Adaptive Importance Sampling for Bit Error Rate Estimation Over Fading Channels

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ABSTRACT: *Computer simulation is an essential approach to access the performance of mobile and portable communications systems. However, in the case of a slowly fading channel (where the number of fading cycles dominantly determines the confidence interval of the simulation results), computer simulation time can be prohibitively long in order to obtain an accurate bit error rate (BER) estimate using the Monte Carlo (MC) method. This paper develops an adaptive importance sampling (AIS) technique for BER estimation over Rayleigh fading channels. The AIS simultaneously biases statistical properties of both channel fading process and input Gaussian noise and adaptively searches for the optimal biased density function during the course of simulation. The AIS technique is applied to analyze the BER performance of QPSK with multiple-symbol differential detection. Computer simulation results show that the AIS technique significantly reduces the simulation time compared with the conventional MC technique, and simplifies the procedure of selecting the optimal biased density function.*

I. Introduction

Computer simulation is an essential tool to assess the bit error rate (BER) performance of digital communications systems where theoretical analysis is very difficult or impossible and to validate theoretical evaluations of the system performance. Using the Monte Carlo (MC) method, the necessary computer simulation time can be prohibitively long for a low BER value or a slowly fading channel. Importance sampling (IS) techniques have been investigated to reduce the simulation time for the BER performance analysis in an additive white Gaussian noise (AWGN) channel (**1–3**). In the modified MC simulation (with IS), the statistical properties of the noise processes driving the system are biased in such a way to make the low probability events occur more frequently, so that the simulation of these events can be made with relatively smaller sample size and with a reduction in time simulation. Since the error events are intentionally increased in a known way, it can be corrected for. However, the biased probability densities (or weighting functions) widely used in IS simulation are still far from optimal ones. In some cases, trial and error method and others are used to search for 'good' densities, which can be very burdensome. Adaptive importance sampling has been investigated to considerably reduce the statistical error of the estimated failure probability in structural reliability analyses (**4, 5**). The technique iteratively adapts the biased probability densities to the optimal ones. Recently a similar approach has been studied for a digital communications

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system in an AWGN channel (**6**). The basic concept of AIS is that the data samples which cause an error event are distributed according to the (unknown) optimal IS density. Therefore, the optimal biased density can be obtained by estimating the probability density of the data samples.

Over the last decade, mobile and portable digital communications systems have experienced tremendous development. Research work on the system performance evaluation is important in developing better system structures and improving system performance. Mobile fading channels are often characterized by fading random processes with AWGN. Due to the complexity of the channel fading characteristics, it can be very difficult (if not impossible) to theoretically evaluate the BER performance of the systems. Computer simulation is a complementary approach. In the simulations, the confidence interval of the BER estimates depends not only on the number of error events that have occurred but also on the number of fading cycles that the system has experienced. In the case of a high received signal-to-noise ratio (SNR), the error events occur only in the deep fading region of each fading cycle. Most of the simulation time is spent in simulating the process where the system experiences non-deep fading and no error events occur. In order to obtain an accurate BER estimate, the system has to undergo a number of fading cycles. As a result, the number of fading cycle is a dominant factor in determining the confidence interval. Generally, the ratio of the maximum Doppler frequency shift f_D of the received signal to the transmission symbol rate $1/T$ (where T is the symbol interval) decides the degree of the signal fading. When $f_D \cdot T \ll 1.0$, the channel exhibits slow fading. For instance, in the European Digital Cordless Telecommunications (DECT) system, T is less than one nanosecond, and f_D is a few Hertz for indoor wireless communications, the value of $f_D \cdot T$ is of the order of 10^{-6}. For such a slow fading channel, it would take a large amount of computer time to estimate the system BER performance. Therefore, IS techniques are necessary to reduce the simulation time.

Previous work investigates the applications of IS and AIS techniques to the BER estimation of a digital communications system in an AWGN channel (**6**). In this paper, the AIS technique is applied to a Rayleigh fading channel. Probability densities of both fading random process and AWGN are biased instead of only the fading process or the noise process, so that more simulation time reduction is possible. An adaptive algorithm is developed to search for the optimal biased probability density during the course of the simulation, instead of using trial and error method. The AIS has advantages over conventional MC and IS techniques in that: (i) the computer simulation time for estimating the BER performance can be dramatically reduced; (ii) the accuracy of the BER estimate can be greatly increased; and (iii) the estimate of the optimal biased density function can be achieved with high accuracy and simplified procedure.

The remainder of this paper is organized as follows. A general system model is described in Section II. Section III investigates the IS technique for fading channels. Section IV develops the adaptive algorithm for the optimal biased probability densities. The AIS technique is applied to analyze the BER performance of QPSK with multiple-symbol differential detection over a slow Rayleigh fading channel in Section V. The conclusions of this work are presented in Section VI.

