# Robust Channel Estimation for OFDM Wireless Communication Systems—An $H_{\infty}$ Approach

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Abstract—In this paper, the joint time-frequency domain channel estimation problem in orthogonal frequency-division multiplexing (OFDM) wireless communication systems is transformed to a set of independent time-domain estimation problems. A robust channel estimation algorithm based on the  $H_{\infty}$  filtering approach is proposed to estimate the channel fading in the time domain. The estimation criterion is to minimize the worst possible amplification of the estimation errors in terms of the exogenous input disturbances such as multiplicative and additive noise. The criterion is different from the traditional minimum estimation error variance criterion for the Kalman estimation algorithm, and requires no a priori knowledge of the disturbance statistics. It is shown that the proposed channel estimation algorithm is more robust compared with the Kalman estimation counterpart in terms of model uncertainty, and is more suitable to practical OFDM wireless communication systems.

Index Terms—Channel estimation, orthogonal frequency-division multiplexing (OFDM),  $H_{\infty}$  filtering, wireless communications.

#### I. INTRODUCTION

T HE high demand for a large volume of multimedia services in wireless communication systems requires high transmission rates. However, high transmission rates may result in severer frequency selective fading and intersymbol interference (ISI) if the bandwidth of the transmitted signal is large compared to the coherence bandwidth of the channel. Orthogonal frequency-division multiplexing (OFDM) has been proposed to combat these types of channel disturbance [1]–[4]. In an OFDM system, the signal is transformed into a number of components, each with a bandwidth narrower than the coherence bandwidth of the propagation channel. Each of the OFDM signal components is modulated onto a distinct subcarrier. With OFDM, the transmission in each subcarrier experiences frequency flat fading, and OFDM is said to have transformed frequency-selective fading to flat fading.

Channel state information is very important to achieve optimal diversity combining and coherent detection at the receiving end. In the absence of channel state information, channel estimators can be used to provide estimates of the channel state information. In OFDM, channel fading information is present in both the time and frequency domains. A proper

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channel estimation algorithm for the OFDM systems should capture both the time- and frequency-domain characteristics. In recent years, channel estimation for OFDM systems has been a very active research area, both in the time domain [6]–[9] and in the frequency domain [4], [10], [11]. By representing the correlation function of the channel fading as the product of the correlation functions in the time domain and the frequency domain, it is possible to perform channel estimation in the time domain alone [5]. To our knowledge, the development of estimation algorithms has been based on known statistics of the fading channel and the additive noise. The criterion used is minimization of the variance of the estimation errors, e.g., the Kalman estimation algorithm. On the basis of known channel statistics, the Kalman estimator is optimal in the sense that the error covariance is minimized. However, in practical systems, channel statistics are not completely known. When the Kalman estimator is not the dual of the channel, the performance of the Kalman estimator may suffer significant degradation [12]. A robust channel estimator for practical OFDM wireless communication systems, which does not depend on a priori knowledge of the channel state information, is desirable. This is the motivation behind the work presented in this paper.

In this paper, the two-dimensional time-frequency channel estimation problem is first transformed to a set of independent one-dimensional time-domain channel estimation problems using the property that the joint time-frequency correlation function of the channel fading can be represented as the product of the correlation functions in the time and the frequency domains. A robust  $H_{\infty}$  channel estimation algorithm is proposed to estimate the channel fading in the time domain. The  $H_{\infty}$  approach differs from the traditional approach such as the Kalman estimation in the following two respects.

- 1) No *a priori* knowledge of the noise source statistics is required. The only assumption is that the noise has finite energy.
- The estimation criterion is to minimize the worst possible effect in the estimation error (including channel modeling error and additive noise).

These two features make the proposed  $H_{\infty}$  estimation algorithm more appropriate for practical OFDM systems where there is significant uncertainty in the statistics of noise and channel fading. Since the proposed  $H_{\infty}$  algorithm has an observer structure similar to that of the Kalman algorithm, the implementation complexity is similar to that of the Kalman algorithm. For this reason, the Kalman algorithm will be used as the benchmark for performance comparison. Simulation results show that the proposed  $H_{\infty}$  estimation approach can

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Fig. 1. Transceiver structure of the MC-CDMA system. (a) The transmitter structure. (b) The receiver structure.

improve both the estimation error and bit error rate (BER) performance compared to the Kalman estimation approach.

The remainder of this paper is organized as follows. Section II presents the OFDM system model used in the derivation of the  $H_{\infty}$  estimation algorithm. In Section III, the two-dimensional joint time-frequency domain channel estimation problem in the OFDM system is decomposed and represented by one-dimensional time-domain estimation problems. Section IV presents the  $H_{\infty}$  algorithm for channel estimation in the OFDM system. In Section V, the performance of the  $H_{\infty}$  estimation algorithm is evaluated by simulation in terms of mean-square-error and BER. Conclusions of this paper are given in Section VI.

## **II. SYSTEM MODEL**

Fig. 1 shows the structure of an OFDM transceiver. The serial data at the input is a sequence of samples occurring at interval  $T_s$ . At the transmitter [Fig. 1(a)], the high-rate serial input data sequence is first serial-to-parallel (S/P) converted into Mlow-rate parallel streams in order to increase the symbol duration to  $T = MT_s$ . The low-rate streams, represented by the symbols  $b_m[k], m = 0, 1, \ldots, M - 1, k = 1, 2, \ldots$ , are modulated onto different subcarriers. In order to eliminate interference between parallel data streams, each of the low-rate data streams is modulated onto a distinct subcarrier belonging to an orthogonal set with subcarrier spacing 1/T. The parallel streams are then multiplexed and a cyclic prefix is added to eliminate the effect of ISI. Thus, the signal transmitted during the kth symbol interval y(t) can be written as

$$y(t) = \sum_{m=0}^{M-1} b_m[k] e^{j2\pi mt/T}, \quad -\Psi + kT \le t \le (k+1)T$$
(1)

where  $b_m[k]$  is the kth data symbol of the mth stream, M is the total number of subcarriers, and  $\Psi$  is the length of the guard interval.

