Chapter 1
Introduction

> *For every complex problem there is an answer that is clear, simple, and wrong*
>
> Paraphrased from H. L. Mencken

The world is a complex place, and simple strategies based on simple assumptions are just not sufficient.

For very good reasons of pedagogy, the vast majority of systems concepts to which undergraduate students are exposed are

LINEAR: Since there exist elegant analytical and algorithmic solutions to allow linear problems to be easily solved;

GAUSSIAN: Since Gaussian statistics emerge from the Central Limit Theorem, are well behaved, and lead to linear estimation problems;

SMALL: Since high-dimensional problems are impractical to illustrate or solve on the blackboard, and large matrices too complex to invert.

Unfortunately, nearly all major environmental, ecological, and social problems facing humankind are *non*-linear, *non*-Gaussian, and *large*.

To be sure, significant research has been undertaken in the fields of large-scale, nonlinear and non-Gaussian problems, so a great deal *is* in fact known, however the analysis of such systems is really very challenging, so textbooks discussing these concepts are mostly at the level of graduate texts and research monographs.

However there is a huge difference between *analyzing* a nonlinear system or deriving its behaviour, as opposed to understanding the *consequences* of a system being nonlinear, which is much simpler. It is the latter consequences which are the scope and aim of this book.

That is, although a detailed analysis of a complex system is, in most cases, too difficult to consider teaching, the consequences of such systems are quite easily understood:

P. Fieguth, *An Introduction to Complex Systems*,
https://doi.org/10.1007/978-3-030-63168-0_1

A NONLINEAR SYSTEM is subject to irreversibility, such that given some change in the inputs to the system, undoing the change does *not* necessarily return the system to its start, whereas all linear systems are reversible. Furthermore nonlinear systems can be subject to discontinuous or catastrophic state changes, which is not possible in linear systems.

NON- GAUSSIAN/POWER- LAW SYSTEMS may be characterized by extreme behaviours which would appear to be unpredictable or unprecedented based on historical data. In contrast, Gaussian statistics converge reliably and effectively assign a probability of zero to extreme events, giving a false sense of security in those circumstances where the underlying behaviour is, in fact, power-law.

LARGE COUPLED NONLINEAR SPATIAL SYSTEMS, also known as complex systems, can give rise to highly surprising macroscopic behaviour that would scarcely be recognizable from the microscopic model, a phenomenon known as emergence. Managing such systems, particularly in response to some sort of failure, is very difficult. In contrast, the nature of linear systems is unaffected by scale.

The sorts of systems that we are talking about are not arcane or abstract, rather there are well-known systems, such as geysers and toy blocks, or indeed everyday systems such as stock markets or weather. It is precisely because such systems are so common and everyday that engineers or other technical professionals are likely to encounter them and need to be informed of the possible consequences of interacting with and influencing such systems.

The context of this text is sketched, necessarily oversimplified, at a high level in Figure 1.1. Essentially we have two interacting classes of systems, the human/societal/economic and natural/ecological/environmental systems, both of which will exhibit one or more elements of nonlinearity and spatial interaction which lead to complex-systems and power-law behaviours. Since a model alone is of limited utility, we are interested in performing inference by combining models with measurements, particularly global-scale remotely-sensed measurements from satellites.

1.1 How to Read This Text

This text is aimed at a target audience of undergraduate engineering students, but is intended to be more broadly of interest and accessible.

Those readers unfamiliar with this text, or with complex systems in general, may wish to begin with the overview in Chapter 2, followed by a survey of the case studies which are presented at the end of every chapter, and which are listed on page XIII.

An explicit goal of this text is not to focus attention on the mathematics behind complex systems, but to develop an understanding of the *interaction* between complex systems theory and the real world, how complex systems properties actually manifest themselves. For this reason there are, in addition to the end-of-chapter case studies, a large number of examples, listed on page XIII, and those readers more

