Semiblind Channel Estimation for Pulse-Based Ultra-Wideband Wireless Communication Systems

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***Abstract*—This paper proposes an H_∞ based semiblind channel estimation algorithm for pulse-based ultra-wideband (UWB) wireless communication systems. In the proposed scheme, sparsely inserted periodic pilot symbols are exploited to adapt to not only the time-varying channel fading and noise processes but to their changing statistics and potential external disturbances, such as interference. While the existing optimal filtering-based channel estimation schemes, which are optimized mostly for traditional narrowband or wideband systems, require *a priori* knowledge of the channel and noise statistics, the proposed scheme does not. By further making full use of the channel characteristics unique in UWB systems, the proposed method is thus especially useful for robust operation in the highly frequency-selective UWB indoor channels for which the channel statistics are environment-dependent, and the noise processes do not necessarily satisfy the white Gaussian distribution in the presence of potential narrowband and multiuser interferences. Performance gain of the proposed scheme over the least square method, an existing technique that could also be applied to UWB channels with unknown statistics, and the Wiener filter-based algorithm is also provided.**

***Index Terms*—Channel estimation, H_∞ filtering, ultra-wideband (UWB), wireless communications.**

I. INTRODUCTION

ULTRA-WIDEBAND (UWB) systems could be implemented using a pulse-based approach [1]–[3] or an orthogonal frequency-division multiplexing (OFDM) based approach [4], [5]. In a pulsed UWB system, bandpass pulses of extremely short duration, typically in the range of a fraction of a nanosecond to a few nanoseconds, are used for information transmission. UWB systems operate in the 3.1–10.6 GHz spectrum allowed by the Federal Communications Commission (FCC) [6] on an unlicensed basis. The ultrawide bandwidth and ultralow transmission power density (-41.25 dBm/MHz for indoor applications) make UWB technology attractive for high-speed, short-range (e.g., indoor) wireless communications. Each of the two leading candidate UWB technologies has its advantages and disadvantages for communications in multipath environments. The advantages of pulsed UWB include an extremely simple transmitter and rich resolvable multipath components that enable the receiver to exploit multipath diversity and capture multipath energy.

To fully exploit multipath diversity and effectively capture multipath energy, however, most viable receivers, such as a RAKE receiver, require a critical process: channel estimation. Channel estimation has been studied extensively in previous research, mainly for narrowband systems or wideband (e.g., direct sequence spread spectrum) systems. Currently, we are aware of only very limited research on channel estimation specifically for UWB systems [7], [8]. In [7], a method based on the maximum likelihood criterion was investigated. In [8], two algorithms (a successive channel estimation algorithm and a sliding window algorithm) were proposed. The basic principle applied in these estimation algorithms is to minimize the variance of the estimation errors, assuming known statistics of the fading channel and the additive noise processes.

Although other existing channel estimation schemes originally proposed for narrowband systems can be modified for UWB systems, the fundamental differences between UWB systems and narrowband or spread spectrum wideband systems significantly degrade the effectiveness and efficiency of such applications, and they must be considered in UWB channel estimation. The first major difference is that for UWB indoor communications, the channel is environment dependent (e.g., geometric arrangement of room, and materials of walls and furniture). Thus, channel statistics such as the channel space-time correlation function will be different in a different indoor environment. Although some research has been dedicated to modeling UWB fading channels [5], [9], [10], currently, there are no commonly accepted time-varying fading models for indoor lognormal fading channels. Moreover, UWB systems are deployed to overlay with legacy narrowband systems, which will introduce narrowband interference (NBI) to the UWB system. Although NBI will appear to be noise-like at the receiver matched filter output, it does change the property of the noise component so that it may not be white Gaussian and is probably difficult to predict. This difference makes most existing optimal filtering based channel estimation schemes that require *a priori* knowledge of statistics of the channel and noise processes unattractive. Although some of the existing algorithms for narrowband systems incorporate procedures to adaptively update the channel statistics so that the channel statistics dependent estimation algorithm will work, significant degradation in

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channel coefficient estimation quality is expected in the presence of channel statistics errors [13].

The second major difference is that a UWB channel is a highly frequency-selective channel with, typically, a large number of multipath components which arrive in cluster [10]. The channel estimator not only needs to (desirably) estimate a large number of coefficients for each symbol but also needs to overcome the potential problems of clustering path arrivals. Therefore, a robust and yet simple channel estimator, which does not depend on *a priori* knowledge of the channel and noise statistics, is very desirable for UWB wireless communication systems.

In this paper, an H_∞ based channel estimation algorithm for pulsed ultra-wideband (UWB) wireless communication systems in slowly fading indoor environments is investigated. This scheme incorporates sparsely inserted periodic pilot symbols in the transmitted data stream for the channel estimation at the receiver end. For each pilot symbol, an H_∞ algorithm is applied to minimize the worst effects of external disturbances (including channel modeling error and additive noise) on the estimation quality. The algorithm is semiblind because, other than the pilot symbols, it does not require any other information such as channel fading and noise statistics. The proposed algorithm exhibits strong robustness in channel estimation, which makes it more suitable for practical UWB wireless communication systems.

The paper is organized as follows. Section II gives the model of a pulsed ultra-wideband (UWB) system that employs pulse position modulation (PPM) for data modulation and time hopping (TH) for multiple access. The derivation of the proposed H_∞ estimation algorithm is discussed in detail in Section III. In Section IV, the performance of the H_∞ estimation algorithm in terms of mean square error (MSE) and bit error rate (BER) is studied via simulation. For comparison purposes, the performance of the least square method, an existing technique that could also be applied to UWB channels with unknown channel statistics, and the Wiener filter-based algorithm, is also provided and compared. Concluding remarks are given in Section V.

