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Simultaneous Acceptance Control Charts for Two Correlated Processes

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Acceptance control charts provide control limits for processes in which the natural dispersion is small and the process mean may be permitted to vary. These control limits are based on specification limits and on permitted probabilities of making incorrect decisions of "acceptance" or "rejection." This article deals with acceptance control charts in which two processes, or two characteristics of a product, are controlled for simultaneous conformance to specification limits or standards. The processes can be positively or negatively correlated. Sample sizes and control limits are derived for each chart, using cost minimization or maximum-sample-size minimization as criteria. The analysis is also applicable to acceptance sampling.

KEY WORDS: Hypothesis testing; Optimization; Quality control.

1. INTRODUCTION

In acceptance control charts, control limits on sample means are derived from "acceptable" and "rejectable" levels for the process mean that are usually calculated using the specification limits. Such charts are unlike Shewhart charts in that the mean may be permitted to drift. Examples of this occur in manufacturing with tool wear or in chemical processes in which the process is not stable in the sense of having a constant mean. The necessary assumption is that the process has a small natural dispersion relative to the specification limits (it has a high process-capability index).

Although these charts are statistically similar to variables-acceptance sampling plans, they are usually a form of process control and not acceptance sampling. A full discussion of acceptance control charts and additional references can be found in Duncan (1986) and Wadsworth, Stephens, and Godfrey (1986). It will be convenient to frame the illustrations for our method in terms of acceptance control charts.

This article deals with the case in which a product may have two characteristics, produced by two separate operations or processes. We define *jointly acceptable* as meaning that the processes producing both characteristics are simultaneously acceptable and *jointly rejectable* as the case in which one or both of these processes are rejectable. It is assumed that a joint rational sample is possible; the means of the characteristics are stable during sampling and during their subsequent incorporation into the product.

Our simultaneous acceptance control charts are different from multivariate control charts such as the Hotelling's T^2 [see Jackson (1985) for a review of such charts], because they seek not to detect changes but to control the changes with reference to product specifications.

As an example, consider the manufacture of electrical watt-hour meters that must have a percentage of reading error within plus or minus 1% under "high" load and plus or minus 2% under "low" load. *To be acceptable, the production process must be producing meters that are acceptable under both high and low load.* The percentage errors for a meter tested under the two power levels are correlated. Although control of accuracy has been done by acceptance sampling—in other words, by testing a sample from a lot and then accepting or rejecting the lot—the use of acceptance control charts moves this control to the assembly line and converts it to process control.

To maintain the charts, samples of, say, 15 meters are taken at intervals. Each meter in a sample undergoes a measurement for accuracy at high power and at low power. Each measurement has its own chart with control limits calculated from the specifications and the permitted risks. Because of the joint nature of the acceptability criterion, however, it would be incorrect to construct the charts as if each were an isolated case. In other words, the two charts (high power and low power) are taken as a system; the control limit for this system is exceeded if the average percentage error of a sample of meters falls outside the control limits on *either* chart. The α (producer's risk) and β (consumer's risk) of the charts are not meaningful unless control limits are calculated in the context of the joint nature of the specifications.

The case in which processes are statistically independent was treated in Wesolowsky (1987). This paper deals with the more difficult case in which the processes are correlated, but it restricts itself to two processes. Another approach was taken by Danzinger and Papp (1988), who also dealt with uncorrelated processes; they provided many examples of simultaneous product criteria. Their objective, however, was different in that they wished to control the total proportion of the product that is nonconforming and did not directly control the quality of each of the characteristics. Their approach did not attempt to optimize sample sizes.

2. PRELIMINARY FORMULATION

Let us first review the acceptance control chart for a single characteristic, j. It is useful to develop our arguments in terms of implied hypothesis testing.

The implied hypothesis test is

$$H_0: \mu = \mu_j^a, \qquad H_1: \mu = \mu_j^r, \tag{1}$$

where μ_j^a is the acceptable process average and μ_j^r is the rejectable process average.

