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Confirmation sample control charts

S. H. STEINER†

Traditional $\bar{X}$ and R control charts are used widely in industry, but do not respond quickly to small or moderate changes in the process output. Confirmation sample control (CSC) charts to detect changes in the process mean and process variability are proposed. These new control charts have substantially better operating characteristics than $\bar{X}$ and R charts. CSC charts require that any unusual observed sample be confirmed through an independent confirmation sample taken from the process. This makes CSC charts appealing to production personnel since the charts require the verification of bad news. The implementation of CSC charts is illustrated, and figures are given that allow the determination of appropriate design parameters.

1. Introduction

Control charts such as $\bar{X}$ and R charts are widely used in industry to monitor quality. These charts are effective at detecting large departures from the in-control condition. Other process monitoring methods, such as Cumulative Sum (CUSUM) and Exponentially Weighted Moving Average (EWMA) charts, are good at detecting more moderate persistent process shifts, but are more complicated to implement since they are sequential in nature accumulating information from previous observations. As a compromise, a number of researchers have investigated adapting the simple Shewhart type control charts to take into account some, but not all, the previous observations.

One approach, which has a long history, is using supplementary runs rules in conjunction with the $\bar{X}$ and R charts. Runs rules were first popularized in the Western Electric Company's *Statistical Quality Control Handbook* (1956). There are a large number of possible runs rules that have been suggested and it is not clear which subset is the best for any particular application. All runs rules try to identify some pattern in the process data that shows evidence of some non-random behaviour. Examples include 2 out of 3 successive points at 2 standard deviations or beyond, and 7 successive points on one side of the centre line. The addition of runs rules to standard $\bar{X}$ and R charts can make the charts more sensitive to process changes, but at the cost of more frequent false alarms. This is especially true when runs rules are applied to R charts since the distribution of the sample range is not symmetric. Also, the addition of runs rules makes the interpretation of the control charts more difficult.

Another approach is the Variable Sampling Interval (VSI) control chart. VSI charts adjust the sampling interval based on the current observation. If there is some evidence (but no out-of-control signal) that the process may have shifted the

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† Department of Statistics and Actuarial Sciences, University of Waterloo, Waterloo, N2L 3G1 Canada.

sampling interval is shortened. This has the effect of reducing the number of non-conforming parts produced if there is a moderate process shift. The VSI approach is most effective when applied to sequential charts such as CUSUM charts since, in that case, sample statistics plotting close to the control limit have accumulated substantial evidence against the in-control hypothesis. CUSUM and EWMA VSI charts are discussed in Reynolds *et al.* (1990), and Saccucci *et al.* (1992) respectively. However, VSI charts have also been created for Shewhart charts (Cui and Reynolds 1988, Reynolds *et al.* 1988), NP charts (Vaughan 1993), and combined Shewhart and CUSUM charts (Amin and Ncube, 1991).

Other related work considered the adjustment of the sample size in response to the observed sample. Variable sample size (VSS) control charts are discussed by Sawalapurkar *et al.* (1990) and Prabhu *et al.* (1994). VSS control charts require samples of different sizes depending on the previously observed sample statistics. By increasing the sample size when the process shows moderate evidence of an out-of-control condition, the sampling effort is more concentrated during out-of-control periods and the expected time to detect process shifts can be reduced. To ease implementation difficulties typically only two different sample sizes are used.

Both the VSS and VSI approaches are fairly complex to implement since either the sample size or the sampling interval changes over time. As a result, although the procedures have been shown to have better run length properties than standard Shewhart charts they have not been widely implemented.

Simpler approaches include Croasdale (1974) and Daudin (1994) who suggest double sampling $\bar{X}$ control charts that occasionally require a second sample. Croasdale and Daudin suggest adding warning limits to the standard control charts in addition to control limits. In their methodology any sample mean that falls between the warning limit and the control limit is considered an indeterminate sample, and in that case a second independent sample is taken to provide more information. In the Croasdale (1974) procedure the final in-control/out-of-control decision after an indeterminate initial sample is based solely on the second sample. Daudin, on the other hand, bases the decision on the combined sample. In both cases the first and second sample sizes need not be the same size.

Double sampling charts, VSI charts, VSS charts, and $\bar{X}$ control charts with supplementary runs rules are compared in terms of power in Costa (1994). Costa concludes that all these methods are superior to standard Shewhart $\bar{X}$ charts, and that Daudin's double sample method is the best at detecting moderate shifts in the process mean. He also concludes that VSS charts detect small shifts of less than one standard deviation unit more quickly than a corresponding VSI scheme, but that this result is reversed for larger mean shifts.

