A Gabor Based Technique for Image Denoising

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Abstract

As an alternative to the wavelet, Gabor function has been used as an efficient representation of two dimensional signals. We are interested in BayesShrink techniques for image denoising, and have shown in our previous work that BayesShrink Ridgelet performs better than VisuShrink Ridgelet and VisuShrink Wavelet. In this paper, a dyadic Gabor filter bank is combined with BayesShrink method for image denoising. In the proposed method, the noisy image is decomposed to different channels in several levels by a dyadic Gabor filter bank. To recover the image, the corrupting noise is removed by applying the proposed BayesShrink method on the noisy Gabor coefficients. The noise variance is estimated in Gabor domain and the estimated noise is then used to dynamically calculate an individual threshold for each spatio-frequency channel. Finally denoised coefficients are transformed back to reconstruct the image.

Keywords—*BayesShrink; VisuShrink; Gabor; Wavelet; denoising.*

1 Introduction

Noise undesirably corrupts the image by perturbations which are not related to the scene under study and ambiguates the underlying signal relative to its observed form. For this reason noise elimination is a main concern in computer vision and image processing. The goal of denoising is to remove the noise and retain important signal features as much as possible. To achieve this goal, traditional approaches use linear processing such as Wiener filtering [1]. In the presence of additive noise, linear filters, which consist of convolving the image with a constant matrix to obtain a linear combination of neighborhood values, can produce a blurred and smoothed image with poor feature localization and incomplete noise suppression. To overcome these shortcomings, nonlinear filters have been proposed. Much research has recently focused on signal denoising using nonlinear techniques in the setting of additive white Gaussian noise. One of the most important denoising techniques is wavelet based denoising [2], [3]. The wavelet transform separates the signal and noise; as a result it can be used to remove the noise while preserving the signal characteristics. Researchers have employed two different approaches to nonlinear wavelet-based denoising: first, known as wavelet thresholding [4], [2], [5], a hard threshold function keeps a coefficient if it is larger than the threshold and sets it to zero otherwise; second, wavelet shrinkage with a soft thresholding function takes the argument and shrinks it toward zero by the threshold. Both approaches are nonlinear and operate on one wavelet coefficient at a time. Different denoising methods have been proposed for signal denoising via wavelet. BayesShrink Wavelet image denoising has been recently introduced [6] as an alternative to the VisuShrink Wavelet image denoising to improve the wavelet based denoising performance. In this paper BayesShrink Gabor image denoising is introduced and the results are compared with those of VisuShrink Wavelet method. The following Section explains Gabor function and VisuShrink thresholding technique for image denoising. Proposed method is presented in Section four. In Section five the results of the proposed method and the VisuShrink Wavelet technique are compared and the paper is concluded in Section six.

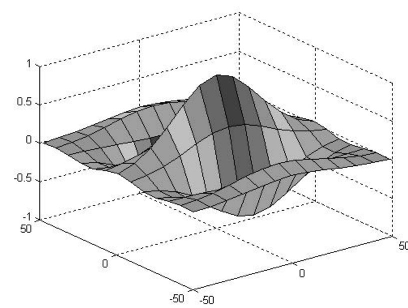


Figure 1: Gabor Filter in Spatial Domain

2 Gabor Function

A typical Gabor filter in the spatial domain is depicted in Fig. 1. A Gabor base function is a Gaussian function modulated with an exponential or sinusoidal function that is defined in terms of the product of a Gaussian and an exponential [7], [8], [9]. Two-dimensional Gabor functions h(x,y) can be written as:

$$h(x,y) = g(x,y) \cdot exp^{-2\pi j f_r x} \quad (1)$$

where,

$$g(x,y) = \frac{1}{2\pi\sigma_x\sigma_y} \cdot exp^{\frac{-1}{2}(\frac{x^2}{\sigma_x^2}+\frac{y^2}{\sigma_y^2})} \quad (2)$$

and its frequency response $H(u,v)$ is:

$$H(u,v) = G(u - f_r, v) = exp^{-2\pi^2[(u-f_r)^2\sigma_x^2 + v^2\sigma_y^2]} \quad (3)$$

where,

$$f_r^2 = u_r^2 + v_r^2 \quad (4)$$

and

$$\theta = tan^{-1}(v_r/u_r) \quad (5)$$

Gabor functions are bandpass filters which are Gaussians, centered on (f_r,θ) in the spatial-frequency domain. The parameters f_r, θ, σ_x and σ_y determine the subband Gabor filter. f_r and θ are center frequency and orientation and σ_x and σ_y are the bandwidth of the filter.