When the sample mean $\overline{X}_j$ falls outside the control limits, this is, in effect, rejection of the null hypothesis in favor of the alternative hypothesis. It is assumed here, without loss of generality, that $\mu_j^r > \mu_j^a$. This indicates that the mean, μ, becomes unacceptable when it is too large. There may be two acceptable means that in fact defined an acceptable range for the mean. There will then be a lower rejectable mean.

This is not material to the analysis, however. In acceptance control charts, it is assumed that the process dispersion is so small that, in effect, only one limit can be active at any one time, so we restrict our analysis to the "upper end."

It has been shown in the previously mentioned references that the required sample size, n, and the control limit, c_j, are

$$n_j = \delta_j(z(\alpha_j) + z(\beta_j))^2, \tag{2}$$

$$c_j = \mu_j^a + z(\alpha_j)\sigma_j/\sqrt{n_j}, \tag{3}$$

and

$$c_j = \mu_j^r - z(\beta_j)\sigma_j/\sqrt{n_j}, \tag{4}$$

where σ_j is the standard deviation of process j, $z(\alpha_j)$ and $z(\beta_j)$ are standardized normal deviates such that the areas under the probability density curve to the right are α_j and β_j, respectively, and

$$\delta_j = [\sigma_j/(\mu_j^r - \mu_j^a)]^2. \tag{5}$$

To summarize, we take a sample of n_j items, calculate the sample mean $\overline{X}_j$, and if $\overline{X}_j > c_j$, then the "alarm" is sounded.

The preceding expressions are in terms of acceptable and rejectable process means. Sometimes these are determined directly; if acceptability and rejectability were given in terms of proportions of product units falling above the upper specification limit, however, then we would convert these requirements to equivalent ones involving means. For example, if process j measurements have a normal distribution and the acceptable fraction nonconforming is called the acceptable process level, APL(j), then

$$\mu_j^a = U_j - z(\text{APL}(j))\sigma_j, \tag{6}$$

where U_j is the upper specification limit. Similarly,

$$\mu_j^r = U_j - z(\text{RPL}(j))\sigma_j \tag{7}$$

for the rejectable process level, RPL(j).

Let us now consider the case in which two processes are being monitored simultaneously. Each process has its own acceptable and rejectable means; we now consider both processes to be part of a joint product, however. This joint product is acceptable if both means are acceptable and rejectable if one or more of the means are rejectable.

As in the single acceptance control chart, there is an implied hypothesis system. The implied hypothesis system is

$$H_0: \mu_1 = \mu_1^a, \mu_2 = \mu_2^a$$

$$H_1: \mu_1 = \mu_1^r, \mu_2 = \mu_2^a$$

$$H_2: \mu_1 = \mu_1^a, \mu_2 = \mu_2^r$$

$$H_3: \mu_1 = \mu_1^r, \mu_2 = \mu_2^r. \tag{8}$$

We wish to design two separate charts—that is, to find n_1, c_1, c_2, and n_2—so that the probability of rejecting on either or both of the charts when H_0 is true is less than or equal to $\alpha^\bullet$ and the probability of accepting on both of the charts simultaneously when H_1, H_2, or H_3 is true is less than or equal to $\beta^\bullet$. The solution to this requirement is not unique, and we will, in addition, minimize a weighted sum of the sample sizes of each characteristic.

An equivalent way of viewing the problem is that we wish to design an operating characteristic (OC) surface for our system of two separate acceptance charts. This surface must be above or equal to $1 - \alpha^\bullet$ when $\mu_1 = \mu_1^a$ and $\mu_2 = \mu_2^a$ and below or equal to $\beta^\bullet$ when either or both of the means are at the rejectable level. Minimizing sampling will "press" this surface against three of these points. The point where both means are rejectable will always be below $\beta^\bullet$, as I shall show with a lemma.