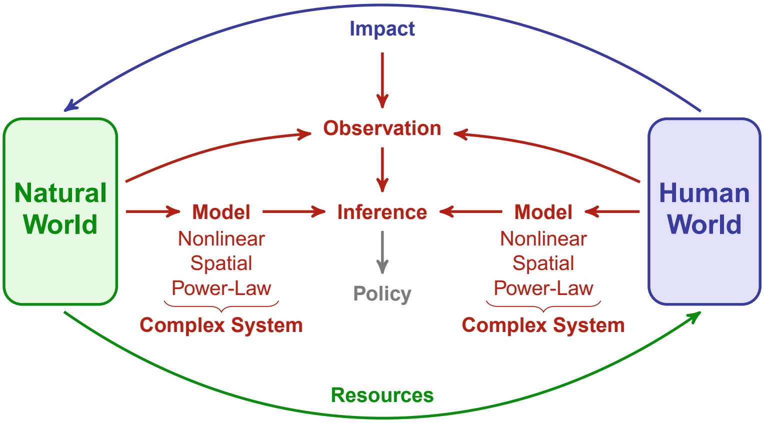


Fig. 1.1 AN OVERVIEW: Human/societal/economic systems (right) draw resources from and have an impact on natural/ecological/environmental systems (left). Both domains contain many examples of systems which exhibit one or more elements of nonlinearity and spatial interaction, leading to complex-systems and power-law behaviours. Observations of one or more systems are combined with models in an inference process, leading to deeper insights and (ideally) better policy. The red portions are the focus of this text, with the blue, green, and grey components illustrating the broader, motivating context.

interested in a high level or qualitative understanding of the book may want to start by focusing on these.

For those readers interested in the mathematics and technical details, the chapters of the book are designed to be read in sequence, from beginning to end, although the spatial and power law chapters can be read somewhat independently from the preceding material, Chapter 4 through Chapter 7, on dynamics and nonlinear systems.

Complex systems can be studied and understood from a variety of angles and levels of technical depth, and the suggested problems at the end of every chapter are intended to reflect this variety, in that there are problems which are mathematical/analytical, computational/numeric, reading/essay, and policy related.

The intent is that most of this text, and nearly all of the examples and case studies, can be understood and appreciated without following the details of the mathematics. The technical details do assume some familiarity with linear algebra and probability theory, and for those readers who need a bit of a reminder, an overview of both topics is presented in Appendices A and B.

This book is, to be sure, only an introduction, and there is a great deal more to explore. Directions for further reading are proposed at the end of every chapter, and the bibliography is organized, topically, by chapter.

References

1. C. Martenson, *The Crash Course: The Unsustainable Future of our Economy, Energy, and Environment* (Wiley, New York, 2011)
2. M. Scheffer, *Critical Transitions in Nature and Society* (Princeton University Press, Princeton, 2009)
3. A. Weisman, *The World Without Us* (Picador, 2007)
4. R. Wright, *A Short History of Progress* (House of Anansi Press, 2004)

Chapter 2
Global Warming and Climate Change

There are few global challenges as widespread, as complex, as significant to our future, and politically as controversial as that of global warming.

The goal of this chapter is, in essence, to motivate this textbook; to convince you, the reader, that very nearly *all* of the topics in this text need to be understood in order to grasp the subtleties of a subject such as global warming.

However the specific problem of global warming is not at all unique, in this regard. That is, after all, the premise of this book: that there is a wide variety of ecological and social challenges, all of which are highly interdisciplinary, and for which a person unfamiliar with one or more of systems theory, nonlinear dynamics, non-Gaussian statistics, and inverse problems is simply ill-equipped to understand.

Other similarly broad problems would include

- Ecological pressures and extinction,
- Human Poverty,
- Energy, and
- Water, to which we shall return in Chapter 12.

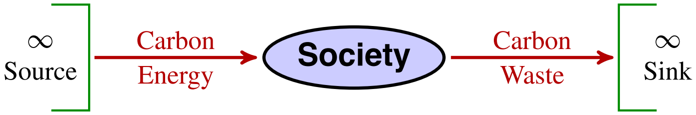
Allow me now to take you on a tour of the entire book, through the lens of global warming.

P. Fieguth, *An Introduction to Complex Systems*,
https://doi.org/10.1007/978-3-030-63168-0_2

Chapter 3: Systems Theory

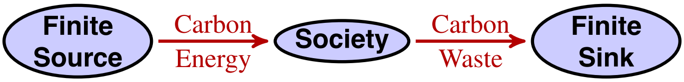
Global warming is the warming of the earth's climate caused by an increase in the concentrations of carbon-dioxide and methane gases in the atmosphere, due to human industry, fossil fuel consumption, and land use.

Suppose we begin with a rather naïve model of carbon flows:



Human society, as a system, interacts with the rest of the world through the boundaries of the system. The naïve model has simplistically assumed fixed boundary conditions, that the sources (energy) and sinks (carbon pollution) are infinite, implying that humans do not influence the global climate.