II. System Model

In binary PPM UWB signaling, information bits $d[i] \in \{0, 1\}$ are transmitted over a train of ultrashort pulses $p(t)$ with pulse width T_p. An FCC spectral compliant pulse $p(t)$ could be obtained by carrier-modulating a baseband pulse, such as a rectangular or a Gaussian pulse [3]. Alternatively, it could be obtained by bandpass filtering a baseband signal of ultrawide bandwidth. We assume that the pulse $p(t)$ has a unit energy, i.e., $\int_{-\infty}^{\infty} p^2(t)dt = 1$. An information bit "0" is represented by a frame of pulses without any delay, while a bit "1" is represented by the same frame of pulses, but with a delay Δ relative to the time reference. The bit interval T_b is much larger than T_p, resulting in a low duty cycle transmission form. Each train of pulses is time hopped (TH) to accommodate multiple access needs. With energy E_p per pulse, the transmitted PPM UWB waveform with TH for a particular user (omitting user index) is written in a general form as

$$s(t) = \sqrt{E_p} \sum_{i=-\infty}^{\infty} \sum_{j=0}^{N_s-1} p(t - iT_b - jT_f - c_jT_c - \Delta d[i]) \tag{1}$$

where $\{c_j\}$ is the N_s-length pseudorandom time-hopping sequence whose sequence element is an integer in the range of $0 \le c_j \le N_h - 1, T_c$ is the duration of addressable time delay bins or chip duration, T_f is the nominal pulse repetition interval, and $T_b = N_sT_f$ is the symbol duration. The ratio T_p/T_f is defined as the duty cycle. In the preceding transmitted signal model, the symbol repetition factor is arbitrarily chosen to be the code sequence length N_s, implying that there are N_s pulses transmitted during each symbol interval. However, this is not a requirement for achieving effective multiple-access capability, as even if pulses were not repeated (i.e., N_s is set to 1), multiple-access capability can be achieved through the use of a unique user-specific pseudorandom hopping sequence. Binary information bits $d[i]$ are modeled as independent and identically distributed (i.i.d.) random variables. There are many appropriate choices of the time shifts Δ in PPM. For orthogonal signaling, Δ is chosen to be such that $\int_{-\infty}^{\infty} p(t)p(t-\Delta)dt = 0$. Such Δ may exist between $0 < \Delta < T_p$, but we will use a value $\Delta \ge T_p$.

Depending on the relative values of T_p, T_c, and T_f, pulse overlap may occur, which will result in undesirable intersymbol interference (ISI) and make the detection process complex. The minimum distance between the last pulse in a symbol interval and the first pulse in the next symbol interval is easily calculated from (1) to be $T_{\min} = T_f - \Delta - T_c(c_{N_s-1} - c_0)$. Thus, we assume that

$$T_c > 2T_p \tag{2a}$$

$$T_{\min} > 2T_p. \tag{2b}$$

Equations (2a) and (2b) guarantee that pulses of adjacent symbols do not overlap at transmission time.

The superfine time resolution in a pulsed UWB system typically results in a much greater number of resolvable paths in the UWB system than in traditional narrowband or wideband systems. Additionally, the fading amplitude of channel coefficients in indoor environments does not follow the well-addressed Rayleigh distribution for a typical mobile channel. Despite the significant differences between UWB and narrowband behaviors, existing work (e.g., [9] and [10]) has shown that a tapped-delay-line model with time-varying coefficients and tap spacing is still appropriate to describe the UWB channels. The signal emerging from the channel after convolving with the transmitted pulse $p(t)$ is expressed as

$$h(t) = \sum_{l=0}^{L-1} h_l(t)p(t - \tau_l) \tag{3}$$

where $h_l(t)$ and τ_l are the channel fading process and the delay of the lth path, respectively, and L is the total number of resolvable multipath components with τ_{L-1} being the maximum delay. A key feature of this paper is that we do not impose any

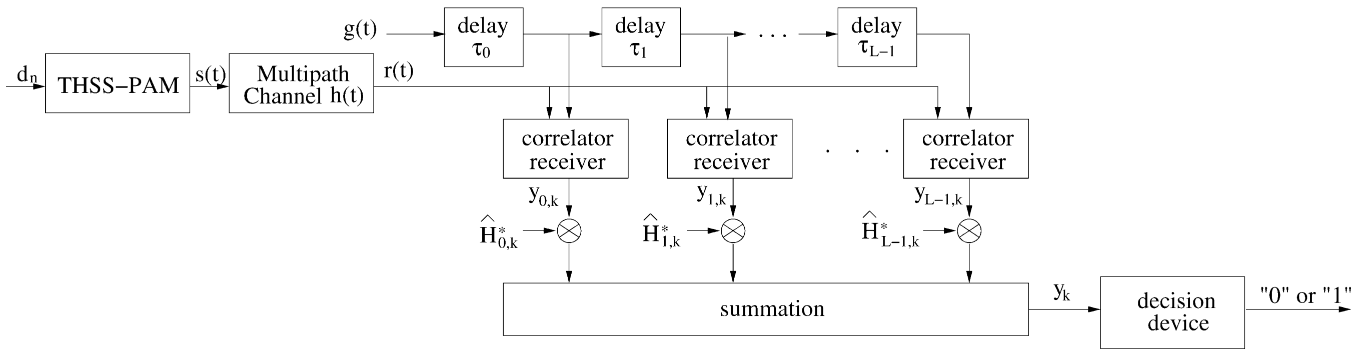

Fig. 1. Transceiver structure of the UWB system.

assumptions on the statistics of fading process $h_l(t)$ in formulating the channel estimation algorithm.

From (1) and (3), the received signal at the receiver input can be expressed as

$$r(t) = \sqrt{E_p} \sum_{i=-\infty}^{\infty} \sum_{j=0}^{N_s-1} \sum_{l=0}^{L-1} h_l(t) \times p(t - iT_b - jT_f - c_jT_c - \Delta d[i] - \tau_l) + n(t) \quad (4)$$

where $n(t)$ is the background noise that may include narrowband and multiuser interferences, if there are any in the system. A RAKE receiver structure is applied to capture multipath energy.