II. System Description

The simplified functional block diagram of a digital communications system over a fading channel is shown in Fig. 1. The data source generates an independent identically distributed (iid) binary data sequence. The transmitter consists of an encoder and a modulator (where the encoder may be omitted depending on the system encoding scheme). The binary sequence is encoded, and modulated into the transmitted signal $s(t)$. The fading channel corrupts the transmitted signal $s(t)$ by introducing a multiplicative envelope distortion $\gamma(t)$ and a carrier phase disturbance $\psi(t)$. The received signal $r(t)$ is also degraded by AWGN $n(t)$ with a one-side spectral density N_0, i.e.

$$r(t) = \gamma(t) \cdot e^{-j\psi(t)} \cdot s(t) + n(t). \tag{1}$$

In the case that the transmitted signal arrives at the receiver antenna through multiple paths and there is no direct path between the transmitter and receiver, the amplitude distortion $\gamma(t)$ has a Rayleigh distribution, and the carrier phase jitter $\psi(t)$ has a uniform distribution over $[0, 2\pi]$. Furthermore, $\gamma(t)$ and $\psi(t)$ are independent random processes. Such a statistical model is widely used for an indoor radio channel (**7**). The autocorrelation coefficient of $\gamma(t)$ can then be derived as (**8**)

$$\rho_\gamma(\tau) = E[\gamma(t)\gamma(t+\tau)] = P \cdot J_0(2\pi f_D \tau) \tag{2}$$

where P is the received signal power and $J_0(\cdot)$ is a zero-order Bessel function. The channel fading has memory depending on the value of f_D. The memory of the system also depends on the structure of the transmitter and receiver, such as those of decoder, modulator and demodulator. The effects of the channel memory on the system BER performance depends on the data transmission rate, and the structure of the transmitter and receiver.

III. Importance Sampling

From equation (1), the system BER performance depends on the transmitted signal $s(t)$, the channel fading $\gamma(t)e^{-j\psi(t)}$ and the Gaussian noise $n(t)$. The function of the receiver can be expressed by a mapper $g(\cdot): \mathscr{R}^M \rightarrow \mathscr{R}$, where M is the system memory. Let $R = \Gamma \cdot S + N$ be the received signal vector with M elements, where $R = \{r_k, r_{k-1}, \ldots, r_{k-M+1}\}$, $\Gamma = \{\gamma_k e^{-j\psi_k}, \gamma_{k-1} e^{-j\psi_{k-1}}, \ldots, \gamma_{k-M+1} e^{-j\psi_{k-M+1}}\}$, $S^T = \{s_k, s_{k-1}, \ldots, s_{k-M+1}\}$ and $N = \{n_k, n_{k-1}, \ldots, n_{k-M+1}\}$. Then the BER can be represented by

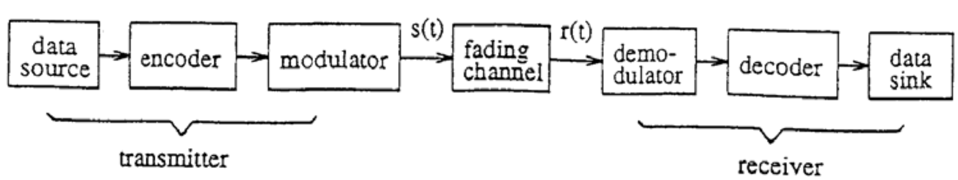

FIG. 1. Functional block diagram of the communications system.

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$$p_e = \int_{\mathscr{R}^M} D[g(R)] f_\gamma(\gamma) f_\psi(\psi) f_n(n) f_s(s)\, d\gamma\, d\psi\, dn\, ds \tag{3}$$

where $f_\gamma(\gamma) f_\psi(\psi)$, $f_n(\mathrm{n})$ and $f_s(n)$ are the probability density functions (pdf's) of the M-dimensional random variables Γ, N and S respectively, and

$$D[g(R)] = \begin{cases} 1, & \text{if an error occurs;} \\ 0, & \text{otherwise.} \end{cases}$$

The MC estimate of p_e is

$$\hat{p}_e = \frac{1}{N} \sum_{i=1}^{N} D[g(R_i)] \tag{4}$$

where N is the sample size of the data bits transmitted and R_i is the ith sample of the received signal. If the original pdf's $f_\gamma(\gamma)$ and $f_n(n)$ are modified to new pdf's $f_\gamma^*(\gamma)$ (>0 for all γ) and $f_n^*(n)$ (>0 for all n), then the IS weighting function is

$$w(\gamma, n) = \frac{f_\gamma(\gamma)}{f_\gamma^*(\gamma)} \cdot \frac{f_n(n)}{f_n^*(n)} = w_\gamma(\gamma) \cdot w_n(n) \tag{5}$$

where $w_\gamma(\gamma) = f_\gamma(\gamma)/f_\gamma^*(\gamma)$ and $w_n(n) = f_n(n)/f_n^*(n)$. Equation (3) can be rewritten as

$$p_e = \int_{\mathscr{R}^M} D[g(R)] w_\gamma(\gamma) w_n(n) f_\gamma^*(\gamma) f_n^*(n) f_s(s) f_\psi(\psi)\, d\gamma\, dn\, ds\, d\psi. \tag{6}$$

