(\mathbf{x}) \ge 0$$

holds for all $\mathbf{x} \in R^n, 1 \le i \le j \le n$ and $\varepsilon, \delta > 0$.

The supermodular order $\le_{\rm sm}$ is defined by substituting the 'increasing' or 'convex' functions in Definition 1 with the 'supermodular' function.

Definition 3: For random vectors $\mathbf{X}$ and $\mathbf{Y}$ with the same marginal distributions, $\mathbf{X}$ is said to be less correlated than $\mathbf{Y}$, written as $\mathbf{X} \le_{\rm c} \mathbf{Y}$, if

$$\operatorname{cov}\big(\phi(X_i)\eta(X_j)\big) \le \operatorname{cov}\big(\phi(Y_i)\eta(Y_j)\big)$$

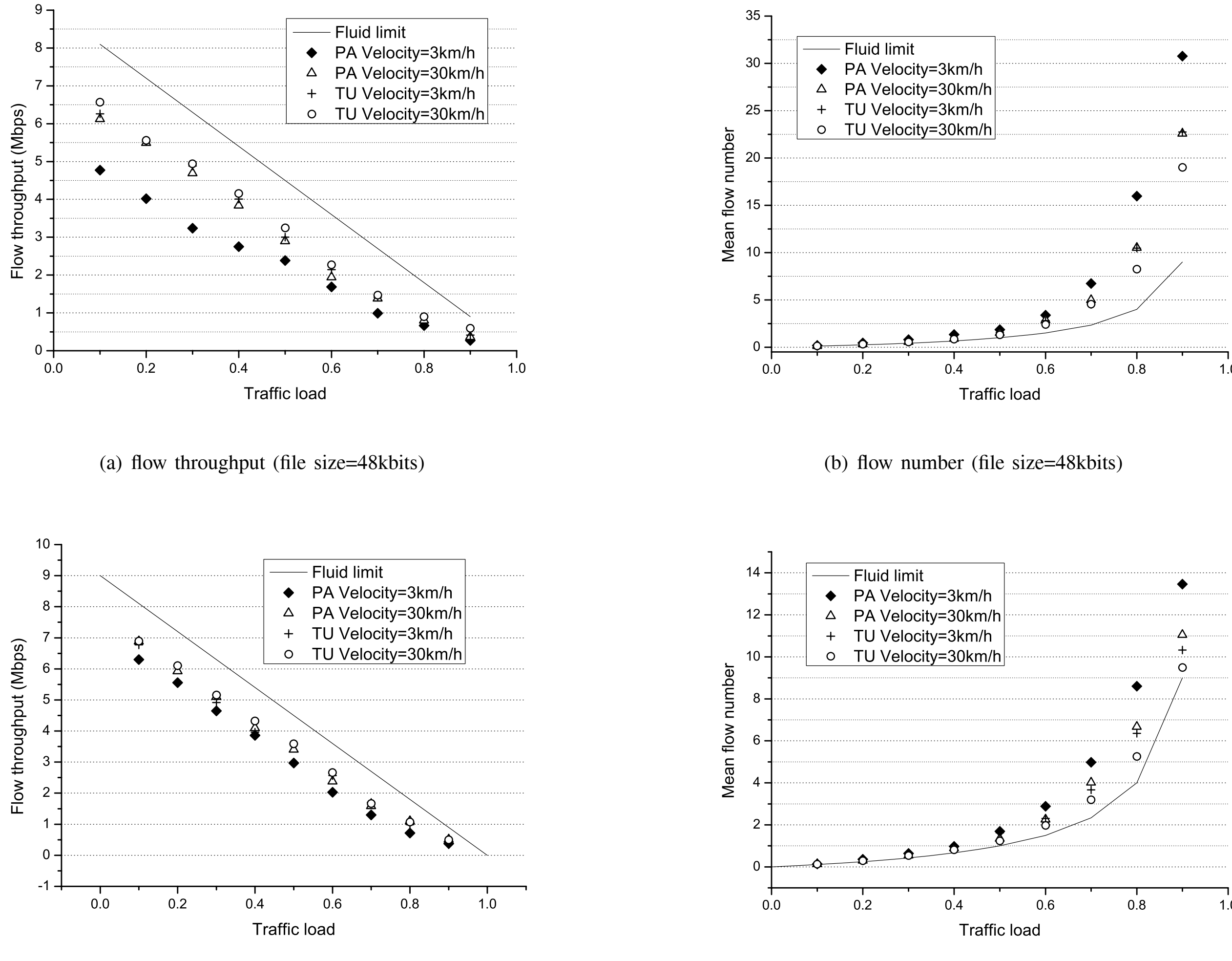
where $\operatorname{cov}(\cdot)$ denotes covariance and both ϕ and η are increasing functions for which the covariance exists.

