

# A Fast Method to Determine Cooccurrence Texture Features Using A Linked List Implementation

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**Abstract:** A linked list approach has been developed to efficiently calculate texture features based on cooccurrence probabilities. The commonly used matrix based approach (the grey level cooccurrence matrix or GLCM) requires an unreasonable amount of computation, especially for image segmentation purposes. The linked list approach calculates exactly the same results while significantly decreasing the time to both generate the cooccurrence data and calculate the texture features. The full dynamic range may be maintained without the dramatic increase in computation time that would be experienced by the GLCM approach, however, the behaviour of the statistics changes with different grey level quantizations. This paper describes the implementation of a linked list algorithm, demonstrates its applicability, and investigates the validity of the cooccurrence texture features.

## I. INTRODUCTION

Texture is an important part of image interpretation. A common approach to texture analysis uses grey level cooccurrence matrix (GLCM) texture features [3]. Although this method has been widely applied to remote sensing image interpretation, it has restrictions. Shokr suggested that a linked list approach may be better suited to generating cooccurrence features than a matrix approach [5]. The primary focus of our paper is to describe a grey level cooccurrence linked list (GLCLL) and to provide insight into its performance.

## II. ALGORITHM DETAILS

### A. GLCM Implementation

A GLCM contains the conditional joint probabilities of all pairwise combinations of grey levels given two pa-

rameters: interpixel distance ( $\delta$ ) and interpixel orientation ( $\theta$ ). Following Barber and LeDrew [2], the probability measure is defined by  $Pr(x) = \{C_{ij} | (\delta, \theta)\}$  where

$C_{ij}$  (the GLCM) is defined by  $C_{ij} = P_{ij} / \sum_{i,j=1}^M P_{ij}$ .  $P_{ij}$

represents the number of occurrences of grey levels  $g_i$  and  $g_j$  and  $M$  is the total possible number of all grey level pairs within a window given a particular  $(\delta, \theta)$ .

Typical grey level shift invariant GLCM texture statistics are presented in Fig. 1. These statistics extract three fundamental characteristics from the cooccurrence matrices. Moments about the main diagonal indicate the degree of smoothness of the texture. Dissimilarity (DIS), contrast (CON), inverse difference (INV), and inverse difference moment (IDM) are statistics of this type. Another fundamental characteristic of the cooccurrence matrix is the uniformity of its entries. The greater the homogeneity, the fewer the number of grey level pairs representing the texture. Maximum probability (MAX), uniformity (UNI), and entropy (ENT) describe homogeneity. The final statistic, correlation (COR), describes the correlation between the grey level pairs  $(g_i, g_j)$ . Note that two of the statistics (INV and IDM) have been normalized to truly reflect the smoothness characteristic. The normalized statistics consistently have a higher classification rate and larger inter-class distances than the unnormalized versions.

In practice, the computational demands of the GLCM texture feature extraction are reduced in a number of ways [2, 5]. The image data is typically quantized from eight bits down to as few as four or five bits. This reduces the size of the GLCMs and causes a dramatic decrease in computational time. Quantization has the potential to remove pertinent information from the image. What happens to the GLCM features if the full dynamic range is used? The number of statis-

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$$\begin{aligned}
\text{MAX: } & \max\{C_{ij}\forall(i,j) \\
\text{UNI: } & \sum_{i=1}^g \sum_{j=1}^g C_{ij}^2 \\
\text{ENT: } & -\sum_{i=1}^g \sum_{j=1}^g C_{ij} \log C_{ij} \\
\text{DIS: } & \sum_{i=1}^g \sum_{j=1}^g C_{ij} |i-j| \\
\text{CON: } & \sum_{i=1}^g \sum_{j=1}^g C_{ij} (i-j)^2 \\
\text{INV: } & \sum_{i=1}^g \sum_{j=1}^g \frac{1}{1+|i-j|/g} C_{ij} \\
\text{IDM: } & \sum_{i=1}^g \sum_{j=1}^g \frac{1}{1+(i-j)^2/g^2} C_{ij} \\
\text{COR: } & \sum_{i=1}^g \sum_{j=1}^g \frac{(i-\mu_x)(j-\mu_y)C_{ij}}{\sigma_x \sigma_y}
\end{aligned}$$

Fig. 1: GLCM texture statistics.

tics and/or the number of  $(\delta, \theta)$  pairs must be limited so that all textures features are calculated within a reasonable duration. Deciding which statistics are the most informative for remotely sensed imagery has been the focus of research efforts. A fixed window size is common when implementing the GLCM, however, if the window contains multiple textures, the cooccurrence measures may become confused. Also, this inherently assumes that each texture has the same resolution. For any image segmentation problem, the GLCM approach is not computationally reasonable using fully overlapped windows.