A slightly more realistic model, but also more complicated, understands energy sources and carbon-dioxide sinks to be *finite*, and therefore subject to influence by human activity:



At least two relatively significant problems remain present in this model:

1. The carbon does not just disappear into some abstract sink: the sink *itself* is subject to important dynamics. Carbon dioxide may be emitted into the atmosphere, however a great deal of it is absorbed into the ocean. Furthermore the ocean, itself, is far from being homogeneous and well-mixed, and the absorbed carbon will concentrate near the surface, whereas most of the potential for storage/sequestration is at the ocean bottom.
2. The *size* of the source or sink is not the only thing which matters: the flow *rates* are crucial to understand as well, in particular the flow rate *relative* to the size of a source or sink, or inconsistencies in flow rates from one system to another.

The latter effect is very much present here, in that the flow rate of carbon from society *into* the atmosphere is controlled by humans, whereas the flow rate of carbon *from* the atmosphere is controlled by physical/natural processes. So although the ocean bottom provides an infinite carbon sink, in principle, inconsistencies in flow rates are causing carbon dioxide to continue to build up:

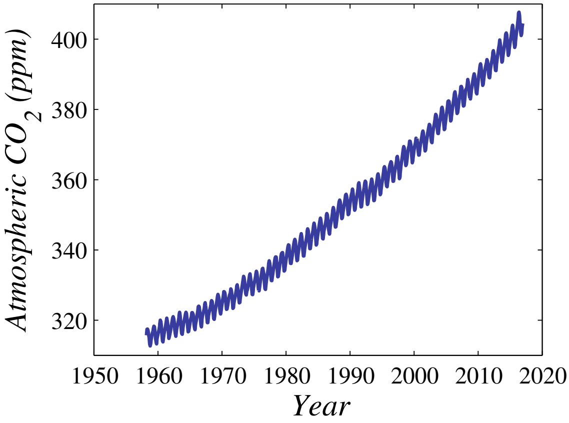
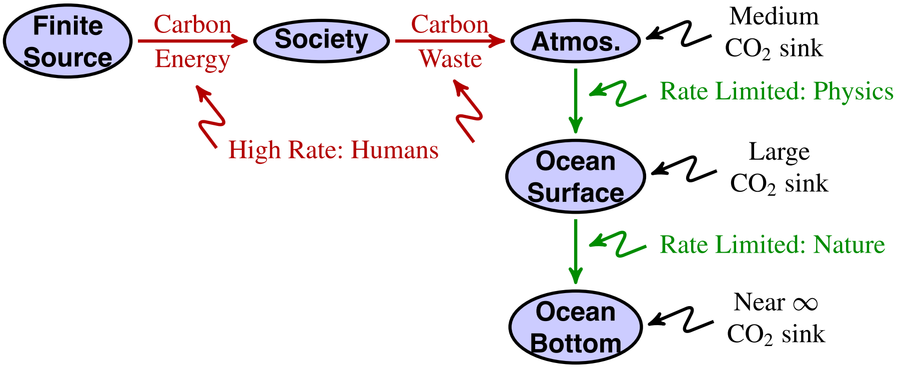


Fig. 2.1 Atmospheric Carbon Dioxide: CO$_2$ levels in the atmosphere have seen a steady and worrying increase over sixty years. The annual variability is a seasonal effect, since there are more forests in the northern hemisphere than in the southern.



So we cannot understand a system, such as a human society, in the absence of the *interactions* that the system has with others around it.

Chapter 4: Dynamic Systems

An additional subtlety is that the latter model is implicitly time-dynamic: there are *rates* of carbon transfer, and a continual transfer of carbon into a finite system implies a system which changes over time.

Two time-dynamic changes associated with carbon are indisputable:

1. CO$_2$ concentrations in the atmosphere are increasing over time, from 315ppm in 1960 to over 400ppm today (Figure 2.1);
2. Since 1800 the world's upper oceans have increased in acidity by 0.1 pH point, corresponding to a 30% increase in acidity, due to the uptake of CO$_2$ into the ocean and the formation of carbonic acid [2], discussed further in Section 12.1.

On the other hand, when we try to articulate questions of global warming, or any other sort of climate change over time, the problem becomes much more slippery:

What is our Baseline? We know that the climate has always been changing, switching between epochs with and without ice ages, for example. Indeed, the earth's climate changes over time periods of all scales:

Tens of millions of years …	Tropical Jurassic (dinosaur) era
Tens of thousands of years …	Ice ages
Hundreds of years …	Medieval warming period (950–1250 AD), So-called little ice age (1550–1850 AD)
Years …	1930's dust bowl, El-Niña / El-Niño

So at what point in time do we actually *start* measuring, in order to know whether the earth is warming or not?

