SYDE 710 Topics in Mathematics:
Algebraic Structure of Discrete Dynamical Systems

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The course lays mathematical foundations for *Algebraic Intelligence* – an approach to Artificial Intelligence that exploits the hidden algebraic structure of any dynamical system with discrete finite spaces of states and events. Examples have multiple types of discrete events (or inputs) transforming their states, and include finite automata, Petri nets, Boolean networks, permutation groups, transformation semigroups, and graph networks. Such discrete-event dynamical systems arise in many branches of science, engineering, artificial intelligence, computer science, and pure mathematics, and have links to invariants and conservation laws in physics, and applications to systems biology, gene regulatory and biochemical networks, reaction graphs, among other areas.

The algebraic analysis of these systems identifies *natural subsystems* ("pools of locally reversible computation", which are certain special permutation subgroups) and allows one algorithmically to derive *hierarchical coordinate systems* (wreath product decompositions). Using this mathematically derived hidden algebraic structure, a human (or an AI) can understand and manipulate the given discrete-event dynamical system.

Target topics include:

* Background from Abstract Algebra (Homomorphisms, Simple Groups, Permutation Groups, Transformation Semigroups, Covering Morphisms, Lagrange's Theorem for Symmetry Structures, Wreath Products)

* Coarse-to-Fine Graining and Hierarchical Manipulation

* Frobenius-Lagrange Coordinates on Permutation Groups, and Applications to Permutation Puzzles (e.g. Rubik's Cube)

* Natural Subsystems of Discrete Event Systems (plus their Permutator Semigroups and Holonomy Groups)

* Global Hierarchical Coordinate Systems (Holonomy Decomposition, Krohn-Rhodes Prime Decomposition)

* Selected Topics from: Computer Algebraic Implementations, Complexity Measures, Complete Invariants for Graphs, Generalizations to Growing and Changing Networks, and AI applications

There is no required textbook for this course. To succeed in this course you will likely need to take notes and participate actively. Students hand in in-class notes to share with the class on a rota.

Grade Weighting: 15% Participation, 50% Assignments, 35% Project. (No Exam)

Auditors will need to attend and complete parts of assignments (problem sets) at a passing level. They need to identify and propose an individual project but will not be required to do the actual project itself.