### B. GLCLL Implementation

One method to improve performance when using sparse matrices is to use a linked list approach. Implementing a grey level cooccurrence linked list (GLCLL) has proven to be very efficient because it does not allocate storage for those grey level pairs that have zero probability, unlike the GLCM approach. Both methods generate the identical texture features.

The linked lists are set up in the following manner. Each node is a structure containing the two cooccurring grey levels, their probability of cooccurrence, and a link to the next node on the list. The linked list is kept sorted based on the cooccurring grey levels. An example of such a sorted list would be  $\{(1,2), (1,4), (1,5), (3,1), (3,3), (3,4), (3,7), (4,0), \dots\}$ . A new grey level pair  $(g_i, g_j)$  is included in a linked list by finding the first instance of  $g_i$  and then proceeding from that point to find  $g_j$ . If the pair is found, then its probability is incremented; otherwise, a new node is added in a sorted fashion. In the traditional GLCM approach, the matrix is symmetric [3]. This would undermine the computational advantages of the linked list approach, hence, the data is stored asymmetrically.

When applying this technique to an image, the GLCLLs

Case	Total Order
GLCM w/o updating	$O(n^2) + O(s^2 g^2)$
GLCM w/ updating	$O(n) + O(s^2 g^2)$
GLCLL w/o updating	$O(n^2 N) + O(s^2 N)$
GLCLL w/ updating	$O(nN) + O(s^2 N)$

Fig. 2: Order of cooccurrence texture extraction methods.

are created when the window is at the top left hand corner. After the features are calculated, the window is moved one column to the right. Instead of recalculating entire GLCLLs, the current GLCLLs are simply updated. The pairs of grey levels introduced by the new column are inserted into the GLCLLs. The pairs associated with the column that the window just passed over are subtracted from the GLCLLs. If the subtraction causes the grey level pair to have a zero probability, then that node is removed. The window moves in a zig-zag fashion until the entire image has been covered. This method will be referred to as “updating”.

## III. METHODS AND RESULTS

In order to directly evaluate sea ice imagery, subimages have been extracted from a validated Limex C-band HH image [4]. This image has three dominant ice classes: brash ice, open water, and first year smooth ice. Two types of evaluations are presented. First, completion times of different scenarios are compared. Second, classification testing is performed to determine the effect of using different grey level quantization levels. Classifications are done using a supervised pairwise Fisher linear discriminant.

### A. Computational Speed Comparisons

Four different scenarios are compared: (1) the GLCM without updating (traditional approach), (2) the GLCM with updating, (3) the GLCLL approach without updating, and (4) the GLCLL approach with updating.

Theoretical orders of the comparative computational speeds are presented in Fig. 2. Computational speeds are dependent on the window dimension  $(n)$ , the number of statistics  $(s)$ , and the number of grey levels  $(g)$ . The GLCLLs are also dependent on the length of the linked lists  $(N)$  which is equal to the total number of distinct grey level pairs found in the window. This value is not only dependent on  $g$  but also the texture characteristics. The computational requirements are split into two aspects: the generation and the calculation of the statistics.

A 32x32 image of brash ice is extracted from the Limex image. Window sizes  $\{5, 10, 20\}$  and quantized grey

	No. Grey Level	GLCM w/o update	GLCM w/ update	GLCLL w/o update	GLCLL w/ update
5	full	10	9.9	0.0059	0.0052
10	full	10	9.8	0.033	0.024
20	full	9.9	9.8	0.26	0.12
5	128	2.5	2.4	0.0059	0.0052
10	128	2.5	2.4	0.032	0.024
20	128	2.5	2.5	0.25	0.11
5	64	0.62	0.61	0.0057	0.0051
10	64	0.63	0.61	0.030	0.022
20	64	0.64	0.62	0.21	0.089

Fig. 3: Completion times (seconds per window sample) to calculate statistics.

levels {full, 128, 64} are used. A total of 28 texture features are determined using  $\{\delta = 1; \theta = 0, 45, 90, 135; \text{statistics} = \text{MAX, UNI, ENT, DIS, CON, INV, IDM}\}$ . The increase in speed is impressive, as presented in Fig. 3. The time per sample of the fastest approach (GLCLL with updating) is always a fraction of the slowest approach (GLCM without updating).

The results match the theoretical orders well. At a fixed window size, doubling the number of grey levels increases the completion time of the GLCM approaches by a factor of four. Calculating the GLCM with updating only improves the computational speed slightly, thus the computational speed of the GLCM approaches is highly dependent on determining the statistics. Modifying the window size for the GLCM approaches has little effect on the computational requirements. In contrast, increasing the window size for a fixed grey level increases the GLCLL completion time because the larger windows have more distinct grey level pairs and this increases the length of the linked lists. The number of grey levels influences the computation times of the GLCLL approach since quantized grey levels shortens the linked lists. Finally, updating the GLCLLs is advantageous, especially with larger window sizes.