Both the supermodular order $\le_{\rm sm}$ and the correlation order $\le_{\rm c}$ are introduced to mathematically describe the property of dependencies among the r.v.'s within a random vector.

B. Proofs of Theorems 2-7

The proofs of Theorems 2-5 are to verify the $\le_{\rm st}$ ordering of the flow numbers $N_p^*(T)$, $p = 1, \ldots, P$, with respect to different physical-layer conditions, which lead to different rate processes $\boldsymbol{S}_j^*$. Here, $\big(N_p^*(T), \boldsymbol{S}_j^*\big)$ represents any pair of $\big(N_p(T), \boldsymbol{S}_j\big)$, $\big(\widetilde{N}_p(T), \widetilde{\boldsymbol{S}}_j\big)$, $\big(\widehat{N}_p(T), \widehat{\boldsymbol{S}}_j\big)$, $\big(N_p^{\rm fl}(T), \boldsymbol{S}_j^{\rm fl}\big)$, $\big(N_p^{\rm qs}(T), \boldsymbol{S}_j^{\rm qs}\big)$, $\big(N_p^{\rm fsel}(T), \boldsymbol{S}_j^{\rm fsel}\big)$, $\big(N_p^{\rm flat}(T), \boldsymbol{S}_j^{\rm flat}\big)$, $\big(N_p^{\rm fl,fsel}(T), \boldsymbol{S}_j^{\rm fl,fsel}\big)$, $\big(N_p^{\rm fl,flat}(T), \boldsymbol{S}_j^{\rm fl,flat}\big)$, $\big(N_p^{\rm qs,fsel}(T), \boldsymbol{S}_j^{\rm qs,fsel}\big)$, or $\big(N_p^{\rm qs,flat}(T), \boldsymbol{S}_j^{\rm qs,flat}\big)$.

According to the definition of $\le_{\rm st}$ ordering, it is sufficient and necessary to prove that for any increasing function f, $\mathbf{E}\big[f\big(N_p^*(t)\big)\big]$ obeys the same ordering. From Theorem 1,



(a) flow throughput (file size=48kbits)

(b) flow number (file size=48kbits)

(c) flow throughput (file size=480kbits)

(d) flow number (file size=480kbits)

Fig. 5. Flow-level performance with different fading speed and frequency selectivity (TU>PA).

since for any increasing function f, $\mathrm{E}_{F_j}\left[f\left(N_p^*(T)\right)\right]$ is a supermodular and convex function of the rate process $\boldsymbol{S}_j^*$. By the property of conditional expectation, we have

$$\mathbf{E}\left[f\left(N_p^*(T)\right)\right] = \mathbf{E}\Big[\mathrm{E}_{F_j}\left[f\left(N_p^*(T)\right)\right](\boldsymbol{S}_j^*)\Big]. \tag{24}$$

Thus, in order to prove the $\leq_{\rm st}$ ordering of the flow numbers, it is sufficient to show that the rate processes hold relative order in terms of supermodular or convex ordering. For example, it is sufficient to show $\widetilde{\boldsymbol{S}}_j \leq_{\rm sm} \boldsymbol{S}_j$ or $\widetilde{\boldsymbol{S}}_j \leq_{\rm cx} \boldsymbol{S}_j$ in order to prove $\widetilde{N}_p(T) \leq_{\rm st} N_p(T)$.

1) Proof of Theorem 2: We first introduce two lemmas, which are on the equality of correlation order and supermoduler order [30], and on the covariance between functions [31].

Lemma 1: Suppose $\mathbf{X}$ and $\mathbf{Y}$ are n-dimensional random vectors. If $\mathbf{X} \leq_{\rm c} \mathbf{Y}$, then $\mathbf{X} \leq_{\rm sm} \mathbf{Y}$.