The transmitted signal y(t) passes through the wireless channel which introduces signal distortion and additive noise. The wireless channel can be modeled as a multipath frequency-selective fading channel using a tapped-delay line with time-varying coefficients and fixed tap spacing [13], which can be represented as

$$h(t,\tau) = \sum_{l=0}^{\chi} h_l(t)\delta(\tau - \tau_l)$$
<sup>(2)</sup>

where  $h_l(t)$  and  $\tau_l$  are the complex amplitude and delay of the *l*th path, respectively.  $\chi$ +1 is the total number of taps.  $\tau_{\chi}$  defines the maximum multipath delay spread. For OFDM to be effective, the length of the cyclic prefix  $\Psi$  should be larger than the maximum multipath delay spread of the channel. In this paper,  $h_l(t)$  is modeled as a wide-sense stationary uncorrelated scattering (WSSUS) process, which has the following correlation function:

$$\phi_h(\Delta t) \triangleq E[h_l(t)h_l^*(t - \Delta t)] \\ = \sigma_l^2 \phi_t(\Delta t)$$
(3)

where \* denotes complex conjugation,  $\sigma_l^2$  is the variance of the channel fading at path l, which is determined by the power delay profile of the channel and satisfies  $\sum_{l=0}^{\chi} \sigma_l^2 = 1$ , and  $\phi_t(\Delta t)$  is the normalized correlation function.

The received signal r(t) in the kth symbol duration can be expressed as

$$r(t) = \int h(t,\tau)y(t-\tau) d\tau$$
$$= \sum_{l=0}^{\chi} h_l(t)y(t-\tau_l) + n(t)$$
(4)

where n(t) is the background noise.

At the receiver [Fig. 1(b)], the received signal is first demodulated after cyclic prefix removal. For practical implementation, modulation and demodulation can be achieved by inverse fast Fourier transform (IFFT) and fast Fourier transform (FFT), respectively. Channel estimation is applied to obtain the estimates of channel fading in each subcarrier such that coherent detection can be achieved. Delay blocks are introduced to synchronize the outputs of the demodulator and channel estimator. In Fig. 1(b), the second index q at the output of the demodulator refers to the qth subcarrier, and  $s_{k,q}$  is the output for the qth subcarrier in the kth symbol interval. By assuming that the channel impulse response is quasi-static during the kth symbol interval so that  $h(t) \approx h(kT)$  for  $kT \leq t < (k+1)T$ , the intercarrier interference can be neglected compared to the background noise. Thus, the qth subcarrier output,  $s_{k,q}, q \in \{0, 1, \dots, M-1\}$ , from the demodulator can be expressed as

$$s_{k,q} = \frac{1}{T} \int_{kT}^{(k+1)T} \left[ \sum_{l=0}^{\chi} h_l(kT) \right] \\ \times \sum_{m=0}^{M-1} b_m[k] e^{j2\pi m(t-\tau_l)/T} + n(t) e^{-j2\pi qt/T} dt \\ = \frac{1}{T} \sum_{l=0}^{\chi} h_l(kT) \sum_{m=0}^{M-1} b_m[k] e^{-j2\pi m\tau_l/T} \\ \times \int_{kT}^{(k+1)T} e^{j2\pi (m-q)t/T} dt \\ + \frac{1}{T} \int_{kT}^{(k+1)T} n(t) e^{-j2\pi qt/T} dt \\ = c_{k,q} H_{k,q} + v_{k,q}$$
(5)

where

$$c_{k,q} = b_q[k]$$

$$H_{k,q} = \sum_{l=0}^{\chi} h_l(kT) e^{-j2\pi q\tau_l/T}$$

$$v_{k,q} = \frac{1}{T} \int_{kT}^{(k+1)T} n(t) e^{-j2\pi qt/T} dt.$$
(6)

If the channel fading characterized by  $H_{k,q}$  were known, then coherent detection and optimum diversity combining would be achievable at the receiver. However,  $H_{k,q}$  is time varying and usually unknown. Hence, an effective channel estimation algorithm is needed to accurately estimate the channel fading parameter  $H_{k,q}$ , given  $s_{i,q}$ ,  $i \le k$  and  $q = 0, 1, \ldots, M - 1$ .

# III. JOINT TIME-FREQUENCY DOMAIN CHANNEL ESTIMATION

Decision-directed [14] and pilot-assisted [15] approaches are two of the most commonly used channel estimation algorithms. Because of the error propagation inherent in the decision-directed approach, the pilot-assisted scheme is preferred. Fig. 2 shows the pilot pattern used in this paper, where the known pilot symbols are inserted in every  $D_t$  OFDM symbols and  $D_f$  subcarriers. In general, the values of  $D_t$  and  $D_f$  may significantly affect the estimation performance and should be selected properly [16]–[18]. Without loss of generality, let

$$c_{k,q} = 1, \quad k \in M_t, \quad q \in M_f$$

where  $M_t$  and  $M_f$  are the sets of pilot positions in the time and frequency domains, respectively. Then, (5) becomes

$$s_{k,q} = H_{k,q} + v_{k,q}, \quad k \in M_t, \quad q \in M_f.$$

$$\tag{7}$$

Since the  $H_{k,q}$  are correlated for different ks and qs, a proper channel estimation algorithm should be carried out jointly in both the time and frequency domains. Directly solving this two-dimensional estimation problem is very difficult. In the following, based on the separation property of the time-frequency correlation function of the channel fading, the two-dimensional estimation problem is decomposed to one-dimensional time-domain estimation problems, which greatly simplifies the original one.