In this article, the confirmation sample control (CSC) chart is proposed and explored in detail. Using CSC charts, whenever a sample of poor or questionable quality is observed an independent confirmation sample is immediately taken to verify that the original observation was not a fluke. The desire to confirm bad news, i.e. out-of-control signals, is well known, and discussed in the context of traditional control charts by Pitt (1987). The CSC procedure is very simple since the second sample is always the same size as the first, and both resulting sample points can be plotted on a single control chart since they are compared to the same control limits. In this article, CSC charts are developed to detect both process mean and process standard deviation shifts.

Confirmation control charts are similar to the double sampling methods proposed by Croasdale and Daudin, but are easier to implement and, unlike the Daudin approach, the proposed procedure has the advantage of being a very simple adaptation of the standard combined $\bar{X}$ and R charts. While CSC charts are slightly more complicated to administer than $\bar{X}$ and R charts they are easier to implement than double sampling charts since the initial and confirmation sample sizes are the same, the control limits used to classify the initial and confirmation samples are the same, and the results of the procedure can be easily summarized on a single chart. Also, unlike double sampling procedures, CSC charts provide an alternative to the traditional pair of $\bar{X}$ and R charts. Compared with $\bar{X}$ and R charts, CSC charts provide an improved ability to detect process shifts, while retaining a small false alarm rate. This increased power arises since the CSC chart is partially sequential in that the total number of observations sampled may be doubled depending on the outcome of the initial sample.

In section 2, two versions of the confirmation sample control chart are defined and appropriate design parameters are derived. A comparison between CSC charts, traditional $\bar{X}$ and R charts, and double sampling charts in terms of operating characteristics and expected sample size is made in section 3. Finally, section 4 discusses the implementation of CSC charts and provides a numerical example.

2. Confirmation sample control charts

The basic idea underlying CSC charts is to allow for a confirmation sample if the initial sample does not lead to a clear conclusion. The CSC chart is designed so that while the process is stable, a confirmation sample is rarely needed. In this way, the sampling requirements of CSC charts are only marginally greater than the traditional Shewhart chart when the process is in-control. However, as a consequence of sampling more extensively only when the process exhibits some evidence of instability, moderate size process shifts are detected more quickly. To avoid a time delay in the detection of a process shift the confirmation sample is obtained as soon as an addition sample from the process can be obtained that is independent of the initial sample.

Two versions of CSC charts are proposed. In both cases confirmation control limits are defined that are used to determine if the initial sample provides clear evidence that the process is still in-control. The chart can be operated using only confirmation control limits, called the stand-alone CSC chart, or it can be designed to have both confirmation control limits and traditional control limits, called the joint CSC chart. Stand-alone CSC charts are easier to administer, but are slightly less sensitive to large process shifts. In most applications, the stand-alone CSC chart is recommended because it is easier to understand and administer. However, if especially good protection against large process changes is required, or if detecting process variability changes is critical, the joint CSC chart is more appropriate.

Both versions of CSC charts are similar to the traditional Shewhart control charts. In both versions a rational sample of size n is taken from the process on a regular basis, say every two hours. For each of these initial samples we calculate a test statistic denoted θ_A. If deemed necessary a second independent confirmation sample of the same size is taken. Denote the test statistic calculated from the confirmation sample as θ_B. The test statistic used depends on the intended purpose of the control chart. For example, the test statistic θ could represent the sample average $\bar{X}$, the sample range R, or the sample standard deviation s. Defining the upper and

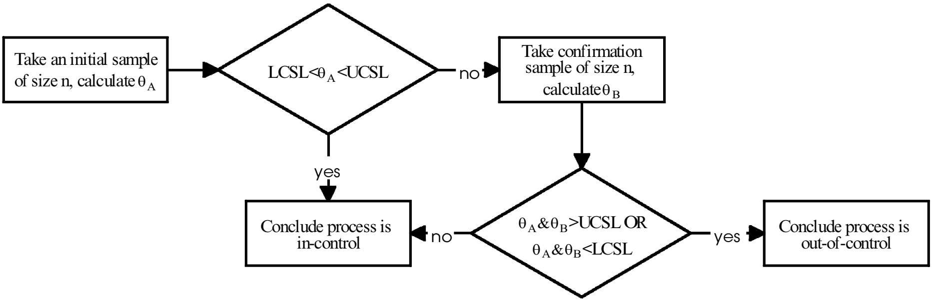

Figure 1. Stand-alone CSC chart decision procedure.

