Recursive approach to optimal control problem of multiarea electric energy system

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Abstract: A recursive fixed-point-type method is presented to find the optimal control of a state-variable model of the megawatt-frequency control problem of multiarea electric energy systems. The results give the numerical decomposition so that only low-order systems are involved in algebraic computations. This approach is conceptually simple and produces considerable savings of computation.

1 Introduction

With the development of the electrical power industry and the interconnection of isolated power systems, very large power systems are formed. The most important contribution that modern optimal theory has made to the control engineers is the ability to handle a large multivariable control problem with ease. The engineer has only to represent the control system in state-variable form and to specify the desired performance mathematically in terms of a cost to be minimised. The application of the optimal control theory to study the stabilisation and optimisation of power systems has been shown in References 1–4. Owing to the high dynamic order of such systems large amounts of computer time and memory capacities are required for adequate solution of even the simplest cases. To decrease the computations and investigate the properties of large scale systems, the numerical power-series expansion method has been used in the past twenty years [4–6]. Because it is nonrecursive in nature, the power-series expansion method becomes very cumbersome and computationally very expensive when a high order of accuracy is required. Since the optimal control regulator is obtained by solving the Riccati equation of the full system, the present paper presents a recursive fixed point type method to obtain the solution of the Riccati equation in terms of reduced-order problems for weakly connected multiarea electric energy system. In addition, if not all of the state variables are available, the optimal control law requires the design of the state estimators. For this case, the estimator Riccati equation can be solved by the same algorithm. This method is conceptually simple and is very suitable for parallel programming, since it produces considerable savings of computation.

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2 Problem formulation

Consider a linear dynamical system composed of two subsystems in the form [5]

$$\dot{x}_1 = A_1 x_1 + \varepsilon A_2 x_2 + B_1 u_1 + \varepsilon B_2 u_2$$
$$\dot{x}_2 = \varepsilon A_3 x_1 + A_4 x_2 + \varepsilon B_3 u_1 + B_4 u_2 \qquad (1)$$

where $x_i \in \mathscr{R}^{ni}$ are state vectors, $u_i \in \mathscr{R}^{mi}$ are control inputs, $i = 1, 2$, and ε is a small coupling parameter. This dynamical system is represented in general by

$$\dot{x} = Ax + Bu \qquad (2)$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad A = \begin{bmatrix} A_1 & \varepsilon A_2 \\ \varepsilon A_3 & A_4 \end{bmatrix} \quad B = \begin{bmatrix} B_1 & \varepsilon B_2 \\ \varepsilon B_3 & B_4 \end{bmatrix} \qquad (3)$$

The linear-quadratic optimal control problem requires to find the control u, which minimises the cost

$$C = \frac{1}{2}\int_0^\infty [x^T Q x + u^T R u]\, dt \quad Q \geqslant 0,\ R > 0 \qquad (4)$$

For the purpose of this paper we assume that the structure of the matrix Q is consistent with the system matrix A, i.e.

$$Q = \begin{bmatrix} Q_1 & \varepsilon Q_2 \\ \varepsilon Q_2^T & Q_3 \end{bmatrix} \qquad (5)$$

All problem matrixes defined in eqns. 1–5 are constant and of appropriate dimensions.

3 Recursive algorithm for optimal control

The optimal controller that minimises the cost C along the trajectories of eqn. 1 weighted by a constant gain matrix F is given by [6]

$$u_{opt} = -F_{opt} x = -R^{-1} B^T P x \qquad (6)$$

where P is the positive semidefinite stabilising solution of the algebraic Riccati equation

$$PA + A^T P + Q - PSP = 0 \quad S = BR^{-1}B^T \qquad (7)$$

It can be shown that the nature of the solution of eqn. 7 is [7–8]

$$P = \begin{bmatrix} P_1 & \varepsilon P_2 \\ \varepsilon P_2^T & P_3 \end{bmatrix} \qquad (8)$$