From (3) and (6), the correlation function of the fading channel  $\{H_{k,q}, k \in M_t \text{ and } q \in M_f\}$  for different times and subcarriers can be written as [5]

$$\phi_{H}[m,n] \triangleq E\left[H_{k,q}H_{k-mD_{t},q-nD_{f}}^{*}\right]$$

$$= E\left[\sum_{l=0}^{\chi} h_{l}(kT)e^{-j2\pi q\tau_{l}/T} \times \sum_{l'=0}^{\chi} h_{l'}^{*}(kT-mD_{t}T)e^{j2\pi (q-nD_{f})\tau_{l}/T}\right]$$

$$= \phi_{t}(mD_{t}T)\sum_{l=0}^{\chi} \sigma_{l}^{2}e^{-j2\pi nD_{f}\tau_{l}/T}$$

$$= \phi_{t}[m]\phi_{f}[n] \qquad (8)$$

where

$$\phi_t[m] \triangleq \phi_t(mD_tT)$$
  
$$\phi_f[n] \triangleq \sum_{l=0}^{\chi} \sigma_l^2 e^{-j2\pi nD_f \tau_l/T}.$$

Equation (8) indicates that the time–frequency correlation function of the fading channel in the OFDM system can be represented as the product of the correlation functions in the time domain and in the frequency domain.



Fig. 2. Configuration of pilot arrangement.

Let

$$\boldsymbol{\phi}_{f}[q] = \begin{bmatrix} \phi_{f}[q] \\ \vdots \\ \phi_{f}[0] \\ \vdots \\ \phi_{f}[-K+1+q] \end{bmatrix}$$
$$\boldsymbol{\Phi}_{f} = [\boldsymbol{\phi}_{f}[0] \quad \boldsymbol{\phi}_{f}[1] \quad \cdots \quad \boldsymbol{\phi}_{f}[K-1]]$$

where  $K = (M)/(D_f)$  is the number of pilot symbols in the frequency domain given the symbol time instant  $k \in M_t$ . Assuming that  $\Phi_f$  is diagonalizable, the eigendecomposition of  $\Phi_f$  is

$$\mathbf{\Phi}_f = \mathbf{U}^H \mathbf{D} \mathbf{U} \tag{9}$$

where the superscript H denotes Hermitian transposition, **U** is a unitary matrix consisting of the eigenvectors of  $\Phi_f$ , and **D** is a  $K \times K$  diagonal matrix with the diagonals consisting of  $K_0$  ( $K_0 \leq K$ ) nonzero eigenvalues  $d_l, l = 0, 1, \ldots, K_0 - 1$ , and  $K - K_0$  zeros. In the absence of knowledge of the channel fading statistics, we choose  $K_0 = \lceil \tau_{\chi}/T_s \rceil$  [5], where  $\lceil x \rceil$  denotes the ceiling function. Alternatively,  $K_0$  may be determined using the approach in [19]. Let

$$\begin{aligned} \mathbf{\bar{s}}_k &= \begin{bmatrix} s_{k,0} & s_{k,D_f} & \cdots & s_{k,(K-1)D_f} \end{bmatrix} \mathbf{U}^H \\ \mathbf{g}_k &= \begin{bmatrix} H_{k,0} & H_{k,D_f} & \cdots & H_{k,(K-1)D_f} \end{bmatrix} \mathbf{U}^H \\ \mathbf{\bar{v}}_k &= \begin{bmatrix} v_{k,0} & v_{k,D_f} & \cdots & v_{k,(K-1)D_f} \end{bmatrix} \mathbf{U}^H. \end{aligned}$$

From (7) and (8), we have

$$\begin{cases} \bar{s}_{k,l} = g_{k,l} + \bar{v}_{k,l}, & l = 0, 1, \dots, K_0 - 1\\ \phi_{g,l}[m] \stackrel{\Delta}{=} E[g_{k,l}g^*_{k-mD_t,l}] = \phi_t[m] \mathbf{U}_l \mathbf{\Phi}_f \mathbf{U}_l^H = d_l \phi_t[m]\\ \sigma_{\bar{v}}^2 \stackrel{\Delta}{=} E[|\bar{v}_{k,l}|^2] = E[|v_{k,l}|^2] = \sigma_v^2 \end{cases}$$
(10)

where  $\bar{s}_{k,l}, g_{k,l}$ , and  $\bar{v}_{k,l}$  are the *l*th elements of  $\bar{s}_k, g_k$ , and  $\bar{v}_k$ , respectively;  $\mathbf{U}_l$  is the *l*th row of  $\mathbf{U}$ . Since the columns of  $\mathbf{U}$  form a unitary system

$$E[g_{k,l}g_{k,l-i}^*] = \mathbf{U}_l \mathbf{\Phi}_f \mathbf{U}_{l-i}^H = d_l \mathbf{U}_l \mathbf{U}_{l-i}^H = 0, \quad \text{for} \quad i \neq 0.$$
(11)

Equation (11) indicates that given time instant k, the  $g_{k,l}$ are uncorrelated for different l, i.e., the estimate of  $g_{k,l}$  only depends on the observation  $\overline{s}_{k',l}, k' \leq k$ . In other words, the original joint time-frequency channel fading estimation problem can be transformed to  $K_0$  one-dimensional time-domain estimation problems shown in (10). Fig. 3 shows the derived channel estimator structure, where the observation vector  $[s_{k,0}, s_{k,D_f}, \ldots, s_{k,(K-1)D_f}]$  is transformed to vector  $\overline{\mathbf{s}}_k = [\overline{s}_{k,0}, \overline{s}_{k,1}, \dots, \overline{s}_{k,K-1}]$  by the matrix  $\mathbf{U}^H$ . Then,  $\{g_{k,l}, l = 0, 1, \dots, K_0 - 1\}$  can be estimated by  $K_0$  one-dimensional time-domain estimators. Let the outputs of the  $K_0$  estimators and  $K - K_0$  zeros form the vector  $\hat{\mathbf{g}}_k = [\hat{g}_{k,0}, \dots, \hat{g}_{k,K_0-1}, 0, \dots, 0]$ . The estimates of  $H_{k,lD_f}$ ,  $l = 0, 1, \dots, K - 1$ , can be obtained by the inverse transforming  $\hat{\mathbf{g}}_k$  using matrix U [5]. Given the estimate of  $H_{k,l}$ at each pilot position,  $H_{k,l}, \forall l \in \{0, 1, \dots, M-1\}$ , can be obtained by interpolation.