We now make observations and define results needed for a precise statement of this design problem. Assume that the measurements in the two processes have a known correlation coefficient ρ. The sample means from the two processes, therefore, also have a correlation of ρ, but only if the two samples are of equal size. For example, if we measure the high-power average percentage of error of 20 meters and the low-power average percentage of error of only the first 10 meters, the correlation of the sample means will not be ρ. As is easily shown, the correlation of the means, ρ, will be

$$\rho_n = \rho(n_{\min}/n_{\max})^{1/2}, \quad (9)$$

where $n_{\min}$ is the minimum of n_1 and n_2 and $n_{\max}$ is the maximum. We adopt the notation ρ_n instead of $\rho(n_1, n_2)$ for simplicity.

Note that the ρ, like the σ's, is estimated from samples once the process is assumed to be stable, which is the usual practice in control charts. Note that a Shewhart chart on the standard deviation is usually maintained in conjunction with the acceptance chart. The correlation should also be monitored. The means are assumed to have an approximately bivariate normal distribution.

We now define notation for the individual charts. Let α_1 be the marginal probability that $\overline{X}_1 > c_1$ when $\mu_1 = \mu_1^a$. Note that

$$\alpha_1 = \Pr(z > (c_1 - \mu_1^a))/(\sigma_1/\sqrt{n_1})$$

and does not depend on ρ or on the marginal distribution of $\overline{X}_2$. Similarly, let β_1 be the marginal probability that $\overline{X}_1 \le c_1$ when $\mu_1 = \mu_1^r$, α_2 be the marginal probability that $\overline{X}_2 > c_2$ when $\mu_2 = \mu_2^a$, and β_2 be the marginal probability that $\overline{X}_2 \le c_2$ when $\mu_2 = \mu_2^r$. Now α, the probability of rejecting process means 1 and 2 when they are jointly acceptable (when H_0 is true), is given by

$$\alpha = \alpha_1 + \alpha_2 - \lambda(\alpha_1, \alpha_2, \rho_n), \quad (10)$$

where

$$\lambda(\alpha_1, \alpha_2, \rho) = L(z(\alpha_1), z(\alpha_2), \rho_n) \quad (11)$$

is the joint probability that $z_1 > z(\alpha_1)$ and $z_2 > z(\alpha_2)$ (or, equivalently, $\overline{X}_1 > c_1$ and $\overline{X}_2 > c_2$) under the assumption of a bivariate normal distribution with correlation ρ_n. The properties of L were given by Abramowitz and Stegun (1970, p. 936). We now examine the probability that our system of two charts will accept the product when $\mu_1 = \mu_1^r$ and μ_1^a:

$$\begin{aligned}\Pr(\text{acceptance} \mid H_1) &= \Pr(\overline{X}_1 \le c_1 \text{ and } \overline{X}_2 \le c_2 \mid H_1) \\ &= \Pr(z_1 < -z(\beta_1) \text{ and } z_2 < z(\alpha_2)) \\ &= \Pr(z_1 > z(\beta_1) \text{ and } z_2 > -z(\alpha_2)) \\ &= L(z(\beta_1), -z(\alpha_2), \rho_n),\end{aligned}$$

which gives (Abramowitz and Stegun 1970, p. 936, ex. 26.3.8):

$$\Pr(\text{acceptance} \mid H_1) = \beta_1 - L(z(\beta_1), z(\alpha_2), -\rho_n). \quad (12)$$

Similarly,

$$\Pr(\text{acceptance} \mid H_2) = \beta_2 - L(z(\alpha_1), z(\beta_2), -\rho_n), \quad (13)$$

and

$$\Pr(\text{acceptance} \mid H_3) = L(z(\beta_1), z(\beta_2), \rho_n). \quad (14)$$

To restrict the probability of Type II error in the system to $\beta^\bullet$, we must, therefore, have

$$\beta^\bullet \le T_1(\rho_n) = \beta_1 - L(z(\beta_1), z(\alpha_2), -\rho_n), \quad (15)$$

$$\beta^\bullet \le T_2(\rho_n) = \beta_2 - L(z(\alpha_1), z(\beta_2), -\rho_n), \quad (16)$$

and

$$\beta^\bullet \le T_3(\rho_n) = L(z(\beta_1), z(\beta_2), \rho_n). \quad (17)$$

We now show that the constraint (17) is always met when (15) and (16) are.

