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PRE-CONTROL AND SOME SIMPLE ALTERNATIVES

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Key Words

Acceptance control charts; Grouped data; Modified Pre-control; Operating characteristic; Process capability; Stoplight control.

Introduction

Pre-control, sometimes called stoplight control, was developed to monitor the proportion of nonconforming units produced in a manufacturing process. Implementation is typically very straightforward. All test units are classified into one of three groups: green, yellow, or red, where the colors loosely correspond to good, questionable, and poor quality products (see Fig. 1). The number of green, yellow, and red units observed in a small sample determines when to stop and adjust the process. The goal of Pre-control is to detect when the proportion of nonconforming units produced becomes too large. Thus, Pre-control schemes monitor the process to ensure that process capability (C_{pk}) remains large.

Pre-control was initially proposed in 1954 (see Refs. 1 and 2 for more details) as an easier alternative to Shewhart charts. However, since that time, at least three different versions of Pre-control have been suggested in the literature. In this article the three versions are called

- Classical Pre-control
- Two-stage Pre-control
- Modified Pre-control

Classical Pre-control refers to the original formulation of Pre-control as described by Shainin and Shainin (1) and Traver (3). Two-stage Pre-control is a modification discussed by Salvia (4) that improves the method's operating characteristics by taking an additional sample if the initial sample yields ambiguous results. Modified Pre-control, on the other hand, as suggested by Gurska and Heaphy (5), represents a departure from the philosophy of Classical and Two-stage Pre-control. Modified Pre-control attempts to compromise between the design philosophy of Shewhart-type control charts and the simplicity of application of Pre-control.

Pre-control schemes are defined by their group classification procedure, their decision criteria, and their qualification procedure. The qualification procedure specifies the required results of an initial intensive sampling scheme used to determine if Pre-control is appropriate for the given application. For all three versions of Pre-control, a process passed the qualification if five consecutive green units are observed. As all three versions of Pre-control have the same qualification procedure, it is not discussed in more detail in this article. The three versions of Pre-control differ most substantially in their group classification method. Classical Pre-control and Two-stage Pre-control base the classification of units on engineering tolerance or specification limits. A unit is classified as green if its quality dimension of interest falls into the central half of the tolerance range. A yellow unit has a quality dimension that falls into the remaining tolerance range, and a red unit falls outside the tolerance range. Assuming, without loss of

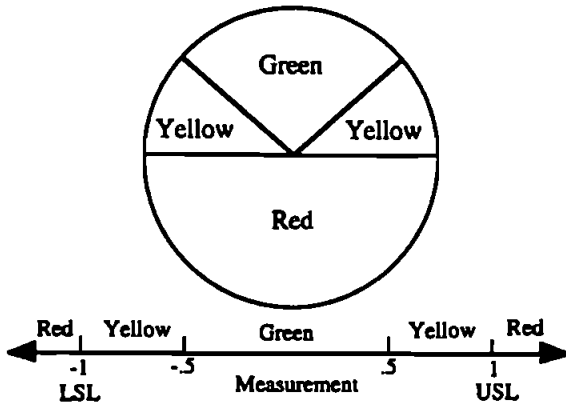


Figure 1. Pre-control classification criteria.

generality, that the upper and lower specification limits (USL and LSL) are 1 and -1 , respectively, group classification is based on the endpoints: $t = [-1, -0.5, 0.5, 1]$. Figure 1 illustrates this classification scheme. The colored circle can be used for the ease of the operators as a dial indicator overlay (1).

Let the quality dimension of interest be Y . Then, given a probability density function for observations $f(y)$, the group probabilities for Classical and Two-stage Pre-control are given by

$$\begin{aligned} P(\text{green}) &= P_g = \int_{y=-0.5}^{0.5} f(y) dy, \\ P(\text{yellow}) &= P_y = \int_{y=-1}^{-0.5} f(y) dy + \int_{y=0.5}^1 f(y) dy, \\ P(\text{red}) &= P_r = \int_{y=-\infty}^{-1} f(y) dy + \int_{y=1}^{\infty} f(y) dy. \end{aligned} \quad (1)$$

Modified Pre-control, on the other hand, classifies units using control limits, as defined for Shewhart charts, rather than tolerance limits. This change indicates a fundamental difference and makes modified Pre-control much more like a Shewhart chart than like Classical Pre-control. Equations (2) give the group probabilities for the Modified Pre-control procedure. Note that to avoid confusion, throughout this article the current process mean and standard deviation are denoted μ and σ , whereas estimates of the in-control mean and standard deviation used to set the control limits for Modified Pre-control and Shewhart-type charts are denoted μ_c and σ_c .

$$\begin{aligned} P_m(\text{green}) &= \int_{\mu_c - 1.5\sigma_c}^{\mu_c + 1.5\sigma_c} f(y) dy, \\ P_m(\text{yellow}) &= \int_{\mu_c - 3\sigma_c}^{\mu_c - 1.5\sigma_c} f(y) dy + \int_{\mu_c + 1.5\sigma_c}^{\mu_c + 3\sigma_c} f(y) dy, \\ P_m(\text{red}) &= \int_{-\infty}^{\mu_c - 3\sigma_c} f(y) dy + \int_{\mu_c + 3\sigma_c}^{\infty} f(y) dy. \end{aligned} \quad (2)$$

The group probabilities given by Eqs. (1) and (2) are the same when $\mu_c = 0$ and $\sigma_c = 1/3$. This makes sense because in that case, the control limits and the tolerance limits are the same. It has been suggested that Classical Pre-control and Two-stage Pre-control are only applicable if the current process spread (six process standard deviations) covers less than 88% of the tolerance range (3). With specification limits at ± 1 , as defined previously, this condition corresponds to the constraint $\sigma < 0.29333$.