For time-domain estimation, it is well known that the lowpass slow-fading channel  $g_{k,l}$  in (10) can be approximated by an autoregressive (AR) process of the form [20], [21]

$$g_{k,l} = \sum_{i=1}^{n} a_{i,l} g_{k-iD_l,l} + w_{k,l}$$
(12)

where  $n, a_{i,l}$ , and  $w_{k,l}$  denote the order, the coefficient (tapgain parameter), and the model noise, respectively. Because the channel fading  $g_{k,l}$  is a stationary stochastic process and  $w_{k,l}$ is a white noise, the tap gain  $a_{i,l}$  and the variance of  $w_{k,l}$  are



Fig. 3. Joint time-frequency channel estimator for OFDM system.

time-invariant. Without loss of generality, let the zeroth timedomain estimator be the reference and omit the second index l. From (10) and (12), the one-dimensional time-domain channel fading estimation problem can be formulated by the following state-space model:

$$\mathbf{X}_{k} = \mathbf{A}\mathbf{X}_{k-1} + \mathbf{B}w_{k} \quad \text{(state equation)} \tag{13}$$

$$\bar{s}_k = \mathbf{C}\mathbf{X}_k + \bar{v}_k$$
 (measurement equation) (14)

where

$$\mathbf{X}_{k} = \begin{bmatrix} g_{k-(n-1)D_{t}}, g_{k-(n-2)D_{t}}, \dots, g_{k} \end{bmatrix}^{T}$$
$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ a_{n} & a_{n-1} & a_{n-2} & \cdots & a_{2} & a_{1} \end{bmatrix}$$
$$\mathbf{C} = \begin{bmatrix} 0, 0, \dots, 0, 1 \end{bmatrix}$$
$$\mathbf{B} = \begin{bmatrix} 0, 0, \dots, 0, 1 \end{bmatrix}^{T}$$

the superscript T denotes matrix transposition and A is the channel state transition matrix. If the channel correlation function  $\phi_q[m]$  in (10) is known, the tap gain parameter  $a_i$  and the variance of  $w_k$  can be calculated using the Yule–Walker equations [22]. If the variances of background noise  $\overline{v}_k$  is also known, the optimal minimum-error-variance-based estimation algorithms, such as the Kalman filter, can be applied to estimate  $\mathbf{X}_k$ . However, the severity of the channel impairments depends on whether it is indoor or outdoor, urban or suburban [5]. In practice, the channel correlation function  $\phi_q[m]$  and the variance of the background noise are not known a priori. The known variance assumptions may provide an estimate that is highly vulnerable to statistical estimation errors, i.e., a small number of measurement errors may have a large effect on the resultant estimate. In the next section, an  $H_{\infty}$ -based channel estimation algorithm for the OFDM system, where knowledge of the variances of  $w_k$  and  $\overline{v}_k$  is not needed, is presented. For comparison purposes, we first briefly review the Kalman estimation algorithm.

# **IV. CHANNEL ESTIMATION ALGORITHMS**

Impairments in a wireless channel are unknown and most likely time-variant. Methods that do not depend on precise knowledge of the channel characteristics should be more effective and robust for performing the channel estimation. The designs of channel estimators in which the estimator gains are optimized using a minimum error variance criterion (the Kalman filtering approach) and a minimum estimation error spectrum criterion (the  $H_{\infty}$  filtering approach) are presented. The Kalman approach is a covariance minimization problem while the  $H_{\infty}$  approach is a minimization problem where the maximum "energy" of the estimation error over all disturbances is minimized.

## A. Kalman Estimation Algorithm: A Brief Review

Assume that both model noise  $w_k$  and background noise  $\bar{v}_k$ in (13) and (14) are uncorrelated white Gaussian processes with zero mean and variances

$$E\{w_k w_k^*\} = W$$
$$E\{\bar{v}_k \bar{v}_k^*\} = V.$$

The design objective of the Kalman estimation algorithm is to determine the optimal estimate  $\hat{g}_k$  at time k based on the observation  $\{\bar{s}_i\}$   $(1 \le i \le k)$  such that the error covariance

$$\varepsilon_k = E\left\{e_k e_k^H\right\} \tag{15}$$

is minimized, where the estimation error  $e_k$  is given by

$$e_k = g_k - \hat{g}_k. \tag{16}$$

For the state-space model (13)and (14), the Kalman estimation algorithm is given by

$$\hat{\mathbf{X}}_{k} = \mathbf{A}\hat{\mathbf{X}}_{k-1} + \mathbf{K}_{k}[\bar{s}_{k} - \mathbf{C}\mathbf{A}\hat{\mathbf{X}}_{k-1}]$$
(17)

with initial condition  $\hat{\mathbf{X}}_0 = [0]_{n \times 1}$ . The estimator gain and error covariance equations are

$$\mathbf{K}_{k} = \mathbf{P}_{k \mid k-1} \mathbf{C} [V + \mathbf{C} \mathbf{P}_{k \mid k-1} \mathbf{C}^{T}]^{-1}$$
(18)

$$\mathbf{P}_{k|k-1} = \mathbf{A}\mathbf{P}_{k-1}\mathbf{A}^T + \mathbf{B}W\mathbf{B}^T$$
(19)

$$\mathbf{P}_{k} = [\mathbf{I} - \mathbf{K}_{k}\mathbf{C}]\mathbf{P}_{k|k-1}$$
(20)

where  $\mathbf{K}_k$  is the Kalman gain vector,  $\mathbf{P}_{k|k-1} = E[(\mathbf{X}_k - \hat{\mathbf{X}}_{k|k-1})^H (\mathbf{X}_k - \hat{\mathbf{X}}_{k|k-1})]$  is the *a priori* error covariance matrix,  $\mathbf{P}_k = E[(\mathbf{X}_k - \hat{\mathbf{X}}_k)^H (\mathbf{X}_k - \hat{\mathbf{X}}_k)]$  is the *a posteriori* error

covariance matrix, with initial condition  $\mathbf{P}_0 = [0]_{n \times n}$ , and **I** is an  $n \times n$  identity matrix.