3 VisuShrink Thresholding Technique

VisuShrink denoising is used to recover the original signal from the noisy one by removing the noise. In contrast with denoising methods that simply smooth the signal by preserving the low frequency content and removing the high frequency components, the frequency contents and characteristics of the signal would be preserved during VisuShrink denoising. VisuShrink Wavelet, proposed by Donoho and Johnstone [2], [4], uses the universal threshold given by:

$$\tau_u = \sigma_n \sqrt{2log(M)} \quad (6)$$

Where σ_n and M are the noise variance and the number of image pixels respectively. Donoho and Jonstone have proven that the maximum of any M independent and identically distributed (iid) values with high probability is less than the universal threshold τ_u. As M is increased the probability will be closer to one, so with a high probability pure noise signals are set to zero. The universal threshold is obtained by considering the constraint that the noise is less than the threshold with high probability as M increases, hence it tends to be high for large values of M. As a result it will shrink many noisy wavelet coefficients to zero and produces smoothed estimated image.

4 Proposed BayesShrink Gabor Method

A dyadic Gabor filter bank is designed to decompose the input image to spatial frequency subbands. Four levels of decomposition is considered such that the radial frequency bandwidth is one octave, hence the frequency difference of f_{r1} and f_{r2} is

$$\log_2 \frac{f_{r2}}{f_{r1}} = 1 \quad (7)$$

Four orientations 0°, 45°, 90° and 135° are used, therefore to cover the frequency domain angular bandwidth is chosen to be 45°. Twenty real channels out of the total forty real and complex decomposed channels are thresholded and used to reconstruct the denoised image.

4.1 Gabor Denoising Concept

To explain the Gabor denoising procedure, assume $I[i,j]$ to be the original M by M image where i and $j = 1, 2, \ldots, M$ and it is corrupted with additive noise $n[i,j]$:

$$S[i,j] = I[i,j] + n[i,j] \quad (8)$$

$n[i,j]$ are identically distributed and independent of $I[i,j]$. The goal of denoising is to estimate $\hat{I}[i,j]$ of $I[i,j]$ by removing the noise $n[i,j]$.

In the first step of Gabor denoising, the observed image S is transformed into the Gabor domain. Then the Gabor coefficients are thresholded and finally the denoised coefficients are transformed back to reconstruct the image. Let G_D and G_R be the forward Gabor decomposition and inverse Gabor reconstruction transforms. Let assume τ and T as threshold and thresolding operator respectively. The Gabor thresholding can be summarized as:

$$\begin{aligned} I_G &= G_D(I) \\ I_T &= T(I_G, \tau) \\ \hat{I} &= G_R(I_T) \end{aligned} \quad (9)$$

The choice of the threshold and the method which is used to calculate the threshold, determine how efficient the denoising technique would be. Although, selecting a small threshold may produce an output image close to the input, the recovered image may still be noisy. On the other hand, a choice of a large threshold may yield a blurred image by setting the most of the wavelet coefficients to zero. The proposed BayesShrink Gabor tresholding technique is explained in the following Section.

4.2 BayesShrink Gabor

The subband Wavelet and Ridgelet coefficients of a natural image can be described by the Generalized Gaussian Distribution (GGD) [6], [10], [11]. A Gabor base function is a Gaussian function modulated with an exponential, hence for most of the natural images Gabor coefficients can be described by the Generalized Gaussian Distribution (GGD) as

$$GG_{\sigma_I,\gamma}(I) = P(\sigma_I, \gamma) exp\{-[\delta(\sigma_I, \gamma)|I|]^\gamma\} \quad (10)$$

where, $-\infty < I < +\infty, \gamma > 0$,

$$\delta(\sigma_I, \gamma) = \sigma_I^{-1} [\frac{\Gamma(3/\gamma)}{\Gamma(1/\gamma)}]^{\frac{1}{2}} \quad (11)$$

TABLE I

Gabor subband coefficients: Each matrix correspond to a specific orientation in different dicomposition level.

