EFFICIENT SIMULATION OF COMPLEX FENESTRATION SYSTEMS IN HEAT BALANCE ROOM MODELS

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ABSTRACT

The solar, longwave, and convective interactions between a window, its shading attachments and its surroundings constitute a complicated coupled heat balance problem that can entail significant computational intensity to simulate in detail. A novel approach represents the fenestration system using several indices of merit – most notably the U-factor and a cross-coupling coefficient. CPU time is reduced without forfeiting features such as the ability to distinguish between air and mean radiant temperatures. The required indices of merit are obtained using thermal network theory and can safely be re-evaluated much less frequently than each time-step, or they can be revised as needed in response to changes in sun angle or shade geometry (e.g., blind slat adjustment). This method has been used to integrate the ASHWAT fenestration model with the California Simulation Engine, a detailed residential model. ASHWAT supports many combinations of glazing and shading layers separated by arbitrary fill gases or by gaps open to outdoor or indoor air. This implementation demonstrates a method that offers generality and detail while providing the input simplicity and computational speed required for practicality.

INTRODUCTION

It is widely understood that the potential for energy conservation in the building sector is enormous. The increased levels of insulation associated with green building design decrease heating loads but, in the presence of heat gains, can exacerbate cooling loads. Solar gain is especially troublesome because it is often the largest and most variable heat gain. A well-designed shading device can be operated to admit solar gain only as it is needed.

This paper describes the integration of the ASHWAT (ASHRAE Window ATtachment) fenestration models with the California Simulation Engine (CSE), a detailed residential full-year simulation model. CSE is being developed to support the 2013 revision of the California Title 24 energy regulations (Wilcox 2011). As of summer 2011, internal CSE versions are undergoing testing; public release (including source code) is planned for mid 2012.

Figure 1 shows a multi-layer fenestration assembly of n=4 layers. Some layers are glazing layers and some are shading layers so this is often called a complex fenestration system. The solar, longwave, and convective interactions between the various layers and their surroundings constitute a complicated and coupled heat balance problem. The ASHWAT models have been specifically developed for the center-glass analysis of complex fenestration systems.

ASHWAT incorporates a significant amount of new research and technical detail that can be traced by starting at two key documents (Barnaby \textit{et al.} 2009, Wright \textit{et al.} 2009). The ASHWAT models support almost any combination of glazing and shading layers, coatings, spacing, fill gases and/or mixtures, and gaps open to outdoor or indoor air. The CSE implementation demonstrates a method that offers generality and detail while providing the input simplicity and computational speed required for practicality. Similar integration projects include the ASHRAE Toolkit (HBX) and ESP-r. HOT3000 implementation is in progress and addition to EnergyPlus is planned. The ASHWAT/HBX version was used to generate the extensive tables of complex fenestration performance data found in Chapter 15 of
the 2009 ASHRAE Handbook – Fundamentals (ASHRAE 2009). Important lessons have been learned about the design, control and placement of shading devices (e.g., Lomaskovski et al. 2009).

The ASHWAT models benefit from two distinct features. First, a multi-layer framework (Wright and Kotey 2006), in which all glazing or shading layers are assigned effective optical properties, is used to track beam and diffuse components of solar radiation. A wide variety of shading layers can be considered and very few input data are needed. Models are currently included for flat-type blinds, pleated drapes, roller blinds and insect screens. Second, new resistance network theory (Wright 2008) used in the energy balance provides two key benefits. (a) This specific network formulation makes it possible to obtain indices of merit – the U-factor for example – for a window with shading attachments exposed to any combination of insulation and indoor/outdoor temperatures. This is a critical part of the ASHWAT-CSE interface. (b) Although the heat transfer modes are coupled and the solver is iterative, none of the heat transfer terms are lagged. This implicit formulation provides exceptional stability and solutions are obtained in very few iterations. In summary, CPU time is used sparingly but results include rich detail regarding the beam/diffuse and radiant/convective splits of solar gain and heat transfer. In addition, on-the-fly operability (e.g., slat angle adjustment) is available at the time-step level and can be used in response to factors such as solar angle or indoor temperature.