The IS estimate of p_e is

$$\hat{p}_e^* = \frac{1}{N^*} \sum_{i=1}^{N^*} D^*[g(R_i)] \tag{7}$$

where N^* is the total data samples in the IS simulation and

$$D^*[g(R_i)] = D[g(R_i)] w_\gamma(\gamma) w_n(n). \tag{8}$$

From equation (7),

$$E[\hat{p}_e^*] = E_*\{D^*[g(R_i)]\} \tag{9}$$

where $E_*(\cdot)$ represents the expectation with respect to the biased densities $f_\gamma^*(\gamma)$ and $f_n^*(n)$. Using equations (7) and (8),

$$\begin{aligned} E[\hat{p}_e^*] &= \int_{\mathscr{R}^M} D^*[g(R)] f_\gamma^*(\gamma) f_\psi(\psi) f_n^*(n) f_s(s)\, d\gamma\, d\psi\, dn\, ds \\ &= \int_{\mathscr{R}^M} D[g(R)] w_\gamma(\gamma) w_n(n) f_\gamma^*(\gamma) f_\psi(\psi) f_n^*(n) f_s(s)\, d\gamma\, d\psi\, dn\, ds \\ &= \int_{\mathscr{R}^M} D[g(R)] f_\gamma(\gamma) f_\psi(\psi) f_n(n) f_s(s)\, d\gamma\, d\psi\, dn\, ds \\ &= E[p_e], \end{aligned} \tag{10}$$

that is, $\hat{p}_e^*$ of equation (7) is an unbiased estimator of p_e. The variance of the IS estimator is

$$\sigma^{*2} = E_*[(\hat{p}_e^* - E(\hat{p}_e^*))^2]$$
$$= \int_{\mathscr{R}^M} \frac{[w_\gamma(\gamma)w_n(n) - p_e]D[g(R)]f_\gamma(\gamma)f_\psi(\psi)f_n(n)f_s(s)}{N^*} \mathrm{d}\gamma\,\mathrm{d}\psi\,\mathrm{d}n\,\mathrm{d}s \quad (11)$$

which can be estimated from simulation results as

$$\hat{\sigma}^{*2} = \frac{1}{N^{*2}} \sum_{i=1}^{N^*} [w_\gamma(\gamma)w_n(n)]^2 D[g(R_i)] - \frac{\hat{p}_e^2}{N^*}. \quad (12)$$

The improvement ratio (IR) of IS to MC simulations can be derived as

$$\beta = \frac{N\sigma^2}{N^*\sigma_*^2} = \frac{\int_{\mathscr{R}^M}(1-p_e)D[g(R)]f_\gamma(\gamma)f_\psi(\psi)f_n(n)f_s(s)\,\mathrm{d}\gamma\,\mathrm{d}\psi\,\mathrm{d}n\,\mathrm{d}s}{\int_{\mathscr{R}^M}[w_\gamma(\gamma)w_n(n)-p_e]D[g(R)]f_\gamma(\gamma)f_\psi(\psi)f_n(n)f_s(s)\,\mathrm{d}\gamma\,\mathrm{d}\psi\,\mathrm{d}n\,\mathrm{d}s} \quad (13)$$

where σ^2 is the variance of MC simulation. When $\sigma^2 = \sigma^{*2}$, β is the factor of the simulation sample size reduction. For AWGN, several types of the weighting function $w_n(n)$ have been investigated, such as optimized variance increasing (**1**) and optimized translations of the original probability densities (**3**). As to Rayleigh fading, the pdf of γ is

$$f_\gamma(\gamma) = \begin{cases} (\gamma/\sigma_\gamma^2)\exp(-\gamma^2/2\sigma_\gamma^2), & \gamma \geqslant 0 \\ 0, & \gamma < 0. \end{cases} \quad (14)$$

The only parameter in the pdf is σ_γ. If the same form of the pdf is desired, then with a modified parameter σ_γ^*, the weighting function for γ is

$$w_\gamma(\gamma) = \frac{f_\gamma(\gamma)}{f_\gamma^*(\gamma)} = \frac{\sigma_\gamma^{*2}}{\sigma_\gamma^2}\exp\left[\frac{\gamma^2(\sigma_\gamma^2 - \sigma_\gamma^{*2})}{2\sigma_\gamma^2\sigma_\gamma^{*2}}\right], \quad \gamma \geqslant 0. \quad (15)$$

The fading process $\Gamma^* = \{\gamma_k^* e^{-j\psi_k^*}, \gamma_{k-1}^* \mathrm{e}^{-j\psi_{k-1}^*}, \ldots, \gamma_{k-M+1}^* \mathrm{e}^{-j\psi_{k-M+1}^*}\}$ has to satisfy the correlation coefficient defined in equation (2). One way to simulate the random process Γ^* is to use the 'sum of sines' method (**8**). The method sums a large number of weighted sinusoids with discrete frequencies spanning the Doppler spectrum to simulate the Rayleigh fading channel. In this way, by changing σ_γ^2 to σ_γ^{*2}, the random process γ is modified to γ^*, while the pdf of ψ^* is still uniformly distributed over $[0, 2\pi]$ (i.e. unchanged). In the IS simulation, the simulation process for each BER value is divided into a number of short subsimulations. Over each subsimulation, the fading process Γ^* is correlated according to equation (2). However, uncorrelated fading processes are generated for the different subsimulations, so that the system can go through all the statistical status of the channel fading in a shorter period than that of an MC simulation. Each subsimulation is long enough to take into account the system memory due to the transmitter and receiver. From equation (15), when $\sigma_\gamma^{*2} < \sigma_\gamma^2$, $w_\gamma(\gamma) < 1$, the system is more likely to experience deep fading and the error events are artificially made more likely to happen. If $f_n(n)$ is biased by increasing the variance σ_n^2 to σ_n^{*2}, the optimal biased variances σ_γ^{*2} and σ_n^{*2} are the values which minimize the estimation variance σ^{*2} of equation (11). The solution cannot be obtained from equation (11) since it contains the item p_e which is unknown and is to be estimated. As in the case of an AWGN channel, trial and error method and others may be used to search for 'good' values of σ_γ^{*2} and σ_n^{*2}, which is time consuming and burdensome; furthermore, the 'good' values may be far from the optimal ones. In the following, an adaptive algorithm is developed to iteratively search for the optimal biased density functions during the course of simulation.