By partitioning eqn. 7 compatible to eqns. 3, 5 and 8 we obtain three algebraic equations

$$P_1 A_1 + A_1^T P_1 + Q_1 - P_1 S_1 P_1$$
$$+ \varepsilon^2 (P_2 A_3 + A_3^T P_2^T) - \varepsilon^2 [(P_1 S_{12} + P_2 Z^T) P_1$$
$$+ (P_1 Z + P_2 (S_2 + \varepsilon^2 S_{21})) P_2^T] = 0 \qquad (9)$$

$$P_3 A_4 + A_4^T P_3 + \varepsilon^2(P_2^T A_2 + A_2^T P_2) + Q_3 - P_3(S_2 + \varepsilon^2 S_{21})P_3 - \varepsilon^2\{[P_2^T(S_1 + \varepsilon^2 S_{12}) + P_3 Z^T]P_2 + P_2^T Z P_3\} = 0 \quad (10)$$

$$P_1 A_2 + P_2 A_4 + A_1^T P_2 + A_3^T P_3 + Q_2 - P_1 S_1 P_2 - P_1 Z P_3 - P_2 S_2 P_3 - \varepsilon^2[(P_1 S_{12} + P_2 Z^T)P_2 + P_2 S_{21} P_3] = 0 \quad (11)$$

where

$$S_1 = B_1 R_1^{-1} B_1^T, \quad S_2 = B_4 R_2^{-1} B_4^T$$
$$S_{12} = B_2 R_2^{-1} B_2^T, \quad S_{21} = B_3 R_1^{-1} B_3^T$$
$$Z = B_1 R_1^{-1} B_3^T + B_2 R_2^{-1} B_4^T \quad (12)$$

Since ε is a small parameter, we can define the $O(\varepsilon^2)$ approximation of eqns. 9–11 as follows:

$$\bar{P}_1 A_1 + A_1^T \bar{P}_1 - \bar{P}_1 S_1 \bar{P}_1 + Q_1 = 0$$
$$\bar{P}_3 A_4 + A_4^T \bar{P}_3 - \bar{P}_3 S_2 \bar{P}_3 + Q_3 = 0 \quad (13)$$

and

$$\bar{P}_2 D_2 + D_1^T \bar{P}_2 = -(\bar{P}_1 A_2 + \bar{A}_3^T P_3 + Q_2 - \bar{P}_1 Z \bar{P}_3) \quad (14)$$

so that the corresponding solution of eqn. 8 is

$$P = \begin{bmatrix} P_1 & \varepsilon P_2 \\ \varepsilon P_2^T & P_3 \end{bmatrix} = \bar{P} + O(\varepsilon^2) \quad (15)$$

where

$$D_1 = [A_1 - S_1 \bar{P}_1], \quad D_2 = [A_4 - S_2 \bar{P}_3] \quad (16)$$

The unique positive semidefinite stabilising solutions of eqn. 13 exist under the assumption that the triples $(A_1, B_1, \sqrt{(Q_1)})$ and $(A_4, B_4, \sqrt{(Q_3)})$ are stabilisable-detectable. Under the assumption, matrixes D_1 and D_2 are stable [9] so that the unique solution of eqn. 14 also exists.

If the errors are defined as

$$P_j = \bar{P}_j + \varepsilon^2 E_j \quad j = 1, 2, 3 \quad (17)$$

then the exact solution will be of the form

$$P = \begin{bmatrix} \bar{P}_1 + \varepsilon^2 E_1 & \varepsilon(\bar{P}_2 + \varepsilon^2 E_2) \\ \varepsilon(\bar{P}_2 + \varepsilon^2 E_2)^T & \bar{P}_3 + \varepsilon^2 E_3 \end{bmatrix} \quad (18)$$

Subtracting eqns. 13 and 14 from the corresponding eqns. 9–11 and using eqn. 17 produces the following equations for the errors:

$$E_1 D_1 + D_1^T E_1 = P_1 S_{12} P_1 + P_2 Z^T P_1 + P_1 Z P_2^T + P_2 S_2 P_2^T - P_2 A_3 - A_3^T P_2^T + \varepsilon^2(E_1 S_1 E_1 + P_2 S_{21} P_2^T) \quad (19)$$

$$E_3 D_2 + D_2^T E_3 = P_3 S_{21} P_3 + P_2^T S_1 P_2 + P_3 Z^T P_2 + P_2^T Z P_3 - P_2^T A_2 - A_2^T P_2 + \varepsilon^2(E_3 S_2 E_3 + P_2^T S_{12} P_2) \quad (20)$$

$$D_1^T E_2 + E_2 D_2 = P_1 S_{12} P_2 + P_2 Z^T P_2 + P_2 S_{21} P_3 - E_1 D_{12} - D_{21}^T E_3 + \varepsilon^2(E_1 S_1 E_2 + E_1 Z E_2 + E_2 S_2 E_3) \quad (21)$$

where

$$D_{12} = A_2 - S_1 \bar{P}_2 - Z \bar{P}_3$$
$$D_{21} = A_3 - S_2 P_2^T - Z^T \bar{P}_1 \quad (22)$$