The received signal $r(t)$ is passed through a single matched filter and then sampled according to the relative delays of different paths. An alternative implementation of such a receiver structure is shown in Fig. 1, where a bank of correlators corresponding to multipath delays $\tau_0, \tau_1, \ldots, \tau_{L-1}$ are applied to capture multipath energy. If the symbol rate is such that the channel excess delay does not cause inter-symbol interference, which is assumed true in this paper, the received signal waveform can be simply described by the following one-shot model with period $t \in [iT_b, (i+1)T_b)$:

$$\begin{aligned} r_i(t) &= r(t + iT_b) \\ &= \sqrt{E_p} \sum_{j=0}^{N_s-1} \sum_{l=0}^{L-1} h_l(t) \\ &\quad \times p(t - jT_f - c_jT_c - \Delta d[i] - \tau_l) + n_i(t) \\ &\quad t \in [0, T_b). \end{aligned} \quad (5)$$

The optimal receiver for the single-user PPM system is a correlation receiver with a template waveform $g(t)$ expressed as

$$g(t) = \frac{1}{\sqrt{N_s}} \sum_{j=0}^{N_s-1} p_r(t - jT_f) \quad (6)$$

where

$$p_r(t) = p(t - c_jT_c) - p(t - c_jT_c - \Delta). \quad (7)$$

Assuming perfect symbol and multipath synchronization [14]–[16], the output at the lth correlator for the ith symbol interval can be obtained as

$$\begin{aligned} y_{l,i} &= \int_{iT_b}^{(i+1)T_b} r_i(t) g(t - \tau_l)\, dt \\ &= \int_{iT_b}^{(i+1)T_b} \sqrt{E_p} \sum_{l'=0}^{L-1} \sum_{j=0}^{N_s-1} h_{l',i} \\ &\quad \times p(t - jT_f - c_jT_c - \Delta d[i] - \tau_{l'}) \\ &\quad \times \left\{ \frac{1}{\sqrt{N_s}} \sum_{k=0}^{N_s-1} [p(t - kT_f - c_kT_c - \tau_l) \right. \\ &\quad \left. - p(t - kT_f - c_kT_c - \Delta - \tau_l)] \right\} dt \\ &\quad + \int_{iT_b}^{(i+1)T_b} n_i(t) g(t - \tau_l)\, dt. \end{aligned} \quad (8)$$

In the discrete-time channel formulation adopted in (8), the fading process is piecewise-constant approximated in each symbol interval. For indoor high-speed communications, it is completely reasonable to assume that the temporal variations of the fading processes $h_l(t)$ are such that the piecewise-constant, discrete-time approximation is valid. Expressed in a mathematical form, this condition is described as

$$h_l(t) = h_l(iT_b) = h_{l,i}, \quad iT_b \le t < (i+1)T_b. \quad (9)$$

As the received ultrashort pulses appearing in $r(t)$ are typically widely separated from one another, even a small time misalignment would make the echoes virtually orthogonal [3], [7]. Thus, the cross-correlation between signal echoes will satisfy the following condition:

$$\int_0^{T_b} p(t - \tau_{l_1}) p(t - \tau_{l_2}) \approx 0, \quad l_1 \ne l_2. \quad (10)$$

Substituting (10) in (8) yields the output signal at the lth correlator, written in a compact form, as

$$y_{l,i} = U_i h_{l,i} + n_{l,i} \quad (11)$$

where

$$U_i = \sqrt{N_s E_p}\,\{R_p[\Delta d[i]] - R_p[\Delta(d[i]-1)]\}$$

$$R_p(\tau) = \int_0^{T_b} p(t)p(t-\tau)\,dt$$

$$n_{l,i} = \int_{iT_b}^{(i+1)T_b} n_i(t)g(t-\tau_l)\,dt.$$

In deriving (11), the conditions given in (2a) and (2b) have already been applied. If bit $d[i] =$ "0" is transmitted, $U_i = \sqrt{N_s E_p}$. On the other hand, if $d[i] =$ "1" is transmitted, $U_i = -\sqrt{N_s E_p}$. In a RAKE receiver, the outputs of all correlators are combined to form the decision variable. If the channel fading coefficients $h_{l,i}, l = 0, 1, \ldots, L-1$, are known at the receiver, an optimal maximal ratio combiner (MRC) can be applied. The decision variable for the ith symbol with MRC can be obtained from (11) as

$$\Gamma_i = \sum_{l=0}^{L-1} h_{l,i}^* y_{l,i} = U_i \sum_{l=0}^{L-1} |h_{l,i}|^2 + n_i \tag{12}$$

where the superscript $*$ denotes complex conjugate, and $n_i = \sum_{l=0}^{L-1} h_{l,i}^* n_{l,i}$. Given Γ_i, the decision rule is that if $\Gamma_i > 0$, a bit decision $\hat{d}[i] =$ "0" is made; otherwise, a bit decision $\hat{d}[i] =$ "1" is made.

In practical systems, $h_{l,i}$ is a time-varying random variable, and is not known *a priori*. Thus, a critical step is to estimate it in the receiver based on the observations $y_{l,i}$. Since the same channel estimator can be applied for all fingers of the RAKE receiver, we will omit multipath index l in the following discussion on the H_∞ channel estimation algorithm.

III. H_∞ Channel Estimation Algorithm

A. Problem Formulation

Consider a pilot-assisted [17] channel estimation algorithm where a known pilot symbol is inserted in the transmitted data stream every D_t symbols. In general, the value of D_t could significantly affect the estimation performance, and thus must be determined properly according to the channel condition [18] such as the fading rapidity and the received signal-to-noise ratio (SNR). In this paper, D_t is chosen to be such that the channel fading coefficients within two adjacent pilot symbols are approximately the same, i.e.,

$$x_k \approx x_{k-1} \tag{13}$$

where $x_k = h_{kD_t}$ and $x_{k-1} = h_{(k-1)D_t}$. Since the indoor channel for high-speed UWB communications exhibits slow fading, a large value of D_t (low pilot overhead) such that $1/D_t \ll 1$ is feasible while maintaining the condition given in (13). Without loss of generality, let bit "0" be used as pilots and the transmission power is normalized to unity. From (11), the correlator output at pilot periods can be written as

$$\bar{y}_k = x_k + \bar{n}_k, \quad k = 0, 1, 2, \ldots \tag{14}$$

where

$$\bar{y}_k = y_{kD_t}/\sqrt{N_s E_p}$$

$$\bar{n}_k = n_{kD_t}/\sqrt{N_s E_p}$$

Combining (13) and (14), the channel estimation problem in the UWB system can be formulated as the following state-space model:

$$x_k \approx x_{k-1} \text{ (state equation)} \tag{15}$$

$$\bar{y}_k = x_k + \bar{n}_k. \text{ (measurement equation).} \tag{16}$$

If the statistics of both x_k and $\bar{n}_k$ were known, the traditional minimum-error-variance based estimation algorithms can be applied to estimate x_k. However, as previously mentioned, this information is not known *a priori* in practice. The known statistics assumptions may provide an estimate which is highly vulnerable to statistical estimation errors, i.e., a small measurement error may have a significant impact on the resultant estimates of the channel coefficients [13]. In the next section, we derive an H_∞ based channel estimation algorithm for the UWB system that does not require the knowledge of the statistics of x_k and $\bar{n}_k$.