Lemma 1. $T_1(\rho_n) \ge T_3(\rho_n)$ and $T_2(\rho) \ge T_3(\rho_n)$.

Proof. $T_1(\rho_n) \ge T_3(\rho_n)$ when $L(z(\beta_1), -z(\alpha_2), \rho_n) > L(z(\beta_1), z(\beta_2), \rho_n)$. This is true when $1 - \alpha_2 > \beta_2$, which is always true, since we assumed that $\mu_r^j > \mu_a^j$. Similarly, $T_2(\rho_n) \ge T_3(\rho_n)$.

Charts meeting the upper limit Type I and Type II error requirements $\alpha^\bullet$ and $\beta^\bullet$ may be designed according to different objectives. I discuss other criteria later, but our basic goal is sample size (sample cost) minimization:

$$\text{Minimize } A_1 n_1 + A_2 n_2 \quad (18)$$

subject to

$$\alpha^\bullet \le \alpha = \alpha_1 + \alpha_2 - L(z(\alpha_1), z(\alpha_2), \rho_n), \quad (19)$$

$$\beta^\bullet \le T_1(\rho_n) = \beta_1 - L(z(\beta_1), z(\alpha_2), -\rho_n), \quad (20)$$

and

$$\beta^\bullet \le T_2(\rho_n) = \beta_2 - L(z(\alpha_1), z(\beta_2), -\rho_n). \quad (21)$$

The A_1 and A_2 in (19)–(21) are constants that must be specified, but n_1 and n_2 are determined by the optimization procedure. We make two observations. First, the $z(\alpha_j)$ and $z(\beta_j)$ can be expressed in terms of n_1, n_2, c_1, and c_2 using (3) and (4); these, therefore, are the real variables in the problem. We will not make this substitution, however, because it will be convenient for our optimization procedure to work with the marginal probabilities α_j and β_j. Second, the preceding inequalities can be replaced by equalities because, as can easily be shown, the inequalities will be tight to minimize sampling cost.

3. OPTIMIZATION PROCEDURE

We now rearrange the constraints (18)–(21), which are converted to equalities, into a form convenient for discussion of our optimization procedure. Minimize

$$\text{Cost}(\alpha_1, \alpha_2, \beta_1, \beta_2) = A_1\delta_1[z(\alpha_1) + z(\beta_1)]^2 + A_2\delta_2[z(\alpha_2) + z(\beta_2)]^2 \quad (22)$$

subject to

$$\alpha_2 = \alpha^\bullet - \alpha_1 + \lambda(\alpha_1, \alpha_2, \rho_n), \quad (23)$$

$$\beta_1 = \beta^\bullet + \lambda(\beta_1, \alpha_2, -\rho_n), \quad (24)$$

and

$$\beta_2 = \beta^\bullet + \lambda(\alpha_1, \beta_2, -\rho_n). \quad (25)$$

Our basic strategy is to convert the problem into a series of easily done univariate searches. We will search for the lowest value of the objective function by varying α_1 over its possible range, while expressing α_2, β_1, and β_2 in terms of α_1.

To put it another way, we will solve (23), (24), and (25) numerically to obtain α_2, β_1, and β_2 for a given α_1 and then substitute them into Cost(α_1, α_2, β_1, β_2) to obtain $C(\alpha_1)$, a function of α_1 only. $C(\alpha_1)$ has no constraints. Note, however, that $C(\alpha_1)$ is not an equation but a numerical procedure. Since α_1 has a well-defined range, the minimum of $C(\alpha_1)$ can be found by a simple search procedure or even plotting a graph, as will be demonstrated.

I shall now discuss the method of constructing $C(\alpha_1)$. The range of possible values for α_1 is (0, $\alpha^\bullet$), where the parentheses indicate that it does not include the specified endpoints. As is easily verified, if $\alpha_1 = 0$, n_1 is infinitely large, and when α_1 is $\alpha^\bullet$, n_2 is infinitely large.