It can be shown easily that the nonlinear eqns. 19–21 have the form

$$E_1 D_1 + D_1^T E_1 = const + \varepsilon^2 f_1(E_1, E_2, \varepsilon^2)$$
$$E_3 D_2 + D_2^T E_3 = const + \varepsilon^2 f_3(E_2, E_3, \varepsilon^2)$$
$$E_2 D_2 + D_1^T E_2 = const + \varepsilon^2 f_2(E_1, E_2, E_3, \varepsilon^2) \quad (23)$$

We can see that all cross-coupling terms and all nonlinear terms in eqns. 19–21 are multiplied by ε^2, so that we propose the following reduced-order parallel algorithm for solving eqns. 19–21

$$E_1^{(i+1)} D_1 + D_1^T E_1^{(i+1)} = P_1^{(i)} S_{12} P_1^{(i)} + P_2^{(i)} Z P_2^{(i)T} + P_2^{(i)} S_2 P_2^{(i)T} - P_2^{(i)} A_3 - A_3^T P_2^{(i)T} + \varepsilon^2(E_1^{(i)} S_1 E_1^{(i)} + P_2^{(i)} S_{21} P_2^{(i)T}) \quad (24)$$

$$E_3^{(i+1)} D_2 + D_2^T E_3^{(i+1)} = P_3^{(i)} S_{21} P_3^{(i)} + P_2^{(i)T} S_1 P_2^{(i)} + P_3^{(i)} Z^T P_2^{(i)} + P_2^{(i)T} Z P_3^{(i)} - P_2^{(i)T} A_2 - A_2^T P_2^{(i)} + \varepsilon^2(E_3^{(i)} S_2 E_3^{(i)} + P_2^{(i)T} S_{12} P_2^{(i)}) \quad (25)$$

$$D_1^T E_2^{(i+1)} + E_2^{(i+1)} D_2 = P_1^{(i+1)} S_{12} P_2^{(i)} + P_2^{(i)} Z^T P_2^{(i)} + P_2^{(i)} S_{21} P_3^{(i)} - E_1^{(i+1)} D_{12} - D_{21}^T E_3^{(i+1)} + \varepsilon^2(E_1^{(i+1)} S_1 E_2^{(i)} + E_1^{(i+1)} Z E_2^{(i)} + E_2^{(i)} S_2 E_3^{(i+1)}) \quad (26)$$

with $E_1^{(0)} = 0$, $E_2^{(0)} = 0$, $E_3^{(0)} = 0$, where

$$P_j^{(i)} = \bar{P}_j + \varepsilon^2 E_j^{(i)} \quad j = 1, 2, 3; \; i = 0, 1, 2, 3, \ldots \quad (27)$$

The following theorem indicates the features of the algorithm of eqns. 24–27.

Theorem: Under the assumption, the algorithm of eqns. 24–27 converges to the exact solution of E with the rate of convergence of $O(\varepsilon^2)$, i.e.

$$\|E - E_j^{(i+1)}\| = O(\varepsilon^2)\|E - E_j^{(i)}\| \quad i = 0, 1, 2, \ldots \quad (28)$$

or equivalently

$$\|E - E_j^{(i)}\| = O(\varepsilon^{2i}) \quad (29)$$

Proof: The Jacobian of eqns. 9–11, at some $\varepsilon = 0$, is given by

$$J = \begin{bmatrix} J_{11} & 0 & 0 \\ J_{21} & J_{22} & J_{23} \\ 0 & 0 & J_{33} \end{bmatrix} \quad (30)$$

where

$$J_{11} = I_{n_1} \oplus D_1^T + D_1^T \oplus I_{n_1}$$
$$J_{22} = J_{n_2} \oplus D_2^T + D_1^T \oplus I_{n_1}$$
$$J_{33} = I_{n_2} \oplus D_2^T + D_2^T \oplus I_{n_2} \quad (31)$$

Since D_1 and D_2 are stable matrixes (by the assumption), J_{ii}, $i = 1, 2, 3$ are nonsingular and hence the Jacobian will be nonsingular at $\varepsilon = 0$. By the implicit function theorem, the existence of the unique bounded solution of eqns. 9–11 is guaranteed for sufficiently small values of ε.