B. H_∞ Estimation Algorithm

Consider a more general state-space model given as

$$x_k = Ax_{k-1} \tag{17}$$

$$\bar{y}_k = Cx_k + \bar{v}_k \tag{18}$$

where all quantities involved in the preceding equations can even be vectors or matrices. When applied to the channel estimation problem for the UWB system described in Section II with state-space models given in (15) and (16), we have $A = C = 1$. Here, we shall not make any assumptions on the disturbance $\bar{n}_k$, except that they have finite energy. The finite-energy assumption is reasonable, since in any practical systems, $\bar{n}_k$ are samples of bandlimited noise or interference processes. Let $z_k = \xi x_k$, where ξ is a linear transformation operator. Since the H_∞ estimation algorithm tries to achieve optimal estimation on a linear combination of the channel state variables, the estimation error is defined as

$$e_k \triangleq z_k - \hat{z}_k \tag{19}$$

where $\hat{z}_k$ is the estimate of z_k. The design criterion of the H_∞ estimator is to minimize the uniformly small estimation error for any $\bar{n}_k$ and the initial condition x_0. The measure of estimation performance is defined as a transfer operator, which transforms the disturbances until the nth pilot symbol $(\bar{n}_k)$ and the uncertainty of the initial condition x_0 to the estimation error e_k, which can be expressed as

$$J \triangleq \frac{\sum_{k=0}^{n} |e_k|_Q^2}{|x_0 - \hat{x}_0|_{p_0^{-1}}^2 + \sum_{k=0}^{n} |\bar{v}_k|_{V_H^{-1}}^2} \tag{20}$$

where $|x|_G^2 \triangleq x^H G x$, $(\cdot)^H$ denotes Hermitian transposition, $\{(x_0 - \hat{x}_0), \bar{v}_k\} \neq 0$, $\hat{x}_0$ is an *a priori* estimate of x_0, $(x_0 - \hat{x}_0)$ represents an unknown initial condition error, and Q, p_0^{-1} and

$V_H > 0$ are the weighting factors chosen according to performance requirements. The optimal estimate of z_k among all possible $\hat{z}_k$s (i.e., the worst case performance measure) should satisfy

$$\|J\|_\infty \stackrel{\Delta}{=} \sup_{\bar{v}_k, x_0} J \leq \gamma^{-1} \tag{21}$$

where "sup" stands for supremum, and $\gamma > 0$ is a predefined level of noise attenuation. The parameter γ should be selected such that the existence of the algorithm is guaranteed, which will be discussed later in this section. Equation (21) shows that the H_∞ optimal estimator guarantees the smallest estimation error energy over all possible disturbances with finite energy.

The discrete H_∞ estimator can also be interpreted as a minimax problem where the estimate $\hat{z}_k$ plays against the exogenous input $\bar{n}_k$ and the uncertainty of the initial state x_0. Using $z_k = \xi x_k$ and $\hat{z}_k = \xi \bar{x}_k$, the performance criterion can be equivalently represented as [19], [20]

$$\min_{\hat{x}_k} \max_{v_k, x_0} J = -\gamma^{-1}|x_0 - \hat{x}_0|^2_{p_0^{-1}} + \sum_{k=0}^{n} \left[|x_k - \hat{x}_k|^2_{\bar{Q}} - \gamma^{-1}|\bar{v}_k|^2_{V_H^{-1}} \right] \tag{22}$$

where $\bar{Q} = \xi^H Q \xi$, and "min" and "max" stand for minimization and maximization, respectively. This minimaximization problem can be solved by using a game theory approach. For the state-space model (17) and (18) with the performance criterion (22), there exists an H_∞ estimator if and only if there exists a stabilizing symmetric solution $P_k > 0$ (positive definite) to the following discrete-time Riccati equation [19], [20]:

$$\begin{aligned} P_{k+1} &= AP_k \left(I - \gamma \bar{Q} P_k + C^H V_H^{-1} C P_k\right)^{-1} A^H \\ P_0 &= p_0 \end{aligned} \tag{23}$$

where p_0 is the initial condition. If a solution P_k exists, then the H_∞ filter is given by

$$\hat{z}_k = \xi \hat{x}_k \tag{24}$$

where

$$\hat{x}_k = A\hat{x}_{k-1} + G_k(y_k - CA\hat{x}_{k-1}), \quad \hat{x}_0 = 0 \tag{25}$$

and G_k is the gain of the H_∞filter given by

$$G_k = AP_k \left(I - \gamma \bar{Q} P_k + C^H V_H^{-1} C P_k\right)^{-1} C^H V_H^{-1}. \tag{26}$$

By letting $P_k^{-1} = R_k^{-1} + \gamma \bar{Q}$, (23)–(26) can be simplified as

$$\hat{x}_k = A\hat{x}_{k-1} + G_k(\bar{y}_k - A\hat{x}_{k-1}), \quad \hat{x}_0 = 0 \tag{27}$$

$$G_k = AR_k \left(I + C^H V_H^{-1} C R_k\right)^{-1} C^H V_H^{-1} \tag{28}$$

$$\begin{aligned} R_{k+1}^{-1} &= \left[A \left(R_k^{-1} + C^H V_H^{-1} C\right)^{-1} A^H\right]^{-1} - \gamma \bar{Q} \\ R_0^{-1} &= p_0^{-1} - \gamma \bar{Q}. \end{aligned} \tag{29}$$