The second important difference between the three Pre-control versions is their decision criteria. Classical Pre-control bases the decision to continue operation or to adjust the process on only one or two sample units. The decision rules are given as follows:

Sample two consecutive parts A and B

- If part A is green, continue operation (no need to measure B).
- If part A is yellow, measure part B. If part B is green, continue operation, otherwise stop and adjust process.
- If part A is red, stop and adjust process (no need to measure B).

The decision procedure for Two-stage Pre-control and Modified Pre-control is more complicated. If the initial two observations do not provide clear evidence regarding the state of the process, additional observations (up to three more) are taken. The decision procedure for Two-stage and Modified Pre-control is given as follows:

Sample two consecutive parts.

- If either part is red, stop process and adjust.
- If both parts are green, continue operation.
- If either or both of the parts are yellow, continue to sample up to three more units. Continue operation if the combined sample contains three green units, and stop the process if three yellow units or a single red unit are observed.

The advantage of this more complicated decision procedure is that more information regarding the state of the process is obtained and, thus, decision errors are less likely. The disadvantage is that, on average, larger sample sizes are needed to make a decision regarding the state of the process. Table 1 summarizes the comparison of the three versions of Pre-control. Clearly, the different versions of Pre-control are not the same in purpose and ease of implementation. By design, Classical Pre-control and Two-stage Pre-control tolerate some deviation in the process mean, so long as the proportion nonconforming does not

Table 1. Comparison of Pre-control Versions

PRE-CONTROL VERSION	GROUP CLASSIFICATION BASED ON	DECISION CRITERIA BASED ON
Classical	Tolerance limits	Two observations
Two-stage	Tolerance limits	Five observations
Modified	Control limits	Five observations

become too large. In this sense, Classical and Two-stage Pre-control are very similar to acceptance control charts. In addition, Classical and Two-stage Pre-control can be quickly implemented because they do not require estimates of the current process mean and standard deviation to set their grouping criterion. By contrast, the goal of Modified Pre-control is to detect deviations from stability. As a result, mean drifts are not tolerated, and Modified Pre-control charts are similar to Shewhart-type charts such as \bar{X} charts. In addition, Modified Pre-control, like a Shewhart chart, requires estimates of the current process parameters to determine the control limits.

Comparison with Traditional Control Charting Methods

In this section, the three Pre-control versions are compared with the appropriate traditional control charting techniques. In addition, the performance of Pre-control under some special situations is explored. Mackertich (6) and Ermer and Roepke (2) compare Classical and Two-stage Pre-control with \bar{X} and R control charts, but this is an inappropriate comparison because, as explained in the previous section, the charts have a different purpose. Here,

Classical and Two-stage Pre-control are compared with acceptance control charts (ACCs) because both these types of monitoring schemes are designed to signal only if the proportion nonconforming becomes unacceptably high. An ACC is designed to monitor a process when the process variability is much smaller than the specification (tolerance) spread (7). Under that assumption, moderate drifts in the mean (from the target value) are tolerable, as they do not yield a significant increase in the proportion of nonconforming units. Like Classical and Two-stage Pre-control, ACCs are based on engineering specification limits. However, ACC limits are derived based on a distributional assumption and require a known and constant process standard deviation. ACC limits are usually set assuming a normal process, although, if justified, other assumptions could be made. Modified Pre-control, on the other hand, is compared with Shewhart \bar{X} charts, because both charts are designed to monitor the process for stability.

All Pre-control schemes differ from traditional control charting techniques in a number of ways. The first obvious difference is that Pre-control uses only information from the classified (or grouped) observations, whereas traditional control charts (\bar{X} charts and ACCs) use variables data. Grouping the data results in decision rules that are easy to implement, but, clearly, the grouping also discards some information. This loss of information can be quantified by calculating the expected statistical (Fisher) information available in the Pre-control grouped observations. Kulldorff (8) and Steiner et al. (9) discuss calculating the Fisher information available in grouped data. Figure 2 shows the expected information about the mean and standard deviation for various parameter values using the Classical Pre-control classification criterion $t = [-1, -0.5, 0.5, 1]$. The plots are scaled so that variables data (infinite