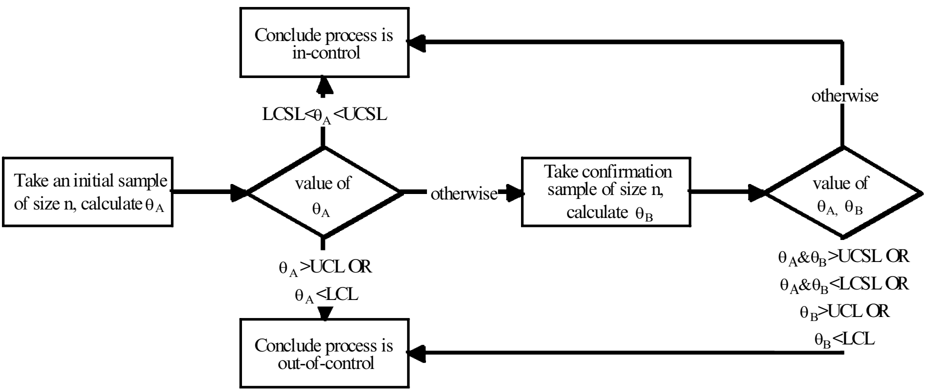

Figure 2. Joint CSC chart decision procedure.

lower control limits as UCL and LCL, and the upper and lower confirmation sample control limits as $UCSL$ and $LCSL$ respectively, the decision procedures for the two versions of CSC are given in flowchart form in figures 1 and 2.

The derivation of appropriate confirmation and control limits depends on the type of chart ($\bar{X}$ or R or s) and the desired operating characteristics. Traditionally, Shewhart control charts have control limits set at plus and minus three sigma. For example, control limits for $\bar{X}$ charts are set at $\bar{\bar{X}} \pm 3\bar{R}/d_2$, where $\bar{R}/d_2$ is an estimate of the variability of $\bar{X}$ derived from subgroup ranges, and $\bar{\bar{X}}$ is an estimate of the true process mean (Montgomery 1991). Using three sigma limits, and assuming that the estimates of the process mean and variability are good, and that $\bar{X}$ follows a normal distribution, leads to an expected false alarm rate of 0.0027. Thus, when the process is in-control (the process mean has not changed from the initial value) the chart will signal a shift 0.27% of the time. This false alarm rate has been found to provide good results in industrial applications.

Based on the decision procedures of the stand-alone CSC chart and the joint CSC charts given in figures 1 and 2, the probability of a signal (concluding the process is out-of-control) assuming fixed values for the confirmation and control limits is given by (1) and (2)

$$P(signal \mid \text{stand-alone}) = P(\theta > UCSL)^2 + P(\theta < LCSL)^2, \tag{1}$$

$$P(signal\,|\,\text{joint}) = P(UCSL < \theta < UCL)[P(\theta > USCL) + P(\theta < LCL)] + P(LCL < \theta < LCS)[P(\theta < LCSL) + P(\theta > UCL)] + P(\theta > UCL) + P(\theta < LCL). \quad (2)$$

Determining reasonable values for the control and confirmation limits for the CSC charts can be accomplished by closely mimicking the traditional control limits design procedure. We set the overall false alarm rate at 0.0027 and assume that false alarms in either direction are equally undesirable. Then, (1) suggests that the stand-alone CSC confirmation limits should be set to yield 0.0367 tail probabilities ($0.0367 = \sqrt{0.0027/2}$) when the process is in-control. In other words, for the stand-alone CSC chart we set the $UCSL$ and $LCSL$ such that $P(\theta > UCSL) = P(\theta < LCSL) = 0.0367$. To derive appropriate control and confirmation limits for the joint CSC chart we assume further that false alarms due to the initial sample and the confirmation sample should be approximately equally likely. Then, the joint CSC confirmation limits and the control limits should be set to yield approximately 0.0256 and 0.0007 tail probabilities. Thus, for the joint CSC chart we set $UCSL$, $LCSL$, UCL and LCL such that $P(\theta < LCSL) = P(\theta > UCSL) = 0.0256$ and $P(\theta > UCL) = 0.0007$. Note that the values of the confirmation limits in the stand-alone and joint versions of the CSC chart are not the same.

When monitoring the process mean we may assume by the central limit theorem that, as long as the distribution of the underlying Xs are close to normal, the sample mean will be approximately normally distributed. In that case, since the normal distribution is symmetrical the confirmation and control limits are set at $\hat{\mu}_\theta \pm \kappa\hat{\sigma}_\theta$, where $\hat{\mu}_\theta$ is an estimate of the mean of θ, and $\hat{\sigma}_\theta$ is an estimate of the standard deviation of θ, and k is chosen to give the desired tail probabilities.