IV. Adaptive Algorithm for Optimal Biased Density Functions

From equation (5), the weighting function consists of two components, one for the biased channel fading, and the other for the biased Gaussian noise. Both biased independent random processes increase the error events in the IS simulation. The adaptive algorithm for estimating the optimal biased pdf of the Gaussian noise $f_n^*(n)$ in the case of non-fading applications has been studied in (**6**). Due to the fact that the Gaussian noise samples are iid random variables, and that the channel fading samples are correlated (equation (2)), the adaptive algorithm developed for the optimal biased pdf of the Gaussian noise is not directly applicable to the case of correlated channel fading samples. In the following, an adaptive algorithm is developed to obtain the optimal biased density $f_\gamma^*(\gamma)$ of the channel fading process. The basic concept underlying the AIS technique is that the channel fading samples which result in error events are distributed according to the optimal IS density. The AIS algorithm consists of sequential simulation runs. During each run, p_e is estimated and when an error event occurs, the sample of the channel fading is recorded. These fading samples are then used to estimate the optimal biased pdf of the channel fading (i.e. the optimal parameter σ_γ^{*2}). The estimated optimal pdf is then used to generate channel fading samples for the next simulation run. Thus, the estimation of the optimal pdf becomes more and more accurate as the number of the simulation runs increases. So does the estimation of the BER value. The advantages of AIS over IS are: with AIS the estimation of the optimal biased density is performed at the same time as the estimation of p_e, and the updated biased density estimate is then used in the next subsimulation runs. With the biased pdf close to the optimal one, substantial simulation time reduction becomes possible.

Since the pdf of a biased Rayleigh amplitude fading is decided by the only parameter σ_γ^{*2}, the approach of a parametric AIS algorithm is considered, instead of non-parametric AIS algorithms which estimate the optimal biased pdf directly. From equation (14),

$$\mu_{\gamma^2} = E(\gamma^2) = \int_{-\infty}^{\infty} \gamma^2 f_\gamma(\gamma)\,\mathrm{d}\gamma = 2\sigma_\gamma^2. \tag{16}$$

Therefore, if we can obtain an estimate of the biased σ_γ^{*2} by calculating the mean of the biased γ^{*2} during the AIS simulation, then we can further obtain the optimal biased pdf. The estimate of the optimal mean μ_γ^* with IS can be achieved by

$$\hat{\mu}^*_{\gamma^2} = E(\gamma^2|\gamma\in\Xi) = \int_{\Xi}\gamma^2\cdot\frac{f_\gamma(\gamma)}{p_e}\,\mathrm{d}\gamma \tag{17}$$

where Ξ is the sample space of γ corresponding to an error event. Equation (17) can be rewritten as

$$\begin{aligned}\hat{\mu}^*_{\gamma^2} &= \frac{p_e^{**}}{p_e}\int_{\Xi}\gamma^2\frac{f_\gamma(\gamma)}{f^*_\gamma(\gamma)}\frac{f^*_\gamma(\gamma)}{p_e^{**}}\,\mathrm{d}\gamma \\ &= \frac{p_e^{**}}{p_e}E_*[\gamma^2 w_\gamma(\gamma)|\gamma\in\Xi] \\ &= \frac{p_e^{**}}{p_e}\bar{\gamma}^2 \end{aligned}\tag{18}$$

where p_e^{**} is the probability of error of the IS simulation and $\bar{\gamma}^2 = E_*[\gamma^2 w_\gamma(\gamma)|\gamma\in\Xi]$. Due to the correlation among channel fading samples in each subsimulation run, not all the samples are suitable to be used to estimate $\bar{\gamma}^2$, which is different from the case of AWGN. Only the independent samples should be used, which is a very small number in each subsimulation run in the case of slow fading channel. One way to increase the sample size is to use all the information available, i.e. to use all independent fading samples in previous subsimulations to estimate $\bar{\gamma}^2$ and then use the estimate to bias the IS density for the next subsimulation. A recursive algorithm is proposed to estimate $\bar{\gamma}^2$:

$$\bar{\gamma}_0^2 = \frac{1}{p_0}\sum_{k=1}^{p_0}\gamma_{0k}^2\cdot w_{\gamma 0}(\gamma_{0k}),\quad m_0 = p_0 \tag{19}$$

$$\bar{\gamma}_i^2 = \frac{m_{i-1}}{m_i}\bar{\gamma}_{i-1}^2 + \frac{1}{m_i}\sum_{k=1}^{p_i}\gamma_{ik}^2\cdot w_{\gamma_i}(\gamma_{ik}),\quad m_i = m_{i-1}+p_i \tag{20}$$