The method used to track solar radiation is documented in (Wright and Kotey 2006). This analysis provides the flux of solar radiation absorbed at each layer, \( S_i \), resulting from the incident solar flux, \( I_{in} \), as well as direct transmittance to the indoor space, \( T_{st} \). The solar gain quantified by \( T_{st} \) is assigned to the various indoor surfaces according to user input and/or a default distribution algorithm; this has no bearing on the subsequent formulations.

The remaining and more interesting development pertains to the way that the ASHWAT heat transfer analysis is linked to the CSE room heat balance and used to track the inward flowing components of absorbed solar radiation as well as heat transfer driven by the indoor/outdoor temperature difference.

**ASHWAT RESISTANCE NETWORK**

Figure 2 shows how the \( i \)th layer in the complex fenestration system can be thermally linked to any other layer, and to the environment, by convective and longwave radiant heat transfer paths. This generality allows for the possibility of heat transfer between non-adjacent nodes that occurs, for example, when a channel is open to airflow between a window and a shading attachment or when the openness of a shading layer allows the transmission of longwave radiation.

The general resistance network consists of resistors that are influenced by environment and layer temperatures. Once the system has been solved (solar and thermal energy balances are applied at each layer – yielding a complete set of temperatures, heat fluxes, heat transfer coefficients, etc.), it is possible to evaluate indices of merit that characterize overall network performance. This is done by using the known resistor values to perform “computational experiments” as described in (Wright 2008). Indices of merit include center-glass U-factor, \( U_{cg} \), Solar Heat Gain Coefficient, \( SHGC_{cg} \), as well as \( f_{t,cb} \) and \( f_{t,cl} \) (weighting factors that relate the air and mean radiant temperatures, \( T_a \) and \( T_{mr} \), and the ambient effective temperature, \( T_{amb} \), on the indoor and outdoor sides). The application of \( f_{t,cb} \) is shown in Equation 1.

\[
T_{real} = f_{t,cb} T_{real} + (1 - f_{t,cb}) T_{real}
\]

These four indices of merit can be used to calculate the total heat gain, \( q_{in} \), as shown in Equation 2.

\[
q_{in} = U_{cg} \cdot \left( f_{t,cb} T_{real} + (1 - f_{t,cb}) T_{real} \right) - \left( f_{t,cl} T_{real} + (1 - f_{t,cl}) T_{real} \right) + SHGC_{cg} \cdot I_{in}
\]

| Figure 2: The general thermal resistance network |

**INWARD FLOWING FRACTION**

The \( i \)th layer of the system absorbs a flux, \( S_i \), of solar radiation and the portion of \( S_i \) that makes its way to the indoor space by means of heat transfer is the inward flowing fraction, \( N_i \). The value of \( N_i \) in a complex fenestration system can also be obtained by computational experiment. In fact, the thermal resistance circuit is sufficiently general that \( N_i \) can be subdivided into two components that apply to heat transfer reaching the indoor space by longwave radiation or by convection, \( N_{a,i} \) and \( N_{c,i} \), respectively. This is done by setting the indoor/outdoor temperature difference to zero, introducing unit sources of flux at each node, one at a time to mimic absorbed solar radiation, and then evaluating the resulting heat flux at the \( T_{in} \) and \( T_{a,in} \) nodes. Equation 3 shows the relationship between the three
inward flowing fractions.
\[ N_i = N_{\text{in},i} + N_{\text{out},i} \]  
(3)

ASHWAT supplies CSE with information that includes \( T_{\text{amb}} \) and the \( N_{\text{in},i} \) and \( N_{\text{out},i} \) sets. The effect of absorbed solar energy is represented by sources of radiant and convective heat flux, \( S_{\text{rad}} \) and \( S_{\text{conv}} \), given by the summations shown in Equations 4 and 5 - energy sources that apply to the \( T_{\text{m,in}} \) and \( T_{\text{a,in}} \) nodes, respectively.

\[ S_{\text{rad}} = \sum_{i=1}^{n} N_{\text{in},i} S_i \]  
(4)

\[ S_{\text{conv}} = \sum_{i=1}^{n} N_{\text{out},i} S_i \]  
(5)

**Simplified Resistance Network**

Equation 2 represents a network connected to two indoor nodes and two outdoor nodes. The outdoor temperatures remain constant during a CSE time-step so Equation 1 is used to collapse the outdoor temperature nodes to an ambient effective temperature node, \( T_{\text{a,out}} \). In contrast, we seek more information about the convective/radiant split on the indoor side so the indoor nodes are left intact. Now, Equation 2 can be rearranged and rewritten as Equation 6, representing a network connected to three environmental nodes.