#### B. $H_{\infty}$ Estimation Algorithm

Consider the state-space model (13)–(14). We shall not make any assumptions on the disturbances  $w_k$  and  $\overline{v}_k$ , except that they have finite energy. The finite energy assumption is reasonable since in any practical system, both  $w_k$  and  $\overline{v}_k$  are samples of bandlimited noise process. Let  $z_k = \boldsymbol{\xi} \mathbf{X}_k$ , where  $\boldsymbol{\xi}$  is a  $1 \times n$ linear transformation operator. Thus, unlike the Kalman estimation approach, the  $H_{\infty}$  estimation approach achieves estimation using a linear combination of the channel state variables. Let  $\hat{z}_k$ be the estimate of  $z_k$ , and the estimation error be

$$e_k \triangleq z_k - \hat{z}_k. \tag{21}$$

The design criterion of the  $H_{\infty}$  estimator is to provide a uniformly small estimation error for any  $w_k, \bar{v}_k$ , and initial condition  $\mathbf{X}_0$ . The measure of performance is defined as the transfer operator which transforms the  $w_k, \bar{v}_k$  and the uncertainty of the initial condition  $\mathbf{X}_0$  to the estimation error  $e_k$ . The objective function is

$$J \triangleq \frac{\sum_{i=0}^{\infty} |e_i|_Q^2}{|\mathbf{X}_0 - \hat{\mathbf{X}}_0|_{\mathbf{p}_0^{-1}}^2 + \sum_{i=0}^{\infty} \left\{ |\bar{v}_i|_{W_H^{-1}}^2 + |w_i|_{W_H^{-1}}^2 \right\}} \quad (22)$$

where  $|\mathbf{x}|_{\mathbf{G}}^2 \triangleq \mathbf{x}^H \mathbf{G} \mathbf{x}, \hat{\mathbf{X}}_0$  is an *a priori* estimate of  $\mathbf{X}_0, (\mathbf{X}_0 - \hat{\mathbf{X}}_0)$  represents unknown initial condition error, and  $Q \ge 0, \mathbf{p}_0 > 0, W_H > 0$ , and  $V_H > 0$  are weighting parameters.  $\mathbf{p}_0$  denotes a positive definite matrix that reflects *a priori* knowledge on how close the initial guess  $\hat{\mathbf{X}}_0$  is to  $\mathbf{X}_0$ .  $W_H$  and  $V_H$  are weighting variables which are left to the choice of the designer and depend on the performance requirement. In practical systems, the values of  $W_H$  and  $V_H$  can be chosen as the estimates of the covariances of the corresponding noises. The optimal estimate of  $z_k$  among all possible  $\hat{z}_k$  (i.e., the worst case performance measure) should satisfy

$$||J||_{\infty} \stackrel{\Delta}{=} \sup_{\overline{v}_k, w_k, \mathbf{X}_0} J \le \gamma^{-1}$$
(23)

where "sup" stands for supremum and  $\gamma$  (>0) is a prescribed level of noise attenuation. The value that  $\gamma$  can take is discussed in the next section. Equation (23) shows that the  $H_{\infty}$  optimal estimator guarantees the smallest estimation error energy over all possible disturbances with finite energy.

The discrete-time  $H_{\infty}$  estimation can be interpreted as a minimax problem where the strategy is to play the estimate  $\hat{z}_k$  against the exogenous inputs  $w_k, \bar{v}_k$  and the uncertainty of the initial state  $\mathbf{X}_0$  [23]. Using  $z_k = \boldsymbol{\xi} \mathbf{X}_k$  and  $\hat{z}_k = \boldsymbol{\xi} \hat{\mathbf{X}}_k$ , the performance criterion can be equivalently represented as

$$\min_{\hat{\mathbf{X}}_{k}} \max_{\bar{v}_{k}, w_{k}, (\mathbf{X}_{0})} J = -\frac{1}{2} \gamma^{-1} |\mathbf{X}_{0} - \hat{\mathbf{X}}_{0}|_{\mathbf{p}_{0}^{-1}}^{2} + \frac{1}{2} \sum_{i=0}^{\infty} [|\mathbf{X}_{k} - \hat{\mathbf{X}}_{k}|_{\bar{Q}}^{2} - \gamma^{-1} \left( |w_{i}|_{W_{H}^{-1}}^{2} + |\bar{v}_{i}|_{V_{H}^{-1}}^{2} \right)] \quad (24)$$

where  $\bar{\mathbf{Q}} = \boldsymbol{\xi}^T Q \boldsymbol{\xi}$ , and "min" and "max" stand for minimization and maximization, respectively. In (24), the maximization is used to calculate the worst case of J over all disturbance, and then, the estimate is obtained by minimizing the worst case of J. This minimax problem can be solved by using a game theory approach [23]–[25]. For the state-space model (13) and (14) with the performance criterion (24), there exists an  $H_{\infty}$  estimator for  $z_k$  if and only if there exists a stabilizing symmetric positive definite solution  $\mathbf{P}_k$  to the following discrete-time Riccati type equation:

$$\mathbf{P}_{k+1} = \mathbf{A}\mathbf{P}_{k} \left(\mathbf{I} - \gamma \bar{\mathbf{Q}}\mathbf{P}_{k} + \mathbf{C}^{T} V_{H}^{-1} \mathbf{C} \mathbf{P}_{k}\right)^{-1} \mathbf{A}^{T} + \mathbf{B} W_{H} \mathbf{B}^{T} \mathbf{P}_{0} = \mathbf{p}_{0}$$
(25)

where  $\mathbf{p}_0$  is the initial condition. If a solution  $\mathbf{P}_k$  exists, then the  $H_{\infty}$  estimator is given by

$$\hat{z}_k = \boldsymbol{\xi} \hat{\mathbf{X}}_k, \quad k = 1, 2, 3, \dots$$
 (26)

where

$$\hat{\mathbf{X}}_{k} = \mathbf{A}\hat{\mathbf{X}}_{k-1} + \mathbf{G}_{k}(\bar{s}_{k} - \mathbf{C}\mathbf{A}\hat{\mathbf{X}}_{k-1}), \quad \hat{\mathbf{X}}_{0} = [0]_{n \times 1} \quad (27)$$

and  $G_k$  is the gain of the  $H_\infty$  estimator given by

$$\mathbf{G}_{k} = \mathbf{P}_{k} \left( \mathbf{I} - \gamma \bar{\mathbf{Q}} \mathbf{P}_{k} + \mathbf{C}^{T} V_{H}^{-1} \mathbf{C} \mathbf{P}_{k} \right)^{-1} \mathbf{C}^{T} V_{H}^{-1}.$$
 (28)

Comparing the Kalman estimation algorithm (17)–(20) and the  $H_{\infty}$  estimation algorithm (25)–(28), we can observe the following.