For the estimation problem defined in (15) and (16), since no information is known on the channel fading and noise, and all components are scalar, in order to further simplify the estimation algorithm, we set the algorithm parameters as

$$\begin{aligned} Q &= 1 \\ V_H &= 1 \\ \xi &= 1 \end{aligned}$$

and p_0 is a large positive number (e.g., 100). Following (27)–(29), the H_∞ estimation algorithm for the UWB system can be represented as

$$\hat{x}_k = \hat{x}_{k-1} + G_k(\bar{y}_k - x_{k-1}) \tag{30}$$

$$G_k = R_k(1 + R_k)^{-1} \tag{31}$$

$$R_{k+1}^{-1} = R_k^{-1} + 1 - \gamma, \quad R_0^{-1} = p_0^{-1} - \gamma. \tag{32}$$

The derived H_∞ algorithm in (30)–(32) is based on the assumption that channel fading is static, as shown in (15). In order to consider the possible variance of the channel fading in time, an exponential forgetting factor $0 < \lambda < 1$ is introduced to the design criterion [21]. From (20), the performance measure with a forgetting factor becomes

$$\begin{aligned} J_\lambda &= \frac{\sum_{k=0}^{n} \lambda^{n-k} |e_k|^2_Q}{\lambda^n |x_0 - \hat{x}_0|^2_{p_0^{-1}} + \sum_{k=0}^{n} \lambda^{n-k} |\bar{v}_i|^2_{V_H^{-1}}} \\ &= \frac{\sum_{k=0}^{n} \lambda^{-k} |e_k|^2_Q}{|x_0 - \hat{x}_0|^2_{p_0^{-1}} + \sum_{k=0}^{n} \lambda^{-k} |\bar{v}_k|^2_{V_H^{-1}}}. \end{aligned} \tag{33}$$

In (33), the term λ^{n-k} gives a larger weight to the more recent estimation error and disturbance. Substituting (33) into (21) leads to the new design criterion

$$\|J_\lambda\| = \sup_{\bar{v}_k, x_0} J_\lambda \leq \gamma^{-1}. \tag{34}$$

Let

$$\begin{aligned} \tilde{x}_k &= \lambda^{-k/2} x_k \\ \hat{\tilde{x}}_k &= \lambda^{-k/2} \hat{x}_k \\ \tilde{v}_k &= \lambda^{-k/2} \bar{v}_k \\ \hat{\tilde{v}}_k &= \lambda^{-k/2} \hat{\bar{v}}_k. \end{aligned}$$

Equations (33)–(34) become the design criterion with respect to the following state-space model:

$$\tilde{x}_k = \lambda^{-1/2} \tilde{x}_{k-1} \tag{35}$$

$$\tilde{y}_k = \tilde{x}_k + \tilde{v}_k \tag{36}$$

where $\tilde{y}_k = \lambda^{-k/2} \bar{y}_k$. Comparing (35) and (36) with (17) and (18), we have $A = \lambda^{-k/2}$ and $C = 1$. Following a similar analysis described previously, the H_∞ algorithm for the state-space model (35) and (36) can be obtained as

$$\hat{\tilde{x}}_k = \lambda^{-k/2} \hat{\tilde{x}}_{k-1} + \tilde{G}_k(\tilde{y}_k - \lambda^{-k/2} \hat{\tilde{x}}_{k-1}), \quad \tilde{x}_0 = 0 \tag{37}$$

$$\tilde{G}_k = \lambda^{-k/2} \tilde{R}_k (1 + \tilde{R}_k)^{-1} \tag{38}$$

$$\tilde{R}_{k+1} = \lambda(\tilde{R}_k^{-1} + 1) - \gamma, \quad \tilde{R}_0^{-1} = p_0^{-1} - \gamma \tag{39}$$

where (37) can be further simplified as

$$\hat{x}_k = \hat{x}_{k-1} + \tilde{G}_k(\bar{y}_k - \hat{x}_{k-1}). \tag{40}$$

A necessary and sufficient condition for the existence of the H_∞ estimator is that the discrete-time Riccati equation (39) should have a positive-definite solution $\tilde{R}_{k+1}$. Thus, the parameter γ should be selected carefully to satisfy this constraint. From (39), in order to guarantee $\tilde{R}_{k+1}$ to be positive, we require

$$\begin{aligned} &\lambda\left(\tilde{R}_k^{-1} + 1\right) - \gamma > 0 \\ &\Rightarrow \gamma < \lambda \leq \lambda\tilde{R}_k^{-1} + \lambda \\ &\Rightarrow \gamma = \eta\lambda \end{aligned} \tag{41}$$

where η takes a value that is less than, but very close to, one.

By examining (38)–(40), it is found that the proposed H_∞ algorithm requires six scalar multiplications and five scalar additions to estimate each multipath. Considering the multiplication only due to its higher complexity, the proposed H_∞ algorithm has an estimation complexity of 6 L scalar multiplications, which is linear with the number of multipaths. By further considering the fact that the proposed algorithm does not require parameter estimation on channel fading and noise statistics, it can be concluded that the proposed H_∞ estimation algorithm is simple to implement, and suitable for UWB systems that require low-power and low-cost receivers.

IV. Simulation Results

In this section, simulation results are presented to demonstrate the performance of the proposed H_∞ channel estimation algorithm. For comparison purposes, the least square (LS) and the Wiener filter based estimation algorithms are also simulated. In the LS algorithm, the received signal $\bar{y}_k$ is directly considered as the channel estimate. LS is another algorithm that can be applied to UWB channels with unknown channel fading and noise statistics. The Wiener filter based algorithm is implemented by letting $\gamma = 0$ in the proposed H_∞ algorithm [13] such that both algorithms are simulated under identical conditions.

A. Simulation Parameters

In the simulation, a single user pulse-based UWB system with TH-PPM is considered. A rectangular pulse shape with pulse duration $T_p = 0.2$ ns is adopted. The chip duration is $T_c = 0.5$ ns and the hopping is carried out in the range between 0 and $9T_c$, i.e., $N_h = 10$. The nominal pulse repetition interval is $T_f = 20$ ns and $N_s = 4$ pulses are used to transmit each bit, so that the system can support bit rate up to 12.5 Mb/s with binary signalling. The delay for transmitting symbol “0” in PPM is set to 0.5 ns. All parameters applied to generate the transmitted and received signals are summarized in Table I. Note that some of the parameter values are chosen to be smaller than those in a practical system to speed up the simulation process. For example, N_h would normally be greater than four when there are more than one user in order to minimize collisions, and the number of pulses modulated by one bit could be larger than four to obtain a larger spreading gain. Since our emphasis is on estimation performance comparison, those choices of system parameters will not affect the observations derived from simulation.