\[ q_{\text{in}} = f_{\text{in}} U_{\text{in}} (T_{\text{m,out}} - T_{\text{a,in}}) + (1 - f_{\text{in}}) U_{\text{in}} (T_{\text{a,ind}} - T_{\text{a,in}}) + \text{SHGC}_{\text{in}} I_{\text{in}} \]  
(6)

The general resistor network can now be reduced to a simple three-resistor \( \Delta \)-circuit using a succession of \( \Gamma \)-\( \Delta \) transformations. The result is shown in Figure 3 where the SHGC term shown in Equation 6 has been replaced by \( S_{\text{rad}} \), \( S_{\text{conv}} \) and \( \text{SHGC}_{\text{in}} \) source terms. The two indoor-outdoor conductances can be seen in Equation 6 and their values are shown in Figure 3. These conductances can also be obtained by more formal and more general derivation. In addition, noting that the analysis of center-glass area and the analysis of total window area can equally well be reduced to a \( \Delta \)-resistance network, the \( \text{cg} \) subscript has been omitted in Figure 3. The distinction between center-glass and total window area is addressed, along with related refinements, in a subsequent section.

The \( C_{\text{S}} \) conductance represents a convective/radiant coupling between the \( T_{\text{m,in}} \) and \( T_{\text{a,in}} \) nodes that is caused by the presence of the complex fenestration. The CSE energy simulation incorporates a cross-coupling conductance at indoor surfaces exposed to \( T_{\text{m,in}} \) and \( T_{\text{a,in}} \). The concept is better understood by considering the case in which solar radiation is absent and the thermal resistance between the layers of a window is so large that \( U_{\text{cg}} \) is extremely small. Even though little indoor/outdoor heat transfer is possible some finite amount of heat transfer will take place between \( T_{\text{m,in}} \) and \( T_{\text{a,in}} \) through the indoor surface conductances as the fenestration system converts energy from one heat transfer mode to the other.

![Figure 3: Simplified three-node resistance network (resistors are labelled as conductances for convenience, with unit area assumed)](image)

A computational experiment can be used to evaluate \( C_{\text{S}} \) because \( C_{\text{S}} \) is uniquely a function of the resistor values. Specifically, the general resistor network is used to calculate the heat flux into the \( T_{\text{m,in}} \) node, \( q_{\text{in}} \), that matches the following set of boundary conditions: \( T_{\text{m,in}} = T_{\text{a,m,in}} = T_{\text{a,m,ind}} = T_{\text{a,in}} = T_{\text{m}} = T_{\text{a}} = 0 \), and \( I_{\text{in}} = 0 \). The "exp" superscript is a reminder that these conditions pertain to the computational experiment.

In terms of the simplified resistor network, Equation 7 is obtained from Figure 3.

\[ q_{\text{in}}^{\text{exp}} = f_{\text{in}} U_{\text{in}} (T_{\text{m,out}}^{\text{exp}} - T_{\text{a,in}}^{\text{exp}}) + C_{\text{S}} (T_{\text{m,m,in}}^{\text{exp}} - T_{\text{m,in}}^{\text{exp}}) \]  
(7)

Examining the right-hand-side of Equation 7, the first term is zero and the temperature difference in the second term is unity. Therefore, the outcome of this specific set of applied boundary conditions is that \( C_{\text{S}} \) and \( q_{\text{in}}^{\text{exp}} \) are numerically equal. Note that because the heat flux \( q_{\text{in}}^{\text{exp}} \) is evaluated using the resistor set obtained for the true environmental condition, \( C_{\text{S}} \) also applies to the true environmental condition. Detail regarding this concept are found in (Wright 2008).