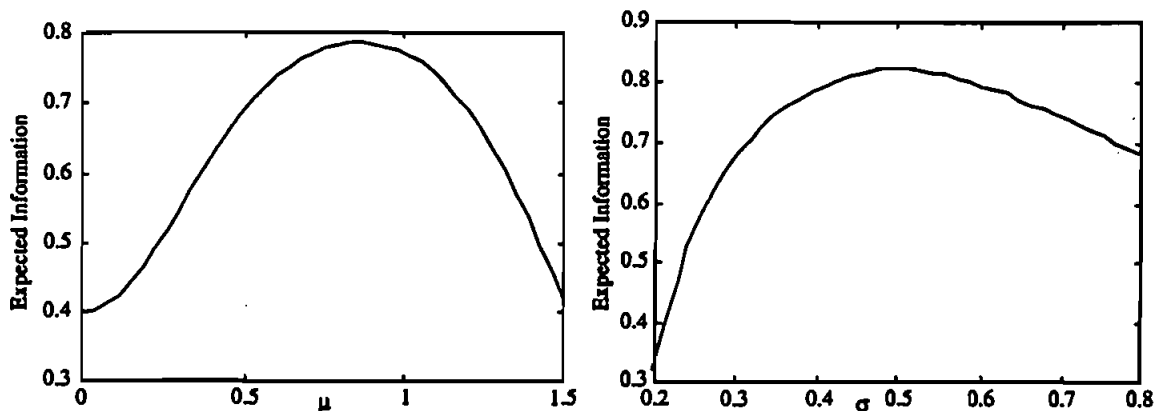


Figure 2. Expected information about μ (left, assume $\sigma = 0.2933$) and σ (right, assume $\mu = 0$), $t = [-1, -0.5, 0.5, 1]$.

number of groups) would provide an expected information content of unity for all values of μ and σ . The expected information about the μ graph is symmetric about $\mu = 0$; thus, negative mean values are not shown. Figure 2 shows that the group classification scheme used in Pre-control is not very efficient when μ is close to zero and/or σ is small. The small amount of information available when μ equals zero is not a major concern however if, at that mean value, it is very easy to conclude that the process should continue operation. This would be the case, for example, if the process standard deviation is small compared with the specification range. The next section explores this issue further.

Pre-control and traditional control charts also differ in their decision procedures. The effectiveness of the decision procedures can be compared through their operating characteristic (OC) curves. An OC curve shows the probability a process monitoring scheme or control chart signals (or fails to signal) for different parameter values. Ryan (7) derives OC curves for ACCs and \bar{X} charts. The probability that Classical and Two-stage Pre-control schemes continue operating, denoted P_{accept} , can be found by calculating the probability of each combination of green and yellow units that leads to acceptance. Salvia (4) gives the probability of acceptance for Classical and Two-stage Pre-control as

$$P_{\text{accept}}(\text{Classical}) = P_g + P_y P_g, \quad (3)$$

$$P_{\text{accept}}(\text{Two-stage}) = (P_g + P_y)^2 - 2P_g P_y(1 - P_g^3 - 3P_g^2 P_y) - P_y^2(1 - P_g^3), \quad (4)$$

where $P_g = P(\text{green})$ and $P_y = P(\text{yellow})$ are given by Eqs. (1).

Figure 3 compares the OC curves for Classical Pre-control and Two-stage Pre-control, as derived from Eqs. (3) and (4), and an ACC with sample size $n = 2$. Figure 3 shows that an ACC with sample size $n = 2$ is superior to Classical Pre-control because it has significantly better power to detect mean shifts. Similarly, the Two-stage Pre-control scheme is superior to an ACC with $n = 2$. Note that different σ values simply shift the OC curves horizontally without changing the ranking. Figure 3 assumes a process whose quality characteristic is approximately normally distributed. A different underlying distribution for the quality characteristic may dramatically change the probabilities of acceptance, but, generally, all the considered procedures are affected similarly.

Two-stage Pre-control has the best operating characteristics and is, thus, preferred over Classical Pre-control. However, in most situations, Two-stage Pre-control re-

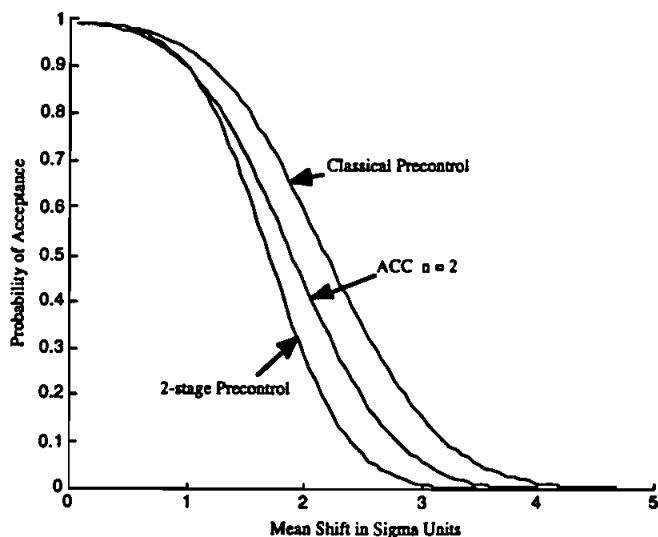


Figure 3. OC curves of Classical and Two-stage Pre-control and ACC with $n = 2$, $N(\mu, \sigma = 0.2933)$ process.

quires sample sizes larger than two to make a decision. As given previously, Two-stage Pre-control uses a partially sequential decision procedure and requires sample sizes of between one and five units. In the Appendix, the average sampling number of the Two-stage Pre-control procedure is derived and denoted by $E(n)$. $E(n)$ depends on the group probabilities, given by Eq. (1), that are, in turn, dependent on the process parameters μ and σ . Figure 4 shows plots of $E(n)$ versus the process mean when the process is assumed to have a Normal distribution.