TABLE I
Signal Simulation Parameters

Pulse rate	5 GHz
Pulse duration (T_p)	2×10^{-10}s
Chip interval (T_c)	5×10^{-10}s
Delay for PPM “0” (δ)	5×10^{-10}s
Chip in one frame (N_h)	10
Pulse repetition interval (T_f)	2×10^{-8}s
Pulses in each bit (N_s)	4
Bit interval (T_b)	8×10^{-8}s

The channel used in simulation is a slow multipath fading channel. The number of resolvable multipath components is four, and the power delay profile satisfies an exponential distribution, i.e., the fading at each multipath has variance

$$\sigma_l^2 = \frac{e^{-l/L}}{\sum_{l=0}^{L-1} e^{-l/L}}. \tag{42}$$

Although log-normal or Nakagami-m distribution is considered to be more suitable for describing the UWB channel [5], Rayleigh fading is still applied in our simulation because of the well-known Jakes model [22] for channel generation. By considering the facts that Rayleigh distribution is a special case of Nakagami-m distribution and that the proposed estimation algorithm does not depend on channel fading statistics, the Rayleigh fading assumption is adequate for the purpose of evaluating the properties and performance of the proposed algorithm. The background noise in the simulation is modeled as a white Gaussian process with variance σ^2, except for the colored noise case. The signal power is normalized to 1 so that the signal-to-noise ratio (SNR) is defined as $1/\sigma^2$. In the simulation, both estimation mean square error (MSE) and bit-error-rate (BER) are considered as performance measures.

B. Simulation Results and Discussions

Figs. 2–4 show the effects of different forgetting factors on the estimation performance. These three figures correspond to three channel models with normalized Doppler frequency among pilots (NDFP) of 0.001, 0.01, and 0.1, respectively, where NDFP is defined as

$$\text{NDFP} = f_d D_t T_b \tag{43}$$

where f_d is the Doppler shift of the system, which is determined by the user movement speed and carrier frequency. For a given channel fading model, NDFP represents the distance between the adjacent pilots, which further represents the system overhead resulting from the pilots. From these figures, it is observed that there exist different optimal forgetting factors for different NDFPs. For NDFP $= 0.001$, the optimal forgetting factor is found to be $\lambda_{\text{opt}} = 0.99$. For NDFP $= 0.01$, in the low-to-medium SNR region (<27 dB), $\lambda_{\text{opt}} = 0.99$, and in the high SNR region (≥27 dB), $\lambda_{\text{opt}} = 0.97$. For NDFP $= 0.1$, $\lambda_{\text{opt}} = 0.9$. The observation indicates that the optimal forgetting factor approaches 1 for a slow fading channel, which is intuitively correct. For all three NDFP values, $\lambda = 1$ always results in the

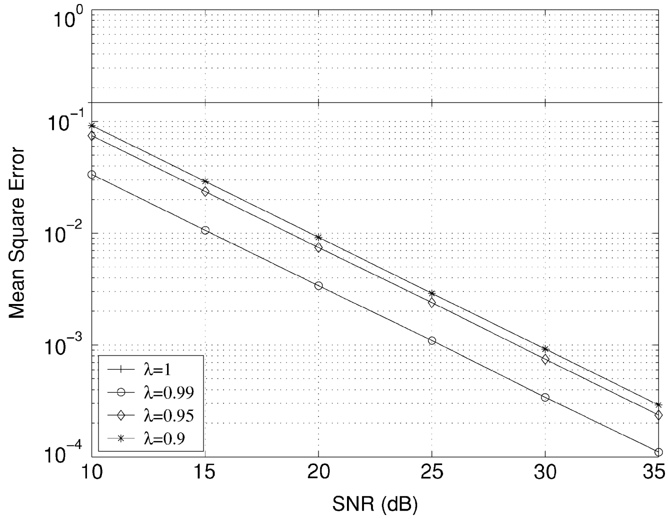

Fig. 2. Mean square error performance of the proposed H_∞ algorithm with NDFP $= 0.001$.

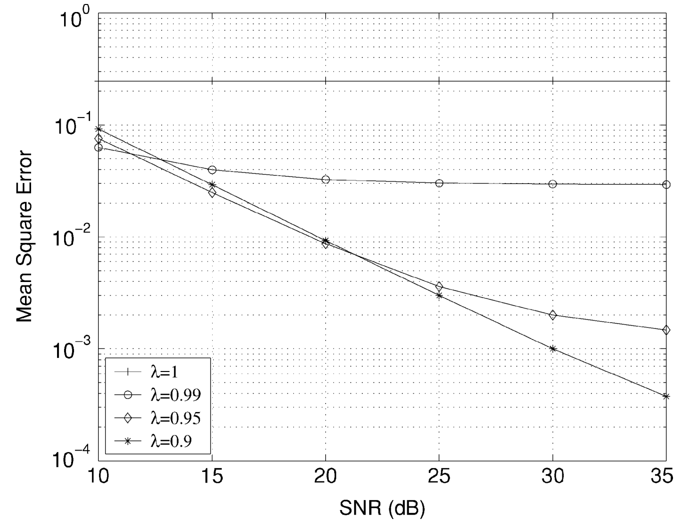

Fig. 4. Mean square error performance of the proposed H_∞ algorithm with NDFP $= 0.1$.

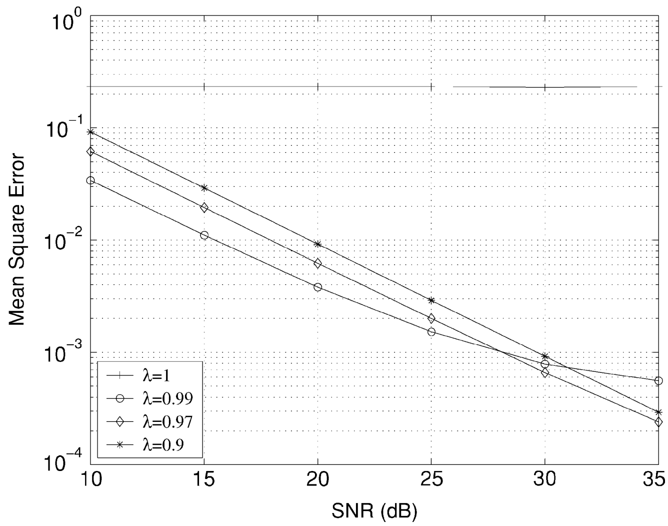

Fig. 3. Mean square error performance of the proposed H_∞ algorithm with NDFP $= 0.01$.