where p_i is the number of the independent fading samples in the ith subsimulation, m_i is the total number of independent fading samples up to the ith subsimulation, γ_{ik} is the kth independent fading sample in the ith subsimulation, all these samples resulting in error events; and $w_{\gamma_i}(\cdot)$ is the weighting density for the ith subsimulation. Since

$$\begin{aligned}E(\bar{\gamma}_0^2) &= \frac{1}{p_0}\sum_{k=1}^{p_0}\int_{r_{0k}\in\Xi}\gamma_{0k}^2 w_{\gamma_0}(\gamma_{0k})f^*_\gamma(\gamma_{0k})\,\mathrm{d}\gamma_{0k} \\ &= \frac{1}{p_0}\sum_{k=1}^{p_0}E_*[\gamma^2 w_\gamma(\gamma)|\gamma\in\Xi] \\ &= \bar{\gamma}^2,\end{aligned}\tag{21}$$

$\bar{\gamma}_0^2$ is an unbiased estimator of $\bar{\gamma}^2$. Assuming that $\bar{\gamma}_{i-1}^2$ is an unbiased estimator of $\bar{\gamma}^2$, then

$$E(\bar{\gamma}_i^2) = \frac{m_{i-1}}{m_i}\bar{\gamma}^2 + \frac{1}{m_i}\sum_{k=1}^{p_i}\int_{\gamma_{ik}\in\Xi}\gamma_{0k}^2 w_{\gamma_i}(\gamma_{ik})f_{\gamma_i}^*(\gamma_{ik})\,\mathrm{d}\gamma_{ik}$$

$$= \frac{m_{i-1}}{m_i}\bar{\gamma}^2 + \frac{1}{p_i}\sum_{k=1}^{p_i} E_*[\gamma^2 w_\gamma(\gamma)|\gamma\in\Xi]$$

$$= \bar{\gamma}^2. \tag{22}$$

Therefore, $\bar{\gamma}_i^2$ of equation (20) is an unbiased estimator of $\bar{\gamma}^2$. In equation (18), substitute $\hat{p}_e$ with its IS estimator p_e^* from (7) and p_e^* with its MC estimate

$$\hat{p}_e^{**} = m_I/N^* \tag{23}$$

at the end of the Ith subsimulation. Then from (19)–(23), the estimate of $\mu_{\gamma^2}^*$ can be derived as

$$\hat{\mu}_{\gamma^2}^* = \frac{\sum_{i=1}^{I}\sum_{k=1}^{p_i}\gamma_{ik}^2 w_{\gamma_i}(\gamma_{ik})}{\sum_{i=1}^{I}\sum_{k=1}^{p_i} w_{\gamma_i}(\gamma_{ik})} \tag{24}$$

at the end of the Ith subsimulation. All the three estimators, $\hat{p}_e^{**}$, $\hat{p}_e$ and $\hat{\gamma}^2$, of equation (18) are unbiased, and the variance of the estimators tends toward zero as the sample size tends to infinity. As a result, the estimator $\hat{\mu}_{\gamma^2}^*$ of equation (24) is a weakly consistent estimator of the optimal $\mu_{\gamma^2}^*$.

In summary, the steps for the AIS algorithm for the fading channel are:

(i) bias the parameter σ_γ^2 to a smaller value $\sigma_{\gamma 0}^{*2}$, the initial IS pdf of the channel fading is determined;

(ii) start a series of short subsimulations with the IS density from the previous step and record the independent fading samples which cause error events;

(iii) calculate the update estimate $\hat{\mu}_{\gamma^2}^*$ according to equation (19)–(20) and (24) to form the update IS pdf of the channel fading;

(iv) run a simulation with $\hat{\sigma}_\gamma^{*2} = \hat{\mu}_{\gamma^2}^*/2$ as the parameter of the biased IS pdf of the channel fading, record the independent channel fading samples which generate error events, and calculate $\hat{p}_e^*$ using equation (7);

(v) if the updated IS pdf of the channel fading from (iii) is still far away from the optimal IS pdf, then repeat (iii)–(v).

It should be mentioned that the optimal σ_γ^{*2} value is a function of p_e. Generally, the optimal value decreases as the SNR value of the received signal increases. The estimated optimal σ_γ^{*2} for a (SNR/bit) value should be used as a reference in choosing the initial biased estimate $\sigma_{\gamma 0}^{*2}$ for a nearby (SNR/bit) value, so that the optimal σ_i^{*2} value can be estimated accurately based on a relatively small size of data samples.