Denote k_{SA}, k_C and k_S as the standard deviation multiples needed for the stand-alone CSC chart confirmation limits, and the joint CSC chart control and confirmation limits respectively. Using the inverse of the standard normal cumulative distribution function gives $k_{SA} = 1.79$. Rounding off this suggests that $k_{SA} = 1.8$ is a good choice for the stand-alone confirmation limits. Thus, to monitor the process mean a stand-alone CSC chart would have a confirmation limit at $\bar{\bar{X}} \pm k_{SA}\hat{\sigma}_{\bar{X}} = \bar{\bar{X}} \pm 1.8\hat{\sigma}_{\bar{X}}$. Similarly, tail probabilities of 0.0256 and 0.0007 respectively suggest $k_C = 1.95$ and $k_S = 3.2$. Thus, for the joint CSC chart the confirmation and control limits would be set at $\bar{\bar{X}} \pm k_C\hat{\sigma}_{\bar{X}} = \bar{\bar{X}} \pm 1.95\hat{\sigma}_{\bar{X}}$ and $\bar{\bar{X}} \pm k_S\hat{\sigma}_{\bar{X}} = \bar{\bar{X}} \pm 3.2\hat{\sigma}_{\bar{X}}$ respectively (see table 1).

A confirmation sample control chart to monitor the process variation (R or s chart) can be designed in a similar way. However, unlike the distributions of the sample mean, the distribution of the sample range and sample standard deviation are skewed. As a result, the use of probability limits (as discussed by Ryan 1989) are

Type of limit	Limits
Stand-alone CSC confirmation limit	$\bar{\bar{X}} \pm 1.8\hat{\sigma}_{\bar{X}}$
Joint CSC confirmation limit	$\bar{\bar{X}} \pm 1.95\hat{\sigma}_{\bar{X}}$
Joint CSC control limit	$\bar{\bar{X}} \pm 3.2\hat{\sigma}_{\bar{X}}$

Table 1. Suggested confirmation and control limits for CSC $\bar{X}$ chart.

	Joint CSC				Stand-alone CSC	
n	$D_{0\cdot0007}$	$D_{0\cdot0256}$	$D_{0\cdot9744}$	$D_{0\cdot9993}$	$D_{0\cdot0367}$	$D_{0\cdot9633}$
2	0.0012	0.0454	0.0651	2.9535	3.1566	4.8326
3	0.0495	0.3067	0.3685	3.4828	3.6708	5.2384
4	0.1748	0.5996	0.6807	3.7941	3.9738	5.4820
5	0.3325	0.8552	0.9446	4.0137	4.1877	5.6560
6	0.4932	1.0717	1.1649	4.1826	4.3522	5.7911
7	0.6454	1.2564	1.3512	4.3192	4.4856	5.9012
8	0.7861	1.4160	1.5112	4.4337	4.5973	5.9939
9	0.9249	1.5557	1.6509	4.5320	4.6933	6.0740
10	1.0328	1.6795	1.7744	4.6180	4.7773	6.1444

Table 2. Percentiles for the distribution of the range.

recommended. To determine appropriate limits, critical values of the appropriate distribution that lead to the desired upper and lower tail probabilities must be determined.

The distribution of the sample range depends on the sample size, and under the assumption that the individual observations are normally distributed has been tabulated by Harter (1960). Barnard (1978) gave a computer routine for the calculation of percentage points of the range distribution assuming the underlying values are normally distributed with mean zero and variance unity. Using the Barnard algorithm (AS 126, 1978) the critical values of the distribution of the sample range for upper and lower tail probabilities of 0.0367, 0.0256 and 0.0007 were derived and are given in table 2, where D_y is defined such that $\Pr(R < D_y) = y$. To set appropriate limits for a CSC range chart we need to use the appropriate values of D_y. For example, the stand-alone Range CSC confirmation limits are set at $LCSL = D_{0\cdot0367}\hat{\sigma}_X$ and $UCSL = D_{0\cdot9633}\hat{\sigma}_X$, where $\hat{\sigma}_X$ is derived as $\bar{R}/d_2$, with $\bar{R}$ calculated from previous data, and d_2 equal to the well known control chart constant (Montgomery 1991). Similarly, the confirmation and control limits of the joint Range CSC chart are set at $LCL = D_{0\cdot0007}\hat{\sigma}_X$, $LCSL = D_{0\cdot0256}\hat{\sigma}_X$, $UCSL = D_{0\cdot9744}\hat{\sigma}_X$ and $UCL = D_{0\cdot9993}\hat{\sigma}_X$

Assuming that the individual observations are normally distributed the appropriate control limits for a CSC standard deviation chart can be determined using the