Let us consider how we find α_2, β_1, and β_2 for a given α_1. If α_1 is given, α_2 is obtained by solving the nonlinear equation (23); then, α_2 being known, β_1 is obtained by solving (24) and β_2 by solving (25). Cost(α_1, α_2, β_1, β_2) can then be evaluated for that α_1, in effect producing $C(\alpha_1)$.

Before we discuss a method for solving the nonlinear equations involved, note that the possible values of α_2, β_1, and β_2 have quite restricted ranges. We will find it useful to explore these. To simplify analysis, assume that $\alpha^\bullet < .5$ and $\beta^\bullet < .5$. This is not an unduly restrictive assumption in practice.

Lemma 2. If $\alpha^\bullet < .5$ and $\beta^\bullet < .5$, then, for α_1 in the range (0, $\alpha^\bullet$), $\alpha_2 \in (\alpha^\bullet - \alpha_1, \alpha^\bullet)$, $\beta_1 \in (\beta^\bullet, \beta^\bullet + \alpha_2)$, and $\beta_2 \in (\beta^\bullet, \beta^\bullet + \alpha_1)$.

Proof. This is readily shown by applying the result that L increases with increasing ρ and that $L(h, k, -1) = 0$ for $h + k \geq 0$ and $L(h, k, 1) = \min\{\Pr(z \geq h), \Pr(z \geq k)\}$ (Abramowitz and Stegun 1970, p. 937) to Equations (23), (24), and (25).

If α_1 is given, the preceding lemma can be used to aid in the solutions of Equations (23), (24), and (25) for α_2, β_1, and β_2. Assume that a routine for evaluating λ is available. There are many such programs; for example, see Drezner (1978) or the IMSL subroutine MDBDOR. There are also simple approximations such as the one by Mee and Owen (1983).

We could use some form of search to solve each of the three equations for α_2, β_1, and β_2, but I suggest an iteration procedure. For example, to solve

$$\alpha_2 = \alpha^\bullet - \alpha_1 + \lambda(\alpha_1, \alpha_2, \rho_n)$$

when α_1 is known, pick a starting value for α_2 and then substitute recursively until the equation is solved to required accuracy. This is known as *one-point* iteration. The procedure will converge because the derivative of λ with respect to α_2 is less than 1 (see Dahlquist and Björck 1974, chap. 6). Similar comments apply to the other two equations.

Experience indicates that the number of iterations required varies from about 3 to 20 if the termination criterion is to make the difference between the two sides of the equation less than .001% of the left side. The number of iterations needed increases when the probability converged to is very small. One can decrease the number of iterations greatly by using the Aitken extrapolation procedure (Dahlquist and Björck 1974, p. 235). There are, of course, many other approaches to solving this kind of nonlinear equation, but this one is very easy to implement.

Use the following procedure to evaluate $C(\alpha_1)$:

Required input: α_1, $\alpha^\bullet$, $\beta^\bullet$, σ_1, σ_2, μ_1^q, μ_1^r, μ_2^q, μ_2^r, A_1, A_2, ρ.
Calculated: δ_1, δ_2 using (5).
Step 1. Set $\rho_n = \rho$.
Step 2. Set $\rho_n^0 = \rho_n$.
Step 3. Find α_2 from (23) by iteration.
Step 4. Find β_1 from (24) by iteration.
Step 5. Find β_2 from (25) by iteration.
Step 6. Find n_1 and n_2 from (2) and hence $n_{\min}$ and $n_{\max}$.
Step 7. Set $\rho_n = \rho(n_{\min}/n_{\max})^2$. If $|\rho_n - \rho_n^0| > \varepsilon$, go to Step 2.
Step 8. Set $C(\alpha_1) = \text{cost}(\alpha_1, \alpha_2, \beta_1, \beta_2)$ using (22).
Step 9. End.

The preceding procedure could be used to search for the best $C(\alpha_1)$ on the range of α_1 : (0, $\alpha^\bullet$) in some formal manner. Simply evaluating enough points for a plot of $C(\alpha_1)$ on this range is probably the best method, however, since the function tends to be rather flat and this will make us aware of trade-offs.