What is Actually Warming? The atmosphere is well-mixed and relatively easy to measure, whereas the heat uptake patterns in the oceans and the ground are far more variable, depending on the presence of subsident currents in the ocean, or geothermal activity underground.

On the other hand, although land and ocean have a far greater mass than the atmosphere, it is primarily the upper surfaces of land and ocean which would be warming, only a small fraction of the total:

Atmosphere	$6 \cdot 10^{14}$ kg per metre at sea level,	$5 \cdot 10^{18}$ kg in total
Ocean	$3 \cdot 10^{17}$ kg per metre of depth,	$1 \cdot 10^{21}$ kg in total
Land	$4 \cdot 10^{17}$ kg per metre of depth,	Ill-defined total mass

Furthermore it is primarily the upper surfaces of land and ocean which are biologically active, and thus a warming of only this top sliver may produce a disproportionate ecological impact. So is *global warming* …

1. a *physical concept*, a warming of the land-oceans-atmosphere in total, which can be measured as increases in global average temperature?, or
2. an *environmental concept*, a disturbance to ecological balance caused by the warming of some *part* of the land-oceans-atmosphere, where the localized warming may produce almost no impact on the global average temperature, and so must be assessed indirectly via some other measurement?

What is Causing the Warming? The increase, over time, in human fossil fuel consumption is well documented. Similarly the increase, over a similar period of time, in atmospheric CO_2 concentration has been accurately measured. However a *correlation*, a statistical relationship, between two events is not the same as *causation*, where one event can be said to cause another.

Measuring a correlation is an exceptionally simple statistical test, whereas causation requires a much deeper understanding. In the case of global warming we need

Table 2.1 SPATIAL AND TEMPORAL SCALES: Typical scales in space and time for structures in ice, water, and air

System	Typical Structure	Spatial Scales	Temporal Scales
Ocean Surface	Eddies	10 km–100 km	Days–Months
Ocean Mid-Depth	Southern Oscillation	100 km–1000 km	Years
Ocean Deep	Thermohaline	1000s km	1000 Years
Atmos. Local	Storms	1 km–100 km	Hours
Atmos. Nonlocal	Pressure Systems	1000 km	Weeks
Ice Local	Cracks	cm–km	Seconds–Years
Ice Nonlocal	Sheets	1km–100 km	1–100 Years

to understand the carbon cycle: the industrial and natural global sources and sinks of carbon.

Chapter 7: Coupled Nonlinear Dynamic Systems

On top of this, in trying to model the presence or flow of heat and energy present in the ocean-atmosphere system, over half of all kinetic energy is not in atmospheric storms or in ocean currents, rather in ocean eddies.

However an eddy, like other similar forms of oceanic and atmospheric turbulence, is one of the hallmarks of coupled nonlinear systems, a nonlinear relationship between multiple elements. No linear system, regardless how complex or large, will exhibit turbulent behaviour, so we cannot fully model the ocean-atmosphere system without studying nonlinear systems.

Chapter 8: Spatial Systems

The atmosphere and oceans are not just nonlinear, or coupled, but indeed very *large* spatially. The governing equation for both air and water flow is the Navier–Stokes nonlinear partial differential equation:

$$\left.\begin{array}{lll} \text{Water/Ocean} & \text{Navier–Stokes} & \textit{Small}\text{ spatial details} \\ & \text{Incompressible} & \textit{Slow}\text{ changes over time} \\ & & \\ \text{Air/Atmosphere} & \text{Navier-Stokes} & \textit{Large}\text{ spatial scales} \\ & \text{Compressible} & \textit{Fast}\text{ changes over time} \end{array}\right\} \begin{array}{c} \text{Challenging} \\ \text{Misfit} \end{array}$$

with interesting challenges due to the different time/space scales of water and air. Really the modelling challenge is much worse, since the range of temporal and spatial scales is actually quite tremendous, as shown in Table 2.1, particularly when the near-fractal[1] behaviour of ocean ice is included.

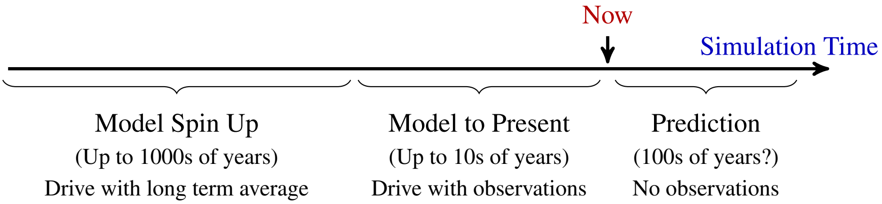
---

[1] Meaning that there is a self-similar behaviour on a wide range of length scales, such as cracks in ice on all scales from nanometre to kilometres.