	Joint CSC				Stand-alone CSC	
n	$G_{0\cdot0007}$	$G_{0\cdot0256}$	$G_{0\cdot9744}$	$G_{0\cdot9993}$	$G_{0\cdot0367}$	$G_{0\cdot9633}$
2	0.0064	0.0321	0.0458	2.0886	2.2322	3.3995
3	0.0259	0.1610	0.1935	1.8177	1.9145	2.7020
4	0.0789	0.2704	0.3069	1.6832	1.7603	2.3873
5	0.1364	0.3503	0.3867	1.5991	1.6649	2.1986
6	0.1891	0.4099	0.4453	1.5402	1.5983	2.0697
7	0.2353	0.4562	0.4902	1.4959	1.5485	1.9745
8	0.2754	0.4934	0.5260	1.4611	1.5094	1.9006
9	0.3103	0.5239	0.5553	1.4327	1.4776	1.8411
10	0.3408	0.5497	0.5798	1.4090	1.4511	1.7918

Table 3. Percentiles for $[\chi^2_{n-1}/(n-1)]^{1/2}$.

chi-square distribution since if $X \sim N(\mu, \sigma_X^2)$ then $(n-1)s^2/\sigma_X^2 \sim \chi_{n-1}^2$, where s is the sample standard deviation. Define G_y such that $\Pr(s < G_y) = y$. The values of G_y for the appropriate critical values given in table 3 were derived using the function 'chi2inv' in the statistics toolbox of MATLAB. Thus, the stand-alone confirmation limits are set at $LCSL = G_{0.0367}\hat{\sigma}_X$ and $UCSL = G_{0.9633}\hat{\sigma}_X$, where, for standard deviation charts, $\hat{\sigma}_X$ equals $\bar{s}/c_4$ with $\bar{s}$ equal to the average subgroup standard deviation derived from previous data, and c_4 is equal to another control chart constant. Similarly the confirmation and control limits for the joint CSC chart are set at $LCL = G_{0.0007}\hat{\sigma}_X$, $LCSL = G_{0.0256}\hat{\sigma}_X$, $UCSL = G_{0.9744}\hat{\sigma}_X$, and $UCL = G_{0.9993}\hat{\sigma}_X$.

3. Comparing CSC charts with $\bar{X}$ and R control charts and double sampling control charts

In the previous section CSC charts were designed to match the false alarm rates of traditional control charts. This section compares the power of the resulting CSC charts to detect process shifts with the power of traditional $\bar{X}$ and R control charts. The results show that the power of the CSC charts to detect moderate process shifts is substantially higher than for $\bar{X}$ and R control charts. In addition, this section compares the expected sample sizes required by both the CSC type charts and Daudin's (1994) double sampling charts

The power of stand-alone and joint CSC charts to detect process shifts is determined from (1) or (2) respectively. The results shown in figure 3 were derived assuming that the individual observations are normally distributed and, without loss of generality, assuming that the in-control process has mean zero and variance one. Figure 3 shows the theoretical probabilities that a standard $\bar{X}$ control chart and the stand-alone and joint versions of a CSC chart will not signal, when the subgroups are of size five. These plots of the power are often referred to as operating characteristic curves, or OC curves. Clearly, the CSC charts are better at detecting deviations from nominal. For example, at a mean shift of one standard deviation unit, the standard $\bar{X}$ chart has a signal probability of 0.22 while both versions of the CSC chart have a

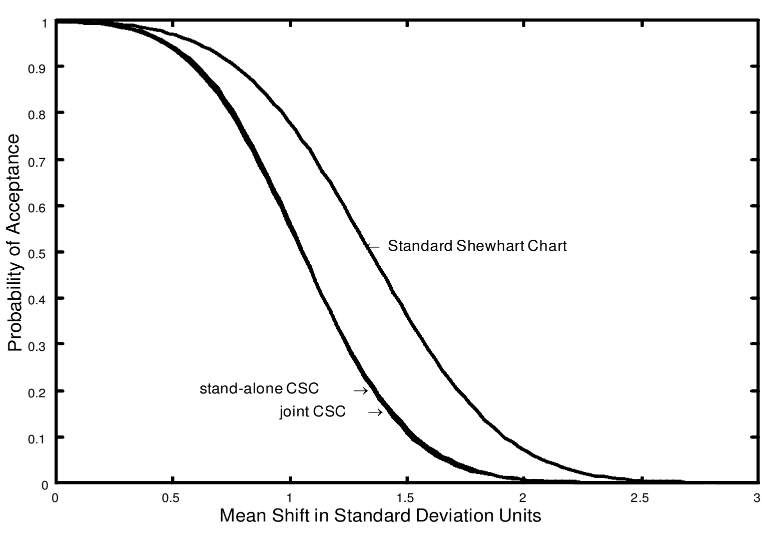

Figure 3. Comparison of the standard $\bar{X}$ and confirmation sample $\bar{X}$ control charts, $n = 5$.

signalling probability twice as large at 0.44. However, this comparison is not completely fair since the CSC chart will occasionally require a second sample, thus increasing the average sample size.