These results suggest a comparison of Two-stage Pre-control with ACCs having larger sample sizes. Figure 5 gives OC curves of Two-stage Pre-control and ACCs with sample sizes of three, four, and five. Clearly, of the compared curves, the Pre-control procedure has the smallest power to detect mean shifts. In addition, ACCs are likely easier to implement because they require fixed sample sizes. On the other hand, the power of Two-stage Pre-control is fairly competitive with an ACC with $n = 3$, and ACCs have the disadvantage of requiring an estimate of σ , an assumption that σ does not change, and precise or variables measurements rather than grouped data. In addition, Two-stage Pre-control requires a smaller sample size, on average, when the process is centered at $\mu = 0$. In conclusion, Two-stage Pre-control appears a reasonable alternative to ACCs when little process knowledge is available or if estimating the process mean and standard deviation is expensive.

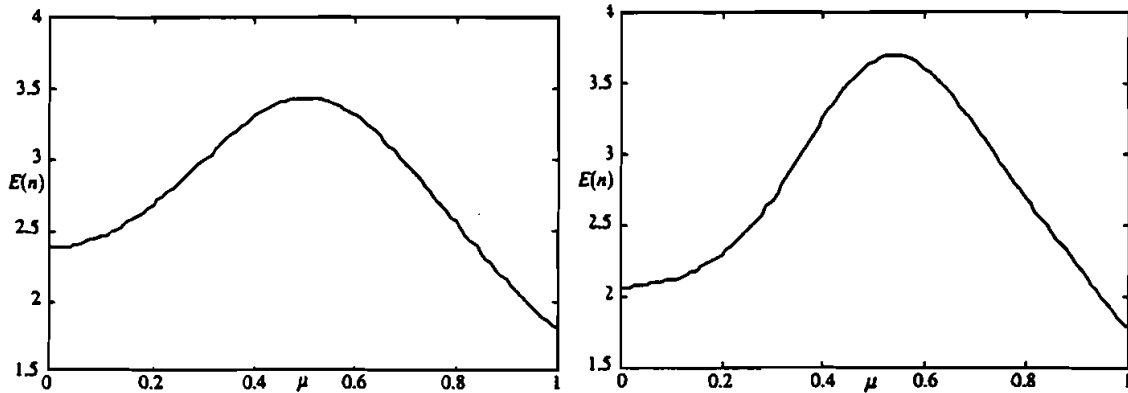


Figure 4. Average sample size for Two-stage Pre-control as a function of the process mean $\sigma = 0.29333$ on left, $\sigma = 0.2$ on right, $N(\mu, \sigma)$ process.

Modified Pre-control, unlike Classical and Two-stage Pre-control, is designed to monitor the stability of a process and, thus, has the same philosophy as an \bar{X} control chart. Modified Pre-control and \bar{X} charts ideally detect any drift in the process mean. The operating characteristics of Modified Pre-control can be determined using Eqs. (4) and (2) with $\mu = 0$ and $\sigma = 1/3$. Table 2 shows a comparison of Modified Pre-control and traditional \bar{X} control charts using various sample sizes. The average sample size of Modified Pre-control can be determined through Eq. (A1) by calculating $E(n; \mu = 0, \sigma = 1/3)$. A plot of the

average sample size for Modified Pre-control would be similar to Figure 4.

Table 2 shows that Modified Pre-control has a very high false alarm rate that is an order of magnitude larger than that for \bar{X} charts. When the process is stable ($\mu = 0$), a Modified Pre-control chart signals a problem on average almost 2.4% of the time. This false alarm rate is too high, as searching for assignable causes is usually time-consuming and expensive, and too many false alarms reduces the credibility of the control chart greatly and may lead to it being ignored.

We now turn to an examination of when Pre-control is applicable. A drawback of Classical and Two-stage Pre-control schemes is that they are designed to prevent defectives, but they do not adapt to different process variabilities. Pre-control is the same for processes with capability C_{pk} equal to 1.33 or 1.67, but, ideally, these processes would be handled differently. Figures 6 and 7 explore the relationship between the probability of a signal using Classical and Two-stage Pre-control and the probability of a defect. Figure 6 is a contour plot showing the probability of a defect for various combinations of μ and σ , assuming $USL = 1$ and $LSL = -1$. The probability of a defect, or a nonconforming unit, is given by $P(\text{red})$ in Eqs. (1). Figure 7 shows contour plots of the probability of a signal for Classical and Two-stage Pre-control [unity minus the probability of acceptance as given by Eqs. (3) and (4)]. Ideally, the contours in Figures 6 and 7 would look similar; however, this is not the case. The Pre-control methods are too conservative when σ is small, because they signal often for many processes that yield a very small proportion of defects. This is because, using Pre-control,