Lemma 2: Let X and Y be two random variables with continuous cumulative distribution function (cdf) $H(x,y)$ and marginal cdf's $F(x)$ and $G(y)$, respectively. Assume ϕ and η are monotonic functions. Then,

$$\mathrm{cov}\big(\phi(X), \eta(Y)\big) = \int \big(H(x,y) - F(x)G(y)\big) d\phi(x)\eta(y). \tag{25}$$

With Lemma 1, it is sufficient to show that

$$\widetilde{\boldsymbol{S}}_j \leq_{\rm c} \boldsymbol{S}_j. \tag{26}$$

Compared with $\mathbf{R}_j$, since the channel frequency selectivity of $\widetilde{\mathbf{R}}_j$ is the same and only the fading speed of flow j is accelerated, we have

$$\{R_j^1(t), \ldots, R_j^K(t)\} =_{\rm st} \{\widetilde{R}_j^1(t), \ldots, \widetilde{R}_j^K(t)\}$$

which means that the marginal distributions of $\boldsymbol{S}_j$ and $\widetilde{\boldsymbol{S}}_j$ are identical, i.e.,

$$\sum_{k=1}^{K} R_j^k(t) =_{\rm st} \sum_{k=1}^{K} \widetilde{R}_j^k(t), \; t = 1, \ldots, T. \tag{27}$$

The covariance of $\boldsymbol{S}_j$ is

$$\mathrm{cov}\Big(\sum_{k=1}^{K} R_j^k(t), \sum_{k=1}^{K} R_j^k(t+\Delta t)\Big)$$
$$= \sum_{k=1}^{K}\sum_{k'=1}^{K} \mathrm{cov}\big(R_j^k(t), R_j^{k'}(t+\Delta t)\big)$$

$$= \sum_{k=1}^{K} \sum_{k'=1}^{K} \Psi_r(\Delta t) \Psi_c(k - k'). \tag{28}$$

The second equality in (28) is based on the factorable form of (3). It can be proved that the covariance function of SNR is simply the square of covariance function of channel gain [34]. Similarly, for $\widetilde{\mathbf{R}}_j$, we have

$$\mathrm{cov}\big(\sum_{k=1}^{K} \widetilde{R}_j^k(t), \sum_{k=1}^{K} \widetilde{R}_j^k(t+\Delta t)\big) = \sum_{k=1}^{K} \sum_{k'=1}^{K} \widetilde{\Psi}_r(\Delta t) \widetilde{\Psi}_c(k-k'). \tag{29}$$

By the properties (ii) and (iii) of $\mathbf{R}_j$ and $\widetilde{\boldsymbol{R}}_j$ in Section IV(B), we have $\Psi_r(\Delta t) \geq \widetilde{\Psi}_r(\Delta t)$ and $\Psi_c(\Delta k) = \widetilde{\Psi}_c(\Delta k)$. Therefore,

$$\begin{aligned} &\mathrm{cov}\big(\sum_{k=1}^{K} R_j^k(t), \sum_{k=1}^{K} R_j^k(t+\Delta t)\big) \\ &\geq \mathrm{cov}\big(\sum_{k=1}^{K} \widetilde{R}_j^k(t), \sum_{k=1}^{K} \widetilde{R}_j^k(t+\Delta t)\big). \end{aligned} \tag{30}$$

For increasing functions ϕ and η, their derivatives $\phi' > 0$ and $\eta' > 0$. According to Lemma 2, (30) leads to

$$\begin{aligned} &\mathrm{cov}\Big(\phi\big(\sum_{k=1}^{K} R_j^k(t)\big), \eta\big(\sum_{k=1}^{K} R_j^k(t+\Delta t)\big)\Big) \\ &\geq \mathrm{cov}\Big(\phi\big(\sum_{k=1}^{K} \widetilde{R}_j^k(t)\big), \eta\big(\sum_{k=1}^{K} \widetilde{R}_j^k(t+\Delta t)\big)\Big). \end{aligned} \tag{31}$$

Combining (27) and (31), (26) can be proved according to Definition 3, and therefore by Lemma 1 we have

$$\widetilde{\boldsymbol{S}}_j \leq \mathrm{sm}\, \boldsymbol{S}_j. \tag{32}$$

The similar analysis procedure can be applied to the proofs of all other theorems, except that each proof may be based on different lemmas. In the following, we omit the detailed analysis procedure and only present the necessary lemmas for each proof.