V. Applications

The AIS technique is applied to study the BER performance of a differentially encoded quadrature phase-shift-keying (QPSK) signal transmitted over a correlated Rayleigh fading channel with multiple-symbol differential detection. The block diagram of the transmitter is shown in Fig. 2, which has a phase mapper, a differential encoder, and a quadrature modulator. The input to the phase mapper is equiprobable, independent m-bit information words $\bar{b}_k^m = [b_k^1, b_k^2, \ldots, b_k^m]$, where $b_k^i \in \{0, 1\}$. The phase mapper converts the uncoded binary sequence $\bar{b}_k^m$ into an M-ary PSK symbol, where M is the number of different symbols $M = 2^m$. The mapping rule can be described in two steps. First, $\bar{b}_k^m$'s are converted into $\Delta\Phi_k$ by

$$\Delta\Phi_k = \frac{2}{M} \sum_{i=1}^{m} 2^{i-1} \pi b_k^i \tag{25}$$

then, a complex valued sequence c_k is obtained by $c_k = e^{j\Delta\Phi_k}$. The differential encoding applied to c_k is also a complex operation, and can be described by $e^{j\phi_k} = c_k e^{j\phi_{k-1}}$,where ϕ_k represents the phase of the transmitted symbol. The relation between ϕ_k and $\Delta\Phi_k$ is

$$\phi_k = \Delta\Phi_k \oplus \phi_{k-1} \tag{26}$$

where $\oplus$ represents mod(2π) addition. For QPSK, $m = 2$. The transmitted signal $s(t)$ is then $s(t) = \mathrm{Re}\{Ae^{w_c t + \phi_k(t)}\}$where A is the amplitude of the signal. The transmitted signal experiences channel fading and is corrupted by AWGN with one-sided spectral density N_0.

The basic units in the receiver include a L-bit differential detector, the block diagram of which is shown in Fig. 3. The signal $y(t)$ is the output of the bandpass filter (BPF). The inphase $(d_i^I(t))$ and quadrature $(d_i^Q(t))$ outputs of the i-bit detector are obtained by low-pass filtering the products of $y(t)$ with an iT seconds delayed and θ_c radian phase shifted version of itself, where $\theta_c = 0$ for inphase channel $(c = I)$ and $\theta_c = \pi$ for quadrature channel $(c = Q)$.

At the time instant kT, the output of the low-pass filter (LPF) can be represented as

$$d_i^c(kT) = \cos(\Delta\Theta_i(k) - \theta_c) + n_i^c(kT), \quad c = I \text{ or } Q \tag{27}$$

where $\Delta\Theta_i(k) = \phi(kT) \ominus \phi(kT - iT)$ ($\ominus$ represents mod(2π) subtraction) and $n_i^c(kT)$ lumps all the noise terms together. Using equation (26), $\Delta\Theta_1(k) = \Delta\Phi_k$ for $i = 1$, $\Delta\Theta_2(k) = \Delta\Phi_k \oplus \Delta\Phi_{k-1}$ for $i = 2$, and in general $\Delta\Theta_i(k) = \Delta\Phi_k \oplus \Delta\Phi_{k-1} \oplus \cdots \oplus \Delta\Phi_{k-i+1}$. The decision rule is: choose the sequence ϕ which maximizes the statistic (**9**)

$$|y((k-L+1)T) + \sum_{i=0}^{L-2} y((k-i)T)\, e^{-j\sum_{m=0}^{L-i-2} \Delta\Phi_{k-i-m}}|^2.$$

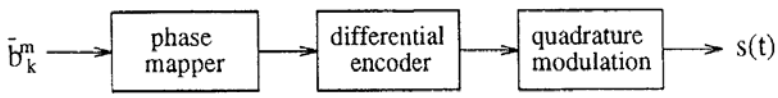

FIG. 2. The differential QPSK modulator.

This rule implies that we observe the received signal over L symbol time intervals and from this observation make a simultaneous decision on $L-1$ data phases. The first received symbol is used to provide a phase reference for the whole block and the last symbol is used as a reference for the next block. The memory length of the modulation scheme is 2 symbols due to the differential encoding, and the memory length of the detector is $L+1$ symbols.

Figure 4 shows the BER performance of the system with $L = 3$ based on (i) theoretical analysis, (ii) MC simulation, and (iii) AIS simulation with biased pdf of channel fading process using the adaptive algorithm to obtain the estimate of the optimal σ_i^{*2} (based on the first 200 data blocks). A channel fading rate of 10^{-4} is selected which characterizes a slowly fading channel. With the adaptive algorithm discussed in Section IV, the biased density function of the channel amplitude fading converges to the optimal biased density functions over relatively small data samples. Figure 5 shows the convergence of the estimated optimal $\hat{\sigma}_i^{*2}$ for the received signal energy per bit (E_b) to the noise spectral density (N_0) ranging from 10 dB to 45 dB. The optimal biased variances σ_i^{*2} are obtained from (11) with p_e values known for the E_b/N_0 ratios. It is observed from the figure that even though the initial σ_{i0}^{*2} values are far from the corresponding optimal ones, they converge to the optimal ones very quickly. With the fast convergence of the $\hat{\sigma}_i^{*2}$ estimate, a large improvement ratio can be achieved. Table I gives the estimated BER performance, estimated variance of $\hat{p}_e$ (obtained according to (12)), $\hat{\sigma}_i^{*2}$ and the corresponding optimal values; Table II gives data sample size and improvement ratio of the simulations. The AIS technique reduces the necessary data sample sizes by 67 times for the SNR/bit values between 10 and 35 dB. The sample size reduction