A sample set of center-glass performance indices is shown in Table 1 for four common systems: single glazed (SG), conventional double glazed (CDG), double-glazed with surface-2 low-e coating plus argon (DGLEA) and CDG with a white venetian blind two inches from the indoor glass (CDG-VB). The Table 1 entries apply to the ASHRAE summer and winter design conditions, as indicated.
Table 1 highlights several influences worth noting. The value of \( \text{SHGC}_{\text{cg}} \) is insensitive to the weather condition. The addition of an indoor venetian blind does not change \( U_{\text{g}} \) appreciably. The venetian blind also reduces \( f_{\text{mv}} \) substantially because the blind roughly triples the area available for convective heat transfer between the room and the fenestration system. The value of \( C_{i} \) increases as layers are added to the system, especially the venetian blind because additional layers add more heat transfer paths to the resistor network.

### Table 1

**Indices of Merit for Various Fenestration Systems. ASHRAE Summer and Winter Design Conditions:**

<table>
<thead>
<tr>
<th></th>
<th>( U_{\text{g}} )</th>
<th>( \text{SHGC}_{\text{cg}} )</th>
<th>( f_{\text{mv}} )</th>
<th>( f_{\text{nv}} )</th>
<th>( C_{i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SUMMER</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conv. coeff., ( h_{i} ) (in/out)</td>
<td>5.0/2.10</td>
<td>3.4/25.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SG (3 mm clear)</td>
<td>7.40/0.86</td>
<td>5.99/0.86</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CDG</td>
<td>0.51/0.21</td>
<td>0.53/0.11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.71</td>
<td>0.39</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>½ inch pane spacing</td>
<td>4.17/0.77</td>
<td>3.30/0.76</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.57/0.28</td>
<td>0.62/0.18</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.65</td>
<td>1.23</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DGLEA</td>
<td>1.70/0.60</td>
<td>1.60/0.59</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fill gas 90% argon/10% air</td>
<td>0.55/0.21</td>
<td>0.61/0.11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coating emissivity = 0.1</td>
<td>2.24</td>
<td>1.68</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CDG-VB</td>
<td>4.08/0.50</td>
<td>3.11/0.49</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slat angle = 45 degrees, solar reflectivity = 0.85</td>
<td>0.27/0.29</td>
<td>0.32/0.18</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.48</td>
<td>2.85</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>WINTER</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: \( h_{i} \), \( U_{\text{g}} \) and \( C_{i} \) have units of W/m\(^2\)K.

### INDOOR RADIANT HEAT TRANSFER

The CSE longwave radiant network uses the Carroll (1980) “MRT view factor” method which incorporates a single mean radiant temperature node, \( T_{\text{m,n}} \), to act as a clearinghouse for radiant exchanges within the room much as \( T_{\text{m,n}} \) does for convective heat transfer. In accord with the Carroll methodology, Figure 4 shows the portion of the Oppenheim radiation network (Oppenheim 1956) between the radiosity of the \( j \)th room surface, \( J_{r} \), and the mean radiant temperature node with black emissive power, \( E_{r} = T_{\text{m,n}} \). All of the indoor surfaces are similarly connected to the \( E_{r} \) node. Thus, \( E_{r} \) floats with a value equal to the \( A_{j} F_{r} \)-weighted radiosity of all the indoor surfaces.

\[
F_{j} = \left(1 - \frac{A_{j}}{A_{\text{total}}} \right)^{i}
\]  

The conductance \( F_{j} \) is a factor, slightly larger than the normal view factor of unity, with the role of raising the conductance between \( J_{j} \) and \( E_{r} \) to compensate for the potential difference \( |J_{j} - E_{r}| \) being smaller than it would have been had \( J_{j} \) not been part of the weighted average used to calculate \( E_{r} \). Carroll (1980) gives an equation to determine the \( F_{j} \) values and shows how the Oppenheim surface resistances are incorporated into the overall conductance between each indoor surface and the MRT node. The Carroll view factor, \( F_{j} \), for the \( j \)th indoor surface, of area \( A_{j} \), is given approximately by Equation 8.

\[
F_{j} = \left(1 - \frac{A_{j}}{A_{\text{total}}} \right)^{i}
\]  