Executing such a model has been and continues to be a huge numerical challenge: dense spatial grids, with layers in depth (ocean) or height (atmosphere) or thickness (ice) and over time.

Chapter 11: System Inversion

Now what would we do with such a model? Ideally we would initialize the model to the earth's current state, and then run it forward to get an idea of future climatic behaviour:



Such a process of coupling a model with real data is known as data assimilation or as an inverse problem. There is the *true* earth state $\underline{z}$, which is unknown, but which can be observed or measured via a forward model $C()$:

True state $\underline{z}(t)$ $\longrightarrow$ Observations $\underline{m}(t) = C\big(\underline{z}(t)\big) + \text{noise}$

In principle what we want to solve is the *inverse* problem, estimating the unknown state of the earth from the measurements:

Inverse Problem: find $\hat{\underline{z}}$, an estimate of $\underline{z}$, by inverting $C()$:

$$\hat{\underline{z}} = C^{-1}(\underline{m}) \tag{2.1}$$

However almost certainly we cannot possibly obtain enough observations, especially of the deep oceans, to actually allow the inverse problem to be solved, analogous to an underconstrained linear system of equations.

Instead, we perform data assimilation, incorporating the observations $\underline{m}$ into a simulation of a climate model having state $\tilde{\underline{z}}(t)$:

We want to push the simulated state towards truth: $\tilde{\underline{z}}(t) \longrightarrow \underline{z}(t)$
The idea is to iteratively nudge $\tilde{\underline{z}}$ in some direction to reduce

$$\left\| \underline{m}(t) - C\big(\tilde{\underline{z}}(t)\big) \right\| \tag{2.2}$$

where $\|\cdot\|$ measures inconsistency. Thus we are trying to push or nudge the simulation to be consistent with real-world measurements, and therefore hopefully towards the true real-world state.

Chapter 11: System Sensing

So measurement is key to successfully modelling and predicting climate. What are the things we can actually measure:

Atmospheric Temperature:
  Weather stations on the earth's surface
  Weather balloons
  Commercial aircraft
  Satellite radiometers

Oceanic Temperature:
  Satellite infrared measurements of ocean surface temperature
  Ocean surface height measurements (thermal expansion)
  Ocean sound speed measurements
  Buoys, drifters, gliders directly taking measurements in the ocean

Temperature Proxies (indirect effects indicative of temperature)
  Arctic ice extent and number of ice-free days
  Arctic permafrost extent
  Date of tree budding / leaf-out / insect appearance / bird migrations

If we consider global remote sensing via satellite, really the *only* possible measurement is of electromagnetic signals. Therefore a satellite is limited to measuring signal *strength* (brightness) and signal *time* (range or distance).

The key idea, however, is that there are a *great* many phenomena $\underline{z}$ which affect an electromagnetic signal via a forward model $C()$, meaning that from the measured electromagnetic signals we can infer all manners of things, from soil moisture to tree species types to urban sprawl to ocean salinity, temperature, and currents.

Chapter 9: Non-Gaussian Systems

Given adequate observations to allow a model to be simulated and run, what do we do with the results?

Understanding climate models and validating the simulated results are huge topics which could certainly fill another textbook. But even something much simpler, such as a time series of historical temperature data, can lead to challenges.

All students are familiar with the Gaussian distribution,[2] a distribution that very accurately describes the number of heads you might expect in tossing a handful of 100 coins. Phenomena which follow a Gaussian distribution, such as human height, are convenient to work with because you can take the average and obtain a meaningful number. This seems obvious.

However most climate phenomena, and also a great many social systems, do *not* follow a Gaussian distribution, and are instead characterized by what is called a *power law*. Examples of power laws include meteor impacts, earthquake sizes, and book popularity. Given power law data (say of meteor impacts over the last ten years), the average is *not* representative of the possible extremes (think dinosaur extinction

---

[2] Or bell curve or normal distribution; all refer to the same thing.