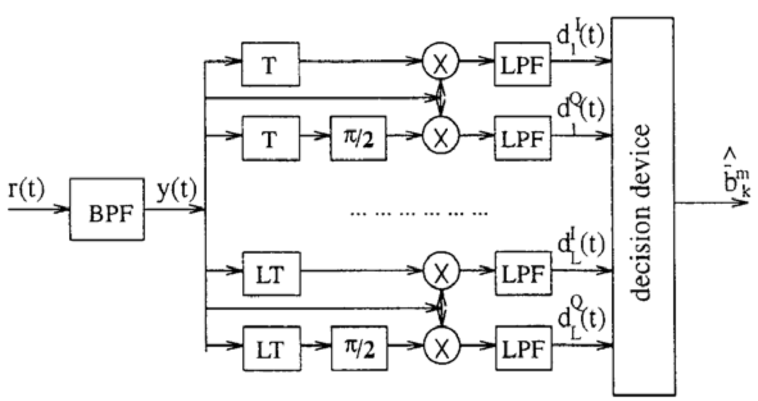

FIG. 3. The multiple-symbol differential detector.

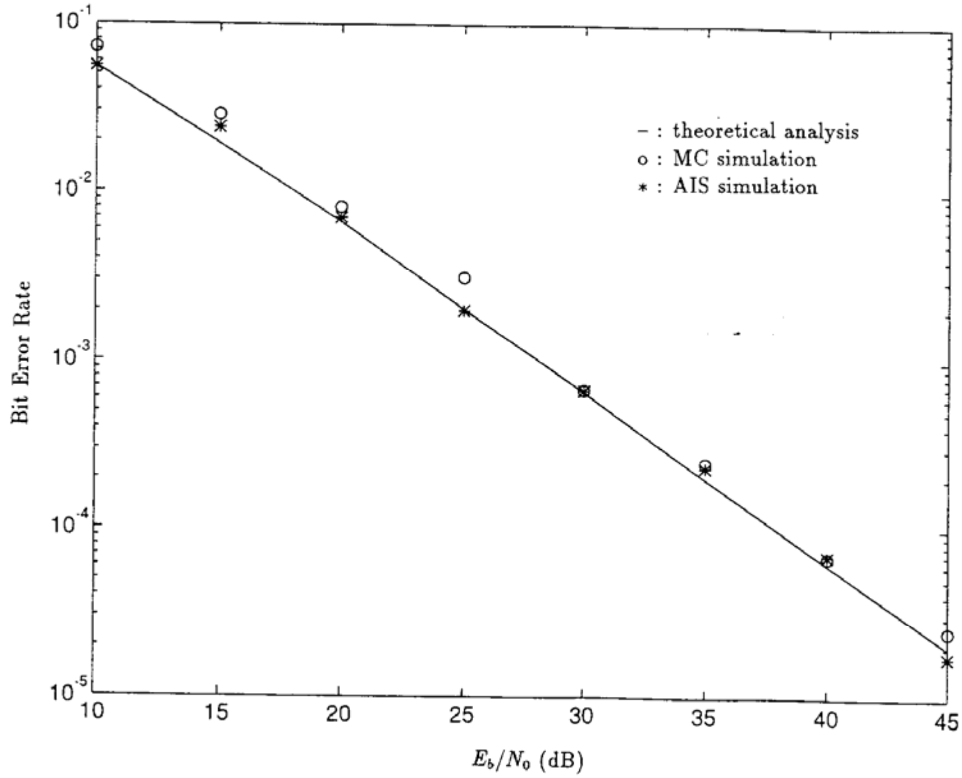

FIG. 4. The BER performance of the differential QPSK.

increases as the SNR/bit increases to 40 dB and 45 dB. The improvement ratio will increase if the memory length of the receiver is reduced or if the channel fading rate decreases. The advantage of AIS simulation over MC simulation is clearly observed from the simulation results (Tables I, II and Fig. 4).

VI. Conclusions

An AIS technique for estimating the BER performance of a mobile or portable communications system over a Rayleigh fading channel has been developed and analyzed. The statistical properties of both channel fading process and input Gaussian noise can be biased simultaneously to increase error events. An adaptive recursive algorithm is proposed to search for the optimal biased density function of the channel fading random process during the course of simulation. The AIS technique is applied to study differential QPSK with multiple-symbol differential detection over a slow Rayleigh fading channel. Computer simulation results show that the AIS technique reduces the simulation time dramatically and simplifies the procedure of choosing an optimal biased density function. The estimate of the optimal biased density function converges very quickly, which results in a significant data sample size reduction (at least 67 times with 4-symbol differential detection).