- 1) The Kalman estimation algorithm minimizes the covariance of the estimation error of the state vector  $\mathbf{X}_k$ based on the  $\{\bar{s}_i\}$   $(1 \le i \le k)$ . The algorithm is independent of  $\boldsymbol{\xi}$ .
- 2) The  $H_{\infty}$  estimation algorithm gives the optimal estimate of  $\boldsymbol{\xi} \mathbf{X}_k$  based on the  $\{\bar{s}_i\}$   $(1 \le i \le k)$  such that the effect of the worst disturbances on the estimation error is minimized.
- 3) The  $H_{\infty}$  and Kalman estimation algorithms have similar observer structure.

Let the weighting parameters  $(W_H, V_H)$  and  $\mathbf{p}_0$  of the  $H_\infty$  estimation algorithm be the same as the covariances (W, V) and  $\mathbf{P}_0$  of the Kalman estimation algorithm. In the limiting case, where the parameter  $\gamma \to 0$ , the  $H_\infty$  estimation algorithm reduces to a Kalman estimation algorithm.

The following observations reveal a glimpse of the implementation complexity of the  $H_{\infty}$  algorithm relative to the Kalman and MMSE algorithms.

- 1) From the similar observer structure between the proposed  $H_{\infty}$  and the Kalman estimation algorithms, the  $H_{\infty}$  estimator has a similar hardware structure and computation complexity as the Kalman estimator.
- 2) For the  $H_{\infty}$  estimation algorithm, different estimation results can be obtained with different vector  $\boldsymbol{\xi}$ . For example, if we choose  $\boldsymbol{\xi} = [1, 0, \dots, 0]_{1 \times n}$ , the  $H_{\infty}$ estimation algorithm is designed to obtain the optimal estimation of  $g_{k-(n-1)D_t}$ . The estimate  $\hat{g}_{k-(n-1)D_t}$

should give a better estimation of channel fading  $g_{k-(n-1)D_t}$  at the *k*th instant since the estimation is based on the  $\{\overline{s}_i\}, 1 \leq i \leq k$ . This estimation is equivalent to the fixed-lag smoothing problem. The only difference from the traditional fixed-lag smoothing problem is that no additional computation is required in this case.

3) Although the MMSE estimation algorithm proposed in [5] can endure some mismatch on the correlation function of the channel fading, information on coherence bandwidth of the channel fading and the variance of the background noise is still required. Obtaining this accurate information may greatly increase the complexity of the receiver design. For the proposed  $H_{\infty}$  algorithm, the inherent robustness reduces the dependence of the estimation performance on the accuracy of the parameter estimation, which significantly reduces the complexity of the receiver design. In addition, because of the recursive property of the  $H_{\infty}$  algorithm, the complexity on the matrix computation is much less than that in the MMSE algorithm since there is no need to store a large number of past measurements.

#### C. $\gamma$ Value Determination

A necessary and sufficient condition for the existence of the  $H_{\infty}$  estimator is that the discrete-time Riccati equation (25) has a positive-definite solution  $\mathbf{P}_k$ . Thus, the parameter  $\gamma$  should be selected carefully to satisfy this constraint. From (25), as long as the parameter  $\gamma$  is small enough, the Riccati equation always has positive definite solutions. On the other hand, in the design criterion (23), it is observed that the larger the  $\gamma$  value, the less effect the interference has on the estimation error. As a result, the  $\gamma$  value at any time instant k + 1 depends on  $\mathbf{P}_k$ ,  $\mathbf{\bar{Q}}$ ,  $\mathbf{C}$ , and  $V_H$ .

From (25), in order to guarantee  $\mathbf{P}_{k+1}$  to be positive definite, it requires

$$\begin{aligned} \mathbf{P}_{k} \left( \mathbf{I} - \gamma \bar{\mathbf{Q}} \mathbf{P}_{k} + \mathbf{C}^{T} V_{H}^{-1} \mathbf{C} \mathbf{P}_{k} \right)^{-1} &> 0 \\ \Rightarrow \mathbf{P}_{k}^{-1} - \gamma \bar{\mathbf{Q}} + \mathbf{C}^{T} V_{H}^{-1} \mathbf{C} &> 0 \\ \Rightarrow \gamma \bar{\mathbf{Q}} &< \mathbf{P}_{k}^{-1} + \mathbf{C}^{T} V_{H}^{-1} \mathbf{C} \\ \Rightarrow \gamma^{-1} \left( \mathbf{P}_{k}^{-1} + \mathbf{C}^{T} V_{H}^{-1} \mathbf{C} \right) > \bar{\mathbf{Q}} \\ \Rightarrow \gamma^{-1} \mathbf{I} > \bar{\mathbf{Q}} \left( \mathbf{P}_{k}^{-1} + \mathbf{C}^{T} V_{H}^{-1} \mathbf{C} \right)^{-1} \\ \Rightarrow \gamma^{-1} > \max \left\{ \operatorname{eig} \left[ \bar{\mathbf{Q}} \left( \mathbf{P}_{k}^{-1} + \mathbf{C}^{T} V_{H}^{-1} \mathbf{C} \right)^{-1} \right] \right\} \end{aligned}$$
(29)

where  $\max\{\operatorname{eig}(\mathbf{X})\}\$  denotes the maximum eigenvalue of the matrix  $\mathbf{X}$ .