In the next step we have to prove convergence of the algorithm of eqns. 24–27 and to give an estimate of the rate of convergence. For $i = 0$, eqns. 19 and 24 imply

$$(E_1 - E_1^{(1)})D_1 + D_1^T(E_1 - E_1^{(1)}) = \varepsilon^2 f_1(E_1, E_2, \varepsilon^2) \quad (32)$$

Since D_1 is stable and E_1 and E_2 are bounded it follows that

$$\|E_1 - E_1^{(1)}\| = O(\varepsilon^2) \tag{33}$$

$$A = \begin{bmatrix} 0 & 0.545 & 0 & 0 & 0 & -0.545 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3.27 & -0.05 & 6 & 0 & 3.27 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3.33 & 3.33 & 0 & 0 & 0 & 0 \\ 0 & 0 & -5.208 & 0 & -13 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 3.27 & 0 & 0 & 0 & -3.27 & -0.05 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3.33 & 3.33 \\ 0 & 0 & 0 & 0 & 0 & 0 & -5.208 & 0 & -12.5 \end{bmatrix} \tag{39}$$

Similarly from eqns. 20 and 25 we have

$$(E_3 - E_3^{(1)})D_2 + D_2^T(E_3 - E_3^{(1)}) = \varepsilon^2 f_3(E_2, E_3, \varepsilon^2) \tag{34}$$

and

$$\|E_3 - E_3^{(1)}\| = O(\varepsilon^2) \tag{35}$$

Using the same arguments in eqns. 21 and 26 produces

$$\|E_2 - E_2^{(1)}\| = O(\varepsilon^2) \tag{36}$$

By continuing the same procedure and by induction we conclude that

$$\begin{aligned} \|E_1 - E_1^{(i)}\| &= O(\varepsilon^{2i}) \\ \|E_2 - E_2^{(i)}\| &= O(\varepsilon^{2i}) \\ \|E_3 - E_3^{(i)}\| &= O(e^{2i}) \end{aligned} \tag{37}$$

with $i = 1, 2, 3, \ldots$, which completes the proof of the theorem. To get a solution of the Riccati equation P, which has dimensions $n \times n = (n_1 + n_2) \times (n_1 + n_2)$, we have only to solve two reduced-order algebraic Riccati equations of dimensions $(n_1 \times n_1)$ and $(n_2 \times n_2)$, respectively. After obtaining the solution of the Riccati equation $P^{(i)}$, the optimal control in eqn. 6 is

$$u_{opt}^{(i)} = -R^{-1}B^T P^{(i)} x \tag{38}$$

When a multiarea system has more than two areas, the algorithm can be used repeatedly. The relationship among each subarea Riccati equation can be found in Reference 6.

4 Multiarea electric energy systems model and numerical solution

The state variable model of the megawatt-frequency control problem of multiarea electric energy systems was developed in References 1 and 2. The model is the multistage decomposition of a two-area, interconnected nonreheat power system ($dimA_{11} = 5 \times 5$, $dimA_{22} = 4 \times 4$). The system description is given in Appendix 8 with the nominal system parameter. The numerical values of the system and input matrixes have been computed as

$$B^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 12.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12.5 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.3 & 0 & 0 & 0 & -0.3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.3 & 0 & 0 & 0 & 1.3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{40}$$

The small coupling parameter ε is built into the problem. The value for ε should be estimated from the strongest coupled matrix; in this case Matrix A. It seems from our experience that the formula

$$\varepsilon = \frac{max(\|A_{12}\|, \|A_{21}\|)}{max\,(\|A_{11}\|, \|A_{22}\|)} = \frac{3.27}{9.32} = 0.351 \tag{41}$$

produces a good estimate for ε, where $\| \; \|$ is any suitable norm. In this example we have used the infinity norm. Simulation results are obtained by using the package L-A-S [10] for the computer aided control system design. After 6 iterations we obtain the solution for the Riccati equation with accuracy of 10^{-6} ($0.351^{12} = 3.496 \times 10^{-6}$) which verifies our theory. The solution of the Riccati equation P is

$$P = \begin{bmatrix} 3.067 & 0.924 & 0.299 & 0.274 & 0.0566 & -0.924 & -0.299 & -0.274 & -0.0566 \\ 0.924 & 1.9035 & 0.350 & 0.177 & 0.240 & -0.469 & 0.0854 & 0.2141 & 0.056 \\ 0.299 & 0.350 & 0.480 & 0.394 & 0.0746 & 0.0854 & -0.009 & 0.0106 & 0.00501 \\ 0.274 & 0.177 & 0.394 & 0.464 & 0.1021 & 0.2141 & 0.0106 & 0.0074 & 0.0024 \\ 0.0566 & 0.024 & 0.0746 & 0.1021 & 0.0237 & 0.056 & 0.0051 & 0.0024 & 0.0005 \\ -0.924 & -0.469 & 0.0854 & 0.2141 & 0.056 & 1.9035 & 0.3495 & 0.1767 & 0.024 \\ -0.299 & 0.0854 & -0.009 & 0.0106 & 0.0051 & 0.3495 & 0.480 & 0.3934 & 0.0746 \\ -0.274 & 0.2151 & 0.0106 & 0.0074 & 0.0024 & 0.1767 & 0.3934 & 0.4644 & 0.1021 \\ -0.0566 & 0.056 & 0.0051 & 0.0024 & 0.0005 & 0.024 & 0.0746 & 0.1021 & 0.0237 \end{bmatrix} \tag{42}$$