The expected sample size for stand-alone and joint CSC charts is given by (3) and (4).

$$E(n|\text{stand-alone}) = n + (P(\theta > UCSL) + P(\theta < LCSL))n, \tag{3}$$

$$E(n|\text{joint}) = n + (P(UCSL < \theta < UCL) + P(LSL < \theta < LCSL))n. \tag{4}$$

Fortunately, for an in-control process the expected increase in sampling effort is quite modest. For example, for the stand-alone CSC chart when the sample mean is centred at the nominal value, only around 7.2% of observations will fall outside the confirmation sample limits and thus require an additional sample. For joint CSC charts, in-control processes lead to a confirmation sample only 5.1% of the time. When the process is out-of-control, on the other hand, the expected additional sampling requirement can be substantial since a confirmation sample would be required more often. However, while the process is out-of-control, typically rapid detection of an out-of-control situation is the highest priority with sampling costs being secondary.

As another comparison, the operating characteristic (OC) curves for stand-alone and joint mean CSC charts with samples of size 3 are almost indistinguishable from the OC curve for a traditional $\bar{X}$ control chart with samples of size 5. Daudin (1994) gives the double sampling control chart that matches the operating characteristics of the traditional Shewhart chart at shifts of zero and 1.79 sigma units and has the minimum expected sample size in-control. The 'optimal' double sampling chart uses an initial sample of size two with a possible second sample of size four. This double sampling chart also has a virtually identical OC curve as the traditional $\bar{X}$ control chart with samples of size 5. As a result, it is feasible to compare these four different charts in terms of their expected sample sizes for different levels of mean shift. Figure 4 shows this comparison. The traditional Shewhart chart always has a sample of size

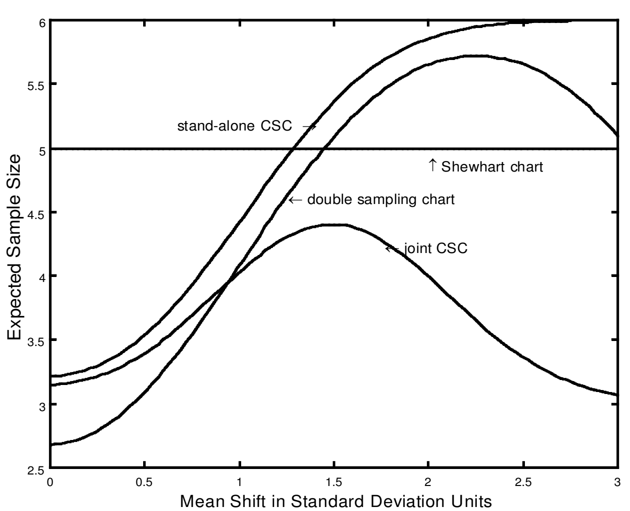

Figure 4. Expected sample size comparison.

five, the stand-alone CSC, joint CSC, and the double sampling chart all have variable expected sample sizes that depend on how likely it is that the confirmation sample will be needed. Figure 4 shows that indeed the double sampling chart has the lowest in-control expected sample size. However, for moderate shifts in the mean, the difference in expected sample size between the double sampling chart and the CSC charts is small, especially when compared with the joint CSC. Also, for these design parameters the joint CSC approach has the smallest expected sample size for large mean shifts. Note that the joint CSC is a special case of double sampling and thus it is possible to define a double sampling procedure that exactly matches the joint CSC shown. Figure 4 shows that there is a trade-off associated with the design parameters of the procedure. By optimizing the in-control performance of the double sampling chart we have derived a procedure that is not optimal, in terms of expected sample size, for large mean shifts.

In figure 5, OC curves for an R chart, and stand-alone and joint range CSC charts are shown for subgroups of size five and ten. Similar results are obtained when comparing traditional Shewhart s charts with standard deviation based CSC charts, but are not shown. The results suggest that for detecting process variability changes the range-based CSC charts are better than traditional R charts, but that the difference is less pronounced than for CSC charts designed to detect process mean shifts. For example, when using subgroups of size 5, the R chart signals with probability 0.31 when the process standard deviation has doubled, while the stand-alone CSC and joint CSC charts signal with probabilities 0.38 and 0.44 respectively. Figure 5 also suggests that for process variability shifts the difference between the stand-alone and joint versions of CSC is larger, with the joint CSC chart performing better. This implies that if detecting process variability changes is critical then the joint CSC chart would be the best choice.