The ASHWAT calculation of longwave exchange, treats the indoor environment as a flat, opaque pseudo-surface parallel to the complex fenestration system. This pseudo-surface is generally treated as black (\( e = 1 \)) on the expectation that none of the radiation leaving the window is reflected back to the window. Also, the shape factor between the window and the indoor pseudo-surface is fixed at unity. Therefore, the ASHWAT formulation precludes the \( F_{j} > 1 \) correction of the conductance attached to \( T_{\text{m,n}} \), but an equally simple adjustment is available. Referring to Figure 1, the implicit radiant network between the radiosity of the window, \( J_{w} \), and the indoor pseudo-surface at \( E_{r} = \sigma T_{\text{m,n}} \) is shown in Figure 5. The quantity \( e_{r}/(1-e_{r}) \) is the “Oppenheim surface conductance” at the pseudo-surface. The conductance “1” is the view factor between the window and the indoor pseudo-surface. This two-conductance network can be reduced to the single conductance shown in Figure 6. Comparing Figures 4 and 6 it is apparent that the heat transfer given by the ASHWAT formulation will equal that of the Carroll model, with its self-weighting factor, simply by assigning a fictitious emissivity to the indoor pseudo-surface:

\[
e_{r} = F_{j}
\]  

Figure 4: Carroll network for radiant flux between surface \( j \) and \( T_{\text{m,n}} \) (TMR) node

Figure 5: Implicit ASHWAT radiant network between window and indoor surface at \( T_{\text{m,n}} \)

Figure 6: Reduced ASHWAT radiant network between window and indoor surface at \( T_{\text{m,n}} \)
This adjustment of $e_r$, although unrealistic in the sense that it exceeds unity, supplies the necessary correction and increases the radiant exchange between the window and the room as desired. Examine Equation 8. As expected, a large window triggers a large adjustment and as window area, $A_t$, approaches zero the adjustment disappears.

LIBRARY OF REPRESENTATIVE GLAZING/SHADING SYSTEMS

The CSE simulation software is used to assess the energy performance of many different buildings with many possible window assemblies. At simulation time very little is known about the windows being proposed for building construction. Two performance indices are available: the NFRC (National Fenestration Rating Council) U-factor and SHGC, denoted here as $U_{LR}$ and SHGC$_{LR}$. These NFRC label ratings present a mismatch with the ASHWA T approach for two reasons. First, the NFRC label ratings apply to the total window (center, edge, and frame) but the ASHWA T results discussed above apply to the center-glass area. Second, ASHWA T requires the input data to describe a specific glazing system but only the two label ratings are available. Several adjustments have been made to alleviate these difficulties.

The ASHWA T indices, $U_{LR}$ and SHGC$_{LR}$ can readily be converted to values that apply to the total window. Evaluating the projected center, edge, and frame areas, $A_{c}$,$ A_{e}$, $A_{f}$, respectively, in the conventional way we can relate the following components: $U_{c}$, $U_{e}$, and $U_{f}$, using Equation 10.

$$A_{c} U_{c} \Delta T = A_{e} U_{e} \Delta T + A_{f} U_{f} \Delta T$$

where $\Delta T$ is the indoor/outdoor temperature difference.

$$\Delta T = T_{i,n} - T_{o,n}$$

and the total product area is:

$$A_{t} = A_{c} + A_{e} + A_{f}$$

Equation 13 is obtained from Equations 10 and 12.

$$U_{t} = \frac{A_{c} U_{c}}{A_{t}} + \frac{A_{e} U_{e}}{A_{t}} + \frac{A_{f} U_{f}}{A_{t}}$$

Several assumptions are made so that $U_{t}$ can be calculated and comparisons can be made. (a) The area ratios used for NFRC rating calculations are also used in Equation 13. (b) The frame is assumed to be reasonably well insulated so $U_{f} = 2.8$ W/m$^2$K is used. This value represents performance typical of a wood, vinyl or a very good thermally broken aluminium frame, but not unbroken aluminium. (c) It is assumed that an insulated edge-spacer (i.e., not an aluminium box spacer (Wright and Sullivan 1989)) is used and the ratio $U_{LR}/U_{c}$ is obtained from (ASHRAE 1989) - re-expressed as Equation 14.

$$U_{c} = \frac{1}{4} + \frac{23}{24} U_{a} \text{ (W/m$^2$K)}$$

Similarly, SHGC$_{t}$ can be obtained from component indices, as shown in Equations 15 and 16.

$$A_{c} U_{c} \Delta T = A_{e} U_{e} \Delta T + A_{f} U_{f} \Delta T$$

$$A_{c} \Delta T = A_{e} \Delta T + A_{f} \Delta T$$

In this case it is assumed that SHGC$_{e} = SHGC_{c}$, a very common simplification, and SHGC$_{f} = 0$, also sound (Wright and McGowan 1999). Equation 16 is replaced by Equation 17.