Using eqn. 6 we can write out the controller

$$u_1 = [-0.707 - 0.3 - 0.932 - 1.28 - 0.296]x_1 + [-0.701 - 0.064 - 0.03 - 0.006]x_2 \quad (43)$$

$$u_2 = [0.707 - 0.701 - 0.064 - 0.03 - 0.006]x_1 + [-0.3 - 0.932 - 1.28 - 0.296]x_2 \quad (44)$$

which is exactly the same as in Reference 2. The physical meanings of the controller were discussed in Reference 2. The Q and R matrixes were varied in Reference 2 to show their effect on the system response (the improvement in damping given by the optimal controller to the system).

5 Conclusion

A recursive reduced-order parallel algorithm has been developed for solving the algebraic Riccati equations of dynamic weakly-coupled multiarea electric energy systems. The algorithm is based on the fixed point approach to a small coupling parameter problem, where the small parameter plays the role of the radius of the convergence. It was shown that the algorithm is computationally very efficient in the study of steady-state linear control problems, especially when a high order of accuracy is required.

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8 Appendix

8.1 Megawatt-frequency control program of multiarea electric energy systems

The state variable model of the megawatt-frequency control problem of multiarea electric energy systems was developed in References 1 and 2.

The system differential equation is

$$\Delta P_{tieiv} = \sum_v T^*_{iv}\left(\int \Delta f_i \, dt - \int \Delta f_v \, dt\right) \quad (45)$$

where

$$T^*_{iv} = \frac{2\pi |V_i| |V_v|}{X_{iv} P_{ri}} \cos(\delta^*_i - \delta^*_v) \quad (46)$$

$$\frac{2H_i}{f^*}\frac{d}{dt}\Delta f_i + D_i \Delta f_i + \sum_v T^*_{iv}\left(\int \Delta f_i \, dt - \int \Delta f_v \, dt\right) = \Delta P_{gi} - \Delta P_{di} \quad (47)$$

$$\frac{d}{dt}\Delta P_{gi} = -\frac{1}{T_{ti}}\Delta P_{gi} + \frac{1}{T_{ti}}\Delta X_{gvi} \quad (48)$$

$$\frac{d}{dt}\Delta X_{gvi} = -\frac{1}{T_{gvi}}\Delta X_{gvi} - \frac{1}{T_{gvi}}\Delta f_i + \frac{1}{T_{gvi}}\Delta P_{ci} \quad (49)$$

For a two-area interconnected system the following state and control variables can be defined

$$X = \begin{bmatrix} \int \Delta P_{tie1} \, dt \\ \int \Delta f_1 \, dt \\ \Delta f_1 \\ \Delta P_{g1} \\ \Delta X_{gvl} \\ \int \Delta f_2 \, dt \\ \Delta f_2 \\ \Delta P_{g2} \\ \Delta X_{gv2} \end{bmatrix}, \quad u = \begin{bmatrix} \Delta P_{cl} \\ \Delta P_{c2} \end{bmatrix} \quad (50)$$

System parameters are taken to be

$$\begin{aligned} &P_{r1} = P_{r2} = 2000\mathrm{MW} \\ &H_1 = H_2 = 5\,s \\ &D_1 = D_2 = 8.33 \times 10^{-3}\ \mathrm{puMW/Hz} \\ &T_{t1} = T_{t2} = 0.3\,s \\ &T_{gv1} = T_{gv2} = 0.08\,s \\ &R_1 = R_2 = 2.4\ \mathrm{Hz/puMW} \\ &P_{tie\,max} = 200\ \mathrm{MW} \\ &\sigma^*_1 - \sigma^*_2 = 30^\circ \\ &T^*_{12} = 0.545\ \mathrm{puMW} \\ &\Delta P_{dl} = 0.01\ \mathrm{puMW} \end{aligned} \quad (51)$$