Once a value of α_1 is selected, the associated n_1 and n_2 are, of course, known, so c_1 and c_2 can be computed from (3) and (4); the solution is then complete.

4. EXAMPLE

We continue our previous watt-hour meter example, in which the percentage error at high power was to be within $\pm 1\%$ and at low power was to be $\pm 2\%$. The standard deviation is .05 at high power and .2 at low power. High-power and low-power error measurements have a correlation of .8. For high power, the acceptable and rejectable proportions nonconforming are .005 and .02, whereas for low power these proportions are .01 and .05, respectively.

Charts are to be designed so that the probability of either or both charts showing sample means "out of control" is less than or equal to .01 when the manufacturing process is producing acceptable meters with respect to both readings and the probability of both charts showing "in control" when the process is producing meters unsatisfactory with respect to either measurement is less than or equal to .05. Hence $\alpha^\bullet = .01$, and $\beta^\bullet = .05$.

Calculate the upper acceptable and rejectable process means (average percentage of error produced by the manufacturing process) for high power as follows:

$$\mu_1^a = 1.0 - z(.005)(.05) = .8711$$

and

$$\mu_1^r = 1.0 - z(.02)(.05) = .8973.$$

The lower aceptable and rejectable means would be $-.871$ and $-.897$, respectively. Hence

$$\delta_1 = (.05/(.897 - .871))^2 = 3.669.$$

Similarly, $\mu_2^a = 1.535$, $\mu_2^r = 1.672$, and $\delta_2 = 2.153$.

We now find the charts that minimize the total number of error measurements necessary. In other words, $A_1 = A_2$. Figure 1 shows a plot of n_1, n_2, and $n_1 + n_2$ as functions of α_1. Minimizing $C(\alpha_1)$ by Golden Sections gave the optimum value $\alpha_1^* = .00708$. Equations (22)–(25) also gave $\alpha_2^* = .00383$ and $\beta_1^* = \beta_2^* = .05$. Moreover, $n_1^* = 62$, and $n_2^* = 40$ after rounding.

The correlation between sample means at these sample sizes was $.8(40/62)^{1/2} = .643$ using (9). This means that we would take samples of 62 meters. High-power measurements are taken on all 62, but low-power measurements are taken on only, say, the first 40.

The control limits from (3) and (4) are then calculated to be $\pm.887$ for high-power average percentage errors and ± 1.62 for low-power average percentage errors. Figure 2 shows the OC surface of the optimal solution.

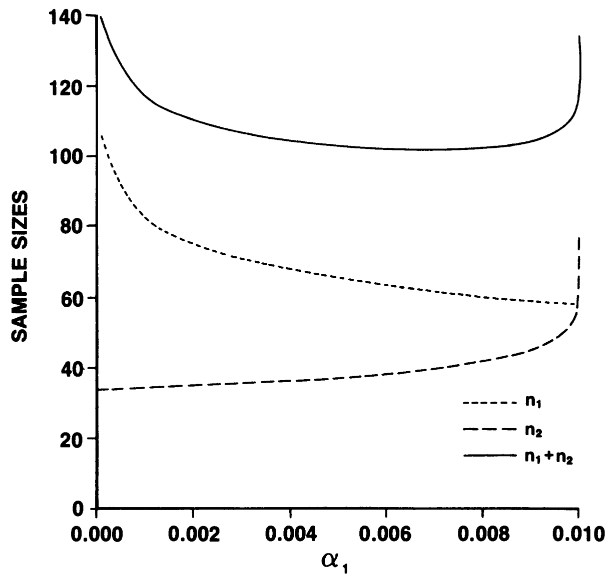

Figure 1. Plot of Sample Sizes for $\rho = .8$.