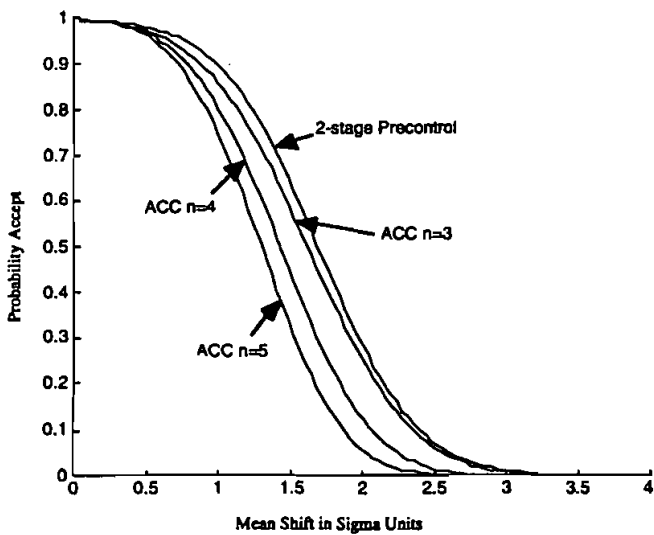


Figure 5. Comparison of Two-stage Pre-control and ACC with $n = 3, n = 4, n = 5, N(\mu, \sigma = 0.29333)$ process.

Table 2. Modified Pre-control Compared with \bar{X} Control Charts

μ	MODIFIED PRE-CONTROL P_{signal}	\bar{X} CHART $n = 3$ P_{signal}	\bar{X} CHART $n = 4$ P_{signal}	\bar{X} CHART $n = 5$ P_{signal}
0	.0238	.0027	.0027	.0027
$\pm 1 \sigma$.2097	.1024	.1587	.2225
$\pm 2 \sigma$.8370	.6787	.8413	.9295

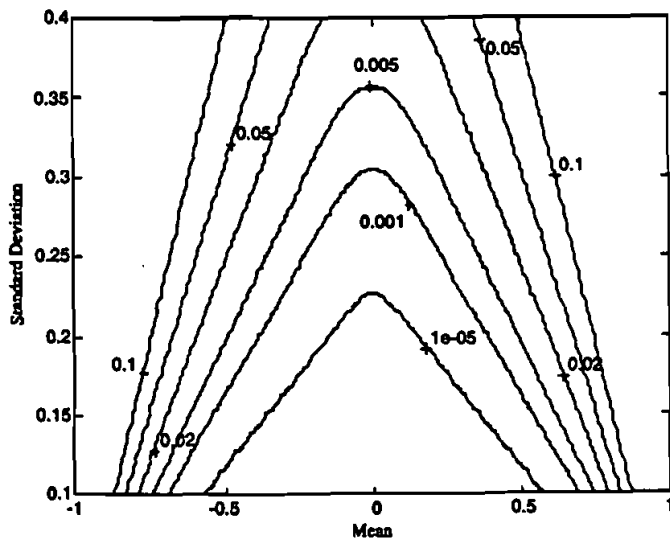


Figure 6. Contour plot of the probability of a defect.

large μ values together with small σ values likely lead to many yellow units and, thus, a signal. As a result, Pre-control is not applicable when σ is very small compared with the specification limits—say, when 6σ covers less than 60% of the tolerance range.

Acceptance control charts do not suffer from this shortcoming because they adjust their control limits for different σ values. Table 3 gives specific values shown on the contour plots and corresponding values from an ACC with $n = 5$ for a direct comparison. The results for ACCs are derived assuming that the estimated mean and standard deviation values are equal to the true values. Most enlightening are the two cases $\mu = 0$, $\sigma = 0.2$, and $\mu = 0.6$, $\sigma = 0.1$. In both cases, the probability of a defect is fairly small and approximately the same. However, when $\mu = 0$ and $\sigma = 0.2$, the probability the Pre-control scheme signals is very small, whereas when $\mu = 0.6$ and $\sigma = 0.1$, the probability of a signal using Pre-control is very large.

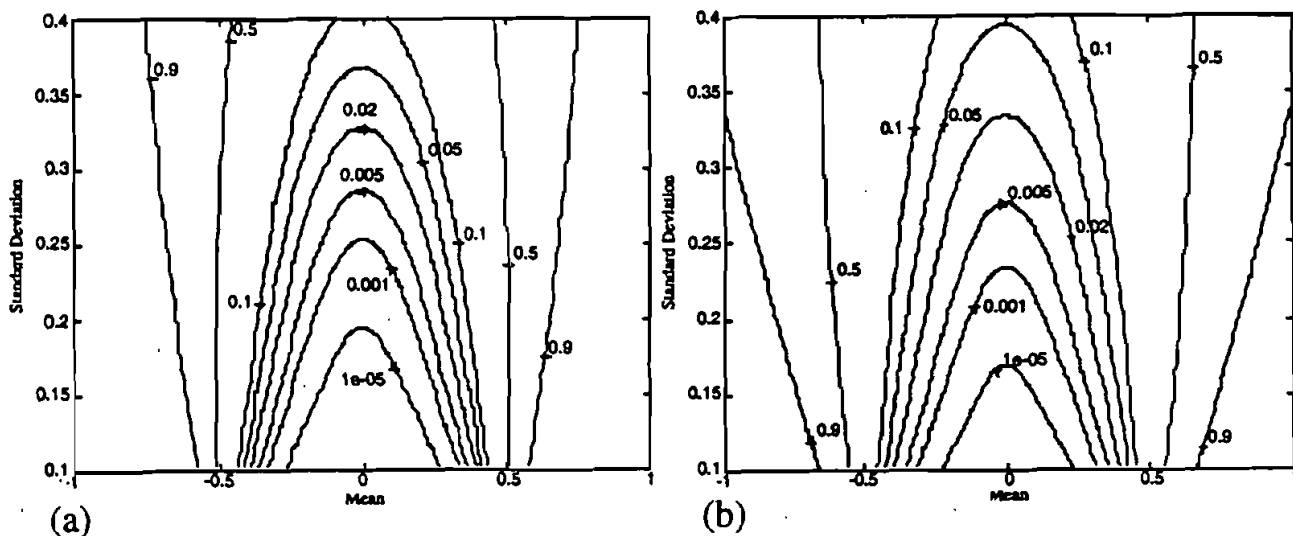


Figure 7. Contour plots of the probability of a signal. (a) Classical Pre-control; (b) Two-stage Pre-control.