2) Proof of Theorem 3: The proof of Theorem 3 is based on the following two lemmas [13]:

Lemma 3: Let $X_1, \ldots, X_n$ be identically distributed random variables. Then $(\mathbf{E}[X_1], \ldots, \mathbf{E}[X_n]) \leq_{\mathrm{sm}} (X_1, \ldots, X_n)$.

Lemma 4: (Lorentz inequality) Let $X_1, \ldots, X_n$ be identically distributed random variables. Then $(X_1, \ldots, X_n) \leq_{\mathrm{sm}} (X_1, \ldots, X_1)$.

3) Proof of Theorem 4: Theorem 4 can be proved based on the following two lemmas on stochastic comparison [32], [33].

Lemma 5: Assume that there are two random vectors $\boldsymbol{X} = (X_1, \ldots, X_n)$ and $\boldsymbol{Y} = (Y_1, \ldots, Y_n)$, we have

$$\boldsymbol{X} \leq_{\mathrm{sm}} \boldsymbol{Y} \Longrightarrow \sum_{i=1}^{n} X_i <_{\mathrm{cx}} \sum_{i=1}^{n} Y_i.$$

Lemma 6: Let $\boldsymbol{X} = (X_1, \ldots, X_n)$ and $\boldsymbol{Y} = (Y_1, \ldots, Y_n)$ be random vectors having multivariate exchangeable distributions with $\mathbf{E}[X_i] = \mu_{\boldsymbol{X}}$, $\mathbf{Var}[X_i] = \sigma_{\boldsymbol{X}}^2$, $\mathrm{cov}[X_i, X_j] = \rho_{\boldsymbol{X}} \sigma_{\boldsymbol{X}}^2$, $\mathbf{E}[Y_i] = \mu_{\boldsymbol{Y}}$, $\mathbf{Var}[Y_i] = \sigma_{\boldsymbol{Y}}^2$, $\mathrm{cov}[Y_i, Y_j] = \rho_{\boldsymbol{Y}} \sigma_{\boldsymbol{Y}}^2$. Then, the following conditions are equivalent:

(i) $\mu_{\boldsymbol{X}} = \mu_{\boldsymbol{Y}}$, $\sigma_{\boldsymbol{X}}^2 \leq \sigma_{\boldsymbol{Y}}^2$, and

$$\frac{\sigma_{\boldsymbol{X}}^2}{\sigma_{\boldsymbol{Y}}^2} \leq \max\left\{\frac{1-\rho_{\boldsymbol{Y}}}{1-\rho_{\boldsymbol{X}}}, \frac{1+(n-1)\rho_{\boldsymbol{Y}}}{1+(n-1)\rho_{\boldsymbol{X}}}\right\};$$

(ii) $\boldsymbol{X} \leq_{\mathrm{cx}} \boldsymbol{Y}$.

4) Proof of Theorem 5 and 6: The proof of Theorem 5 is based on the following lemma on supermodular comparison [32].

Lemma 7: Let $\boldsymbol{X} = (X_1, \ldots, X_n)$ be a random vector and let $\boldsymbol{Y} = (Y_1, \ldots, Y_n)$ be a vector of independent random variables such that, marginally, $X_i =_{\mathrm{st}} Y_i$, $i = 1, \ldots, n$. If $X_1, \ldots, X_n$ are weakly positively associated, then $\boldsymbol{X} \geq_{\mathrm{sm}} \boldsymbol{Y}$.

The proof of Theorem 6 can be obtained by combining the proofs of Theorem 3 and 5.

5) Proof of Theorem 7: First assume that the transmission time of flow j is T time slots. Consider the case when the fading speed is relatively slow and the rate process within T time slots can be considered constant as in the quasi-stationary limit regime, i.e., the worst case scenario. Then we increase the transmission time of flow j to $2T$ time slots. Assume that the rate process within duration $\{T, \ldots, 2T\}$ varies from those within duration $\{1, \ldots, T\}$. Therefore, the supermoduler order of the rate process of flow j is smaller than that of the quasi-stationary limit regime, which means that the performance of flow j is better than the worst case. Therefore, when the transmission time of flow j is larger, the performance difference between flow j and the fluid limit regime becomes smaller.

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