If the correlation coefficient were $-.8$ instead of .8, then $n_1^* = 63$, $n_2^* = 39$, $\alpha_1^* = .00584$, $\alpha_2^* = .00416$, $\beta_1^* = .0526$, and $\beta_1^* = .0535$. Moreover, for $\rho = 0$: $n_1^* = 63$, $n_2^* = 40$, $\alpha_1^* = .00636$, $\alpha_2^* = .00371$, $\beta_1^* = .0502$, and $\beta_2^* = .0502$.

5. SPECIAL CASES AND EXTENSIONS

One alternative to minimizing the sum of weighted measurements is to minimize the largest sample that is taken. For example, we could wish to take the smallest possible sample of meters from the production line.

This would mean minimizing

$$\max\{\delta_1[z(\alpha_1) + z(\beta_1)]^2, \delta_2[z(\alpha_2) + z(\beta_2)]^2\}.$$

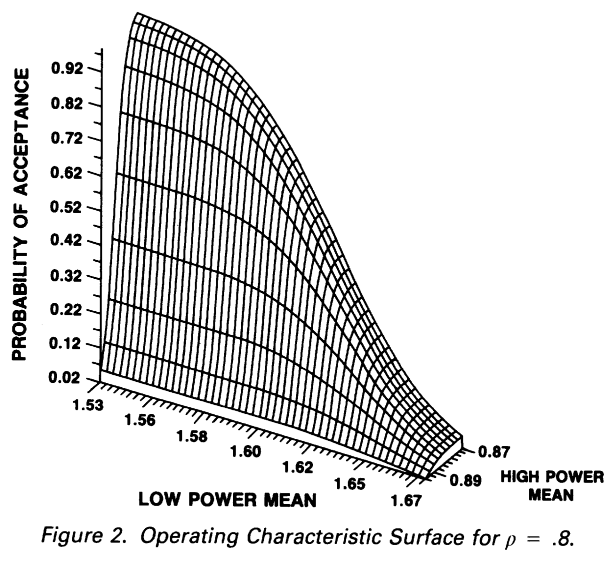

Figure 2. Operating Characteristic Surface for $\rho = .8$.

Our method remains unchanged except for the quantity being calculated for each α_1. When this objective is used in the preceding example with $\rho = .8$, we obtain $\alpha_1^* = .00997$, $\alpha_2^* = .000199$, and $n_1^* = n_2^* = 58$ after rounding.

Analysis of Figure 1 shows why this minimum occurs at $n_1 = n_2$; n_1 decreases with α_1 and n_2 increases simultaneously because α_2 decreases. This is because variation in sample size is, in this example, determined primarily by the $z(\alpha_j)$ terms; β_1 and β_2 remain relatively stable around .05 with changes in α_1. This means that sample-size changes are dominated by the $z(\alpha_j)$'s. This will be true in most practical situations because usually $\beta^\bullet$ is chosen to be much larger than $\alpha^\bullet$, and, as is seen from Lemma 2, this restricts the range of the β_j's.

It is also interesting to consider the height of the OC surface at $H_3 : \mu_1 = \mu_1', \mu_2 = \mu_2'$. We know from Lemma 1 that this will be below $\beta^\bullet$, but this probability of acceptance will vary. One could ask the question of whether an alternative design criterion could be forcing the OC surface through a prespecified point at H_3.

If such a specified probability is β^0, it is easy to adjust our procedure to minimize $|\beta^0 - L(z(\beta_1), z(\beta_2), \rho_n)|$ [see (14)]. $L(z(\beta_1), z(\beta_2), \rho_n)$ has only a limited possible range of values, however, because β_1 and β_2 have limited ranges, as is evident from Lemma 2. Hence it may not be possible to force the surface through a desired point. Probably the best procedure, if the probability of acceptance at H_3 is of concern, is to plot this probability along with the sample sizes and to use it as an auxiliary criterion.

6. SUMMARY AND CONCLUSIONS

This article has derived relatively simple computational procedures for determining the optimal sample sizes and control limits when acceptance control charts must reflect a commitment to joint, or total, acceptability with respect to two correlated processes. A search method was developed that minimizes the cost-weighted combined sample size or the maximum sample size. It may be used to fit an OC surface through four points if the points are such as to make this possible.

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