Table 3. Probability of Defect and Signals

μ	σ	$P(\text{defect})$	CLASSICAL	TWO-STAGE	ACC $n = 5$
			PRE-CONTROL	PRE-CONTROL	
			P_{signal}	P_{signal}	P_{signal}
0	0.1	~ 0	3.2E-13	1.7E-18	~ 0
0	0.2	5.7E-5	1.5E-4	1.8E-5	2.6E-7
0	0.3	8.6E-4	.0099	.0088	Not applicable
0.5	0.1	2.9E-7	.25	.4688	1.3E-7
0.6	0.1	3.2E-5	.7079	.9540	.0018
0.7	0.1	.0013	.9550	.9994	.2489

In contrast, for ACCs, the probability of a signal is very small in both cases.

In summary, Classical and Two-stage Pre-control are fairly competitive with ACCs, except under certain circumstances. When the process variation is very small compared with the tolerance range ($6\sigma < 0.6T$, where T is the tolerance range), Classical and Two-stage Pre-control lead to undesirable results such as rejecting a process that is producing virtually all parts within specification. Also, as discussed in Ref. 3, when the process variation is relatively large compared to the tolerance range ($6\sigma > 0.88T$), Classical and Two-stage Pre-control yield excessively large false alarm rates. However, if 6σ lies between 60% and 88% of the tolerance range, Classical or Two-stage Pre-control yield good results. Modified Pre-control, on the other hand, is similar to Two-stage Pre-control with 6σ equal to 100% of the tolerance range. As a result, Modified Pre-control is a poor method because it yields too many false alarms.

Alternatives to Traditional Pre-control

As discussed in the previous section, Pre-control has a number of important advantages over traditional control charts, mainly in terms of simplicity of implementation. However, its design choices, in particular the grouping criteria and the decision rules, appear quite ad hoc. We may ask if the performance of Pre-control could be improved while retaining its simplicity? This section considers three simple variations of Pre-control called Ten Unit Pre-control, Mean Shift Pre-control, and Simplified Pre-control. Each of these proposed variations is very similar to Two-stage Pre-control and requires only minor modifications. The goal is not to develop the optimal approach under particular assumptions but rather to consider simple changes that retain the flavor of Pre-control. Optimal ACCs and Shewhart-type charts for grouped data under distribu-

tional assumptions are developed by Steiner et al. (9,10), utilizing the likelihood ratio.

One reason why two-stage Pre-control is fairly competitive with ACCs in terms of power is due to its partially sequential decision procedure. As a result, one possible improvement approach is to make the decision procedure more sequential. A totally sequential procedure is feasible theoretically but would be difficult to implement because it would require large sample sizes occasionally and defining an appropriate acceptance/rejection criterion would be difficult. A version of Pre-control that allows a maximum of ten units at each decision point is a compromise. The decision rules for proposed Ten Unit Pre-control are given below. To monitor the process, this decision procedure should be repeated periodically.

Take one sample unit at a time.

Define #G and #Y as the cumulative number of green and yellow units, respectively.

- Stop the process if
 - There are any red units, or
 - At any time, $\#Y - \#G \geq 2$ together with $\#Y \geq 3$, or
 - At any time, $\#Y \geq 5$
- Continue operation of the process, and stop sampling, if at anytime $\#G - \#Y \geq 2$.
- Otherwise, continue to sample.

Expressions for the probability of a signal and the average sample size of Ten Unit Pre-control are derived by enumerating all the possible paths to acceptance and rejection. Table 4 shows a comparison of the operating characteristics of the proposed Ten Unit Pre-control scheme and Two-stage Pre-control for various combinations of the process mean and standard deviation. Table 4 shows that Ten Unit Pre-control has lower false alarm rates and more power to detect large shifts in the mean than Two-stage Pre-control. In addition, although the Ten Unit Pre-control

Table 4. Comparison of Ten Unit and Two-stage Pre-control

μ	σ	TWO-STAGE PRE-CONTROL	TWO-STAGE PRE-CONTROL	TEN UNIT PRE-CONTROL	TEN UNIT PRE-CONTROL
		P_{signal}	$E(n)$	P_{signal}	$E(n)$
0	0.2933	.0069	2.37	.0045	2.41
$\pm 1 \sigma$	0.2933	.1058	2.96	.0895	3.27
$\pm 2 \sigma$	0.2933	.7155	3.32	.7427	3.74
0	1/3	.0238	2.55	.0174	2.65
$\pm 1 \sigma$	1/3	.2097	3.11	.1959	3.52
$\pm 2 \sigma$	1/3	.8370	2.95	.8513	3.17

procedure occasionally requires samples of size 10 to make a decision, on average it requires approximately the same number of units as Two-stage Pre-control. As a result, Ten Unit Pre-control is recommended whenever Two-stage Pre-control is applicable, and the potential additional sampling and the slightly more complicated decision rules are not problematic or very expensive.