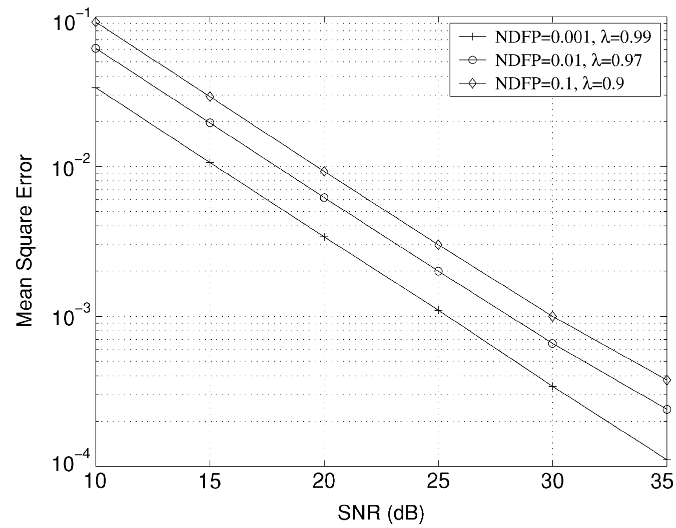

Fig. 5. Mean square error performance of the proposed H_∞ algorithm with different NDFPs and associated optimal forgetting factor λ.

poorest estimation performance, as $\lambda = 1$ implies that channel fading is a constant, and the algorithm does not take time variations of the channel into account. From Figs. 3 and 4, it is also observed that a bad choice of λ values causes an estimation error floor.

Fig. 5 summarizes the estimation performance of the proposed H_∞ algorithm under different NDFPs with optimal forgetting factor λ. Obviously, a smaller NDFP gives a better estimation performance. For example, for MSE $= 10^{-3}$, the SNR required for channel estimation with NDFP $= 0.001$ is about 2.7 dB less than that with NDFP $= 0.01$ and about 4.4 dB less than that with NDFP $= 0.1$. These observations show that the proposed H_∞ algorithm works better for a slower fading channel. Given the same Doppler shift, a small NDFP means a high system overhead. Therefore, a properly chosen value of D_t should consider the tradeoff between estimation performance and system overhead.

Fig. 6 shows a comparison of BER performance among the proposed H_∞ algorithm, the LS algorithm, and the Wiener filter based algorithm. The channel is assumed to have a normalized Doppler frequency $f_d T_b = 0.00001$ with $D_t = 100$, i.e., NDFP $= 0.001$. The interpilot symbol channel coefficients are estimated using linear interpolation based on the estimates on pilots. Performance improvement of the proposed H_∞ algorithm over the LS algorithm is clearly observed. At BER $= 10^{-5}$, the proposed H_∞ algorithm achieves a nearly 1.8 dB gain over the LS algorithm. Compared with the Wiener filter based algorithm, the proposed H_∞ algorithm performs better over almost all SNR values considered, except in the low SNR region (<7 dB). Specifically, at BER $= 10^{-5}$, the proposed algorithm performs about 3 dB better than the Wiener filter based approach.

Performance of the proposed H_∞ algorithm in colored noise is shown in Fig. 7. In this simulation, colored noise is generated by passing a white Gaussian noise through a linear filter that

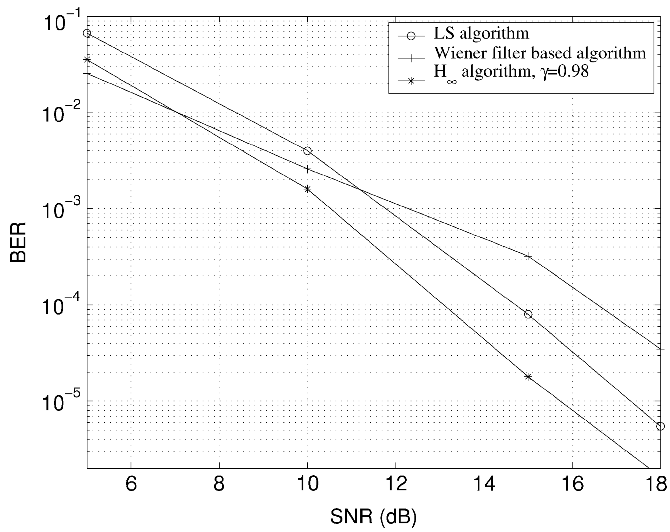

Fig. 6. BER performance of the proposed H_∞, the LS, and the Wiener filter-based algorithms.

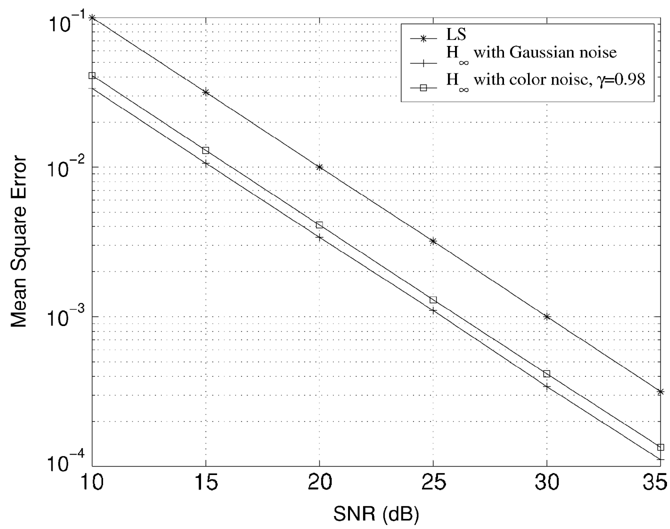

Fig. 7. Mean square error performance of the proposed H_∞ algorithm under colored noise.

has a transfer function $(1)/(1-0.2z^{-1})$. For comparison, the estimation performance of the proposed H_∞ algorithm with white Gaussian noise is also shown in the same figure. With colored noise, performance of the proposed algorithm is only sightly worse than the case of the white noise, which is less than 0.8 dB at MSE $=10^{-3}$. Even in colored noise, the proposed H_∞ algorithm can still achieve more than 3.7 dB SNR gain over the LS algorithm to achieve an MSE of 10^{-3}.