We may also compare the average run length of the combined $\bar{X}$ and R charts and CSC charts for detecting both process mean and process standard deviation shifts. Figure 6 shows average run length contours for the combined $\bar{X}$ and R charts and the combined mean and range-based stand-alone CSC charts. Each individual chart is designed to have an ARL of 370 when the process mean equals zero and the process standard deviation is unity. This corresponds to the point on each contour plot marked with a cross, where the ARL of the combined charts is approximately

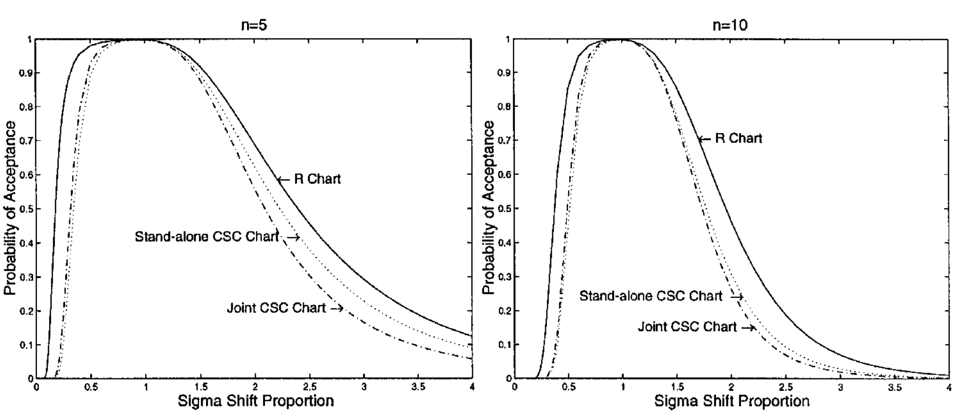

Figure 5. Operating characteristic curves for R charts.

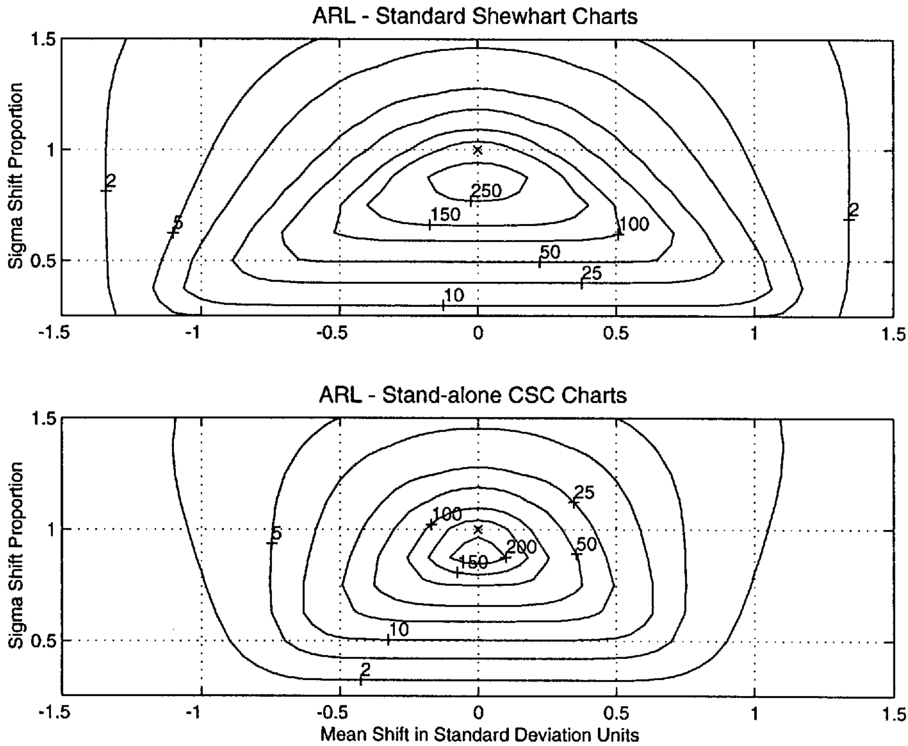

Figure 6. Contour plots of average run length, $n = 5$.

185. Figure 6 clearly illustrates the effectiveness of the combined mean and range stand-alone CSC Charts in comparison to $\bar{X}$ and R Shewhart charts. Notice that when the process standard deviation is slightly smaller than the in-control value both procedures have ARLs that are longer than the in-control ARL. This is undesirable, but is hard to avoid since detecting a small decrease in the variability is difficult.