A second improvement idea stems from the realization that, using Pre-control, units are classified into one of five groups (see Fig. 1), but only three groups (green, yellow, and red) are distinguished during the decision procedure. As a result, traditional Pre-control methods ignore important information pertaining to the process mean. The proposed Mean Shift Pre-control scheme utilizes this information. The Mean Shift Pre-control method classifies observations into one of four groups: green, upper yellow, lower yellow, and red, where the two yellow groups correspond to the upper and lower quarter of the specification range, respectively. The red group is not divided into units falling outside the upper and lower specification limits because any kind of red unit should be very rare and lead to a signal automatically in any case. The Mean Shift Pre-control decision rules, given below, are similar to those used in Two-stage Pre-control:

Sample two consecutive parts.

- If either unit is red, stop the process.
- If both units are green, continue operation.
- If either or both units are yellow (upper or lower), sample three additional units one at a time. If, in the combined sample, three upper yellows, three lower yellows, or a single red are observed, stop the process. Otherwise, continue operation of the process.

Table 5 compares Mean Shift Pre-control and Two-stage Pre-control in terms of their power to detect process

changes. The results show that Mean Shift Pre-control has a much smaller false alarm rate than Two-stage Pre-control and virtually identical power to detect one or two sigma unit shifts in the mean. It should be noted, however, that Mean Shift Pre-control is not as sensitive to process variation shifts. As a result, Mean Shift Pre-control is a good alternative to Two-stage Pre-control only when a stable process variation can be assumed or if the process dispersion is simultaneously monitored in some other way.

Simplified Pre-control, the third variation, is designed to be simpler than Two-stage Pre-control. Using Simplified Pre-control, units are classified only as green or yellow. Simplified Pre-control is also an attractive alternative, as it can be adapted easily to handle situations where σ is small and traditional Pre-control schemes are not appropriate. The group probabilities and decision process for Simplified Pre-control are given below:

$$P(\text{green}|\text{Simplified Pre-control}) = \int_{y=-c}^c f(y) dy, \quad (5)$$

$$P(\text{yellow}|\text{Simplified Pre-control}) = \int_{y=-\infty}^{-c} f(y) dy + \int_{y=c}^{\infty} f(y) dy,$$

where c is a constant.

Table 5. Comparison of Two-stage Pre-control and Mean Shift Pre-control

μ	σ	TWO-STAGE PRE-CONTROL	MEAN SHIFT PRE-CONTROL
		P_{signal}	P_{signal}
0	1/3	.0238	.0120
$\pm 1 \sigma$	1/3	.2097	.2031
$\pm 2 \sigma$	1/3	.8370	.8369
0	.2933	.0069	.0031
$\pm 1 \sigma$.2933	.1058	.1029
$\pm 2 \sigma$.2933	.7155	.7154

The Simplified Pre-control decision procedure is as follows:

Sample five consecutive parts.

- If three or more units are 'yellow', then stop the process.
- Otherwise continue operation.

The loss of efficiency using Simplified Pre-control rather than Two-stage Pre-control is very slight when the process variation is in the range where Pre-control is appropriate. The loss of efficiency is small because, using Pre-control, the probability of a red unit is usually very small; thus, the red classification provides very little information. In fact, setting $c = 0.5$ in Eqs. (5), Simplified Pre-control has a slightly smaller false alarm rate than Two-stage Pre-control, although not quite as much power to detect mean shifts. Table 6 shows a comparison for selected values of μ and σ . Compared with Two-stage Pre-control, Simplified Pre-control has the advantage of requiring less effort in the group classification, and the decision rules are more straightforward. However, Simplified Pre-control requires a larger average sample size than Two-Stage Pre-control.

The most important benefit of Simplified Pre-control, however, is that it can be adapted fairly easily when the process variation is much smaller than the tolerance range. As shown in the previous section, Pre-control is not appropriate if the process standard deviation is less than around one-tenth the tolerance range. When σ is small, observing yellow units with the traditional grouping criterion as defined by Eqs. (1) is not necessarily evidence of a problem. See Figures 6 and 7. In this circumstance, the Simplified Pre-control group limits $[-c, c]$ can be set with c closer to the tolerance limits ($0.5 < c < 1$). By adjusting the group limits, it is possible to detect only process mean shifts that yield many nonconforming units.