$$SHGC_{t} = \frac{A_{c} U_{c}}{A_{t}}$$

The same manoeuvre can be used to obtain total window values corresponding to $S_{LR}$ and $\tau_{LR}$.

$$S_{t} = \frac{1 - \frac{A_{f}}{A_{t}}}{S_{c}}$$

$$\tau_{t} = \frac{1 - \frac{A_{f}}{A_{t}}}{\tau_{c}}$$

Again, the same area ratio used for NFRC rating calculations was used to apply Equations 17 to 20.

At this stage ASHWA T and NFRC label values can be directly compared so it is possible to choose a specific glazing system to be used in the CSE simulation. The selection is made from a library of representative glazing systems chosen to span the range of products available for installation in California. Selection is made first by number of glazings and then by closest match of SHGC.

Recognizing that a perfect match will not be found between $U_{LR}$ and $U_{c}$, and between SHGC$_{LR}$ and SHGC$_{c}$, two adjustment factors are defined:

$$B_{LF} = \frac{U_{LR}}{U_{c}}$$

$$B_{LF} = \frac{SHGC_{LR}}{SHGC_{c}}$$

where the LRDC subscript indicates that both numerator and denominator are evaluated at the NFRC label rating design condition.

These factors, $B_{LF}$ and $B_{LF}$, are required to be within a few percent of unity and as such can be used to make small adjustments to the indices supplied by
ASHWAT so that ASHWAT results are brought inline with the known label ratings. Specifically, CSE is supplied with seven results that are used to assemble the simplified circuit (Figure 3): \( f_{\text{int}} \times f_{\text{ext}} \), and \( C_x \) unaltered, plus the results of Equations 23 through 26.

\[
U = U_{\text{int}}(BF_s) \quad (23)
\]

\[
S_x = S_{\text{int}}(BF_s) \quad (24)
\]

\[
S_m = S_{\text{mmt}}(BF_s) \quad (25)
\]

\[
\tau = \tau_{\text{int}}(BF_s) \quad (26)
\]

**STRATEGIES TO MINIMIZE CPU TIME**

The analysis presented above shows that the behavior of a complex fenestration system and its coupling to the indoor and outdoor environment can be represented by a simplified thermal circuit characterized by a few indices of merit. This immediately suggests that computational efficiency could be achieved by caching precalculated values for the indices.

Sensitivity analysis has confirmed the trends implied by the values in Table 1: the indices are strongly influenced by the inside convection coefficient \( h_{\text{int}} \) but change little in response to modest changes in adjacent temperatures or incident radiation. The CSE simulator exploits this by re-using prior indices unless conditions change "enough" to warrant a full ASHWAT calculation. CSE typically uses 2 minute time steps and thus can potentially require 262,800 ASHWAT calculations per fenestration for an annual simulation. Table 2 summarizes results from a number of calculation-trigger algorithms for a typical fenestration system. It is evident that essentially equivalent results can be obtained even when a high proportion of time steps re-use saved indices. In addition, cooling and heating requirements (not shown) change minimally as the number of ASHWAT calculations is reduced.

The current CSE scheme caches only one set of indices; this set is reused until a new calculation is triggered. It may be possible to achieve additional efficiency by retaining additional sets and selecting among them. This approach remains to be pursued.

<table>
<thead>
<tr>
<th>Change to trigger calculation</th>
<th>Calc fraction</th>
<th>Air node gain MBE</th>
<th>RMSE</th>
<th>Radiant node gain MBE</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{\text{int}} \times T_{\text{ext}} )</td>
<td>frac</td>
<td>0.01</td>
<td>0.01</td>
<td>0.045</td>
<td>0.045</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>0.02</td>
<td>0.02</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>(once per hour)</td>
<td>0.03</td>
<td>0.02</td>
<td>0.02</td>
<td>0.05</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Calc fraction: (ASHWAT calculations) / (time steps)

MBE: mean bias error, W/m²

RMSE: root-mean-square error, W/m²

In addition to the prior strategies, it is useful to exploit the linear nature of solar flux absorbed at each layer. That is, for a given system configuration and solar position, the layer absorbed fractions are independent of incident intensity. Thus the fractions for diffuse can be derived once for an entire simulation. Beam absorbed fractions can be derived for multiple days grouped by solar declination. Fractions for various shade alternatives (e.g. slat angle) can also be cached. During the simulation, the appropriate fractions are chosen and scaled by actual incident intensity to yield the required layer heat gains. Strategies of this sort can reduce the required number of solar analyses by a factor of 10 to 30.