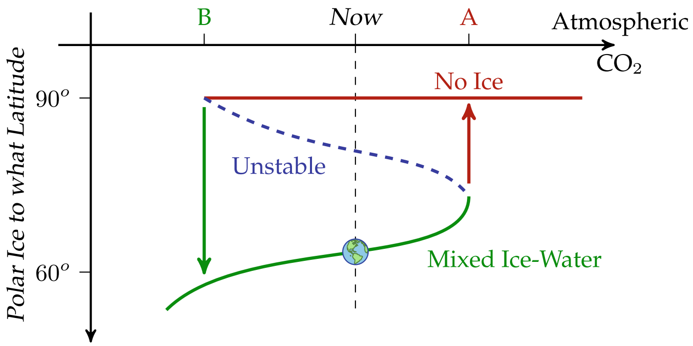


Fig. 2.2 STABLE CLIMATE STATES AS A FUNCTION OF ATMOSPHERIC CARBON DIOXIDE: The earth may have no ice (top) or may have polar ice to some latitude (middle); the arrows indicate the discontinuous climatic jumps ("catastrophes"). Although obviously an oversimplification of global climate, the effects illustrated here are nevertheless real. A more complete version of this diagram can be seen in Figure 6.19.

…). In fact, taking an average over longer and longer periods of time still fails to converge. That seems strange.

We humans only barely learn from history at the best of times; learning from historical power law data is even worse, because it is difficult to know how much data we need to reach a conclusion.

Chapter 5: Linear and Nonlinear Systems

Lastly, it is important to understand the macro-behaviour of climate as a nonlinear system:

> **Linear systems** are subject to *superposition* and have no *hysteresis*.
> **Nonlinear systems** are subject to *catastrophes* and *hysteresis*.

These concepts are most effectively explained in the context of a plot of stable climate states, shown in Figure 2.2.

The principle of superposition says that if increasing CO_2 by some amount leads to a reduction in ice, then twice the CO_2 leads to twice the reduction.

Superposition is highly intuitive, very simple, and usually wrong. We are presently on the lower curve in Figure 2.2, a planet with a mixture of ice and water. As CO_2 is increased, the amount of ice indeed slowly decreases, until point "A", at which point an *infinitesimal* increase in CO_2 leads to the *complete* disappearance of *all* ice. We have here a bi-stable nonlinear system with a bifurcation at point "A", leading to a discontinuous state transition known as a "catastrophe."

In a linear system, to undo the climate damage we would need to reduce the CO_2 level back to below "A". A nonlinear system, in contrast, has memory, what is called hysteresis. Reducing CO_2 to just below "A" has no effect, as we are stuck on the ice-free stable state; to return to the mixed water-ice stable state we need to reduce CO_2 much, *much* further, to "B".

Chapter 10: Complex Systems

Climate, ecology, human wealth and poverty, energy, water are all complex systems: nonlinear, non-Gaussian, coupled, poorly-measured, spatial dynamic problems.

Viewing any of these as systems with isolated or fixed boundaries, subject to superposition and Gaussian statistics, is to misrepresent the problem to such a degree as to render useless any proposed engineering, social, economic, or political solution.

This book cannot pretend to solve global warming or any of many other complex problems, but perhaps we can take one or two steps towards understanding the subtleties of complex systems, as a first step in identifying meaningful solutions.

Further Reading

The references may be found at the end of each chapter. Also note that the textbook further reading page maintains updated references and links.

Wikipedia Links: Global Warming, Climate Change

A challenge, particularly with global warming, is that there is an enormous number of books, most polarized to the extremes of either denial or despair. Books such as *What We Know About Climate Change* [1], *Global Warming Reader* [5], or the personable *Walden Warming* [6] offer a broad spectrum on the subject.

Good starting points for global warming science would be the respective chapters in earth systems books, such as [3, 4].

Regarding the role of nonlinear systems in climate, the reader is referred to Case Study 6.5, the book by Scheffer [7], or the recent paper by Steffen et al. [8].

References

1. K. Emanual, *What We Know About Climate Change* (MIT Press, 2012)
2. A. Johnson, N. White, Ocean acidification: The other climate change issue. Am. Sci. **102**(1) (2014)
3. L. Kump et al., *The Earth System* (Prentice Hall, 2010)
4. F. Mackenzie, *Our Changing Planet* (Prentice Hall, 2011)
5. B. McKibben, *The Global Warming Reader* (OR Books, 2011)
6. R. Primack, *Walden Warming* (University of Chicago Press, Chicago, 2014)
7. M. Scheffer, *Critical Transitions in Nature and Society* (Princeton University Press, Princeton, 2009)
8. W. Steffen et al., Trajectories of the earth system in the anthropocene. Proc. Natl. Acad. Sci.**33**(115), (2018)