Levl 4, 0°	Levl 4, 45°	Levl 4, 90°	Levl 4, 135°
Levl 3, 0°	Levl 3, 45°	Levl 3, 90°	Levl 3, 135°
Levl 2, 0°	Levl 2, 45°	Levl 2, 90°	Levl 2, 135°
Levl 1, 0°	Levl 1, 45°	Levl 1, 90°	Levl 1, 135°

and,

$$P(\sigma_I,\gamma) = \frac{\gamma.\delta(\sigma_I,\gamma)}{2\Gamma(1/\gamma)} \quad (12)$$

σ_I is the standard deviation of subband Gabor coefficients, γ is the shape parameter and Gamma function Γ is defined as:

$$\Gamma(x) = \int_0^\infty exp\{-y\}y^{x-1}dy \quad (13)$$

Moreover the distribution of the Gabor coefficients in a subband can be described by a shape parameter γ in the range of [0.5,1]. Considering such a distribution for the Gabor coefficients and estimating γ and σ_I for each subband, the soft threshold τ_s which minimizes the Bayesian Risk [6], [10], can be obtained by:

$$\Re(\tau_s) = E(\hat{I}-I)^2 = E_I E_{J|I}(\hat{I}-I)^2 \quad (14)$$

where $\hat{I}$ is $\tau_s(J)$, $J|I$ is $N(I,\sigma)$ and I is $GG_{\sigma_I,\gamma}$. Then the optimal threshold τ_s^* is given by:

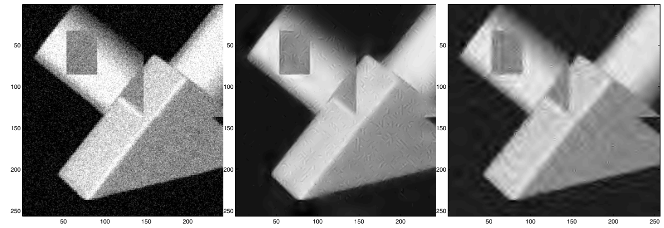
$$\tau_s^*(\sigma_I,\gamma) = arg \ \min_{\tau_s} \ \Re(\tau_s) \quad (15)$$

It does not have a closed form solution and numerical calculation is used to find τ_s^*. A proper estimation of the value τ_s^* is concluded by setting the threshold as:

$$\hat{\tau}(\hat{\sigma_I}) = \frac{\hat{\sigma_n}}{\hat{\sigma_I}} \quad (16)$$

4.3 Calculating the BayesShrink threshold

Subband dependent threshold is used to calculate BayesShrink Gabor threshold. The estimated threshold is given by (16) where σ_n and σ_I are noise and signal standard deviations respectively. Gabor coefficients corresponding to different orientations are depicted in Tab. I. In this table each matrix is corresponded to a specific orientation, hence the number of columns determines the number of orientations and each row contains subband coefficients for a specific decomposition level.



(a) (b) (c)

Figure 2. (a) Original image. (b) Denoised image using VisuShrink Gabor denoising SNR=16.09. (c) Denoised image using VisuShrink Wavelet denoising SNR=17.37.

To estimate the noise variance σ_n^2 from the subband coefficients, the median estimator is used on each subband matrix coefficient:

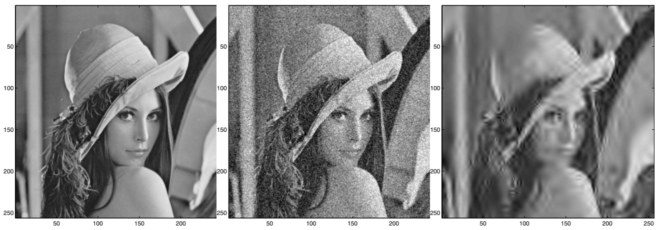
$$\hat{\sigma_n} = median(|subband \ cooeficients|)/0.6745 \quad (17)$$