V. Conclusion

In this paper, an H_∞ based channel estimation algorithm is proposed for pulse-based UWB systems. The proposed H_∞ algorithm inserts pilots periodically at the transmitter end, and introduces a forgetting factor to track the channel variation in time. Since no statistical information of channel fading and noise is required, the proposed H_∞ algorithm exhibits robustness in channel estimation. Simulation results demonstrate that the proposed H_∞ algorithm achieves much better performance in terms of MSE and BER over the LS and the Wiener filter based algorithms in both white and colored background noise. By further considering its simple implementation, the proposed H_∞ algorithm is very attractive for practical UWB systems to achieve low-cost and low-power consumption.

References

[1] M. Z. Win and R. A. Scholtz, "Ultra-wide bandwidth time-hopping spread-spectrum impulse radio for wireless multiple-access communications," *IEEE Trans. Commun.*, vol. 48, no. 4, pp. 679–691, Apr. 2000.

[2] G. B. Giannakis, "Ultra-wideband communications: An idea whose time has come," in *Proc. IEEE Workshop on SPAWC 2003*, Rome, Italy, Jun. 2003, pp. 15–18.

[3] W. Zhuang, X. Shen, and Q. Bi, "Ultra-wideband wireless communications," *Wireless Commun. Mobile Comput.*, vol. 3, pp. 663–685, 2003.

[4] E. Saberinia and A. H. Tewfik, "Pulsed and non-pulsed OFDM ultra wideband wireless personal area networks," in *Proc. IEEE UWBST 2003*, Reston, VA, Nov. 2003, pp. 275–279.

[5] A. Batra, J. Balakrishnan, G. R. Aiello, J. R. Foerster, and A. Dabak, "Design of a multiband OFDM system for realistic UWB channel environment," *IEEE Trans. Microw. Theory Tech.*, vol. 52, no. 9, pp. 2123–2138, Sep. 2004.

[6] "First report and order, revision of part 15 of the commission's rules regarding ultra-wideband transmission systems," FCC, Washington, DC, ET Docket 98–153, Feb. 2002.

[7] V. Lottici, A. D'Andrea, and U. Mengali, "Channel estimation for ultra-wideband communications," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 9, pp. 1638–1645, Dec. 2002.

[8] B. Mielczarek, M. Wessman, and A. Svensson, "Performance of coherent UWB Rake receivers with channel estimators," in *Proc. IEEE Vehicular Technology Conf.*, Orlando, FL, Fall, Oct. 2003, pp. 1880–1884.

[9] C. Dajana, M. Z. Win, and A. F. Molisch, "The ultra-wideband indoor channel: From statistical model to simulations," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 6, pp. 1247–1257, Aug. 2002.

[10] A. F. Molisch, J. R. Foerster, and M. Pendergrass, "Channel models for ultrawideband personal area networks," *IEEE Wireless Commun.*, vol. 10, no. 6, pp. 14–21, Dec. 2003.

[11] A. Saleh and R. Valenzuela, "A statistical model for indoor multipath propagation," *IEEE J. Sel. Areas Commun.*, vol. SAC-5, no. 2, pp. 128–137, Feb. 1987.

[12] H. Hashemi, "Impulse response modeling of indoor radio propagation channels," *IEEE J. Sel. Areas Commun.*, vol. 11, no. 7, pp. 967–978, Sep. 1993.

[13] W. Wong, J. Cai, X. Shen, and J. W. Mark, "Robust H_∞ channel estimation algorithm for ultra-wideband wireless communication systems," in *Proc. Globe Mobile Congr.*, Shanghai, China, Oct. 2004, pp. 342–347.

[14] H. Bing, X. Hou, X. Yang, T. Yang, and C. Li, "A two-step synchronous sliding method of sub-nanosecond pulse for ultra-wideband (UWB) radio," in *Proc. IEEE Int. Conf. Communications, Circuits and Systems and West Sino Expositions*, Chengdu, China, Jun. 2002, pp. 142–145.

[15] R. Blazquez, P. Newaskar, and A. Chandrakasan, "Coarse acquisition for ultra wideband digital receivers," in *Proc. IEEE Int. Conf. Acoustics, Speech, Signal Processing*, Hong Kong, Apr. 2003, pp. 137–140.

[16] C. Carbonelli, U. Mengali, and U. Mitra, "Synchronization and channel estimation for UWB signals," *Proc. IEEE Globecom'03*, pp. 764–768, Dec. 2003.

[17] P. Schramm and R. R. Muller, "Pilot symbol assisted BPSK on Rayleigh fading channels with diversity: Performance analysis and parameter optimization," *IEEE Trans. Commun.*, vol. 46, no. 12, pp. 1560–1563, Dec. 1998.

[18] P. Hoeher, S. Kaiser, and P. Robertson, "Two-dimension pilot-symbol-aided channel estimation by Wiener filtering," *Proc. ICASSP 1997*, pp. 1845–1848, Munich, Germany, Apr. 1997.

[19] U. Shaked and Y. Theodor, "H_∞-optimal estimation: A tutorial," in *Proc. 31st IEEE CDC*, Tucson, AZ, Dec. 1992, pp. 2278–2286.

[20] K. M. Nagpal and P. P. Khargonekar, "Filtering and smoothing in an H_∞ setting," *IEEE Trans. Autom. Contr.*, vol. 36, no. 2, pp. 152–166, Feb. 1991.

[21] W. Zhuang, "Adaptive H_∞ channel equalization for wireless personal communications," *IEEE Trans. Veh. Technol.*, vol. 48, no. 1, pp. 126–136, Jan. 1999.

[22] W. C. Jakes, *Microwave Mobile Communications*. New York: IEEE Press, 1993.

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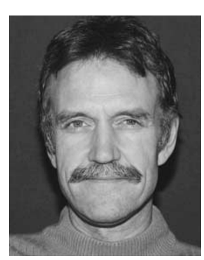

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