#### D. Tap-Gain Parameter Estimation

The channel estimation algorithm proposed in Section IV-B needs the information on the tap-gain parameters,  $a_i, i = 1, 2, ..., n$ , of the AR model. In this section, an  $H_{\infty}$  algorithm is proposed to estimate the tap-gain parameters from the observations.

Since the low-pass slow-fading channel  $g_k$  can be approximated by an AR model described in (12), from (10), the received signal at the pilot position can be written as

$$\bar{s}_{k} = g_{k} + \bar{v}_{k}$$

$$= \sum_{i=1}^{n} a_{i}g_{k-iD_{t}} + w_{k} + \bar{v}_{k}$$

$$= \sum_{i=1}^{n} a_{i} (\bar{s}_{k-iD_{t}} - \bar{v}_{k-iD_{t}}) + w_{k} + \bar{v}_{k}$$

$$= \sum_{i=1}^{n} a_{i}\bar{s}_{k-iD_{t}} + u_{k}$$
(30)

where  $u_k = w_k + \overline{v}_k - \sum_{i=1}^n a_i \overline{v}_{k-iD_t}$ . For the stationary stochastic process  $g_k, \{a_i, i = 1, 2, ..., n\}$  is time-invariant. Here, we need to estimate  $a_i$  given the observation  $\overline{s}_k$ . Let

$$\boldsymbol{\alpha} = [a_n, a_{n-1}, \dots, a_1]^T$$
$$\boldsymbol{\theta}_k = [\bar{s}_{k-n}, \bar{s}_{k-n+1}, \dots, \bar{s}_{k-1}]^T$$
(31)

and  $\hat{\alpha}_k$  be the estimate of  $\alpha$  at time instant k. Then the measurement and estimation error equations can be written as

$$\bar{s}_k = \boldsymbol{\theta}_k^T \boldsymbol{\alpha}_k + u_k \tag{32}$$

$$e_k^t = \boldsymbol{\theta}_k^T \boldsymbol{\alpha} - \boldsymbol{\theta}_k^T \hat{\boldsymbol{\alpha}}_k \tag{33}$$

where the superscript t denotes that the error is due to the tapgain estimation. The performance criterion can be represented as

$$\min_{e_k^t} \max_{u_k, \boldsymbol{\alpha}_0} J^t = -\frac{1}{2} (\gamma^t)^{-1} |\boldsymbol{\alpha} - \hat{\boldsymbol{\alpha}}_0|^2_{(\mathbf{p}_0^t)^{-1}} \\
+ \frac{1}{2} \sum_{i=0}^{\infty} \left[ |e_k^t|^2_{(Q^t)} - \gamma^{-1} |u_i|^2_{(V^t)^{-1}} \right] \quad (34)$$

where  $\hat{\alpha}_0$  is an *a priori* estimate of  $\alpha$ , and  $Q^t$ ,  $(\mathbf{p}_0^t)$ , and  $V^t$  are weights. Following a similar approach as in Section IV-B, the  $H_\infty$  estimation algorithm to estimate the optimal  $\alpha_k$  can be obtained as

$$\hat{\boldsymbol{\alpha}}_{k} = \hat{\boldsymbol{\alpha}}_{k-1} + \mathbf{K}_{k}^{t} (\bar{s}_{k} - \boldsymbol{\theta}_{k} \hat{\boldsymbol{\alpha}}_{k-1}), \quad \hat{\boldsymbol{\alpha}}_{0} = [0]_{n \times 1} \quad (35)$$

$$\mathbf{K}_{k}^{t} = \mathbf{P}_{k}^{t} \left( \mathbf{I} - \gamma^{t} \boldsymbol{\theta}_{k}^{T} Q^{t} \boldsymbol{\theta}_{k} \mathbf{P}_{k}^{t} + \boldsymbol{\theta}_{k}^{T} (V^{t})^{-1} \boldsymbol{\theta}_{k} \mathbf{P}_{k}^{t} \right)^{-1} \times \boldsymbol{\theta}_{k}^{T} (V^{t})^{-1} \qquad (36)$$

$$\mathbf{P}_{k}^{t} = \mathbf{P}_{k}^{t} \left( \mathbf{I} - \gamma^{t} \boldsymbol{\theta}_{k}^{T} O^{t} \boldsymbol{\theta}_{k} \mathbf{P}_{k}^{t} + \boldsymbol{\theta}_{k}^{T} (V^{t})^{-1} \boldsymbol{\theta}_{k} \mathbf{P}_{k}^{t} \right)^{-1} \qquad (36)$$

$$\mathbf{P}_{0}^{t} = \mathbf{p}_{0}^{t}.$$
(37)

In order to guarantee the existence of the algorithm,  $\gamma^t$  should satisfy (29). In practical systems, the tap-gain parameters can be estimated by transmitting a training sequence with high signal-to-noise ratio (SNR). Simulation results in the next section show that for high SNR training sequences, the estimator can provide fairly accurate estimates of the tap-gains.



Fig. 4. Performance comparison between one-dimensional estimation and joint two-dimensional estimation.

### V. SIMULATION RESULTS AND DISCUSSION

In this section, simulation results are presented to evaluate the performance of channel estimation with both the  $H_{\infty}$  and Kalman approaches.