Table 6. Comparison of Two-stage Pre-control and Simplified Pre-control with $c = 0.5$

μ	σ	TWO-STAGE PRE-CONTROL	SIMPLIFIED PRE-CONTROL
		P_{signal}	P_{signal}
0	1/3	.0238	.0193
$\pm 1 \sigma$	1/3	.2097	.1831
$\pm 2 \sigma$	1/3	.8370	.8258
0	.2933	.0069	.006
$\pm 1 \sigma$.2933	.1058	.0972
$\pm 2 \sigma$.2933	.7155	.7103

The best value for c depends on the process standard deviation and the acceptable and rejectable process levels (APL and RPL, respectively). A good choice for c is the midpoint between the APL mean and the RPL mean, where APL and RPL mean values can be determined from acceptable and unacceptable process capability values. Process capability is defined as $C_{pk} = \min((USL - \mu)/3\sigma, (\mu - LSL)/3\sigma)$, where LSL and USL are the lower and upper specification limits, respectively (7). For example, assume the process standard deviation σ equals $7/10$ and that an acceptable process capability value is $C_{pk} = 4/3$ (an average of 6 defects out of 100,000 units), whereas $C_{pk} = 2/3$ (an average of 94 defects out of 10,000 units) is unacceptable. Then, looking only at the upper specification limit (the problem is symmetric on the lower limit), the corresponding APL and RPL mean values are $\mu = 0.6$ and $\mu = 0.8$, respectively. In this case, choosing $c = 0.7$ is reasonable. Figure 8 illustrates the operating characteristics obtained in this example for Two-stage Pre-control and Simplified Pre-control. For example, using Simplified Pre-control with $c = 0.7$, the probability of a signal when $\mu = 0.6$ is .031 and the probability of a signal when $\mu = 0.8$ is .969. If this false alarm rate is too large, adjusting the decision barrier c slightly higher may be appropriate. The OC curve for Two-stage Pre-control, in Figure 8, shows that even at the APL, $\mu = 0.6$, the scheme results in an almost guaranteed signal. As a result, Simplified Pre-control with an adjusted value of c is recommended for any process whose process standard deviation is too small for effective use of Two-stage Pre-control.

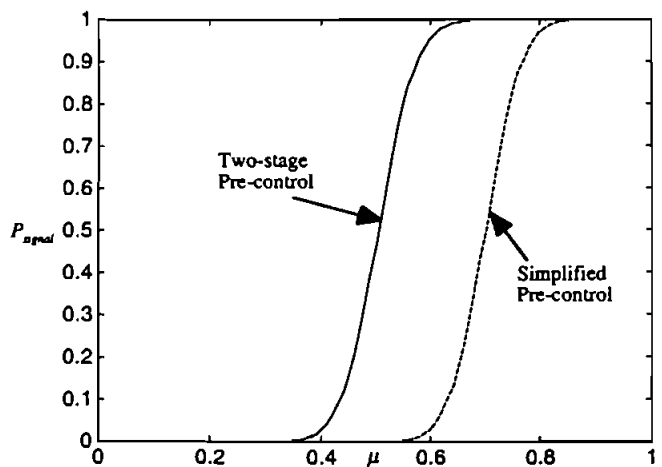


Figure 8. OC curves for the adapted Simplified Pre-control procedure with $c = 0.7$, and Two-stage Pre-control.

Conclusions

This article compares and contrasts three previously recommended Pre-control approaches and suggests three simple new variations that improve performance under certain situations. Classical Pre-control and Two-stage Pre-control are compared with acceptance control charts rather than \bar{X} charts as in previous articles, as this is a more appropriate comparison. It is concluded that Classical Pre-control and Two-stage Pre-control are good methods when the process standard deviation σ lies in the range $T/10 \leq \sigma \leq 11T/75$, where T is the tolerance range. Two-stage Pre-control is preferred over Classical Pre-control unless the additional sampling required is very onerous. Modified Pre-control, on the other hand, has the same goal as an \bar{X} chart but is shown to have an excessively large false alarm rate and is thus not recommended.

Three new alternatives to Pre-control are suggested that retain the simplicity of Pre-control. The Ten Unit approach is recommended when improved operating characteristics are desired and taking some additional independent observations is not difficult or expensive. The Mean Shift Pre-control is better than Two-stage Pre-control in detecting process mean shifts. However, Mean Shift Pre-control is only recommended when the process dispersion is constant or is being monitored by some other method. Simplified Pre-control utilizes a simplified classification and decision procedure while providing similar operating characteristics as Two-stage Pre-control. Simplified Pre-control is recommended when the process standard deviation is smaller than $T/10$ because it can be adapted easily to handle arbitrarily small process standard deviation values.

Appendix

In this appendix, an expression for the average sampling number $E(n)$ of the Two-stage Pre-control scheme, discussed in the second section, is derived. $E(n)$, given in Eq. (A1), is determined by calculating the probability of each possible path to either acceptance or rejection of the process. In Eq. (A1), p_i , $i = 1, \dots, 5$, equals the probability that the Two-stage Pre-control reaches a decision in i units. For example, Two-stage Pre-control would stop after two units if we observe either two green units (process

acceptable) or if the first observation is yellow or green and the second observation is red (process rejectable).

$$E(n) = \sum_{i=1}^5 ip_i, \quad (\text{A1})$$

where

$$\begin{aligned} p_1 &= p_r, & p_2 &= p_g^2 + p_r(p_y + p_g), \\ p_3 &= p_r p_y^2 + 2p_g p_y p_r + p_y^3, \\ p_4 &= 3p_y^2 p_g p_r + 2p_g^2 p_y p_r + 2p_g^3 p_y + 3p_y^3 p_g, \\ p_5 &= 5p_y^2 p_g^2 p_r + 5p_y^3 p_g^2 + 5p_g^3 p_y^2. \end{aligned}$$

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