4. Implementation of CSC charts

CSC charts are easy to implement in practice. However, the idea that the confirmation sample is an additional independent sample drawn from the production process rather than a re-measurement of the initial sample should be emphasized when explaining the procedure to production personnel. Since the confirmation (and control) limits are the same for both the initial and confirmation samples the results of the procedure can be displayed on a single chart. To keep track of initial and confirmation sample results we use '○' to denote initial samples, and '×' for confirmation samples. Whenever both an initial and confirmation sample are required the results are plotted at the same time period and are joined together with a line. To create a reasonable run chart, points from different time periods are connected using the midpoint between the initial and confirmation sample results.

This is illustrated for a combination stand-alone version of the mean and standard deviation CSC chart in figure 7. For this example the process is in-control, normal with mean zero and variance unity. Using subgroups of size five the confirmation limits are set at $-0{\cdot}8$ and 0.8 for the mean chart and 0.39 and 1.6 for the standard deviation CSC chart. In the generated sample of 50 time periods five

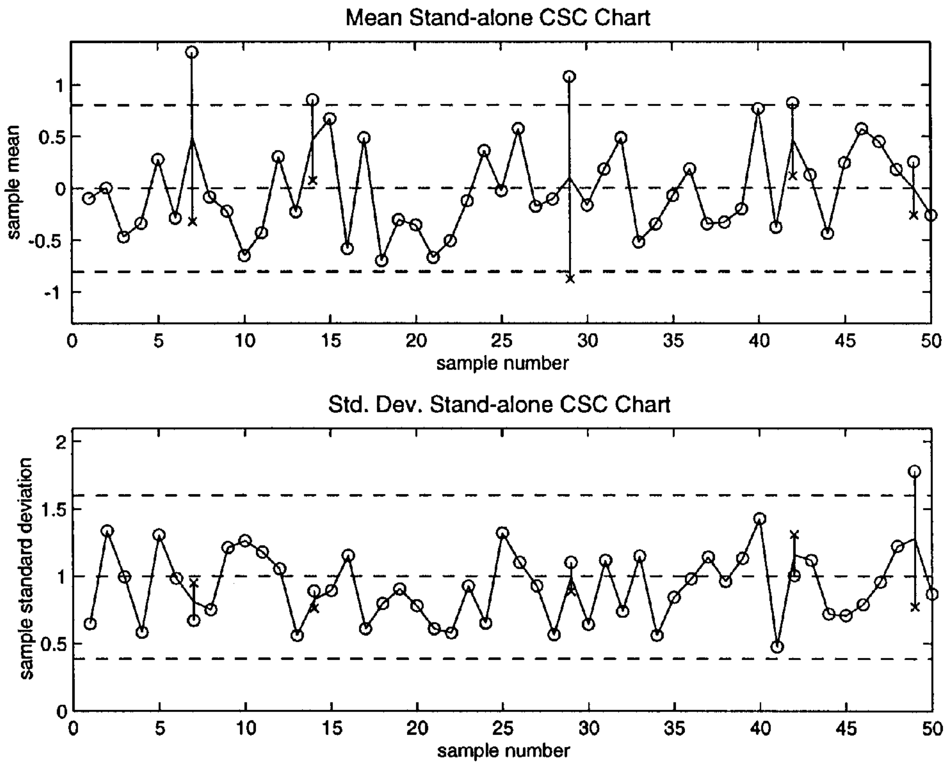

Figure 7. Example of an in-control stand-alone CSC chart.

confirmation samples were required, four times due to a point outside the confirmation limit on the mean chart, and once due to a point outside the confirmation limits on the standard deviation chart. There are no signals in the charts of figure 7. At subgroup number 29 the initial and confirmation subgroup means are outside the confirmation limits on opposite sides, but from figure 1 this does not lead to a signal.

To implement a joint CSC chart the procedure is similar, but there are four limits on each chart. To aid the interpretation, different colours for the confirmation and control limits are recommended.

5. Conclusions

Confirmation sample control charts are a good alternative to traditional $\bar{X}$ and R control charts in situations where an additional independent confirmation sample can be quickly obtained if necessary. Confirmation sample control charts have better power to detect changes in either the process mean or the process variability than traditional $\bar{X}$ and R charts. This is accomplished by taking an additional sample only when there is some evidence of instability. As a result, CSC charts have an intuitive appeal to production personnel who often wish to confirm bad news before acting upon it.

Confirmation sample control charts are very easy to implementation in practice. Since the scheme restricts the initial and confirmation samples to be the same size the results of the procedure can be summarized in a single control chart.

Acknowledgments

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