**CONCLUSION**

There are complicated interactions between a complex fenestration system (a window with shading attachment(s)) and its room, involving (at least) short wavelength (solar) radiant gains, long-wavelength (thermal) radiant exchanges, and convective transfers. Modeling challenges include flow of room air between a shade and glazing, diathermanous layers (shades are generally not opaque to thermal radiation), and the temperature dependence of both radiant and convective transfers. Rigorous modeling of all these effects within a short-time-step room heat balance simulation imposes a significant computational burden.

A novel approach represents a complex fenestration system as several overall conductances and heat gains that are included in the room heat balance. The
conductances are uniquely described by several indices of merit – notably the U-factor and a cross-coupling coefficient that represents a heat transfer path between the indoor air and radiating surfaces via the fenestration system. This approach reduces computation without forfeiting features such as the ability to distinguish between air and mean radiant temperatures. The required indices of merit are derived using thermal network theory and are sufficiently stable that they can be re-evaluated much less frequently than each time-step. On the other hand, they can be revised as needed in response to changes in sun angle or shade geometry (e.g. blind slit adjustment). The same theory yields heat gains that represent the direct and redirected components of solar gain.

The new approach has been used to integrate the ASHWA1 fenestration model with CSE. ASHWA1 supports essentially any combination of glazing and shading layers separated by arbitrary fill gases or by gaps open to outdoor or indoor air. Simple normal-incidence materials are required to characterize shade materials and shade geometry is easily specified. Models are included that calculate off-normal properties.

Anticipated future work focuses on adapting the approach presented here to room models that do not rely on an explicit mean radiant temperature. In particular, planning is underway for integration of ASHWA1 into EnergyPlus. The EnergyPlus zone model performs surface-by-surface heat balances, so the radiative-convective intermediation captured by $C_\alpha$ is handled implicitly. It remains to be determined whether the efficiencies offered by the novel approach presented here can be fully exploited in a "classic" heat balance model such as EnergyPlus.

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NOMENCLATURE

- $A$: area
- $BF$: adjustment factor, solar terms
- $BF_u$: adjustment factor, U-factor terms
- $C_\alpha$: black emissive power
- $f_r$: weighting factor relating $T_a$, $T_m$ and $T_{in}$
- $h_c$: convective heat transfer coefficient
- $I_{sol}$: incident solar flux
- $J$: radiosity
- $n$: number of layers
- $N_i$: inward flowing fraction, layer $i$
- $N_{in}$: inward flowing fraction, layer $i$ to $T_{in}$ node
- $N_a$: inward flowing fraction, layer $i$ to $T_a$ node
- $S_i$: solar flux absorbed at layer $i$
- $S_{in}$: inward flowing convective source at $T_{in}$
- $S_{in}$: inward flowing longwave source at $T_{in}$
- $SHGC$: Solar Heat Gain Coefficient
- $q_{in}$: heat gain (flux) to room
- $q_{in}$: heat flux to $T_{in}$ node
- $T_a$: air temperature
- $T_{in}$: ambient effective temperature
- $T_m$: mean radiant temperature
- $U$: U-factor

Greek characters

- $\tau$: solar transmittance
- $\Delta$: describing a Delta-circuit (e.g., Figure 3)
- $\Delta T$: temperature difference
- $\varepsilon$: emissivity

Subscripts

- $a$: air
- all surfaces: all surfaces exposed to room
- $eg$: pertaining to center-glass area
- $eg$: pertaining to edge-glass area
- $fr$: pertaining to frame area
- $i$: layer index
- $in$: indoor side
- $j$: room surface index
- $LR$: label rating
- $LRDC$: NFRC label rating design condition
- $m$: mean radiant
- $out$: outdoor side
- $r$: room
- $tot$: pertaining to total window area
- $w$: window area exposed to room
- $exp$: pertaining to a computational experiment

Superscripts

see (Wright 2008)

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