### B. Classification Testing

Sixty-four and 100 8x8 samples each of brush ice, first year smooth ice, and water are selected from the Limex image to represent the training and classification data sets. Sixteen texture features are selected using  $\{\delta = 1; \theta = 0, 90; \text{statistics} = \text{MAX, UNI, ENT, DIS, CON, INV, IDM, COR}\}$ . In order to determine the effect of quantization on the classification, grey levels {full, 128, 64, 32, 16} are used. The classification accuracy of the testing is presented in Fig. 4. Surprisingly, the grey level quantization has little effect on the classification ability. A contributing factor could be the inherent distinctiveness of the three texture classes. Another factor is provided in Section IV following investigation

No. Grey Levels	Training Results (%)	Test Results (%)
full	98.4	89.7
128	98.4	89.3
64	98.4	86.0
32	96.4	89.0
16	97.4	89.0

Fig. 4: Classification accuracy for different grey level quantizations.

Grey Levels	MAX	UNI	ENT	DIS	CON	INV	IDM	COR
full	62	65	67	91	91	91	91	55
128	71	77	78	91	90	91	91	56
64	77	87	87	91	88	91	89	56
32	83	90	92	91	89	92	90	60
16	87	90	91	91	84	91	84	70

Fig. 5: Classification accuracy of individual statistics based on test data across different grey level quantizations.

of the classification success of the individual statistics.

Fig. 5 represents the classification percentage of the test data for individual statistics given  $\{\delta = 1; \theta = 0, 90\}$ . The homogeneity features (MAX, UNI, and ENT) all have significantly increasing classification occurring with coarser quantization. Quantization is required for these features so that a reasonable number of grey level pairs are repeated within the same window to provide a good estimate of that statistic. With full dynamic range, few grey level pairs are repeated within the same window and a high state of entropy exists in the cooccurrence data. Discrimination is difficult since all classes tend to a near maximum state of entropy generating clusters that overlap in the feature space. As a result, these measures actually rely on the quantization in order to be effective. The smoothness statistics (DIS, CON, INV, and IDM) tend to decrease classification accuracy with increasing quantization. Quantization tends to smooth the data preventing the smoothness statistics from performing optimally. Compared to the other statistics, COR is quite ineffective, but tends to improve with coarser quantization.

Cooccurrence texture features tend to be highly correlated. In this study, the average correlation across all selected texture features for each class significantly increases with coarser quantization. A high correlation between the features is expected for DIS, CON, INV, and IDM since they are all functionally the same. MAX, UNI, and ENT also display the same type of behaviour. COR did not show any correlated behaviour with any other statistics. Thus, only one of (DIS, CON, INV, IDM) and only one of (MAX, UNI, ENT) and

perhaps COR should be used as statistics. This is essentially the same conclusion that Baraldi and Parmiggiani [1] determine.

#### IV. DISCUSSION AND CONCLUSIONS

The linked list approach for the generation of cooccurrence image data is greatly preferred to the traditional matrix approach since it calculates texture features orders of magnitude faster. However, in general, generation of cooccurrence data does not lend itself to rapid feature extraction. The full dynamic range of the image can be utilized using GLCLLs, with only a modest increase in computation time. The degree of this increase is texture dependent.

Textures that have noticeable but subtle differences at full dynamic range may become statistically similar under coarse quantization. This may be very important when classifying remotely sensed imagery. A SAR sea ice image often contains many different types of ice types as well as transitions between these ice types and quantizing the imagery can remove the subtle differences between two similar ice classes that must be segmented. It would be much safer and consistent if the full dynamic range is used for feature extraction.

The classification studies reveal that grey level quantization does not have a significant effect on the class assignment accuracy using all the available statistics and the given data set. This is probably due to a trade-off in the effectiveness of the individual statistics with changing quantization. At full dynamic range, homogeneity statistics are poor and smoothness statistics are quite strong. At coarse quantizations, smoothness statistics are still quite good and the homogeneity statistics have increased their effectiveness.

A major stumbling block is the difficulty selecting window sizes and pixel separations to uniquely identify a particular class. Also, different textures within the same image often have different spatial resolutions. Picking multiple window sizes and multiple pixel separations may solve this problem and generate a robust feature set, however, the computational cost would increase dramatically. If a window is too large, too much blurring between boundaries can occur and the computational times are higher. Smaller windows tend to be more erroneous but generate better accuracy near boundaries.

There is a possibility that only limited information can be derived from the cooccurrence data for the purposes of interpreting SAR sea ice imagery. The important features are based on two sources of information:

smoothness and homogeneity. We are in the process of comparing the features generated by the cooccurrence probabilities to features generated by digital filtering techniques. We expect that similar improved statistics can be generated in a more computationally favourable manner using such methods.

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