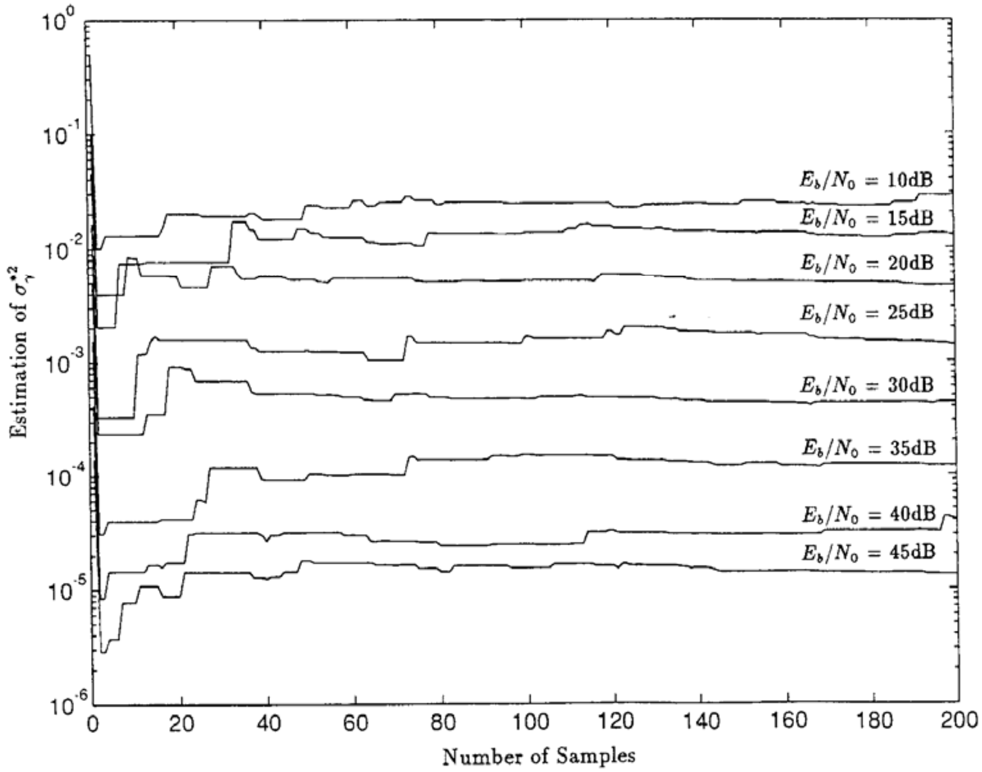

FIG. 5. Convergence of σ_γ^{*2} as a function of sample size.

TABLE I
Estimates of p_e and σ_γ^{*2}, and their performance

SNR/bit (dB)	Bit error rate		Biased density	
	$\hat{p}_e$	$\hat{\sigma}^{*2}$	$\hat{\sigma}_\gamma^{*2}$	σ_γ^{*2}
10	5.50×10^{-2}	4.6×10^{-6}	2.86×10^{-2}	3.29×10^{-2}
15	2.08×10^{-2}	6.4×10^{-7}	1.28×10^{-2}	1.25×10^{-2}
20	7.86×10^{-3}	1.1×10^{-7}	4.65×10^{-3}	4.71×10^{-3}
25	1.98×10^{-3}	1.5×10^{-8}	1.39×10^{-3}	1.50×10^{-3}
30	5.91×10^{-4}	1.3×10^{-9}	4.39×10^{-4}	4.71×10^{-4}
35	2.34×10^{-4}	1.2×10^{-10}	1.20×10^{-4}	1.36×10^{-4}
40	6.98×10^{-5}	1.0×10^{-11}	4.06×10^{-5}	4.25×10^{-5}
45	1.77×10^{-5}	1.1×10^{-12}	1.32×10^{-5}	1.41×10^{-5}

TABLE II
The data sample size and improvement ratio

SNR/bit (dB)	Sample size		Improvement ratio
	MC	AIS	
10	6.0×10^5	9.0×10^3	67
15	6.0×10^5	9.0×10^3	67
20	6.0×10^5	9.0×10^3	67
25	6.0×10^5	9.0×10^3	67
30	6.0×10^5	9.0×10^3	67
35	6.0×10^5	9.0×10^3	67
40	9.0×10^5	9.0×10^3	100
45	1.8×10^6	9.0×10^3	200

References

(1) K. S. Shanmugan and P. Balaban, "A modified Monte Carlo simulation technique for the evaluation of error rate in digital communication systems", *IEEE Trans. Commun.*, Vol. COM-28, pp. 1916–1924, 1980.

(2) M. C. Jeruchim, "Techniques for estimating the bit error rate in the simulation of digital communication systems", *IEEE J. Select. Areas Commun.*, Vol. SAC-2, pp. 153–170, January 1984.

(3) D. Lu and K. Yao, "Improved importance sampling technique for efficient simulation of digital communications systems", *IEEE J. Select. Areas Commun.*, Vol. SAC-6, pp. 67–75, 1988.

(4) C. G. Bucher, "Adaptive sampling—an iterative fast Monte Carlo procedure", *Structural Safety*, Vol. 5, pp. 119–126, 1988.

(5) R. E. Melchers, "Search-based importance sampling", *Structural Safety*, Vol. 9, pp. 117–128, 1990.

(6) J. S. Stadler and S. Roy, "Adaptive importance sampling", *IEEE J. Select. Areas Commun.*, Vol. SAC-11, pp. 309–316, 1993.

(7) A. A. M. Saleh and R. A. Valenzuela, "A statistical model for indoor multipath propagation", *IEEE J. Select. Areas Commun.*, Vol. SAC-5, pp. 128–137, 1987.

(8) W. C. Jakes, Jr, "Microwave Mobile Communications", John Wiley & Sons, 1974.

(9) D. Divarsalar and M. K. Simon, "Multiple-symbol differential detection of MPSK", *IEEE Trans. Commun.*, Vol. COM-38, pp. 300–308, 1990.

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