#### A. System Parameters

Consider an OFDM system using binary phase-shift keying with 32 subcarriers. The channel used in the simulation is a twopath Rayleigh-fading channel model with delay zero and  $T_s$ . The power spectral density satisfies Jakes model, i.e., the time correlation function is of the form

$$\phi_t[m] = J_0(2\pi f_d T m D_t) \tag{38}$$

where  $f_dT$  is the normalized Doppler frequency and is set to 0.05 to characterize a slowly fading channel. The power delay profile is assumed exponentially distributed, i.e.,

$$\sigma_l^2 = \frac{e^{-l/(\chi+1)}}{\sum_{l=0}^{\chi} e^{-l/(\chi+1)}}.$$
(39)

The background noise is modeled as a zero-mean independent identically distributed complex Gaussian random sequence with variance  $\sigma^2$ . The signal power is normalized to 1 so that the input SNR is defined as  $1/\sigma^2$ . The length of the time window n is three and the vector  $\boldsymbol{\xi}$  of the  $H_{\infty}$  estimation algorithm is[1, 0, 0].  $\gamma = 1$  is obtained from (29). For performance comparison, without loss of generality, we choose  $D_t = 1$ . Since the focus is robustness of the channel fading estimation algorithm to the errors on W and V, accurate tap-gain parameters  $a_i, i = 1, 2, \ldots, n$ , are used in the simulation. The tap-gain parameters  $a_i$ , i = 1, 2, ..., n, are also estimated based on (35)–(37) by transmitting a training sequence with SNR = 40 dB. It is shown the performance with the estimated tap-gain parameters is similar to that with the accurate tap-gain parameters.

## B. Effect of Number of Pilots in the Frequency Domain

Fig. 4 shows the mean-square-error of the  $H_{\infty}$  algorithm with different values of  $D_f$ . For performance comparison, the one-dimensional time-domain estimation algorithm proposed in [17], which only uses the time correlation of the channel fading, is also simulated. To make a fair comparison between one-dimensional and two-dimensional algorithms, in our simulation, the one-dimensional  $H_{\infty}$  estimation algorithm is used in place of the one-dimensional least square (LS) algorithm in [17]. The simulation results show that joint estimation in both the time and frequency domains outperforms the one-dimensional time-domain estimation. Decreasing the value of  $D_f$ , i.e., increasing the number of pilots in the frequency domain, can further improve the estimation performance. However, further increasing the number of pilots can only yield marginal improvement on the mean-square-error since the improvement saturates at about  $D_f = 4$ . In the following simulation, without loss of generality, we choose  $D_f = 2$ .

## C. Effect of Input SNR

Fig. 5 shows the mean-square-error versus SNR [no interference (intracell or intercell interference)]. The simulation results show the following.

1) With an increase in input SNR, the mean-square-error performance of both the  $H_{\infty}$  and Kalman estimation algorithms improves.



Fig. 5. Mean-square-error performance over different input SNR.



Fig. 6. Effects of errors on background noise covariance V.

- 2) The  $H_{\infty}$  estimation algorithm outperforms the Kalman estimation algorithm over all the SNR range considered.
- 3) At very high SNR, the performance of the  $H_{\infty}$  and Kalman estimation algorithms merges because the signal component tends to swamp out the channel noise.

# D. Effect of Model Parameter (V and W) Errors

Figs. 6 and 7 show the effects of the background noise and the model noise covariance errors on the estimation performance, respectively. Note that the accurate values of W and V are used in Figs. 6 and 7, respectively. In the simulation, the input SNR is chosen to be 15 dB. Let the channel noise covariance and



Fig. 7. Effects of errors on model noise covariance W.

the model noise covariance used for estimator design be  $\rho V$ and  $\rho W$ , respectively, where  $\rho$  is a multiplier to represent the deviation of the design parameters from the true values. In the simulation,  $\rho$  takes value in the range from -10 to 10 dB and  $\rho = 0$  dB means no deviation. From the figures, it can be seen that model parameter errors can considerably degrade the performance of the Kalman estimation algorithm. The  $H_{\infty}$  estimation algorithm outperforms the Kalman estimation algorithm over the whole error range considered. The larger the error, the larger is the performance gain of the  $H_{\infty}$  algorithm over the Kalman algorithm. Furthermore, as the errors increase, the performance degradation of the  $H_{\infty}$  estimation algorithm is more gradual compared to that of the Kalman estimation algorithm. For example, in Fig. 6, the mean-square-error using the Kalman estimation algorithm changes from 0.0038 at 0 dB to 0.009 at 10 dB, while the mean-square-error using the  $H_{\infty}$  estimation algorithm changes from 0.0015 at 0 dB to 0.0028 at 10 dB. The variation of mean-square-error for the Kalman estimation algorithm is four times larger than that of the  $H_{\infty}$  estimation algorithm. The smaller variation indicates that the  $H_{\infty}$  estimation algorithm is more robust against the parameter errors compared to the Kalman estimation algorithm.

#### E. BER Performance

At the receiver, the received signal is multiplied by the conjugate of the channel estimate to compensate for the phase offset introduced by the fading channel, and the data symbols are recovered by coherent detection. Fig. 8 shows the BER performance of the OFDM system using the  $H_{\infty}$  and Kalman channel estimation algorithms. The following is observed.

- 1) The BER performance based on the  $H_{\infty}$  estimation algorithm outperforms that based on the Kalman estimation algorithm over all the SNR range considered. The reason is that the more accurate channel estimate obtained by the  $H_{\infty}$  estimation algorithm can provide more accurate phase information about the channel fading. More accurate phase information can provide better coherent detection performance. For example, at a BER of  $10^{-4}$ , the input SNR of the system with  $H_{\infty}$ estimation is 27.2 dB, while it is 31.7 dB for the system with Kalman estimation. The improvement is 4.5 dB.
- 2) At high SNR, the BER characteristics of both the  $H_{\infty}$  and Kalman estimation algorithms are close to each other. The reason is that, at high SNR, both estimation algorithms can achieve nearly the same channel estimation accuracy.

## VI. CONCLUSION

A robust channel estimation algorithm based on the  $H_{\infty}$  approach has been proposed for OFDM wireless communication systems. The proposed  $H_{\infty}$  algorithm minimizes the effect of worst disturbance (including both background noise and channel model noise) on the estimation error and, therefore, is less sensitive to the uncertainty of the channel statistics. Simulation results indicate that the  $H_{\infty}$  estimation algorithm



Fig. 8. BER performance comparison between the  $H_{\infty}$  and Kalman estimation algorithms.

has superior performance to the Kalman estimation counterpart, while keeping the similar implementation complexity.

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