Signal standard deviation is calculated for each orientation in each subband detail individually. Thus having 4 orientations and L subband, $4 \cdot L$ different σ_I must be estimated corresponding to $4 \cdot L$ subband-orientations coefficients. Note that in BayesShrink Wavelet denoising, σ_I is estimated on vertical, horizontal and diagonal subbands [6], thus having L decomposition levels, $3 \cdot L$ different σ_I must be estimated to calculate the thresholds for the different subbands. To estimate the signal standard deviation (σ_I), the observed signal S is considered to be $S = I + n$, while signal (I) and noise (n) are assumed to be independent. Therefore, $\sigma_S^2 = \sigma_I^2 + \sigma_n^2$ where σ_S^2 is the variance of the observed signal. So $\hat{\sigma_I}$ is estimated by

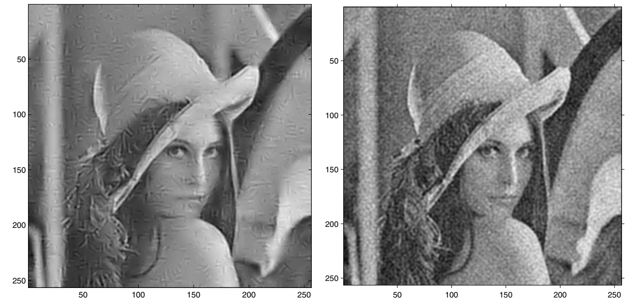
$$\hat{\sigma_I} = \sqrt{max((\hat{\sigma}_S^2 - \hat{\sigma}_n^2), 0)} \quad (18)$$

5 Results

In this section the proposed Gabor denoising technique is used to recover the noisy images which are corrupted with additive white noise. BayesShrink and VisuShrink Gabor image denoising methods are implemented and compared with VisuShrink Wavelet denoising. Some natural images and images with straight regions are used to test the proposed method in the following experiments. Denoised images depicted in Fig. 2(b) and (c) are derived using the VisuShrink Gabor and VisuShrink Wavelet thresholding methods respectively. The wavelet results are obtained based on three different wavelet bases including Daubechies, Symlets and Biorthogonal and the best one among them is chosen to be compared with the proposed VisuShrink



(a) (b) (c)



(d) (e)

Figure 3. (a) Original lena image. (b) Noisy image SNR=5.50. (c) Denoised image using VisuShrink Wavelet denoising SNR=9.43. (d) Denoised image using the proposed VisuShrink Gabor denoising SNR=10.23. (e) Denoised image using proposed BayesShrink Gabor denoising SNR=9.95.

Gabor method. As we can observe despite having somewhat poorer performance in comparison with VisuShrink Wavelet based on SNR, the result obtained by VisuShrink Gabor denoising is visually better and smoother. Fig. 3(a) and (b) show original and noisy lena image respectively while Fig. 3(c) shows the recovered image using VisuShrink Wavelet. The result obtained by applying the proposed VisuShrink Gabor is depicted in Fig. 3(d) and denoised image using the proposed BayesShrink Gabor is depicted in Fig. 3(e). As we can observe proposed VisuShrink and BayesShrink Gabor denoising methods perform better than VisuShrink Wavelet and produce better and smoother results, both visually and in terms of SNR.

6 Conclusions

In this paper the Gabor transform for image denoising was addressed by introducing VisuShrink and BayesShrink Gabor denoising techniques. The proposed method was applied on natural images and images with straight regions. The denoising performance of the results was compared with that of the VisuShrink Wavelet image denoising method. The experimental results by the proposed method showed the superiority of the image quality and its higher SNR in comparison with VisuShrink Wavelet technique. Furthermore we observed that regardless of the selected wavelet basis, VisuShrink and BayesShrink Gabor perform better than VisuShrink Wavelet. However, the choice of the wavelet bases might effect on the performance of the VisusShrink Wavelet. Although the proposed methods perform better than VisuShrink Wavelet, these methods perform somewhat poorer in comparison with BayesShrink Wavelet, based on SNR. However the proposed BayesShrink Gabor produces less local artifact than wavelet counterpart. Future work is conducted to improve the performance of these methods. Our previous ridgelet-based denoising methods will also be compared with these BayesShrink Gabor denoisisng methods.

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